Complexe (partie géométrique)

Q1 Les ABC

→ on appelle affixe (les coordonnées)

$z_A = 3 - i \Rightarrow A(3,-1)$

→ affixe d'un vecteur $\vec{AB}$: $aff(\vec{AB}) = z_B - z_A$

→ distance AB : $AB = |z_B - z_A| = \sqrt{(\text{Réel})^2 + (\text{Im})^2}$

→ affixe du milieu $I = A*B$: $z_I = \dfrac{z_A + z_B}{2}$

→ pour montrer que $\vec{u} \perp \vec{v} \iff \dfrac{aff(\vec{u})}{aff(\vec{v})} \in i\mathbb{R}$

→ pour montrer que $\vec{u} \parallel \vec{v} \iff \dfrac{aff(\vec{u})}{aff(\vec{v})} \in \mathbb{R}$

exemple $z_A = 1+i$; $z_B = 3+i$; $z_C = 1-2i$

* Montrer que $\vec{AB} \perp \vec{AC}$

$$\frac{aff(\vec{AB})}{aff(\vec{AC})} = \frac{z_B - z_A}{z_C - z_A} = \frac{(3+i)-(1+i)}{(1-2i)-(1+i)} = \frac{(2)\times 3i}{(-3i)\times 3i} = \frac{6i}{9} \in i\mathbb{R}$$

donc $\vec{AB} \perp \vec{AC}$

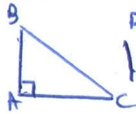
$z_A = 3 - i$ $z_B = -1 - 2i$

$aff(\vec{AB}) = z_B - z_A = (-1-i) - (3-2i)$
$= -4 + i$

$AB = |z_B - z_A| = |-4+i| = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$

Q2 : Déterminer la nature du triangle ABC :

→ rectangle en A

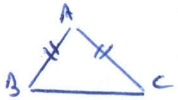


$BC^2 = AB^2 + AC^2$

$|z_C - z_B|^2 = |z_B - z_A|^2 + |z_C - z_A|^2$

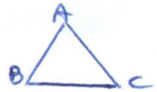
Ou bien $\dfrac{aff(\vec{AB})}{aff(\vec{AC})} \in i\mathbb{R}$

→ isocèle en A



$AB = AC$

→ équilatéral



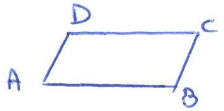
$AB = AC = BC$

Q3 : Nature de quadrilatère ABCD

→ parallélogramme #

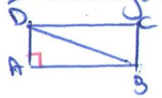
$\vec{AB} = \vec{DC}$

$aff(\vec{AB}) = aff(\vec{DC})$



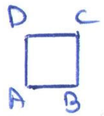
$z_B - z_A = z_C - z_D$

→ rectangle



| → #
| → angle droit ($DB^2 = AB^2 + AD^2$)

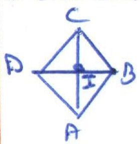
→ carré



$\iff$ | → rectangle
| → $AB = BC$

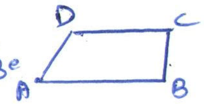
→ losange $\iff I = A*C = D*B$

$z_I = \dfrac{z_A + z_C}{2} = \dfrac{z_D + z_B}{2}$



Ou bien | → #
| → $AB = AD$

→ Trapèze



$\vec{AB} = \alpha\,\vec{DC}$

$z_B - z_A = \alpha(z_C - z_D)$