Roots – rational or not

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Conjecture. *A natural root of a natural number is either an integer or irrational.*

Proof. Let's assume $a^{1/b}$ is rational, i.e.,

$$a^{1/b} = x/y$$

Raising both sides to the power b,

$$\begin{aligned} a &= x^b/y^b \\ \implies x^b &= ay^b \end{aligned}$$

x^2 has pairs of prime factors (each contributing to the individual x). Therefore if x^2 is even, there must be another 2 as its factor. Hence x must be even also.

All of x's prime factors get "tripled" when when calculate x^3. Therefore if x^3 is a multiple of 3, it must be a factor of x as well.

If b is prime: Let all of x's prime factors be $\{p_1, p_2, \ldots, p_x\}$. If x^b is a multiple of b, it must be a factor of x as well.

$\square$