(4) 棒の両端で棒の応力は常にゼロであるから、

$\sigma(x) = \frac{F}{A} = -E\varepsilon = -E\frac{\partial u}{\partial x}$ より、

$\sigma(0) = 0$

$\Rightarrow \frac{\partial u}{\partial x}(0, t) = 0$

$\sigma(l) = 0$

$\Rightarrow \frac{\partial u}{\partial x}(l, t) = 0$

(2) 変数分離法より、

$u(x, t) = X(x)\,T(t)$

①に代入すると、 $X T'' = \left(\frac{E}{\rho}\right) X'' T$

$\therefore \frac{X''}{X} = \left(\frac{\rho}{E}\right)\left(\frac{T''}{T}\right) \cdots ②$

② $= -\lambda^2$ とおく。 $X'' + \lambda^2 X = 0$

$\therefore X = A\cos(\lambda x) + B\sin(\lambda x)$

$\therefore \frac{dX}{dx} = -\lambda A\sin(\lambda x) + \lambda B\cos(\lambda x)$

よし、境界条件

$\frac{dX}{dx}(0) = 0 \quad \therefore B = 0$

$\frac{dX}{dx}(l) = 0 \quad \therefore \sin(\lambda l) = 0 \Rightarrow \lambda l = n\pi \Rightarrow \lambda = \frac{n\pi}{l}$

$\therefore X(x) = A\cos\left(\frac{n\pi x}{l}\right)$

$T'' + \lambda^2\left(\frac{E}{\rho}\right)T = 0$

$\therefore T(t) = C\cos\left(\frac{n\pi c}{l}t + \varphi\right), \; c^2 = \frac{E}{\rho}$

波動方程式： $u(x, t) = P\cos\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi c t}{l} + \varphi\right)$

ただし、 $c = \sqrt{\frac{E}{\rho}}$

(5) $\lambda = \frac{l}{n\pi}$, $A = P$

$u(x, t) = P\left[\cos\frac{n\pi}{l}(x + ct) + \cos\frac{n\pi}{l}(x - ct)\right]$