

Q.1 Prove by induction: $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
 For **all** n .

A) If we can prove that if the above statement is true for a given n , it is also true for $n + 1$. We would have proven it for **all** numbers!

basis step: Let $n = 1$ The sum $1 + \cdots + 1 = 1 = \frac{1(2)}{2}$

induction step: Assume that the statement is true for a given k . This means $1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$. Now we prove this to be true for $k + 1$.

Let's consider $1 + 2 + 3 + \cdots + (k + 1)$

$$\begin{aligned} 1 + 2 + 3 + \cdots + (k + 1) &= 1 + 2 + 3 + \cdots + k + (k + 1) \\ &= \frac{k(k + 1)}{2} + (k + 1) \\ &= \frac{k(k + 1)}{2} + \frac{2(k + 1)}{2} \\ &= \frac{(k + 1)(k + 2)}{2} \\ &= \frac{(k + 1)((k + 1) + 1)}{2} \end{aligned}$$