MATH 312 Final Project Proposal

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1 Background

1.1 The Three Body Problem

The three body problem is a classic application of Newtonian physics wherein the position and velocity of three bodies at some time are determined by their initial positions and velocities, their masses, and the gravitational constant G. In order to simplify this problem for an undergraduate project, we made the following simplifying assumptions: that all bodies have the same mass ($m = 1$), that all planets start from rest, and that all three are acted upon by the same gravitational constant $G = 6.67 * 10^{-11}$. Additionally, we will only be considering the three body problem in two dimensions, which also simplifies it for our purposes.

1.2 Equations

Taking $m_1 = m_2 = m_2 = G = 1$, the following vector equation describes the motion of a single planet in the three body problem.

$$\ddot{\boldsymbol{r}} = \begin{pmatrix} \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{pmatrix} = \begin{pmatrix} -\frac{x_1-x_2}{((x_2-x_1)^2-(y_2-y_1)^2)^{3/2}} + -\frac{x_1-x_3}{((x_3-x_1)^2-(y_3-y_1)^2)^{3/2}} \\ -\frac{y_1-y_2}{((x_2-x_1)^2-(y_2-y_1)^2)^{3/2}} + -\frac{y_1-y_3}{((x_3-x_1)^2-(y_3-y_1)^2)^{3/2}} \end{pmatrix}$$

Here x_1 and y_1 are the x and y coordinates of the planet in question, x_2 and y_2 are the x and y coordinates of the second planet, and x_3 and y_3 are the x and y coordinates of the third planet. The same equation can be written for the second and third planets by swapping the subscripts.

2 Goal

Our intended goal is to model the three body problem in the 2D plane for a range of initial conditions, using Heun's Method. Once every approximation is complete, we would create a "moment map" of the result by averaging the amount of times a planet appears at a particular pixel and giving that pixel a corresponding RGB value. For example, a pixel where planet 1 (red) passes through numerous times would appear very dark red in the moment map, while a pixel where planet 1 (red) and 3 (blue) pass through a lot would appear purple.The steps towards this goal fall into two categories: those that we are certain we can achieve and those that are more ambitious.

3 Minimum Viable Product

The minimum viable product (MVP) of our project entails (1) rewritten equations for the three-body problem in two dimensions, (2) code in python to compute Heun's equations for all 12 of the three-body equations, and (3) plots of three-body motion featuring all three bodies and the center of mass in Python.

4 Overall Goal

Our more ambitious goals entail the creation of the RGB map synthesizing the motion of the three bodies over a variety of initial conditions in order to achieve a better understanding of how the bodies tend to move.