probabilité continue :

I) densité de probabilité $f(t)$ sur $[a,b]$

→ d.d.p

- pour montrer que f est une d.d.p il faut 3 conditions :
  - continue sur $[a,b]$
  - positive sur $[a,b]$
  - $\int_a^b f(t)\,dt = 1$

exemple : mq $f(t) = e^{2t}$ n'est pas une d.d.p sur $[0,2]$:

f est continue et positive sur $[0,2]$.

$$\int_0^2 e^{2t}\,dt = \frac{1}{2}\left[e^{2t}\right]_0^2 = \frac{1}{2}\left(e^4 - 1\right) \neq 1$$

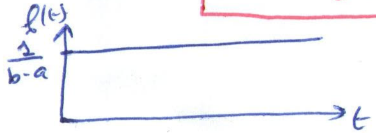
donc f n'est pas une d.d.p.

- Th :

$$\boxed{p([c,d]) = p(c \leq X \leq d) = \int_c^d f(t)\,dt}$$

II) Loi uniforme sur $[a,b]$: (au hasard).

→ $\boxed{\text{d.d.p}}$: $\boxed{f(t) = \frac{1}{b-a}}$



si $[c,d] \subset [a,b]$.

$$p(c \leq X \leq d) = p([c,d]) = \int_c^d \frac{1}{b-a}\,dt$$

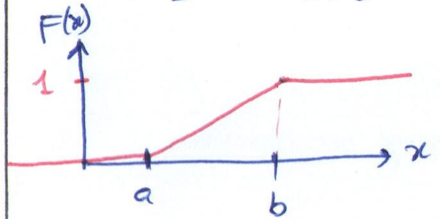
$$\boxed{p(c \leq X \leq d) = \frac{d-c}{b-a}}$$

$$\boxed{p(X \geq c) = p(c \leq X \leq b) = \frac{b-c}{b-a}}$$

$$\boxed{p(X \leq d) = p(a \leq X \leq d) = \frac{d-a}{b-a}}$$

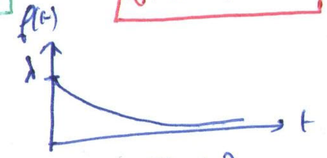
→ fonction de répartition $F(x)$:

$$F(x) \begin{cases} 0 & x < a \\ p(a \leq X \leq x) & \text{si } x \in [a,b] \\ 1 & x > b \end{cases}$$



III) Loi exponentielle de paramètre λ sur $[0,+\infty[$

→ $\boxed{\text{d.d.p}}$ $\boxed{f(t) = \lambda e^{-\lambda t}}$ sur $[0,+\infty[$



si $[c,d] \subset [a,b]$

$$p(c \leq X \leq d) = \int_c^d f(t)\,dt = \int_c^d \lambda e^{-\lambda t}\,dt$$
$$= -\left[e^{-\lambda t}\right]_c^d = -\left(e^{-\lambda d} - e^{-\lambda c}\right)$$
$$= e^{-\lambda c} - e^{-\lambda d}$$

$$\boxed{p(c \leq X \leq d) = e^{-\lambda c} - e^{-\lambda d}}$$

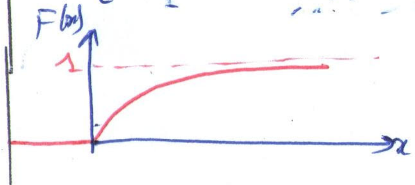
$$\boxed{p(X \geq c) = p([c,+\infty[) = e^{-\lambda c}}$$

$$\boxed{p(X \leq d) = p(0 \leq X \leq d) = 1 - e^{-\lambda d}}$$

→ fonction de répartition $F(x)$:

$F : \mathbb{R} \to [0,1]$

$$\begin{cases} 0 & \text{si } x < 0 \\ p(0 \leq X \leq x) = 1 - e^{-\lambda x} & \text{si } x \geq 0 \end{cases}$$



B.M Taki Eddine
23.390.248