Dougy's question on money

7/16/2025

Problem. *Doug asked a very interesting question:* ***How many coins do we need to carry if we were to be able to pay any denomination from 0 to 99 cents?***

I took the liberty to change his question into something completely different: ***Given we have enough coins of each kind (Quarters, Dimes, Nickles, and Pennies) in how many different ways can we represent money from 1 to 100 cents?***

So here we will examine my question, and ignore his.

Solution. *We introduce a function,* $C(100, Q, D, N, P)$*, representing the number of such combinations, using quarters (Q), dimes (D), nickles (N), and pennies (P)*

We make some simple observations. One is that $C(0, \cdots) = 1$*: There is one combination to pay 0 cents (that is to use no coins!)*

Also $C(100, Q) = 1$ *(use 4 quarters), and so on...*

1 A simple example

As we begin to dive deeper, we see that $C(5, N, P) = 2$*, the two combinations that can be visualized with the table:*

N	*P*
0	*5*
1	*0*

2 Using only Nickles and Pennies

The following tables will come in handy later:

$C(100, N, P) = 21$

N	*P*
0	*100*
1	*95*
2	*90*
...	...
20	*0*

$C(75, N, P) = 16$

N	*P*
0	*75*
1	*70*
2	*65*
...	...
15	*0*

$C(50, N, P) = 11$

N	*P*
0	*50*
1	*45*
2	*40*
...	...
10	*0*

$C(25, N, P) = 6$

N	*P*
0	*25*
1	*20*
2	*15*
...	...
5	*0*

Finally, $C(0, N, P) = 1$

3 Introducing the Dime!

Given the previous explorations with just Nickles and Pennies, we can build larger tables using the new Dime. Let's start with $C(100, D, N, P)$*:*

D	*N*	*P*	*Notice*
0	*0*	*100*	$C(100, N, P) = 21$ *combinations*
0	*1*	*95*	
0	*2*	*90*	
0	...	...	
0	*20*	*0*	
1	*0*	*90*	$C(90, N, P)$ *combinations*
1	*1*	*85*	
1	...	...	
2	$C(80, N, P)$		
3	$C(70, N, P)$		
...	...		
10	*C(0, N, P)=1*		

Hence we that $C(100, D, N, P)$ *depends on various* $C(100, N, P)$*,* $C(90, N, P)$*, etc. Similarly, it's easy to see that:*

Q	*D*	*N*	*P*
0	*C(100, D, N, P)*		
1	*C(75, D, N, P)*		
2	*C(50, D, N, P)*		
3	*C(25, D, N, P)*		
4	*C(0, D, N, P)=1*		

4 Trees with Dimes, Nickles, & Pennies

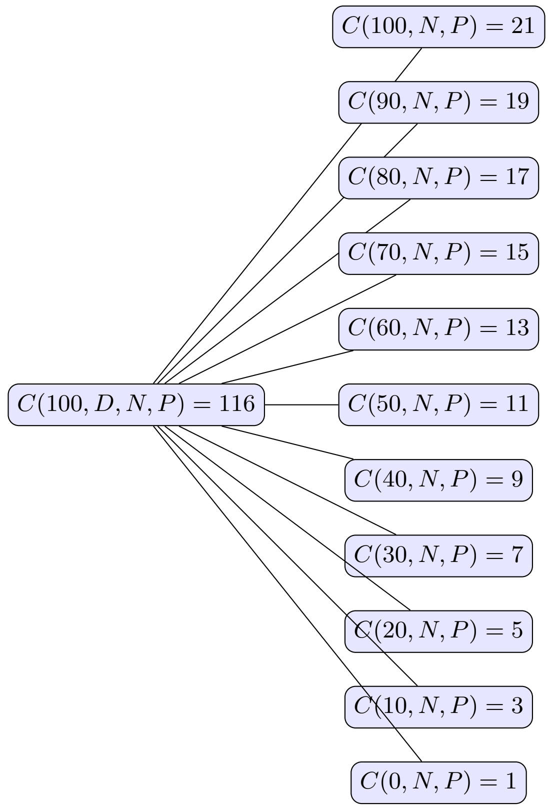

Figure 1: 116 conbinations (100 cents using D, N, and P)

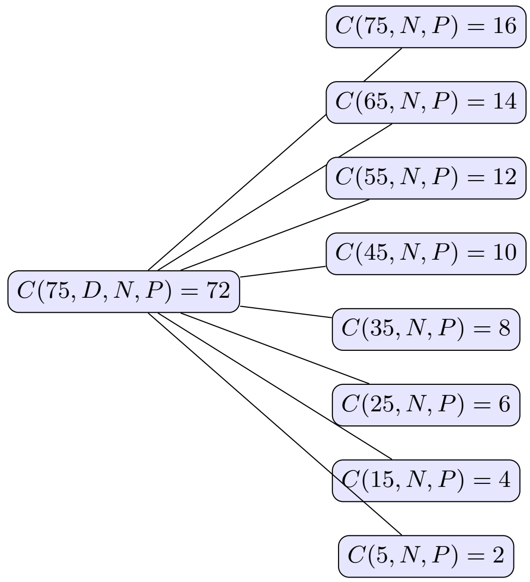

Figure 2: 72 combinations (75 cents using D, N, and P)

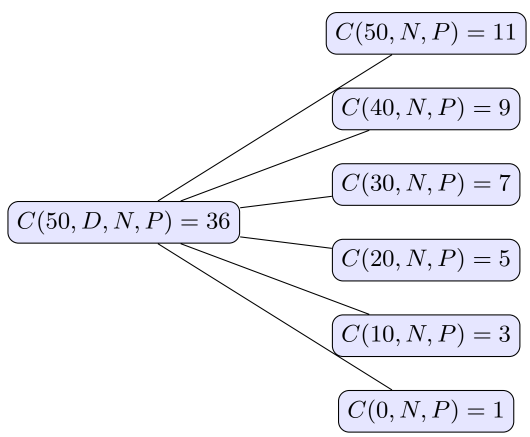

Figure 3: 36 combinations (50 cents using D, N, and P)

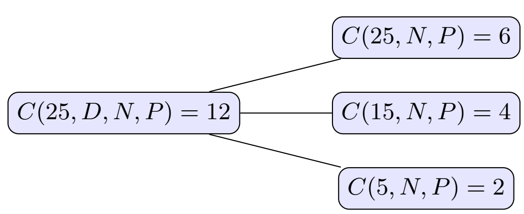

Figure 4: 12 combinations (25 cents using D, N, and P)

5 Putting it all together

The last coin, the Quarter, can be used 0, 1, 2, 3, or 4 times. The following tree shows the desired root node, and its dependency on the five previously established values:

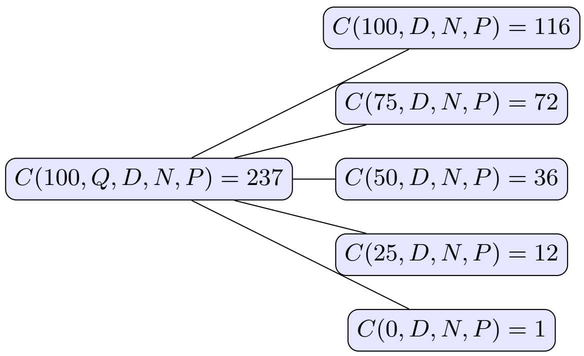

Figure 5: 277 combinations (100 cents using Q, D, N, and P)

Thus we see that using all four kind of coins, we have 237 *combinations to represent any value from 0 to 100 cents.*