Question
What's the minimum number of people, such that **at least 2** of them have the same birthday (date and month)?

Answer
We will start with the fact that there are 366 unique days on a calendar. Hence when there are 367 people, the desired probability equals 1.

So let there be $n \leqslant 366$ people under observation. We will first calculate the probability, P that **none of them** have the same birthday. This is calculated as follows:

- The first person has an arbitrary birthday, b_1
- AND the second person has a distinct birthday, b_2 (this probability is $365/366$)
- AND the third person has a distinct birthday, b_3 (this probability is $364/366$)
- AND the forth person has a distinct birthday, b_4 (this probability is $363/366$)
- AND so on $\ldots$

On multiplying all these probabilities (because ANDs), we get

$$P = \frac{365}{366} \cdot \frac{364}{366} \cdot \frac{363}{366} \cdot \ldots \ (n-1 \text{ times})$$

To make life easier for ourselves, we multiply both the numerator and the denominator by 366, simplify the fraction a little, and get

$$P = \frac{366!}{(365-n)! \cdot 366^n}$$

Hence the desired probability that at least 2 of them have the same probability is the compliment of P

$$\boxed{P' = 1 - \frac{366!}{(365-n)! \cdot 366^n}}$$

When calculated for different values of n, we have the following probabilities of at least 2 people with the same birthday:

n	P'
10	0.116

20	0.410
22	0.474
23	0.506
30	0.706
40	0.890
50	0.970
60	0.994
70	0.999

Conclusions

- Among every 23 people, the chances that at least 2 have the same birthday is more than 50%.
- Among every 70 people, the chances that at least 2 have the same birthday is more than 99.9%.

□