Synthesis of Star-Junction Multiplexers

Giuseppe Macchiarella, *Senior Member, IEEE*, and Stefano Tamiazzo

***Abstract*—This paper illustrates a general approach to the synthesis of microwave multiplexers (RF combiners) presenting a star-junction topology (with a resonating junction). The channel filters composing the multiplexer can be arbitrarily specified; in particular, the attenuation characteristic may include transmission zeros (imaginary and/or complex). The synthesis procedure, developed in a low-pass frequency domain (suitably defined), is based on an iterative algorithm which evaluates the characteristic polynomials of both the overall multiplexer and the composing filters. Using these latter polynomials, the normalized coupling matrices of the filters can be computed using one of the methods available in the literature and the dimensioning of the channel filters is carried out through the usual de-normalized parameters (coupling coefficients and external Q). The detailed design steps illustrating the synthesis of a triplexer for base stations of mobile communications are presented. The designed device was then built, and the comparison between measurements and simulations has validated the new synthesis approach.**

***Index Terms*—Circuit synthesis, microwave filters, multiplexers.**

I. Introduction

In this paper, the class of multiplexers realizing RF combiners is considered. These devices are widely employed in most communications systems to combine signals in various RF channels (both from the transmitter and/or from the antenna). When high selectivity for each channel is required, transmission zeros are introduced in the attenuation characteristic (this allows a reduction of the overall number of resonators, simultaneously improving the overall insertion losses). Moreover, it is often not possible (or convenient), in several applications, to include isolators or circulators in the multiplexer configuration, so the input ports of the filters are directly connected to each others through a combining structure (manifold or star junction). In this case, there is a strong interaction among the filters, and the synthesis of the overall device may be very difficult and time consuming to accomplish. In practice, the design today is based mainly on optimization, exploiting techniques like space mapping [1]. Nevertheless, some works have recently illustrated how to approach the synthesis of a class of RF combiners (duplexers and triplexers) in a fashion similar to that used for classical coupled-cavity filters [2], [3]; in fact, this approach is based on the characterization of the combiner through suitable characteristic polynomials which are associated with (and define) the scattering parameters of the overall device (assumed to be lossless). Once these polynomials have been evaluated (using suitable procedures), it is also possible to obtain the characteristic polynomials which characterize the channel filters separately from the multiplexer; the synthesis of these filters can be then carried out, using the same techniques employed for narrowband filters (based on the normalized coupling matrix [6]). This approach is very effective because it does not resort to optimization for the initial synthesis of the combiner, allowing a normalized prototype which satisfies the specifications (almost exactly) to be obtained very quickly. The dimensioning of the physical device can be then carried out employing the parameters obtained from the synthesized prototype (i.e., the coupling coefficients and external Q); in many cases, this dimensioning produces acceptable results (especially when tuning elements are provided in the realized structure). In the cases where a very accurate dimensioning is requested, the design obtained with the proposed approach represents a very good starting point for a subsequent numerical optimization (based on EM modeling of the physical structure of the multiplexer).

The purpose of this work is to present a general synthesis procedure for star-junction RF combiners (characterized by a resonant junction), which extends the approach introduced in [2] and [3] to an arbitrary number of channel filters. It should be noted that, to the authors' knowledge, there are no synthesis procedures in the literature for a similar class of combiners, which do not use optimization techniques and which employ filters with arbitrary topology (in particular, the procedures introduced by Rhodes and Levy in [4] and [5] refer to inline Chebyshev filters without transmission zeros).

II. Representation of the Multiplexer

The multiplexer here considered belongs to the category of RF combiners with a resonant star-junction topology [6, ch. 18.4]; a general schematization of this multiplexer type is reported in Fig. 1. The following parameters characterize the kth channel filter: number of resonators np_k; passband limits $(f_{i,k}-f_{s,k})$; return loss RL_k; number of transmission zeros n_{zk}; and transmission zeros $\{fz_k\}$. It is also assumed that the filters are not fully canonic (i.e., $np_k > nz_k$). The resonant junction is represented by a shunt resonator with resonant frequency $f_{\mathrm{ris},0}$ and external Q_e.

The synthesis of this structure is carried out in a normalized low-pass frequency domain (represented by $s=\Sigma+j\Omega$), which is related to the actual bandpass domain $p=\sigma+j\omega$ through the well-known relation

$$s=\left(\frac{f_0}{B}\right)\left(\frac{p}{f_0}+\frac{f_0}{p}\right) \tag{1}$$

which, for $s=j\omega=j2\pi f$, becomes

$$\Omega=\left(\frac{f_0}{B}\right)\left(\frac{f}{f_0}-\frac{f_0}{f}\right) \tag{1'}$$

Manuscript received May 06, 2010; revised August 26, 2010; accepted August 29, 2010. Date of publication November 09, 2010; date of current version December 10, 2010.

G. Macchiarella is with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, 20133 Milan, Italy (e-mail: macchiar@elet.polimi.it).

S. Tamiazzo is with Andrew Telecommunication Products, 20041 Milan, Italy (e-mail: stefano.tamiazzo@andrew.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TMTT.2010.2086570

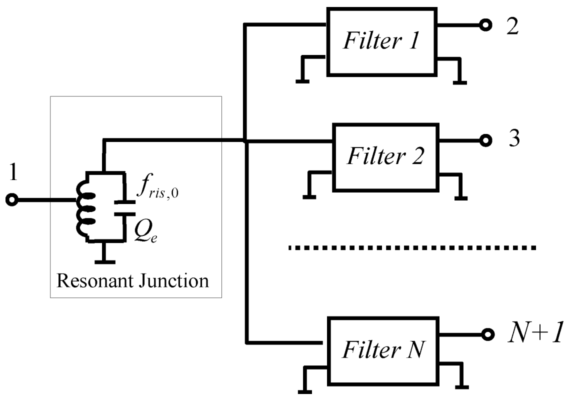

Fig. 1. General architecture of the resonant star-junction multiplexer.

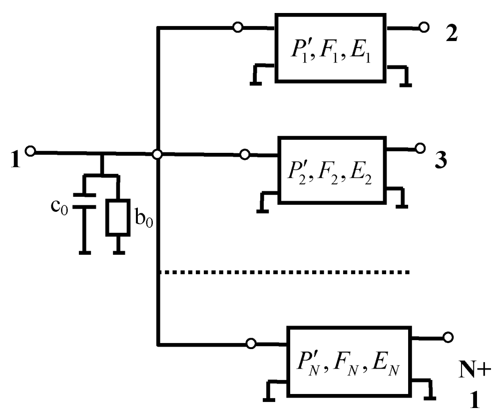

Fig. 2. Normalized prototype circuit of the multiplexer.

where B and f_0 are, in this case, defined as

$$B = f_{s,N} - f_{i,1} \quad f_0 = \sqrt{f_{s,N} \cdot f_{i,1}} \tag{2}$$

where $f_{i,1}$ and $f_{s,N}$ are the lower frequency of the first passband and the upper frequency of the last passband, respectively.

An equivalent circuit of the multiplexer in the normalized domain s is reported in Fig. 2.

The frequency-invariant susceptance b_0 appears in the normalized representation of the junction because the resonant frequency $f_{ris,0}$ does not coincide, in general, with f_0 [defined in (2)]. The channel filters in the domain s are represented through their characteristic polynomials, which define the scattering parameters of each filter when it is separated from the multiplexer [6, ch. 6], and are given as

$$S_{k,11} = \frac{F_k(s)}{E_k(s)} \quad S_{k,21} = \frac{P_k'(s)}{E_k(s)} = \frac{p_k P_k(s)}{E_k(s)}. \tag{3}$$

The roots of $P_k(s), F_k(s)$, and $E_k(s)$ are the transmission zeros, the reflection zeros, and the poles, respectively, of the filter k; moreover, E_k and F_k are monic polynomials (i.e., their highest degree coefficient is unitary). After the transmission zeros of the channel filters are assigned, the roots zP_k of P_k' are also known [evaluated through (1)]; this means that the monic polynomials $P_k(s)$ in (3) are known, while the constants p_k (which are the highest degree coefficients of P_k') depend on the multiplexer design. We also assume that the roots zP_k obey the constraints requested for the synthesis of the channel filters in the form of coupled lossless resonators, i.e., they must be a purely imaginary or complex pair with an opposite real part [6, ch. 6].

The approach here proposed to the synthesis of this class of multiplexer consists in the evaluation of the polynomials P_k, F_k, and E_k for all of the channel filters (together with the parameters of the resonant junction) which determine an equiripple response in each defined passband (with the assigned parameters np_k, RL_k, and $nz_k, \{fz_k\}$). Note the flexibility of such an approach: once the characteristic polynomials are known, several degrees of freedom are still available for the selection of the filters topology; moreover, using the polynomials, we can compute the multiplexer response without actually carrying out the synthesis of the overall network, which allows, for instance, knowing the actual out-of-band selectivity presented by the multiplexer independently of its final topology.

Once the characteristic polynomials of the channel filters have been determined, the subsequent step in synthesis consists of the evaluation of the normalized coupling matrix $(\mathbf{M_k})$ associated with a specific filter topology; this task is performed by using one of several techniques available in the literature, developed for the synthesis of narrowband coupled-cavity filters [6, ch. 8–10]. Finally, the dimensioning of the actual structure implementing the multiplexer is realized by using the de-normalized parameters obtainable from the matrices $\mathbf{M_k}$; such parameters are represented by the cavities' resonant frequencies, the coupling coefficients, and external Q, which can be computed with the formulas reported in Section III

III. Characteristic Polynomials of the Multiplexer

In the normalized domain s, the multiplexer is a lossless, $(\mathrm{N}+1)$-port network, which can be described by suitable characteristic polynomials associated with the scattering parameters

$$S_{11} = \frac{U'(s)}{D(s)} = \frac{u_o U(s)}{D(s)} \quad S_{k1} = \frac{T_k'(s)}{D(s)} = \frac{t_k T_k(s)}{D(s)} \tag{4}$$

where $U(s)$ and $D(s)$ are monic polynomials whose roots represents the reflection zeros at port 1 (assumed in the following as the input port) and the poles of the multiplexer, respectively, and $T_k(s)$ are the transmission polynomials (between ports 1 and k), whose roots define the transmission zeros between the two ports. The order *Np* of polynomials U and D is obtained by summing the number of all poles in the multiplexer (including the resonant junction contribution) to yield

$$Np = \sum_{k=1}^{N} np_k + 1. \tag{5}$$

The coefficients u_0 and t_k (assumed to be real in the following) represent the highest degree coefficients of the polynomials $U'(s)$ and $T_k'(s)$.

As will we verify in the following, the degree of polynomial $T_k'(s)$ is

$$Nz_k = nz_k + \sum_{i \neq k}^{N} np_i = nz_k + Np - np_k - 1. \tag{5'}$$

A. Evaluation of $U'(s), D(s), T_k'(s)$ as a Function of the Polynomials of Channel Filters

Let consider in Fig. 2 the input admittance Y_{in} at port 1 with all of the other ports closed on unit loads: it has

$$Y_{\text{in}} = sc_0 + jb_0 + \sum_{1=1}^{N} y_{\text{in},k}. \tag{6}$$

The admittance $y_{\text{in},k}$ at the input of each channel filter can be expressed as a function of the polynomials E_k and F_k as

$$y_{\text{in},k} = \frac{1 - S_{k,11}}{1 + S_{k,11}} = \frac{E_k - F_k}{E_k + F_k} = \frac{D_k}{S_k} \tag{7}$$

with

$$D_k = \frac{E_k - F_k}{2} \quad S_k = \frac{E_k + F_k}{2}. \tag{8}$$

Let observe that S_k is a monic polynomial of degree np_k; D_k has instead degree $np_k - 1$ and it is not monic.

Substituting (7) into (6), we obtain

$$Y_{\text{in}} = \frac{(sc_0 + jb_0) \cdot \prod_k^{1,\ldots N} S_k + \sum_k^{1,\ldots N} \left(D_k \prod_{i \neq k}^{1,\ldots N} S_i \right)}{\prod_k^{1,\ldots N} S_k}. \tag{9}$$

The scattering parameter S_{11} of the multiplexer then results in (10), shown at the bottom of this page. From (10) and (4), the polynomials $U(s)$ and $D(s)$ and the coefficient u_0 are immediately derived as

$$\begin{aligned} D(s) &= s \cdot S + \frac{1}{c_o} \left[(1 + jb_0) \cdot S + \sum_k^{1,\ldots N} (D_k \cdot W_k) \right] \\ U(s) &= s \cdot S - \frac{1}{c_o} \left[(1 - jb_0) \cdot S - \sum_k^{1,\ldots N} (D_k \cdot W_k) \right] \\ u_0 &= -1 \end{aligned} \tag{11}$$

where $S(s)$ and $W_k(s)$ are defined as follows:

$$S(s) = \prod_k^{1,\ldots N} S_k \quad W_k(s) = \prod_{i \neq k}^{1,\ldots N} S_i. \tag{12}$$

Note that $S(s)$ is a monic polynomial of order $Np - 1$; the order of $W_k(s)$ is $Np - np_k - 1$.

Consider now the scattering parameter S_{k1} defined in (4); it can be expressed as function of $S_{k,21}, Y_{\text{in}}$, and $y_{\text{in},k}$ as

$$S_{k1} = \frac{t_k T_k(s)}{D(s)} = \frac{S_{k,21}(1 + y_{\text{in},k})}{1 + Y_{\text{in}}}. \tag{13}$$

After substituting (7) and (9) into (13), with some manipulations, the following expressions for $T_k(s)$ and t_k are found:

$$T_k(s) = P_k(s) \cdot W_k(s), \quad t_k = p_k / c_0 \tag{14}$$

The number of transmission zeros reported by (5') (i.e., the order of $T_k(s)$) is then confirmed; the roots of the polynomial $W_k(s)$ represent the additional $Np - np_k - 1$ zeros produced on the transmission between ports 1 and k by the loading, at the junction, of all of the filters except for the kth.

B. Derivation of the Channel Filters Polynomials Given $U'(s), D(s), T_k'(s)$

Once the characteristic polynomials $U'(s), D(s), T_k'(s)$ of the multiplexer are known, the computation of the characteristic polynomials of the channel filters can be carried out (also the junction parameters b_0 and c_0 are obtained during this derivation).

From (11), subtracting $U(s)$ from $D(s)$, we have

$$[D(s) - U(s)] = \frac{2}{c_0} \cdot S(s) = \frac{2}{c_0} \cdot \prod_{k=1}^{N} S_k. \tag{15}$$

Because $S(s)$ is a monic polynomial, it can be immediately obtained from the roots of $[D(s) - U(s)]$; moreover, from (15), is also evident that the roots of $S_k(s)$ belong to those of $[D(s) - U(s)]$. We must then devise a criterion for distributing the roots of $[D(s) - U(s)]$ among the polynomials $S_k\ (s)$; specifically, we assume that the roots zS_k of $S_k(s)$ are the closest to the kth passband among all of the roots of $[D(s) - U(s)]$. In practice, considering that also the polynomials $S_k(s)$ are monic, they can be found by sorting the roots of $[D(s) - U(s)]$ with increasing imaginary part and assigning the first np_1 roots to zS_1, the following np_2 to zS_2, and so on.

Let us now evaluate $D(s)$ at $s = zS_k$; from (11), we have

$$D(zS_k) = \frac{1}{c_o} [D_k(zS_k) \cdot W_k(zS_k)]. \tag{16}$$

Then, the polynomials $D_k(s)$ evaluated for $s = zS_k$ are given by

$$D_k(zS_k) = c_o \cdot \frac{D(zS_k)}{W_k(zS_k)} \quad zS_k = [zS_1, \ldots, zS_{np_k}]. \tag{17}$$

Note that both $D(s)$ and $W_k(s)$ are known once $S_k(s)$ has been evaluated as above illustrated.

$$S_{11} = \frac{1 - Y_{\text{in}}}{1 + Y_{\text{in}}} = \frac{\prod_k^{1,\ldots N} S_k - (sc_0 + jb_0) \cdot \prod_k^{1,\ldots N} S_k - \sum_k^{1,\ldots N} \left(D_k \cdot \prod_{i \neq k}^{1,\ldots N} S_i \right)}{\prod_k^{1,\ldots N} S_k + (sc_0 + jb_0) \cdot \prod_k^{1,\ldots N} S_k + \sum_k^{1,\ldots N} \left(D_k \cdot \prod_{i \neq k}^{1,\ldots N} S_i \right)} \tag{10}$$

One can now observe that each $D_k(s)$ is a nonmonic polynomial of order $np_k - 1$, so it depends on np_k unknown coefficients; imposing the np_k values obtained from (17), these coefficients can be found by solving the following linear system:

$$\mathbf{d_k} = \mathbf{V_k^{-1}} \cdot \mathbf{p_k} \tag{18}$$

with

$$\begin{aligned} V_k(i,j) &= [zS_k(i)]^{(np_k - j)} \\ p_k(i) &= c_o \cdot \frac{D(zS_k(i))}{W_k(zS_k(i))}, \qquad i,j = 1, \dots np_k. \end{aligned} \tag{19}$$

The parameter c_0 is computed by observing from (15) that the highest degree coefficient δ_1 of $[D(s) - U(s)]$ is $2/c_0$, and then

$$c_0 = 2/\delta_1. \tag{20}$$

b_0 can be obtained from the polynomial $D(s)$; in fact, from (11), the second highest degree coefficient d_2 of $D(s)$ is given by $d_2 = s_2 + (1 + jb_0)/c_0$, where s_2 is the second highest degree coefficient of $S(s)$. Thus

$$jb_0 = c_0(d_2 - s_2) - 1. \tag{21}$$

Once $D_k(s)$ and $S_k(s)$ are known, the characteristic polynomials E_k and F_k are computed from (8) to yield

$$F_k = \frac{S_k - D_k}{2} \quad E_k = \frac{S_k + D_k}{2}. \tag{22}$$

The monic polynomials $P_k(s)$ are known because their roots zP_k are obtained with (1) from the assigned transmission zeros $\{fz_k\}$ of each filter; using (14), the coefficients p_k are computed from the coefficients t_k of the transmission parameters S_{k1} to yield

$$p_k = c_0 \cdot t_k. \tag{23}$$

IV. Synthesis of the Multiplexer

In the previous sections, the polynomial characterization of the multiplexer in the normalized frequency domain s has been developed. Suitable expressions have also been derived relating the characteristic polynomials of the channel filters with the polynomials of the whole multiplexer. Here, we will illustrate how to compute the multiplexer polynomials which meet the following design goal.

"Assign the channel filters passbands $(f_{i,k} - f_{s,k})$ and the parameters $np_k, RL_k, nz_k, \{fz_k\}$, and find the polynomials $U'(s), D(s), T'_k(s)$ which determine an equiripple response in the passbands of channel filters."

The response is here intended as the reflection coefficient $|S_{11}|$ at the input port of the multiplexer. It must be immediately said that an exact solution to this problem was not found; what is illustrated in the following is, in fact, a suboptimal solution, where $|S_{11}|$ exhibits a "quasi-equiripple" behavior in the passbands (i.e., a response with a very small deviation with respect to the ideal equiripple condition around the imposed return loss level in each channel filter).

The developed procedure requires that the reflection zeros at the input port of the multiplexer (i.e., the roots zU of $U'(s)$) are assigned. It is then hypothesized that the quasi-equiripple response is obtained with the following assignment:

$$zU = \{zU_0, \{z\bar{F}_k\}\} \tag{24}$$

where zU_0 is a real constant with a value between 1 and 2 (typically 1.5) and $\{z\bar{F}_k\}$ are the roots of polynomials $\bar{F}_k(s)$ obtained by synthesizing all the filters separately from the multiplexer (with the assigned parameters $np_k, RL_k, nz_k, \{z\bar{P}_k\}$ and the generalized Chebyshev characteristic [6, ch. 6]. The value of the constant zU_0 is not critical, and whatever real value in the specified range can be used.

We wish to emphasize that this assignment of the reflection zeros is not based on a theoretical assumption; however, its validity is verified *a posteriori* by the evidence of the produced results.

Let $\bar{F}_k(s), \bar{P}_k(s), \bar{E}_k(s)$ be the characteristic polynomials of the channel filters synthesized separately from the multiplexer; note that the unknown polynomials $P_k(s)$ actually coincide with $\bar{P}_k(s)$ because they are monic and their roots are the assigned transmission zeros $\{z\bar{P}_k\}$ of the channel filters.

The evaluation of the unknown multiplexer polynomials is carried out through an iterative procedure, which is initialized by assigning the polynomial $S(s)$, defined in (12), as follows:

$$S^{(0)}(s) = \prod_{k=1}^{N} \frac{\bar{E}_k + \bar{F}_k}{2}. \tag{25}$$

The iterations (represented below with the superscript m) begin with the evaluation of the polynomials $S_k^{(m)}$ through the ordering and allocation of the roots of $S^{(m)}$, as seen in Section III-B (the roots of $S(s)$ are the same as those of $[D(s) - U(s)]$); the polynomials $T_k(s)$ are then evaluated with (14), with $P_k(s)$ known as observed before, to yield

$$T_k^{(m)}(s) = P_k(s) \cdot \prod_{i \neq k}^{N} S_i^{(m)}(s). \tag{26}$$

For computing the coefficients $t_k^{(m)}$, the requested return loss (RL_k) is imposed at N frequencies $j\Omega_k$, suitably selected in the passbands, to yield

$$\begin{aligned} |S_{11}(j\Omega_k)|^2 =& \frac{|U(j\Omega_k)|^2}{\left|D^{(m)}(j\Omega_k)\right|^2} \\ =& \frac{|U(j\Omega_k)|^2}{|U(j\Omega_k)|^2 + \sum_1^N \left|t_k^{(m)}\right|^2 \cdot \left|T_k^{(m)}(j\Omega_k)\right|^2} \\ =& 10^{-RL_k/10}. \end{aligned} \tag{27}$$

Note that the Feldkeller equation (unitarity of the scattering matrix) has been used for expanding $|D^{(m)}(j\Omega_k)|^2$. From (27), the coefficients t_k are obtained by solving the following system of N linear equations:

$$\left|\mathbf{t^{(m)}}\right|^2 = \mathbf{A}^{-1} \cdot \mathbf{b} \tag{28}$$

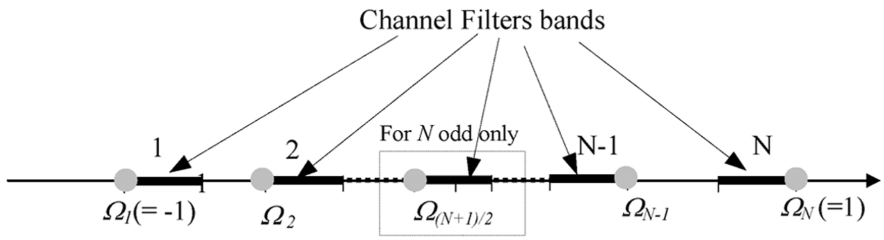

Fig. 3. Frequencies $j\Omega_k$ where the return loss RL_k is imposed.

with

$$
\begin{aligned}
A_{kq} &= \left| \frac{T_q^{(m)}(j\Omega_k)}{U(j\Omega_k)} \right|^2 \\
b_q &= (10^{RL_q/10} - 1), \qquad k, q = 1, \dots N \qquad (29) \\
t_k^{(m)} &= \sqrt{\left| t_k^{(m)} \right|^2}.
\end{aligned}
$$

Note that the solution (29) implies that $t_k^{(m)}$ is real and positive.

The assignment of the frequencies $j\Omega_k$ is illustrated in Fig. 3. One frequency per channel is selected; for the first $N/2$ (N even) or $(N+1)/2$ (N odd) channels, the selected Ω_k is the lower limit of the passband, while, for the last $N/2$ (N even) or $(N-1)/2$ (N odd) channels, Ω_k is the upper limit of the passband.

At this point, the conservation of power in the s domain is used for relating $U(s)$ and $T_k'(s)$ with the unknown polynomial $D(s)$ as follows:

$$
\begin{aligned}
D_2^{(m)}(s) &= D^{(m)}(s) D^{*(m)}(-s) \\
&= U(s)U^*(-s) + \sum_1^N \left| t_k^{(m)} \right|^2 \cdot T_k^{(m)}(s) T_k^{*(m)}(-s)
\end{aligned} \qquad (30)
$$

where the * at superscript denotes the operation of *para-conjugation* [6, ch. 6]. The monic polynomial $D(s)$ is then obtained from the roots of $D_2(s)$, by selecting those with negative real part (*spectral factorization*).

A new estimation of $S(s)$ is then computed from the roots of $[D^{(m)}(s) - U(s)]$, and the iterative procedure can start again from the beginning; the iterations continue until the results obtained become sufficiently stable. The convergence is evaluated on the roots $zS^{(m)}$ of $S^{(m)}$ as follows:

$$
\max\left(\frac{zS^{(m)} - zS^{(m-1)}}{zS^{(m)}} \right) < \varepsilon. \qquad (31)
$$

Typically, less than ten iterations are requested to get convergence (with $\varepsilon = 10^{-3}$).

The flowchart of the iterative algorithm is shown in Fig. 4.

As an example of the synthesis procedure, let consider a multiplexer of order five ($\mathrm{N} = 5$), with the following assigned parameters:

- Channel bands: $[(-1, -0.7); (-0.5, -0.3); (-0.1, 0.05); (0.25, 0.55); (0.8, 1)]$.

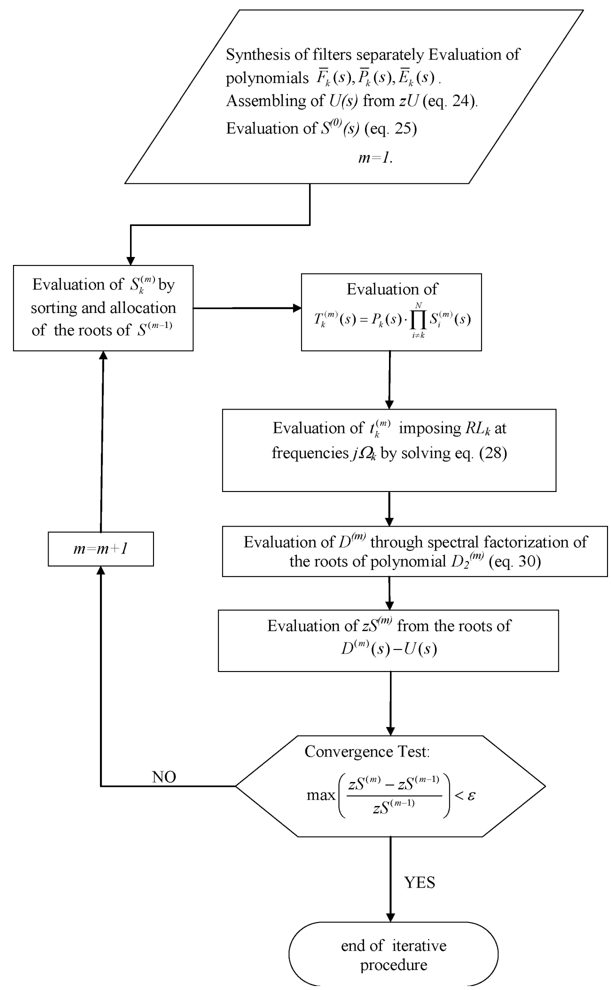

Fig. 4. Flowchart of the synthesis algorithm.

- np_k: $[5, 4, 3, 3, 4], RL_k : [25, 25, 25, 25, 25]$ dB.
- nz_k: $[2, 1, 0, 0, 1]$, $\{z\bar{P}_k\} = \{(-1.12\mathrm{i}, -0.66\mathrm{i}), (-0.17\mathrm{i}), (), (), (0.75\mathrm{i})\}$.

The iterative procedure converges after four iterations (the computed polynomials are not reported due to lack of space). The transmission and reflection scattering parameters computed from these polynomials are shown in Fig. 5. It can be observed that, in addition to the assigned transmission zeros of the channel filters, other zeros appear in the transmission characteristic, which are mostly complex; as said before, these additional zeros are the roots of polynomials $W_k(s)$ and are determined by the reciprocal loading of the channel filters

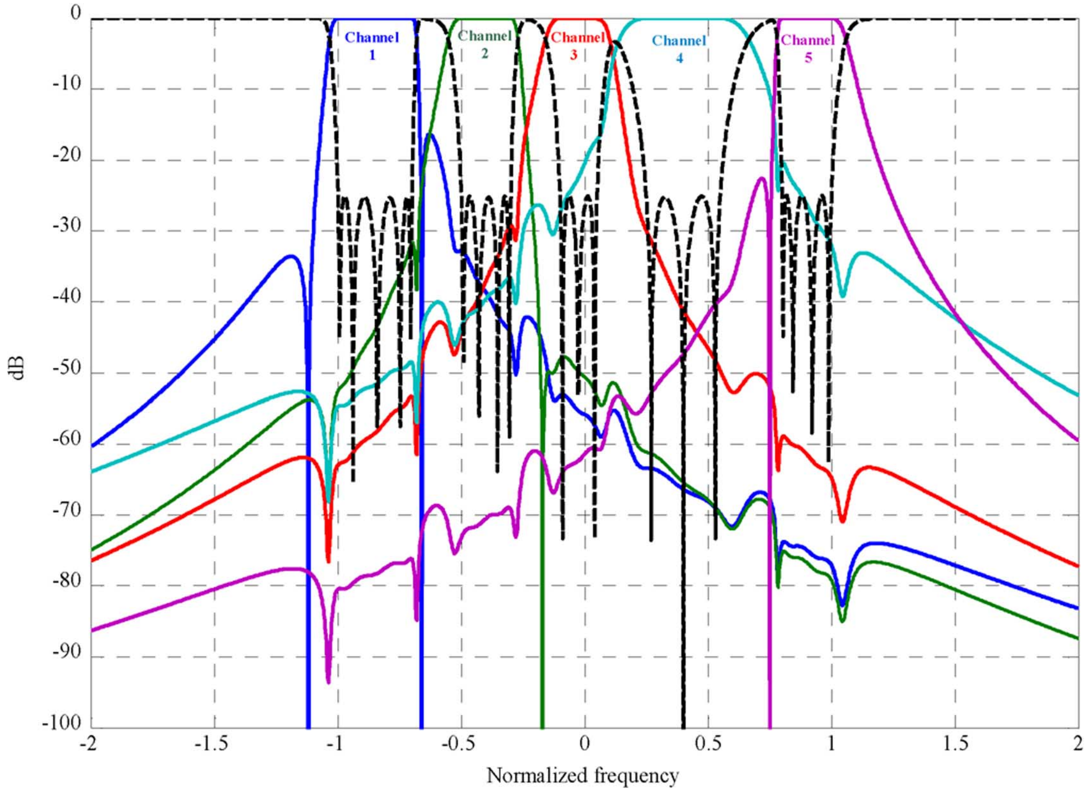

Fig. 5. Response of the multiplexer of order five from the computed polynomials. The dashed line is the return loss at port 1 (resonant junction); the other lines represent the transmission parameters $|S_{k1}|$.

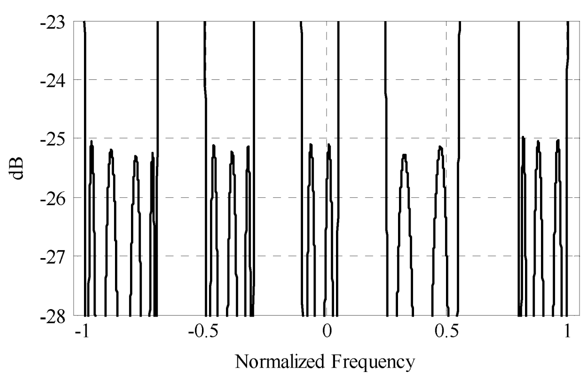

Fig. 6. "Quasi-equiripple" response in the channel filters' passbands.

through the star-junction connection (note that these zeros may give an additional contribute to selectivity of the multiplexer).

In Fig. 6, an expanded view of the return loss in the frequency range $(-1, 1)$ is reported to put into evidence the "quasi-equiripple" response attainable with the novel synthesis technique; note that the deviation relative to the ideal "equiripple" response is less than 0.5 dB.

One may wonder how to satisfy the out-of-band attenuation requirements for each channel filter using the proposed design approach. As is known, the attenuation produced by a filter is primarily determined by the number of resonators and by the assigned transmission zeros; the selection of these parameters in order to fit the selectivity requirements of the multiplexer channels can be initially made by considering the response of the filters synthesized separately from the multiplexer, with the generalized Chebyshev characteristic. The attenuation then produced in each channel once the multiplexer has been designed is always larger or at least equal to the one obtained with the separated filters. This result is illustrated in Fig. 7, where the attenuation of channels 1 and 5 of the previously synthesized multiplexer (of order five) is compared with the attenuation produced by the corresponding filters separately synthesized. As can be observed, the attenuation just outside the passbands (and close to the transmission zero frequencies) is practically coincident in the two cases; away from the passbands, the attenuation obtained from the multiplexer becomes larger than that produced by the filters alone. In conclusion, for satisfying the selectivity requirements, the response of the filters separated from the multiplexer can be used, having in addition a further attenuation increase once the multiplexer is synthesized.

Another aspect concerning the requirements is how much closer can the passbands of the channel filters be. In principle, there is no limit on the closeness of the passbands; however, it has been found that, by assigning passbands too close, (28) may produce imaginary solutions, determining the fail of the synthesis algorithm. As an example, Fig. 8 shows the response of multiplexers of order three and four with the closest passbands which the synthesis algorithm allowed.

V. Complete Design Example

Here, the complete design of a triplexer starting from the specifications in the real frequency domain is illustrated. After the evaluation of the characteristic polynomials (both of the

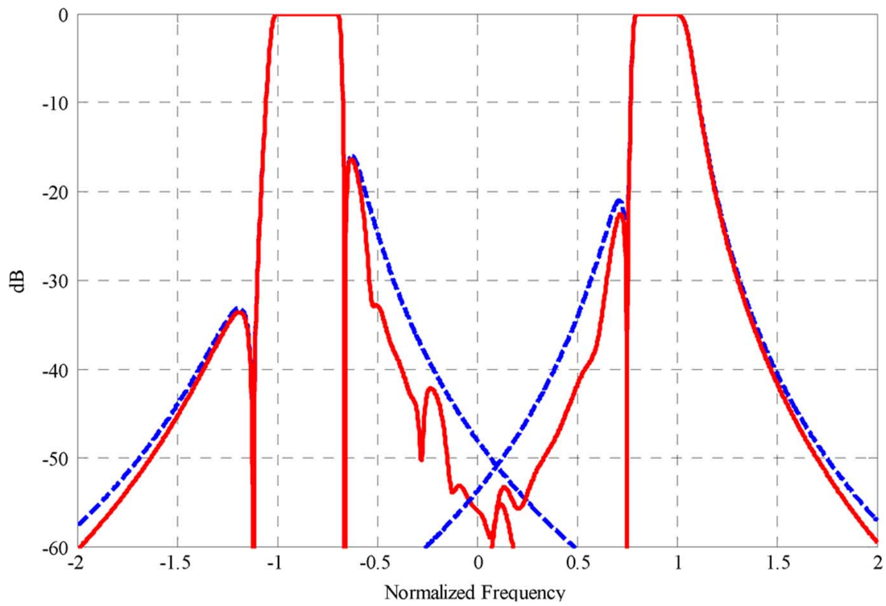

Fig. 7. Attenuation produced by the multiplexer in channels 1 and 5 (continuous lines) compared with the attenuation evaluated from the separately synthesized filters (dashed lines).

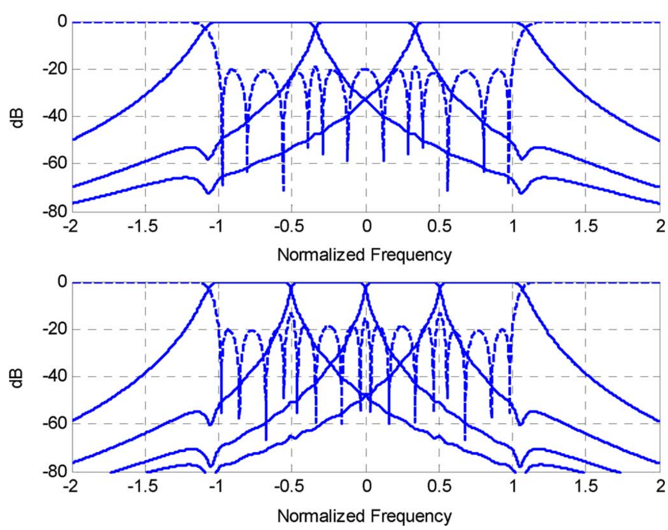

Fig. 8. Response of multiplexers of order three and four (imposed return loss: 20 dB, no transmission zeros) with the closest passbands allowed by the synthesis algorithm.

triplexer and the channel filters), the normalized coupling matrices are evaluated for the specified filters' topology. Then, using suitable relations, the de-normalized parameters (coupling coefficients and external Q) are obtained. Finally, these parameters are used for the dimensioning of a sample device, which has been fabricated for validating the new design approach.

A. Specifications and Polynomials Evaluation

The specifications refer to a triplexer combiner used in base stations for mobile communications. The following parameters are assigned.

- Passbands (MHz): (697–717), (727–769), (776–799)
- Filters' order (np_k): [7, 10, 8]
- Return loss (RL_k): 22 dB (all of the channels).
- Transmission zeros fz_k (MHz): (728); (714.5, 778); (767).

With (1) and (2), the passbands and transmission zeros are evaluated in the normalized domain using $f_0 = 746.26$ MHz and $\mathrm{B} = 82$ MHz. Then, the characteristic polynomials of the triplexer are evaluated using the iterative procedure illustrated in Section IV; the characteristic polynomials of the channel filters (with the parameters of the resonant junction) are then computed with the formulas of Section III-B (all of these polynomials are not reported here due to lack of space).

B. Synthesis of the Normalized Prototype Network of the Triplexer

The next step of the design is the assignment of the channel filters' topology; assuming cascaded-blocks configuration, the imaginary transmission zeros in the three filters are extracted by means of triplet blocks. The overall topology assigned to the triplexer is illustrated in Fig. 9, which represents schematically the low-pass equivalent circuit of the triplexer.

The synthesis consists of the evaluation of the normalized coupling matrices $\mathbf{M_k}$ of the three filters, starting from their characteristic polynomials previously computed (the order of $\mathbf{M_k}$ is np_k+2, including the coupling with the resonant junction and the load). There are well-known techniques in the literature for carrying out this task; here, the method based on the synthesis of a transverse canonical prototype followed by suitable matrix transformations (Given's transform) has been used [7]. The elements of the computed coupling matrices are reported in the scheme in Fig. 9 (the numbers in normal characters are the elements $M_k(i,i)$, while the ones in bold characters are the $M_k(i,j)$). It must be remarked that the evaluation of the matrices $\mathbf{M_k}$ from the characteristic polynomials of the channel filters is carried out as for a single filter (i.e., without considering the connection to the junction); in fact, the reciprocal interaction among the filters has been taken into account during the polynomials computation.

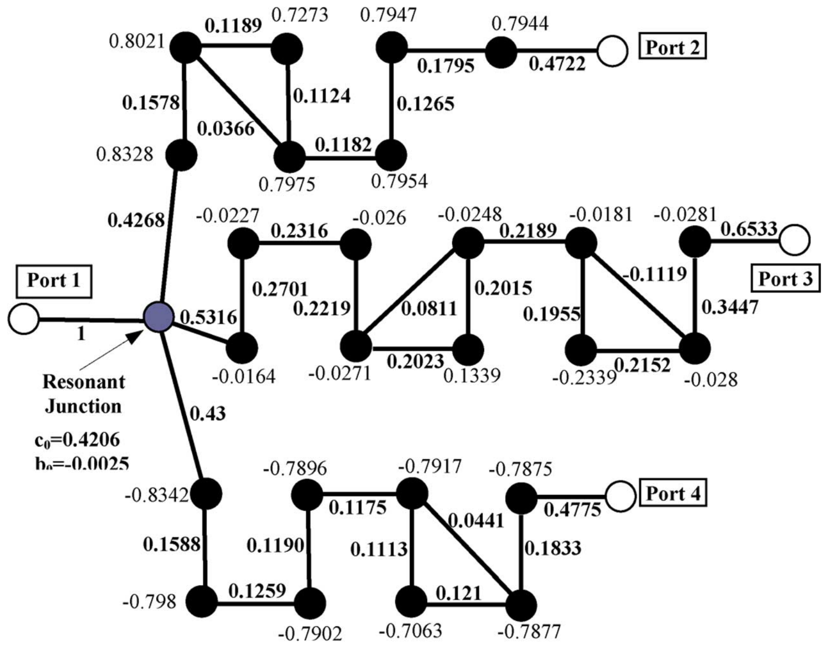

Fig. 9. Topology of the triplexer low-pass prototype. Each black node represents a unit capacitance in parallel with a frequency-invariant susceptance whose value is reported in the figure (normal character); the segments represent admittance inverters whose parameters are the numbers in bold character. The resonant junction (gray node) is the parallel of c_0 with b_0; the input and output white nodes are unit conductances.

C. De-Normalization of the Prototype Network

The circuit in Fig. 9 must be de-normalized in order to obtain the parameters needed for the geometrical dimensioning of the structure implementing the triplexer. As is well known, these parameters are the coupling coefficients and the external Q, which can be valuated with the following formulas [8]:

$$k_{i,j}^k = B_n \cdot M_k(i,j)$$
$$Q_e^k = \frac{1}{B_n \cdot (M_k(np_k+1, np_k+2))^2} \tag{32}$$

where $B_n = B/f_0$ [B and f_0 are defined in (2)]. The above expressions refer to the coupling inside the filters; for the coupling involving the resonant junction, it has instead

$$k_{0,1}^k = B_n \cdot \frac{M_k(1,2)}{\sqrt{c_0}} \qquad Q_e = \frac{c_0}{B_n} \tag{33}$$

where Q_e represents the external Q of the junction connected to the input load (port 1).

In addition to the above parameters, the resonant frequencies of the de-normalized resonators must also be evaluated; in fact, the resonance of the cavities composing the channel filters does not occur at f_0 for the presence of frequency-invariant susceptances in the normalized prototype. For the resonators inside the filters, we have

$$f_{\mathrm{ris},i}^k = f_0 \left[\sqrt{1 + \left(\frac{B_n \cdot M_{i,i}^k}{2}\right)^2} - \left(\frac{B_n \cdot M_{i,i}^k}{2}\right)\right] \tag{34}$$

while, for the resonant junction, the following expression must be used:

$$f_{\mathrm{ris},0} = f_0 \left[\sqrt{1 + \left(\frac{B_n \cdot b_0}{2}\right)^2} - \left(\frac{B_n \cdot b_0}{2}\right)\right]. \tag{35}$$

Note that, using resonators tuned at the above frequencies, the response of the de-normalized equivalent circuit of the triplexer is no longer exactly the same as the prototype transformed with (1); in fact, the frequency-invariant susceptances are not actually introduced in the de-normalized circuit (their effect is approximated by detuning the resonators with respect to f_0).

The following results have been obtained for the designed triplexer.

Filter 1:
- $f_{\mathrm{ris},i}$ (MHz): [704.99, 706.47, 710.09, 706.7, 706.79, 706.83, 706.84].
- $k_{i,i+1}$: [0.08995, 0.02156, 0.016252, 0.01536, 0.016162, 0.017287, 0.024529].
- $k_{2,4}$: 0.0050067.
- Q_e: 32.81.

Filter 2:
- $f_{\mathrm{ris},i}$ (MHz): [747.09, 747.42, 747.58, 747.64, 739.46, 747.53, 747.18, 758.28, 747.687, 747.694].
- $k_{i,i+1}$: [0.112, 0.0369, 0.03165, 0.03033, 0.02765, 0.02755, 0.02991, 0.026716, 0.02941, 0.04711].
- $k_{4,6}, k_{7,9}$: [0.011084, 0.0153].
- Q_e: 17.14.

Filter 3:
- $f_{\mathrm{ris},i}$ (MHz): [790.01, 788.07, 787.65, 787.62, 787.74, 783.15, 787.51, 787.5].
- $k_{i,i+1}$: [0.0906, 0.0217, 0.0172, 0.0162, 0.016, 0.0152, 0.0165, 0.025].
- $k_{5,7}$: -0.00603.
- Q_e: 32.09.

Resonant Node:
- $f_{\mathrm{ris},0}$ (MHz): 746.39.
- Q_e: 3.077.

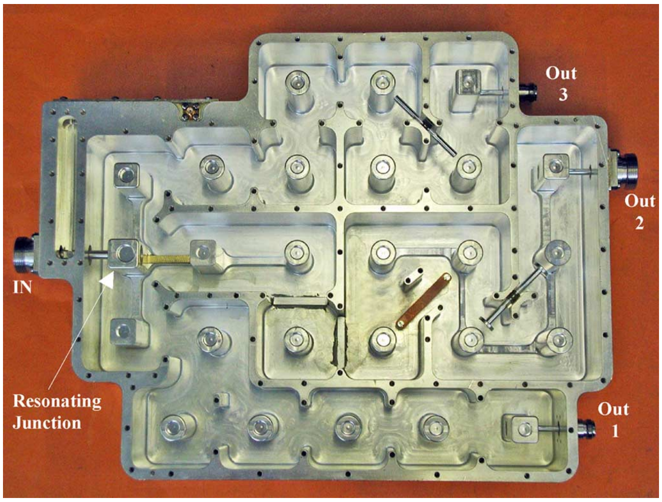

Fig. 10. Photograph of the manufactured triplexer (top cover removed).

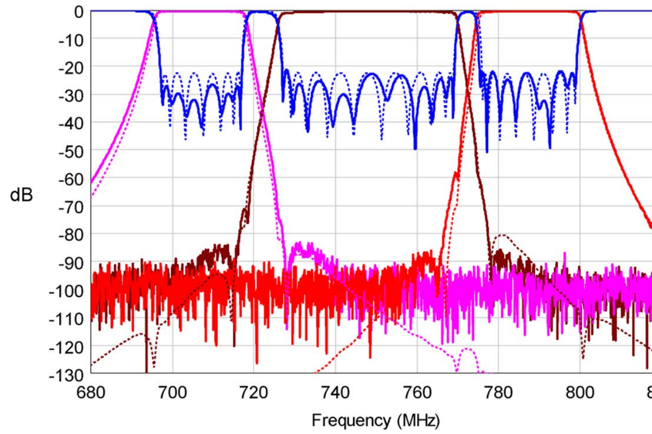

Fig. 11. Measured attenuation and return loss of the triplexer after a first alignment (continuous lines); simulated response (dashed lines) is reported for comparison.

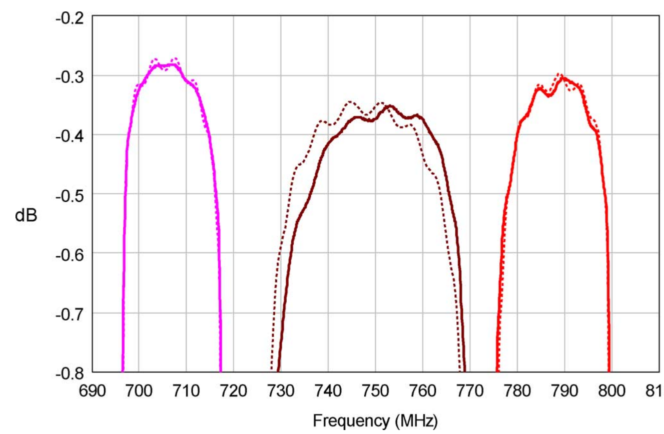

Fig. 12. Measured (continuous lines) and simulated (dashed lines) passbands attenuation of the fabricated triplexer. The following unloaded Q has been assigned to the equivalent resonators in the simulation: $Q_{0,RX} = 6020, Q_{0,MX} = 3640, Q_{0,TX} = 6230$.

D. Fabrication of the Triplexer

The coaxial technology is adopted for the realization of the triplexer; the resonators are constituted by coaxial cavities with an inner round (or square) conductor (rod) of length $\lambda/8$ at the center frequency of each channel passband and resonating with a screw penetrating inside the rod. The dimensions of the cavities are selected in order to obtain an unloaded Q of about 6000. The main coupling are realized by removing part of the walls between the cavities, while the cross coupling are implemented through suitable probes (inductive or capacitive). Note that the large value of the coupling coefficients between the resonant junction and the first resonator in each filter has requested additional probes connecting the lower part of the rods.

The dimensioning of the triplexer has been carried out by imposing the coupling coefficients and the external Q previously computed to the physical structure (advanced techniques based on electromagnetic simulations and parameters extraction [8] have been used to this purpose); note, however, that tuning screws have been provided both for resonators and for coupling structures in order to relax the requested fabrication accuracy and to allow, at the same time, an accurate alignment of the triplexer. Fig. 10 shows the fabricated structure (top view with the cover removed).

In Figs. 11 and 12, the measured response of the fabricated triplexer (after a first alignment) is compared with the simulated response (computed through the de-normalized network). A satisfactory agreement between the curves can be observed.

VI. Comments About the Limits of the Novel Synthesis Method

Here, some limits of the novel technique for the synthesis of multiplexers are discussed. These limits mainly concern the numerical accuracy requested for obtaining meaningful results with the implemented procedure. In fact, it must be observed that the overall number of poles in the low-pass prototype of the multiplexer increases very rapidly with the number of channel filters and with the order of each filter. For example, the prototype of the triplexer before designed is a 26° network (i.e., the order of polynomials $U(s)$ and $D(s)$ is 26); during the synthesis procedure (Section IV), the evaluation of the polynomial $D(s)$ requires finding the roots of $D_2(s)$ (30), which is a polynomial of twice order respect to $D(s)$ (then 52 in this case). It is therefore comprehensible as the round-off errors arising from the finite number of digits used by the computer may produce inaccurate results in the iterative procedure (which could even not converge in the most serious cases). It has been found that, with double precision arithmetic (which allows about 15-digit accuracy), the maximum degree of the multiplexer prototype should not exceed about 25–30. Moreover, this limit is influenced by the frequency separation between the channels: in fact, the maximum allowed degree of the polynomials tends to decrease when a channel is far from all others (this is due to the crowding of the polynomials roots in a very small range).

It must be finally remarked that the proposed technique belongs to the category of narrowband synthesis methods for microwave selective networks. As is well known, these methods are based on the assumption of small relative bandwidth (i.e., $B_n \ll 1$). This means that the proposed design approach gives the best performances when the channel filters have a narrow bandwidth and the separation between adjacent channels is relatively small.

VII. Conclusion

In this work, a novel method for the polynomial synthesis of microwave star-junction multiplexers with a resonating junction has been presented. The channel filters can be arbitrarily specified, including the assignment of transmission zeros (which must be pure imaginary or in complex pairs with opposite real part). An iterative procedure has been developed for the evaluation of the characteristic polynomials of the multiplexer, which are subsequently used for computing the polynomials associated with the channel filters; these polynomials are then employed for synthesizing the filters like they were detached from the multiplexer. The proposed technique allows a noticeable flexibility in the design process of the considered devices; in fact, it is possible to verify, for a given set of parameters assigned to the channel filters, the actual response presented by the multiplexer (through its characteristic polynomials) without carrying out the complete design process. Moreover, the topology of the channel filters can also be selected after the evaluation of the multiplexer polynomials; in this way, the results of the synthesis process are not constrained to a specific configuration, which must be only compatible with the assigned transmission zeros. This latter aspect represents a definite advantage of the proposed approach with respect to optimization techniques applied to the synthesis of a specific topological configuration of the multiplexer (which require to restart the optimization from the beginning for every change in the filter's topology).

The main drawback of the proposed synthesis technique has been also pointed out; it is represented by the round-off errors arising in fixed-length numerical computations, whose effect becomes more and more relevant with the growth of the overall number of poles in the low-pass prototype.

The design of a triplexer used in base stations for mobile communications has been reported for illustrating the complete design process based on the proposed approach, including the evaluation of the de-normalized parameters requested for the physical dimensioning of coupling structures (i.e., the resonant frequencies of the cavities, the coupling coefficients and the external Q). This device has been fabricated, and the measured response is in good agreement with the expected one (obtained from the de-normalized prototype).

Concluding this work, it is worthwhile to observe that the polynomial synthesis technique presented here could be very attractive also for manifold-coupled multiplexers; a completely different formulation of the synthesis procedure (with respect to the one presented here) should be devised, however, due to the phase rotations between the input ports of the filters introduced by the manifold. This topic will be the subject of future research work.

References

[1] J. W. Bandler *et al.*, "Space mapping: The state of art," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 1, pp. 337–361, Jan. 2004.

[2] G. Macchiarella and S. Tamiazzo, "Novel approach to the synthesis of microwave duplexers," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 12, pp. 4281–4290, Dec. 2006.

[3] G. Macchiarella and S. Tamiazzo, "Design of triplexer combiners for base stations of mobile communications," in *IEEE MTT-S Int. Microw. Symp. Dig.*, Anheim, CA, May 2010, pp. 429–432.

[4] J. D. Rhodes and R. Levy, "A generalized multiplexer theory," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-27, no. 2, pp. 99–111, Feb. 1979.

[5] J. D. Rhodes and R. Levy, "Design of general manifold multiplexer," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-27, no. 2, pp. 99–111, Feb. 1979.

[6] R. J. Cameron, C. M. Kudsia, and R. R. Mansour, *Microwave Filters for Communication Systems*. Hoboken, NJ: Wiley, 2007.

[7] S. Tamiazzo and G. Macchiarella, "An analytical technique for the synthesis of cascaded N-tuplets cross-coupled resonators microwave filters using matrix rotations," *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 5, pp. 1693–1698, May 2005.

[8] D. Swanson and G. Macchiarella, "Microwave filter design by synthesis and optimization," *IEEE Microw. Mag.*, vol. 8, no. 2, pp. 55–69, Apr. 2007.

Giuseppe Macchiarella (SM'06) received the Laurea degree in electronic engineering from the Politecnico di Milano, Milan, Italy, in 1975.

From 1977 to 1987, he was a Researcher with the National Research Council of Italy, where he was involved in studies on microwave propagation. In 1987, he became an Associate Professor of microwave engineering with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Milan, Italy. He has authored or coauthored more than 100 papers and conference presentations. His current research work is in the field of microwave filters with special emphasis on microwave filter synthesis and power amplifiers linearization.

Stefano Tamiazzo received the Laurea degree in telecommunication engineering from the Politecnico di Milano, Milan, Italy, in 2002, and the M.S. degree in information technology from CEFRIEL, Milan, Italy, in 2003.

He is currently with Andrew Telecommunication Products, Milan, Italy, where he is involved in the design of microwave filters and combiners for wireless applications.