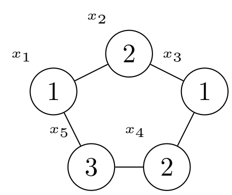
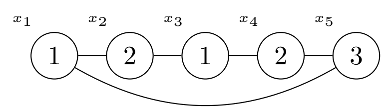
Theorem. *For every $t \geq 3$, there existsa graph G_t so that $\chi(G_t) = t$ and $\omega(G_t) = 2$.*

Proof. By induction on n, the number of people.
Basis: For $t = 3$, consider the cycle C_5:

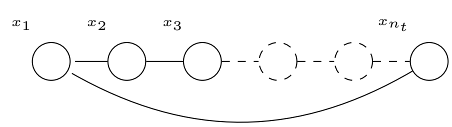


Which can also be drawn as:

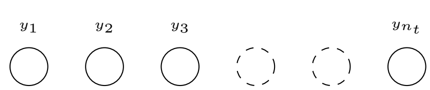


As we see, $\chi(G_3) = 3$ and $\omega(G_3) = 2$.

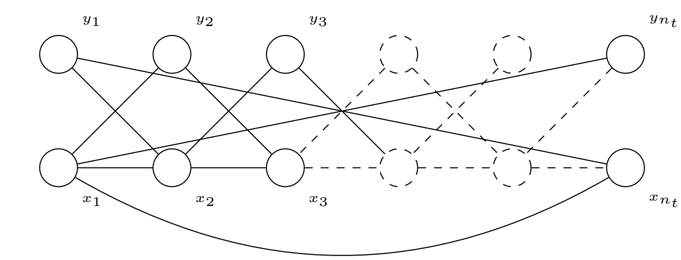
Induction hypothesis: Assume
Induction: Assume that for $t \geq 3$, we have determined the graph G_t. Suppose that G_t has n_t vertices. Lebel the vertices $x_1, x_2, x_3, \ldots, x_{n_t}$.



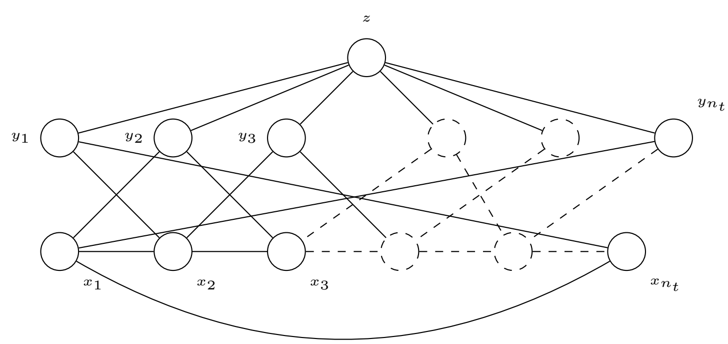
To construct G_{t+1} we begin by copying the vertices, from G_t (no edges) into an independent set and calling it $I = \{y_1, y_2, y_3, \ldots, y_{n_t}\}$.



We then add a copy H_t of G_t with y_i adjacent to x_j if and only if x_i is adjacent to x_j. In other words



Finally we attach a new vertex z adjacent to all vertices in I



Now to determine $\omega(G_{t+1})$ let's look at the cases that there might be a triangle (C_3) present in G_{t+1}:

Case I The traingle comprises of z and 2 nodes from I:
This case is **not possible** since the nodes in I are not adjacent to each other.
Case II The traingle comprises of 3 nodes from I:
This case is **not possible** since the nodes in I are not adjacent to each other.
Case III The traingle comprises of 3 nodes from H_t:
This case is **not possible** since there is no triangle in H_t ($\omega(H_t) = \omega(G_t) = 2$, from the premise).
Case IV The traingle comprises of z, a node from I and another, from H_t:
This case is **not possible** since z is is not adjacent to any node in H_t.

Case V The traingle comprises of z, a 2 nodes from H_t:
This case is **not possible** since z is is not adjacent to any node in H_t.

Case VI The traingle comprises of one node from H_t and 2 nodes from I:
This case is **not possible** since the nodes in I are not adjacent to each other.
Case VII The traingle comprises of one node from I and 2 nodes from H_t:
This case is **not possible** since every node in I is adjacent To exactly 2 nodes in H_t that are not adjacent.

Therefore, $\omega(G_{t+1}) = 2$

$\square$