$\mathbb{R}, \mathbb{Q}, \mathbb{N}, \mathbb{Z}$

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Conjecture. *The sum and product of two rational numbers is also rational number.*

Proof. If $a = \frac{a_1}{a_2}, b = \frac{b_1}{b_2} \in \mathbb{Q}, a_2 \neq 0, b_2 \neq 0,$

$a + b = \frac{a_1 b_2 + b_2 a_1}{a_2 b_2}$. Since $a_2 b_2 \neq 0, (a_2 b_2), (a_1 b_2 + b_2 a_1) \in \mathbb{N}$, $a + b$ is rational.

$ab = \frac{a_1 b_1}{a_2 b_2}$. Since $a_2 b_2 \neq 0, (a_1 b_1), (a_2 b_2) \in \mathbb{N}$, ab is rational. □

Conjecture. *If $a > b$ and $b > c$ then $a > c$.*

Proof. Let $\mathbb{P}$ be the set of positive real numbers. $a > b, b > c \implies (a - b), (b - c) \in \mathbb{P}$. **The Ordered Properties of** $\mathbb{R}$ hold that $a, b \in \mathbb{P} \implies (a + b) \in \mathbb{P}$.

Therefore from the hypothesis $(a - b) + (b - c) = (a - c) \in \mathbb{P}$. In other words, $a > c$. □