

# Documentation of MRs for Testing Constraint Checking Implementation

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## 1 Symbol Conventions

For ease of narrative, in the following sections, we use  $D_0$  and  $S_0$  to denote the context pool and the constraint in source test input,  $Rt_0$  and  $Rl_0$  to denote the truth value and the link set in the source test output. Similarly, we use  $D_1$ ,  $S_1$ ,  $Rt_1$  and  $Rl_1$  to denote corresponding variables in the follow-up test.

## 2 MR-Data

There are two different MRs in this part, and they only transform the data under checking and keep the constraint unchanged.

### 2.1 Link

Before introducing the MR-Data, let us talk more about links.

Link explains how a constraint is violated or satisfied. A link is composed of two parts *type* and *bindings*. Type of a link is either *vio* or *sat*, indicating that this link is a violated link or a satisfied link respectively. Bindings of a link is a set of variable assignment, which can help explain how a constraint is violated or satisfied. For example,  $l_1 :< vio, \{x = 1, y = 2\} >$  and  $l_2 :< vio, \{x = 2, y = 2\} >$  are two different links. The type of  $l_1$  is *vio* and the bindings of  $l_1$  is  $\{x = 1, y = 2\}$ .

For ease of description, we use the following notation in this documentation. Let  $l$  be a link and  $ls = \{l_1, l_2, \dots, l_n\}$  be a link set. We use  $type(l)$  to denote the type of  $l$  and  $bindings(l)$  to denote the bindings of  $l$ . If  $type(l_1) == type(l_2) \wedge bindings(l_1) \subseteq bindings(l_2)$ , we say  $l_2$  "covers"  $l_1$  and denote it as  $l_1 \preceq l_2$ . If  $\forall l_i \in ls_1 (\exists l_j \in ls_2 (l_i \preceq l_j))$ , we say  $ls_2$  "covers"  $ls_1$  and denote it as  $ls_1 \preceq ls_2$ . It is easy to see that  $\preceq$  is partial order relation and the inclusion relation of link sets is a special case of the "cover" relation. Besides that, we use  $var(l)$  and  $var(ls)$  to denote the set of the variables appeared in a link or a link set. And we use  $val(ls, x)$  to denote all the value assigned to the variable  $x$ .

To illustrate these concepts more clearly, let's look at the example in Fig 1.  $ls_1 \preceq ls_2$  is easy to understand because the only link  $l_1$  in  $ls_1$  is also in  $ls_2$  and  $l_1 \preceq l_1$ . On the other hand,  $l_1 \preceq l_3$  and  $l_2 \preceq l_4$ , so  $ls_2 \preceq ls_3$ .

For more details of the rules for calculating links, please refer to the existing work[1].

### 2.2 Impact of data changes

Then, we talk about the impact of data changes.

According to the existing work[2], impact of data changes can divide the into two categories: inconsistency incurring and inconsistency-hidden(also named as inc+/inc-). The inconsistency incurring changes leads to more possibilities for incurring violation and inconsistency hidden changes

Let:

$$\begin{aligned}
l_1 &= \langle vio, \{x = 1\} \rangle \\
l_2 &= \langle vio, \{y = 4\} \rangle \\
l_3 &= \langle vio, \{x = 1, y = 3\} \rangle \\
l_4 &= \langle vio, \{x = 2, y = 4\} \rangle \\
ls_1 &= \{l_1\} \\
ls_2 &= \{l_1, l_2\} \\
ls_3 &= \{l_3, l_4\}
\end{aligned}$$

then:

$$\begin{aligned}
ls_1 &\leq ls_2 \leq ls_3 \\
var(ls_3) &= \{x, y\} \\
val(ls_3, x) &= \{1, 2\}
\end{aligned}$$

Figure 1: An example of cover relation among link sets

has the opposite effect. These two kinds of impacts of data changes depend on the constraint and can be derived from the constraint in a static way.

For example, constraint  $S : \forall x \in X (not(\exists y \in Y (same(X, Y))))$  has the inconsistency-incurring changes set  $\{\langle +, X \rangle, \langle +, Y \rangle\}$ , which means if we add any element to X or Y, constraint  $s$  would tend to generate more violation.

For more details of the derivation rules for two impact of data changes, please refer to the existing work[2].

### 2.3 MR-Data 1

Now we can elaborate our MR-Data 1 as follow:

**MR-Data 1:** If  $Rt_0 == F$ , we apply any change with an inconsistency incurring impact on  $S_0$  to  $D_0$  to obtain  $D_1$ . Then, in the follow-up execution,  $Rt_1$  should still be  $F$  and  $Rl_1$  should satisfy the condition:  $Rl_0 \preceq Rl_1$ .

### 2.4 MR-Data 2

MR-Data 2 is the other side of MR-Data 1:

**MR-Data 2:** If  $Rt_0 == T$ , we apply any change with an inconsistency hidden impact on  $S_0$  to  $D_0$  to obtain  $D_1$ . Then, in the follow-up execution,  $Rt_1$  should still be  $T$  and  $Rl_1$  should satisfy the condition:  $Rl_0 \preceq Rl_1$ .

## 3 MR-Cons

There are nine different MRs in this part, and they only transform the constraint for checking and keep the data unchanged.

### 3.1 MR-Cons 1

**MR-Cons 1:** If the root of the constraint  $S_0$  is  $\forall v \in C$ , we transform it to  $\exists v \in C$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 3 cases:

Case 1: If  $Rt_0$  is  $T$ ,  $Rt_1$  should be  $T$  and  $val(Rl_1, v) == C$ .

- Case 2: If  $Rt_0$  is  $F$  and  $val(Rt_0, v) == C$ ,  $Rt_1$  should be  $F$  and  $Rl_1$  should be empty.  
Case 3: If  $Rt_0$  is  $F$  and  $val(Rt_0, v) \subset C$ ,  $Rt_1$  should be  $T$  and  $val(Rt_1, v) == C - val(Rt_0, v)$ .

### 3.2 MR-Cons 2

**MR-Cons 2:** If the root of the constraint  $S_0$  is  $\exists v \in C$ , we transform it to  $\forall v \in C$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 3 cases:

- Case 1: If  $Rt_0$  is  $F$ ,  $Rt_1$  should be  $F$  and  $val(Rl_1, v) == C$ .  
Case 2: If  $Rt_0$  is  $T$  and  $val(Rt_0, v) == C$ ,  $Rt_1$  should be  $T$  and  $Rl_1$  should be empty.  
Case 3: If  $Rt_0$  is  $T$  and  $val(Rt_0, v) \subset C$ ,  $Rt_1$  should be  $F$  and  $val(Rl_1, v) == C - val(Rt_0, v)$ .

### 3.3 MR-Cons 3

Before we introduce more MRs, let us talk more about the links of binary quantifier. Let  $S : f_l \text{ op } f_r$  be a constraint, where  $op$  is a binary quantifier (e.g. *and*, *or*, *implies*) and  $f_l/f_r$  is the left/right sub-constraint of  $op$ . According to the rules of link generation [1], the links of  $S$  (denoted as  $ls$ ) is one of the following 5 cases: (1).  $ls$  is empty. (2).  $ls$  only contains links from left sub-constraint (denoted as  $ls_{f_l}$ ). (3).  $ls$  only contains links from right sub-constraint (denoted as  $ls_{f_r}$ ). (4).  $ls$  is the union of  $ls_{f_l}$  and  $ls_{f_r}$ . (5).  $ls$  is the cartesian product of  $ls_{f_l}$  and  $ls_{f_r}$ . For the convenience of description, we use the  $flag(ls)$  to denote these cases. For example, if  $flag(ls) == 2$ , it means links composition of constraint  $S$  is case (2) above, i.e.  $ls$  only contains links from its left sub-constraint.

Now we can explain our MR-Cons 3 as follows:

**MR-Cons 3:** If the root of the constraint  $S_0$  is *and*, which can be formed of  $f_1$  and  $f_2$ , we transform it to  $f_1 \text{ or } f_2$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .  
Case 2: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .  
Case 3: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3$ .  
Case 4: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 4$ .  
Case 5: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 5$ .  
Case 6: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ , either  $Rt_1 == F \wedge flag(Rl_1) == 2$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ .  
Case 7: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ , either  $Rt_1 == F \wedge flag(Rl_1) == 3$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ .  
Case 8: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == F \wedge flag(Rl_1) == 1$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2 \vee flag(Rl_1) == 3)$ .

### 3.4 MR-Cons 4

**MR-Cons 4:** If the root of the constraint  $S_0$  is *or*, which can be formed of  $f_1$  or  $f_2$ , we transform it to  $f_1 \text{ and } f_2$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 1$ .  
Case 2: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 2$ .  
Case 3: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 3$ .  
Case 4: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 4$ .  
Case 5: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 5$ .  
Case 6: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ , either  $Rt_1 == T \wedge flag(Rl_1) == 2$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ .  
Case 7: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ , either  $Rt_1 == T \wedge flag(Rl_1) == 3$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ .

Case 8: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == T \wedge flag(Rl_1) == 1$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2 \vee flag(Rl_1) == 3)$ .

### 3.5 MR-Cons 5

**MR-Cons 5:** If the root of the constraint  $S_0$  is *or*, which can be formed of  $f_1$  or  $f_2$ , we transform it to  $f_1$  implies  $f_2$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 2: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .
- Case 3: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 4: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .
- Case 5: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3$ .
- Case 6: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3 \vee flag(Rl_1) == 4$ .
- Case 7: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ , either  $Rt_1 == T \wedge flag(Rl_1) == 1$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 2 \vee flag(Rl_1) == 5)$ .
- Case 8: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ .

### 3.6 MR-Cons 6

**MR-Cons 6:** If the root of the constraint  $S_0$  is *implies*, which can be formed of  $f_1$  implies  $f_2$ , we transform it to  $f_1$  or  $f_2$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 2: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .
- Case 3: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 4: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .
- Case 5: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3$ .
- Case 6: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3 \vee flag(Rl_1) == 4$ .
- Case 7: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ , either  $Rt_1 == T \wedge flag(Rl_1) == 1$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 2 \vee flag(Rl_1) == 5)$ .
- Case 8: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ .

### 3.7 MR-Cons 7

**MR-Cons 7:** If the root of the constraint  $S_0$  is *and*, which can be formed of  $f_1$  and  $f_2$ , we transform it to  $f_1$  implies  $f_2$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 2: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 3: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3$ .
- Case 4: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3$ .
- Case 5: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .
- Case 6: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2 \vee flag(Rl_1) == 4$ .
- Case 7: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ , either  $Rt_1 == T \wedge flag(Rl_1) == 1$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 3 \vee flag(Rl_1) == 5)$ .
- Case 8: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ , or  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ .

### 3.8 MR-Cons 8

**MR-Cons 8:** If the root of the constraint  $S_0$  is *implies*, which can be formed of  $f_1$  *implies*  $f_2$ , we transform it to  $f_1$  and  $f_2$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 1$ .
- Case 2: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 1$ .
- Case 3: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 3$ .
- Case 4: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 3$ .
- Case 5: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 2$ .
- Case 6: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 2 \vee flag(Rl_1) == 4$ .
- Case 7: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ , either  $Rt_1 == F \wedge flag(Rl_1) == 1$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 3 \vee flag(Rl_1) == 5)$ .
- Case 8: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == F \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ .

### 3.9 MR-Cons 9

**MR-Cons 9:** If the root of the constraint  $S_0$  is *implies*, which can be formed of  $f_1$  *implies*  $f_2$ , we transform it to  $f_2$  *implies*  $f_1$  to obtain  $S_1$ . Then, in the follow-up execution,  $Rt_1$  and  $Rl_1$  should satisfy different conditions according to the following 8 cases:

- Case 1: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 1$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 1$ .
- Case 2: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 2$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 2$ .
- Case 3: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 3$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 3$ .
- Case 4: If  $Rt_0$  is  $F$  and  $flag(Rl_0) == 5$ ,  $Rt_1$  should be  $T$  and  $flag(Rl_1) == 4$ .
- Case 5: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 4$ ,  $Rt_1$  should be  $F$  and  $flag(Rl_1) == 5$ .
- Case 6: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 2$ , either  $Rt_1 == F \wedge flag(Rl_1) == 2$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 3)$ .
- Case 7: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 3$ , either  $Rt_1 == F \wedge flag(Rl_1) == 3$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2)$ .
- Case 8: If  $Rt_0$  is  $T$  and  $flag(Rl_0) == 1$ , either  $Rt_1 == F \wedge flag(Rl_1) == 1$ , or  $Rt_1 == T \wedge (flag(Rl_1) == 1 \vee flag(Rl_1) == 2 \vee flag(Rl_1) == 3)$ .

## 4 MR-All

There are two different MRs in this part, and they transform the data under checking and the constraint simultaneously.

### 4.1 Positive quantifier and negative quantifier

Let's start with an in-depth analysis of the quantifier (e.g.  $\forall$ ,  $\exists$ , *and*, *or*) in the constraint. A constraint's quantifiers can be into two parts: *positive* and *negative*, according to their impact to the whole constraint. A positive quantifier supports the root's violation or satisfaction while a negative quantifier supports it in the opposite way. It means if a positive quantifier is  $F$  under a given data, the whole constraint tends to be  $F$  while if a positive quantifier is  $F$ , the whole constraint tend to be  $T$ . In brief, a positive quantifier with  $T$  or a negative quantifier with  $F$  contributes to the whole constraint taking the truth value of  $T$ ; a positive quantifier with  $F$  or a negative quantifier with  $T$  contributes to the whole constraint taking the truth value of  $F$ .

Let us consider the example in the Fig. 2. In the figure, quantifiers colored with green stands for positive quantifiers and quantifiers colored with red stands for negative quantifiers. Quantifier *not* in constraint  $S$  is positive, which means if it is  $T$ , it tends to make  $S$  be  $T$  while if it is  $F$ , it tends to make  $S$  be  $F$ .

In definitions, only the quantifier that is transitively through *implies* from the left or *not* from the root can reverse its supporting direction initialized to be positive, which has been clearly illustrated in Fig. 2.

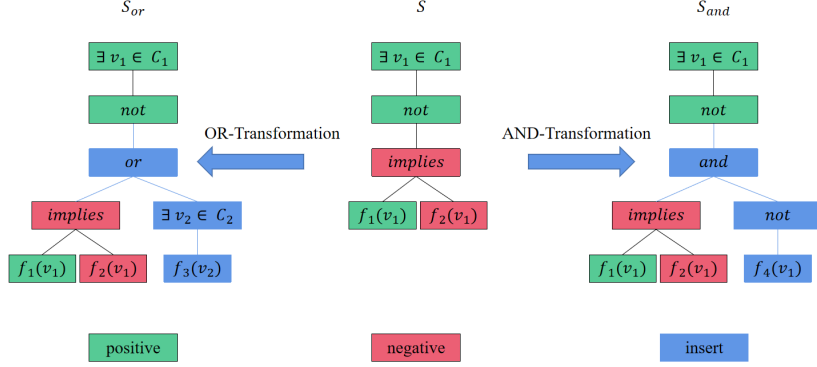


Figure 2: An example of constraint transformation

## 4.2 Constraint transformation

Since we want to transform both the data part and constraint part at the same time. To control the transformation impact, we control to insert a new formula which naturally requires new data being added into the data part. We propose two kinds of transformations: *AND-Transformation* and *OR-Transformation*. *AND-Transformation* means linking a quantifier with a newly added *and*, which links to a newly added constraint as another branch. Similarly, *OR-Transformation* means linking quantifier with a newly added *OR*. Note that the newly added constraint can be arbitrary. Fig 2 has clearly shown two kinds of transformations.

## 4.3 MR-All 1

Now, we can elaborate our MR-All 1 using concepts above as follow:

**MR-All 1:** If  $Rt_0 == F$ , we apply *AND-Transformation* to any positive quantifier or apply *OR-Transformation* to any negative quantifier in  $S_0$  to obtain  $D_1$  and  $S_1$ . Then, in the follow-up execution,  $Rt_1$  should still be  $F$  and  $Rl_1$  should satisfy the condition:  $Rl_0 \preceq Rl_1$ .

## 4.4 MR-All 2

MR-All 2 is the other side of MR-All 1:

**MR-All 2:** If  $Rt_0 == T$ , we apply *OR-Transformation* to any positive quantifier or apply *AND-Transformation* to any negative quantifier in  $S_0$  to obtain  $D_1$  and  $S_1$ . Then, in the follow-up execution,  $Rt_1$  should still be  $T$  and  $Rl_1$  should satisfy the condition:  $Rl_0 \preceq Rl_1$ .

## 5 MR-CCT

If more internal states can be observed, e.g., CCT status like substantial parts accessible in existing work[3], we can additional design MRs with respect to measure the impact for such detailed CCT statuses under different transformations between source and follow-up executions.

## 5.1 CCT and SCCT

Normally, a constraint for checking can be expanded to a *Consistency Computation Tree* (CCT) based on both its static constraint structure and the current data under checking. SCCT [3] is substantial parts in CCT, which is the minimum set of nodes contributing to the link generation. Fig. 3 has shown an example of CCT and SCCT. In the figure, the nodes colored with purple are SCCT nodes and the links of this constraint is entirely up to them. By comparing the SCCT in the source output and the follow-up output through different transformations, we can design the MR-CCT.

In the next section, we will use  $SCCT_0$  and  $SCCT_1$  to denote the set of SCCT nodes in the source output and follow-up output separately.

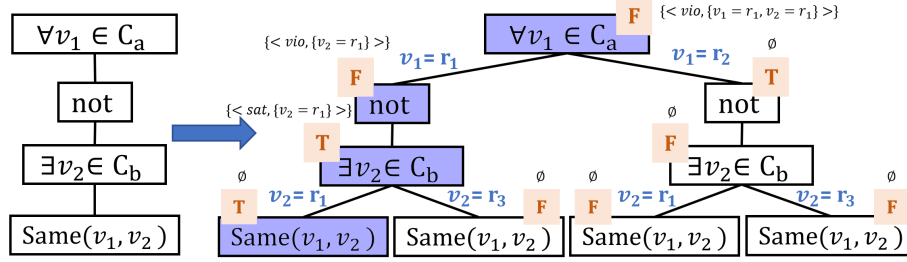


Figure 3: An example of CCT and SCCT

## 5.2 MR-CCT 1

MR-CCT 1 uses the transformation similar to MR-Data 1 and can be elaborated as follow:

**MR-CCT1:** If  $Rt_0 == F$ , we apply any change with an inconsistency incurring impact on  $S_0$  to  $D_0$  to obtain  $D_1$ . Then, in the follow-up execution,  $SCCT_1$  should contain  $SCCT_0$ , i.e.  $SCCT_0 \subseteq SCCT_1$ .

## 5.3 MR-CCT 2

Similarly, if we use the transformation mentioned in MR-All 1, then we can derive the MR-CCT 2:

**MR-CCT2:** If  $Rt_0 == F$ , we apply AND-Transformation to any positive quantifier or apply OR-Transformation to any negative quantifier in  $S_0$  to obtain  $D_1$  and  $S_1$ . Then, in the follow-up execution,  $SCCT_1$  should contain  $SCCT_0$ , i.e.  $SCCT_0 \subseteq SCCT_1$ .

## References

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