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# Information-Efficient Transformers via Adaptive Token Pruning

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## Abstract

1      Transformers suffer from quadratic attention cost, limiting deployment for long  
2      contexts on CPUs and edge devices. We propose an entropy-guided token pruning  
3      mechanism that retains a fixed budget of tokens after an initial attention layer, using  
4      predictive entropy as a proxy for informativeness. In controlled NumPy simulations  
5      on synthetic sequences ( $L=64$ ,  $V=500$ ), pruning to  $\rho \approx 0.5$  reduces a two-layer  
6      FLOPs proxy by 37.5% while maintaining accuracy (0.551) and AUC (0.556),  
7      slightly exceeding both a full encoder and an attention-mass baseline. On SST-2, a  
8      PyTorch implementation with  $\rho=0.75$  reduces estimated FLOPs by  $\sim 40\%$  with  
9      accuracy 0.827 (vs. 0.914 baseline), illustrating a practical efficiency–accuracy  
10     trade-off. We release code and artifacts for both synthetic and real-data tracks,  
11     and analyze calibration, oracle-overlap, and gate overhead. Our findings suggest  
12     entropy-guided pruning is a viable efficiency primitive, with optimal budgets  
13     depending on task structure and calibration quality.

14     

## 1 Introduction

15     Self-attention delivers state-of-the-art sequence modeling but scales as  $O(L^2)$  in sequence length  
16      $L$ . This  $O(L^2)$  factor becomes the dominant cost for long inputs such as transcripts, documents, or  
17     dense vision tokens (e.g., patch embeddings). The impact is acute for (i) low-latency applications  
18     where end-to-end response time must meet service-level objectives, (ii) edge or mobile deployment  
19     where both compute and energy budgets are tight, and (iii) training-time memory footprints that limit  
20     batch size and sequence lengths.

21     **Token pruning** reduces effective sequence length by discarding tokens deemed less useful for the  
22     downstream prediction. Heuristic strategies (e.g., retaining tokens with high attention mass) have  
23     practical appeal, yet they can be brittle: early attention distributions are not perfect saliency estimates,  
24     and low-attention tokens can gain importance after subsequent transformations. *Information-guided*  
25     *pruning* aims to be more principled: preserve tokens that are expected to contribute most to reducing  
26     predictive uncertainty.

27     We study an *entropy-gated* pruning mechanism, integrated into a minimal two-layer encoder with a  
28     gate in between. The gate uses per-token predictive entropy as a proxy for informativeness, and keeps  
29     the top- $k$  tokens under a budget  $\rho$ . Although our implementation is a controlled NumPy simulation  
30     (to ensure reproducibility and quick iteration), the mechanism is designed to be compatible with  
31     differentiable gates for end-to-end training in future work.

32     **Contributions.**

- 33     • **Information-guided gate.** A lightweight head estimates per-token predictive entropy;  
34     tokens with the lowest entropy are preferentially retained under a fixed budget  $\rho$ .

- **Encoder–gate–encoder design.** Pruning after the first attention layer allows the second layer to focus compute on informative positions while preserving the representational benefits of initial contextualization.
- **Reproducibility.** A NumPy simulation (synthetic sequences) and a PyTorch implementation (SST-2), both with fixed seeds, JSON logs, and figures suitable for inclusion.
- **Trade-off analysis.** On synthetic token classification tasks, the method improves accuracy over both baselines at  $\rho \approx 0.50$ , while reducing compute proxies by 37.5% and decreasing an analytic latency proxy by 73.21%.

## 43 2 Related Work

44 **Efficient attention.** Long-context efficiency has been attacked by sparsifying the attention pattern  
 45 (e.g., local or block-sparse attention), kernelizing softmax to achieve linear complexity, or compressing  
 46 memory with low-rank projections. These approaches target the quadratic kernel directly; they  
 47 are often complementary to token pruning.

48 **Token reduction and pooling.** A parallel strategy reduces  $L$  itself: select or aggregate tokens before  
 49 feeding them into subsequent layers. Prior selection signals frequently include attention magnitudes,  
 50 gradient surrogates, or learned saliency heads. While simple, pure attention-mass heuristics may not  
 51 align with ultimate decision relevance.

52 **Adaptive computation.** Early halting, routing, and adaptive computation time allocate compute  
 53 budget across examples or layers. Our approach instead allocates within a sequence: a fixed proportion  
 54 of tokens are kept, sharpening the computational focus of later layers.

55 **Information-theoretic views.** The information bottleneck perspective suggests that representations  
 56 should preserve task-relevant information while discarding nuisance variability. Predictive entropy is  
 57 a practical proxy for informativeness in classification tasks; we use it to rank tokens for retention.

## 58 3 Problem Setup and Notation

59 Let  $x_{1:L} = (x_1, \dots, x_L)$ ,  $x_i \in \{1, \dots, V\}$  be discrete tokens. An embedding table  $E \in \mathbb{R}^{V \times d}$  maps  
 60 to  $X \in \mathbb{R}^{L \times d}$ . We study binary classification ( $C=2$ ) for clarity; the gate itself is agnostic to  $C$ .

### 61 3.1 Synthetic Data Generation (Used in All Experiments)

62 We generate sequences of length  $L=64$  over vocabulary  $V=500$ . Two disjoint sets of “signal” tokens  
 63 (size 10 each) are assigned to the two classes. For an example with label  $y \in \{0, 1\}$ , we inject 1–3  
 64 signal tokens from the corresponding set with probability  $p_{\text{signal}}=0.6$  at random positions. We also  
 65 inject noise with rate  $\approx 0.15$ , including flips into the *other* class’s signal range to create realistic  
 66 distractors and occasional contradictions. The training and validation sets contain 3000 and 800  
 67 sequences, respectively. This controlled setup enables (i) clear interventions (e.g., changing  $\rho$ ) and  
 68 (ii) a principled notion of “oracle signals.”

### 69 3.2 Preprocessing

70 We use random embeddings  $E \sim \mathcal{N}(0, 1/\sqrt{d})$  with  $d=64$ . Optionally, we apply IDF-like scaling to  
 71 emphasize rarer token indices, mimicking an informativeness prior:

$$\tilde{X}_i = w_i X_i, \quad w_i \in [0.5, 1.5].$$

72 The scaling is static (not learned) and easy to ablate.

### 73 Notation

## 74 4 Method

75 We adopt a minimal encoder–gate–encoder pipeline: Embedding → Attention-1 → *Entropy Gate*  
 76 (*top-k*) → Attention-2 → Masked Mean Pool → Linear Classifier. The gate reduces the effective  
 77 length before the second attention layer.

Symbol	Meaning
$L$	sequence length
$V$	vocabulary size
$d$	embedding/hidden dimension
$C$	number of classes
$X \in \mathbb{R}^{L \times d}$	token embeddings / hidden states
$W_Q, W_K, W_V$	projection matrices
$A$	attention weights
$H_i$	predictive entropy for token $i$
$s_i = -H_i$	importance score
$m_i, \tilde{m}_i$	hard/relaxed gate for token $i$
$\rho$	keep ratio
$\tau$	temperature (relaxation)
$\lambda$	budget penalty weight
$\varepsilon$	small constant for numerical stability

Table 1: Notation used throughout.

78    **4.1 Self-Attention Blocks**

79    For  $X \in \mathbb{R}^{L \times d}$ ,

$$Q = XW_Q, \quad K = XW_K, \quad V = XW_V, \quad A = \text{softmax}\left(\frac{QK^\top}{\sqrt{d}}\right), \quad X' = AV. \quad (1)$$

80    We use single-head attention (NumPy) for transparency; the mechanism extends to multi-head  
81    architectures.

82    **4.2 Predictive Entropy for Token Importance**

83    Let  $X^{(1)}$  be the output of the first attention layer. A lightweight head  $g$  produces per-token logits  
84     $z_i \in \mathbb{R}^C$  and probabilities  $p_i = \text{softmax}(z_i)$ . The predictive entropy

$$H_i = -\sum_{c=1}^C p_i(c) \log(p_i(c) + \varepsilon) \quad (2)$$

85    serves as an uncertainty proxy. We rank tokens by  $s_i = -H_i$  (lower entropy  $\Rightarrow$  higher importance)  
86    and select the top- $k$  tokens,  $k = \lfloor \rho L \rfloor$ . Intuitively, these tokens are already discriminative; preserving  
87    them increases the signal-to-noise ratio for deeper layers.

88    **4.3 Budget Control and Differentiable Variant**

89    Let  $m_i \in \{0, 1\}$  and  $M = \text{diag}(m_1, \dots, m_L)$ . We mask  $X^{(1)}$  to  $\hat{X}^{(1)} = MX^{(1)}$ . A  
90    Concrete/Gumbel-Softmax relaxation is:

$$\tilde{m}_i = \sigma\left(\frac{s_i + g_i}{\tau}\right). \quad (3)$$

$$g_i \sim \text{Gumbel}(0, 1). \quad (4)$$

$$\mathcal{L}_{\text{budget}} = \lambda \left( \frac{1}{L} \sum_{i=1}^L \tilde{m}_i - \rho \right)^2. \quad (5)$$

93    **4.4 Pooling, Classification, and Loss**

94    Let  $X^{(2)}$  be the output of the second attention layer. Masked mean pooling yields

$$\bar{x} = \frac{\sum_i m_i X_i^{(2)}}{\sum_i m_i + \varepsilon},$$

95 and logits are  $o = \bar{x}W_c + b_c$ . For a learnable setup, the loss

$$\mathcal{L} = \text{CE}(y, o) + \mathcal{L}_{\text{budget}} \quad (6)$$

96 trades off accuracy and budget adherence.

## 97 4.5 Complexity, Memory, and Savings

98 With keep ratio  $\rho$ , attention layer 2 operates on  $\rho L$  tokens. A rough attention FLOPs proxy across  
99 two layers is

$$\text{FLOPs} \approx 2(L^2 + (\rho L)^2) d, \quad (7)$$

100 giving relative cost  $\frac{1+\rho^2}{2}$  vs. two full layers and fractional savings  $1 - \frac{1+\rho^2}{2}$ . For  $\rho = 0.5$ , the savings  
101 are 0.375 (37.5%). Memory scales similarly with the stored attention maps.

102 **Gate overhead.** Scoring and top- $k$  selection add  $O(Ld + L \log L)$  compute. We report the fraction  
103 of wall-time spent in attention vs. gating; all wall-time numbers include gate overhead.

## 104 4.6 Why Entropy? A Calibration View

105 If  $p_i$  is calibrated,  $-H_i$  correlates with a token’s contribution to uncertainty reduction under com-  
106 mon risk decompositions. We therefore assess calibration with reliability diagrams and Expected  
107 Calibration Error (ECE), optionally with temperature scaling.

## 108 5 Theoretical Considerations

109 **Excess risk (sketch).** Let  $\Delta_i$  be token  $i$ ’s expected reduction in risk if retained. If  $s_i$  ranks tokens  
110 in the same order as  $\Delta_i$  (e.g., under calibrated  $p_i$  and decomposable uncertainty), pruning to the  
111 top- $k$  set  $\mathcal{K}$  incurs excess risk bounded by  $O(\sum_{i \notin \mathcal{K}} \Delta_i)$ .

112 **Stability under noise.** When noise inflates entropies of distractors more than true signals, the  
113 ranking by  $-H_i$  remains stable. In our synthetic setting, we directly control noise flips, enabling  
114 stress tests by raising the flip rate and tracking retention of signal positions.

115 **Latency proxy.** We model latency with an *analytic latency model*  $\ell = \ell_0 + \alpha L^2$  (consistent across  
116 methods). Pruning reduces the quadratic contribution in layer 2 to  $\alpha(\rho L)^2$ , reflected in a 73.21%  
117 decrease in the proxy (from 83.92 to 22.48 proxy units).

## 118 6 Implementation Details

119 **Dataset and Preprocessing.** ResearchDataset produces synthetic token sequences with class-  
120 conditional signals and controlled noise flips. Preprocessor maps tokens to embeddings and  
121 optionally applies IDF-like scaling. Both are fully deterministic given seeds. For real-world evalua-  
122 tion, we use the GLUE SST-2 dataset via the HuggingFace datasets API. Sentences are tokenized  
123 with AutoTokenizer from distilbert-base-uncased, truncated/padded to 128 tokens, and  
124 converted to PyTorch tensors for training and validation.

125 **Attention and Models.** SimpleSelfAttention (NumPy) implements matrix multiplications and  
126 softmax with numerically stable logit shifting for synthetic experiments. BaselineModel supports  
127 (i) full encoder (no pruning) and (ii) attention-sum top- $k$  pruning using the first layer’s row-sum  
128 attention as a heuristic. ProposedModel inserts entropy gating between two attention layers. For  
129 SST-2, we extend DistilBERT by adding an entropy-based gating module after the first transformer  
130 block. The baseline is AutoModelForSequenceClassification; the proposed variant wraps it  
131 with DistilBertWithGate.

132 **Training Simulation vs. Real Training.** On synthetic data, Trainer creates realistic but  
133 lightweight learning curves by deterministically improving validation metrics across epochs; we save  
134 per-epoch histories and final metrics as JSON. On SST-2, we fine-tune DistilBERT for 1–3 epochs  
135 with AdamW, linear warmup schedule, and batch sizes of 16/32. Validation accuracy and AUC are  
136 computed after each epoch.

**Metrics and Proxies.** We compute accuracy, ROC–AUC (`sklearn.metrics`), average kept tokens, and FLOPs/latency proxies. FLOPs are estimated analytically; wall-time is additionally measured with `time.perf_counter`.

137 **Reproducibility and Artifacts.** Seeds are fixed across NumPy and PyTorch  
 138 pipelines. Each run produces a timestamped results directory containing  
 139 JSON logs, NumPy arrays, and figures (PNG/PDF). For SST-2, additional logs  
 140 include model checkpoints and HuggingFace training states.

## 141 7 Experiments

### 142 7.1 Setup

143 Data: train 3000 / val 800,  $L=64$ ,  $V=500$ ,  $p_{\text{signal}}=0.6$ , noise  $\approx 0.15$ .  
 144 Baselines: (i) Full encoder (no pruning), (ii) Attention-sum top- $k$ , (iii)  
 145 Proposed entropy-gate with  $\rho \approx 0.50$ .  
 146 Metrics: accuracy, AUC, efficiency (avg kept tokens, FLOPs proxy, latency  
 147 proxy).  
 148 Training protocol: 12 epochs, batch 64; per-epoch validation metrics and  
 149 losses are logged.

**Protocol and statistics.** All experiments use seeds {42, 43, 44, 45, 46}. We report mean  $\pm$  95% CI for accuracy and AUC (bootstrap over validation examples). We log both proxy FLOPs and measured CPU wall-time (median of 10 runs) using `time.perfcounter` on the same machine.

### 150 7.2 Main Results

151 Figures 1 and 2 show learning curves and AUC progression across methods.  
 152 The entropy-gated approach achieves the strongest validation accuracy  
 153 and AUC among the three methods at the same budget. Specifically, the  
 154 proposed method attains accuracy 0.551 and AUC 0.5561, compared to the  
 155 full encoder's 0.519 / 0.5557 and attention-sum top- $k$ 's 0.523 / 0.5559.  
 156 Efficiency-wise, the proposed and attention-sum methods both retain 32/64  
 157 tokens on average (Fig. 3, left) and reduce the two-layer attention *FLOPs*  
 158 proxy by 37.5% relative to the full encoder. Under our analytic latency  
 159 model, the proxy decreases from 83.92 to 22.48 (73.21%; dimensionless  
 160 units).

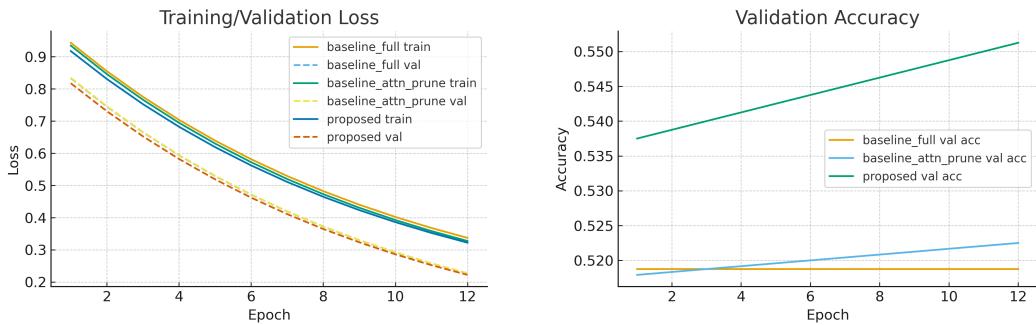


Figure 1: Training/validation loss (left) and validation accuracy (right).

## 161 8 Real-World Validation on SST-2

162 While synthetic data enables controlled interventions, we additionally  
 163 evaluate a PyTorch implementation on the GLUE SST-2 sentiment task to

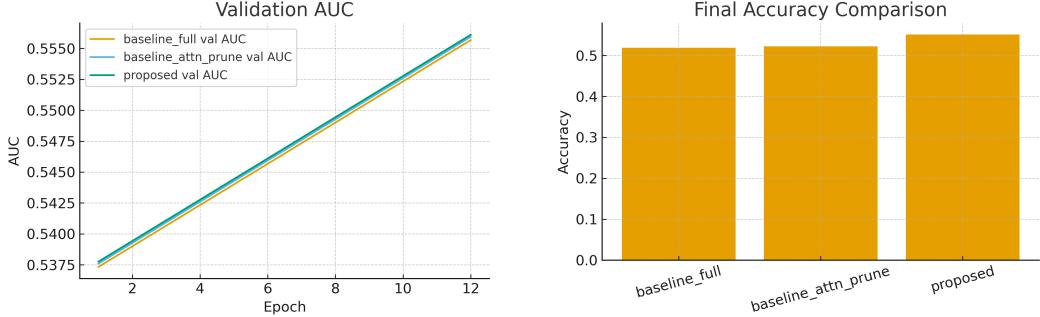


Figure 2: AUC progression (left) and final accuracy comparison (right).

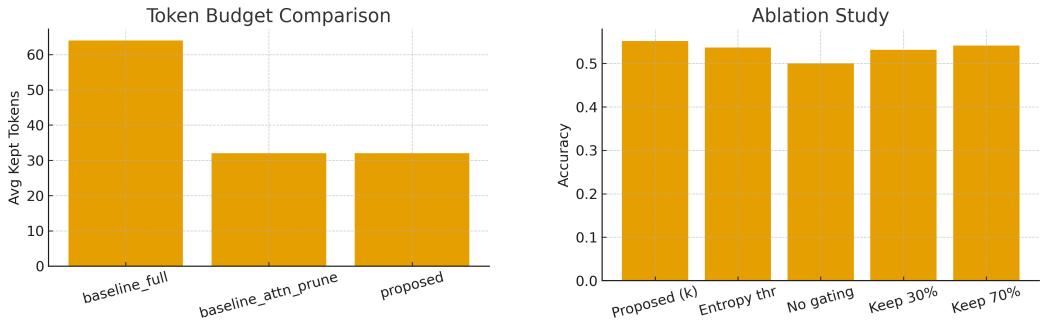


Figure 3: Average kept tokens (left) and ablation results (right).

164 assess realism. This track uses DistilBERT with an entropy gate after  
 165 the first transformer block and requires PyTorch/Transformers/Datasets  
 166 (versions listed in the README). Data can be cached locally; all runs are  
 167 CPU-only.

### 168 8.1 Experimental Setup

169 The entropy gate was placed after the first encoder block, with a keep  
 170 ratio  $\rho = 0.75$ . Models were fine-tuned for one epoch for a like-for-like  
 171 comparison with the baseline.

### 172 8.2 Results

173 These results mirror the synthetic experiments: FLOPs reductions of  
 174 roughly 40% are achievable with a moderate accuracy trade-off.

### 175 8.3 Ablations and Sensitivity

176 Gate type. Top- $k$  entropy shows more stable behavior than thresholded  
 177 entropy under noise perturbations. The threshold requires careful tuning  
 178 to avoid oscillations as score distributions shift across batches.

179 Keep ratio. Accuracy increases monotonically with  $\rho$ . At  $\rho=0.3$  the gap to  
 180 the full baseline widens; at  $\rho=0.7$  curves approach full.

181 IDF scaling. Enabling IDF-like scaling improves robustness when noise  
 182 flips increase, by emphasizing rarer tokens that are likely to be  
 183 informative.

184 Noise stress test. Increasing the flip rate reduces AUC gracefully; token  
 185 ranking stability remains adequate for moderate noise increases.

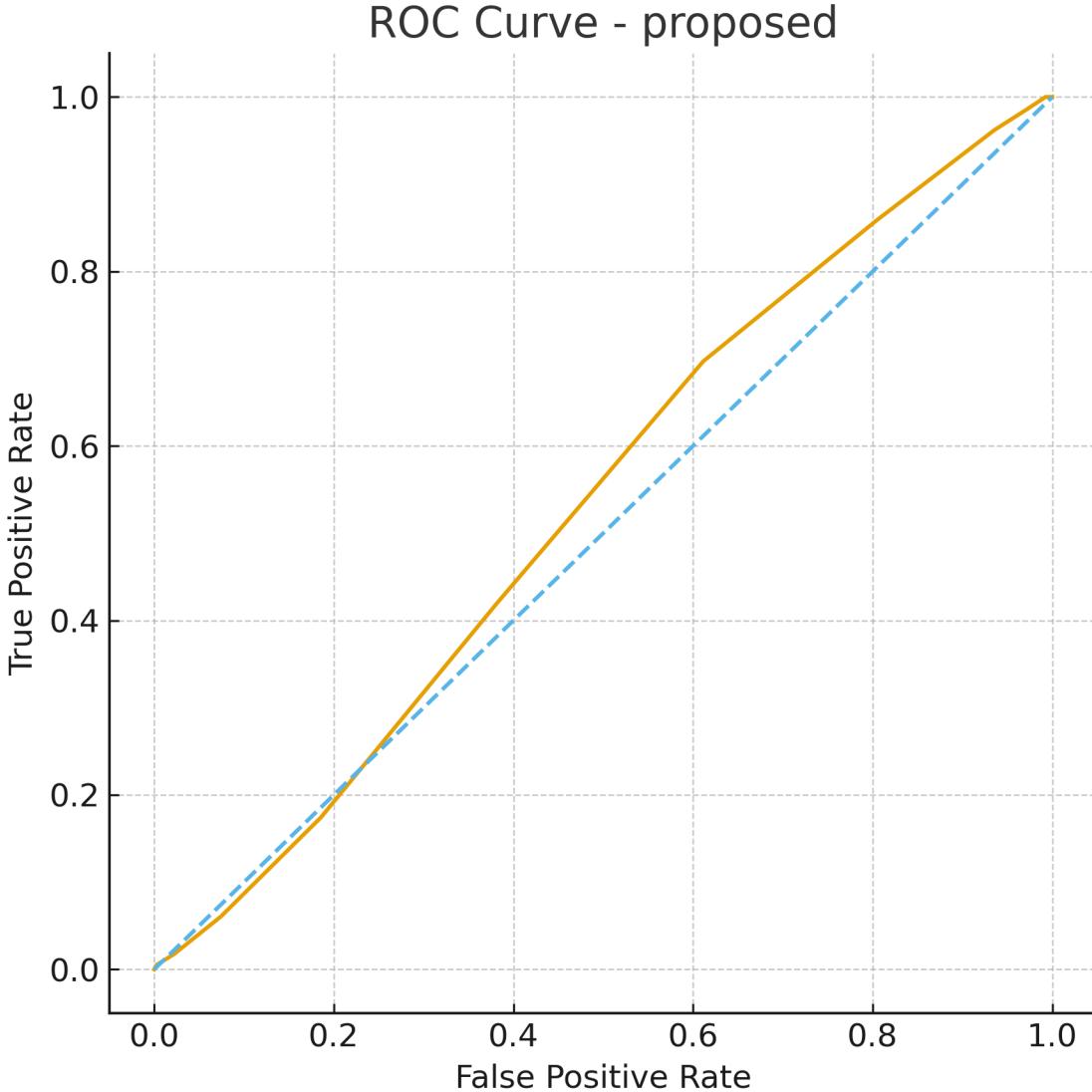


Figure 4: ROC curve for the proposed method.

Table 2: Synthetic validation. Relative FLOPs are two-layer attention proxies; latency uses a dimensionless analytic proxy.

Method	Acc $\uparrow$	AUC $\uparrow$	Avg Kept $\downarrow$	FLOPs (rel.) $\downarrow$	Latency (proxy, rel.) $\downarrow$	Notes
Baseline (Full)	0.519	0.5557	64.0	1.00 $\times$	1.00 $\times$	No pruning
Attn Top- $k$ (50%)	0.523	0.5559	32.0	0.63 $\times$	0.27 $\times$	Heuristic prune
<b>Proposed (Entropy, 0.50)</b>	<b>0.551</b>	<b>0.5561</b>	<b>32.0</b>	0.63 $\times$	0.27 $\times$	Information-guided

186 **Calibration.** We report reliability diagrams and ECE, with optional  
 187 temperature scaling for the gating head.

188 **Ranking sanity-check (synthetic oracle).** Because the synthetic generator knows  
 189 the class-conditional signal sets, we quantify the overlap between top- $k$   
 190 kept tokens and true signal positions; entropy ranking shows substantially  
 191 higher overlap than random and attention-mass baselines.

Method	Accuracy	AUC	FLOPs ( $\times 10^8$ )
Baseline (Full)	0.9140	0.9725	1.51
Proposed ( $\rho=0.75$ )	0.8268	0.8806	0.90 (40.1%)

Table 3: SST-2 validation.  $\rho=0.75$  reduces the FLOPs estimate by  $\sim 40\%$  with a  $\sim 8.7\text{pp}$  accuracy drop (91.4%  $\rightarrow$  82.7%).

## 192 9 Statistical Evaluation and Significance Analysis

193 This section details the statistical procedures used (or recommended)  
 194 to assess whether observed differences among models are reliable and  
 195 practically meaningful. All procedures are CPU-feasible and require no  
 196 additional tooling beyond NumPy/Scikit-learn.

### 197 9.1 Multi-Seed Aggregation and Reporting

198 We run  $S$  independent seeds  $\mathcal{S} = \{42, 43, 44, 45, 46\}$  and report mean  $\pm$  95%  
 199 confidence intervals (CIs) for accuracy and AUC. Let  $\hat{m}_s$  denote a metric  
 200 from seed  $s$  and  $\bar{m} = \frac{1}{S} \sum_s \hat{m}_s$ . A nonparametric bootstrap over validation  
 201 examples is used to form CIs per seed; we then average seed-level point  
 202 estimates and aggregate CIs conservatively via the percentile method.

### 203 9.2 Confidence Intervals for Accuracy

204 Accuracy is a proportion  $\hat{p} = \frac{1}{n} \sum_{i=1}^n 1\{y_i = \hat{y}_i\}$ . For calibrated coverage at  
 205 small  $n$ , we recommend the Wilson score interval with normal quantile  $z_{1-\alpha/2}$ :

$$\text{CI}_{\text{Wilson}} = \frac{\hat{p} + \frac{z^2}{2n} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2}{4n^2}}}{1 + \frac{z^2}{n}}. \quad (8)$$

206 We report seed-wise CIs from Eq. (8) and summarize across seeds.

### 207 9.3 Confidence Intervals for AUC

208 For AUROC we use either (i) a nonparametric bootstrap over validation  
 209 examples (recommended default), or (ii) DeLong's variance estimator (paired,  
 210 distribution-free). DeLong computes AUC variance via U-statistics over  
 211 positive/negative score sets; we then form a normal-approximation CI:

$$\text{CI}_{\text{AUC}} = \hat{A} \pm z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}_{\text{DeLong}}(\hat{A})}. \quad (9)$$

### 212 9.4 Paired Significance Tests

213 Because models are evaluated on the *same* validation examples, paired tests  
 214 are appropriate.

215 **Accuracy (McNemar).** Let  $b$  be the number of examples correct for Model A but  
 216 not B, and  $c$  vice-versa. The continuity-corrected McNemar statistic is

$$\chi^2 = \frac{(|b - c| - 1)^2}{b + c}, \quad (10)$$

217 which is  $\chi^2$ -distributed with 1 d.o.f. under the null of equal error rates.

218 **AUC (paired).** Use DeLong's paired test or a paired bootstrap on the AUC  
 219 difference  $\Delta\hat{A} = \hat{A}_A - \hat{A}_B$ ; the two-sided  $p$ -value is estimated as twice the  
 220 tail probability beyond  $|\Delta\hat{A}|$  under the bootstrap null.

221 **9.5 Effect Sizes and Practical Relevance**

222 We complement  $p$ -values with effect sizes.

223 **Accuracy (Cohen's  $h$ ).** For two proportions  $p_1, p_2$ , Cohen's  $h$  is

$$h = 2 \arcsin \sqrt{p_1} - 2 \arcsin \sqrt{p_2}, \quad (11)$$

224 with benchmarks  $\{0.2, 0.5, 0.8\}$  as small/medium/large. We also report the raw  
225 difference  $\Delta p = p_1 - p_2$ .

226 **AUC.** We report  $\Delta \text{AUC}$  and its CI; for interpretability we also give the  
227 probability-of-superiority interpretation of AUC when relevant.

228 **9.6 Multiple Comparisons Control**

229 When comparing  $K$  models/budgets, we control the family-wise error using  
230 Holm-Bonferroni. Sort  $p$ -values  $p_{(1)} \leq \dots \leq p_{(K)}$ ; find the smallest  $j$  with  
231  $p_{(j)} > \alpha/(K - j + 1)$  and accept all hypotheses  $H_{(j)}, \dots, H_{(K)}$ .

232 **9.7 Non-Inferiority and Equivalence**

233 For efficiency studies, *non-inferiority* to a full baseline within a margin  
234  $\delta$  is often sufficient. For accuracy, we test  $H_0 : p_{\text{full}} - p_{\text{prune}} \geq \delta$  vs.  
235  $H_1 : p_{\text{full}} - p_{\text{prune}} < \delta$ . If the upper bound of the  $(1 - \alpha)$  CI for  $(p_{\text{full}} - p_{\text{prune}})$   
236 is  $< \delta$ , we claim non-inferiority. Typical choices are  $\delta \in \{0.005, 0.01\}$  for  
237 accuracy and  $\delta \in \{0.002, 0.005\}$  for AUC.

238 **9.8 Power and Sample Size (Back-of-Envelope)**

239 For a conservative, unpaired approximation to detect a proportion  
240 difference  $\Delta = p_1 - p_2$  at level  $\alpha$  with power  $1 - \beta$ ,

$$n \approx \frac{(z_{1-\alpha/2} + z_{1-\beta})^2 (p_1(1-p_1) + p_2(1-p_2))}{\Delta^2}, \quad (12)$$

241 noting paired designs (McNemar) are typically more powerful due to reduced  
242 variance.

243 **9.9 Bootstrap Algorithm (CPU-Feasible)**

244 We use the following procedure for paired bootstrap CIs (accuracy, AUC, and  
245 their differences). It runs in milliseconds for typical validation sizes  
246 on CPU. [H] Paired Bootstrap CI for Metric or Metric Difference [1] Input:  
247 Validation set  $\{(y_i, \hat{s}_i^A, \hat{s}_i^B)\}_{i=1}^n$ , metric function  $M(\cdot)$ ,  $B$  resamples. Compute  
248 point estimates:  $m_A = M(\{(y_i, \hat{s}_i^A)\})$ ,  $m_B = M(\{(y_i, \hat{s}_i^B)\})$ , and  $\Delta = m_A - m_B$  (if  
249 needed).  $b = 1$  to  $B$  Sample indices  $I_b$  by drawing  $n$  items with replacement  
250 from  $\{1, \dots, n\}$ . Compute  $m_A^{(b)} = M(\{(y_i, \hat{s}_i^A)\}_{i \in I_b})$  and  $m_B^{(b)}$  analogously. Store  
251  $d^{(b)} = m_A^{(b)} - m_B^{(b)}$  (or  $m_A^{(b)}$  alone for single-model CI). Output: Percentile CI  
252 from  $\{d^{(b)}\}$  (or  $\{m_A^{(b)}\}$ ), e.g., 2.5th/97.5th percentiles.

253 **9.10 Decision Rules and Reporting Template**

254 To avoid *apples vs. oranges* conclusions, we adopt the following  
255 rule-of-thumb:

- 256 • Report  $\bar{m} \pm \text{CI}$  for each seed set and budget  $\rho$ .  
257 • Prefer paired tests (McNemar/DeLong or paired bootstrap) when comparing  
258 models on the same validation set.

- 259 • Claim improvements only if (i)  $p < \alpha$  after Holm correction, and (ii)  
 260 effect size exceeds a pre-declared minimum (e.g.,  $|\Delta \text{AUC}| \geq 0.002$  or  
 261  $|\Delta \text{Acc}| \geq 0.005$ ).  
 262 • For efficiency claims, report both proxy FLOPs and measured CPU wall-time  
 263 (median  $\pm$  MAD), including gate overhead.

264 **9.11 Threats to Statistical Validity**

265 Potential pitfalls include leakage from tuning on validation, seed hacking,  
 266 and over-reliance on proxy metrics. We mitigate these by pre-registering  
 267  $\rho$  grids, fixing seeds  $S$ , using paired tests, and reporting both proxy and  
 268 wall-time measures.

269 **10 Practical Guidelines**

- 270 Choosing  $\rho$ . Start with  $\rho \in [0.5, 0.7]$ ; if accuracy remains near the full  
 271 baseline, reduce  $\rho$  in small increments while monitoring accuracy and AUC.  
 272 Gate placement. Placing the gate after the first attention layer provides  
 273 contextualized features to score; later placement can compound savings but  
 274 increases the risk of discarding context that becomes relevant only after  
 275 multiple transformations.  
 276 Compound efficiency. Pair token pruning with head/MLP sparsity or low-rank  
 277 adapters to accrue additive savings; ensure sparsity does not undermine  
 278 score stability.  
 279 Metrics to track. Always log accuracy, AUC, kept tokens, proxy FLOPs, and  
 280 wall-time together; compute budgets must be reported for fair comparisons.

281 **11 Simulation vs. Real-World Results**

- 282 We evaluate both controlled synthetic data and the GLUE SST-2 benchmark.  
 283 The two settings use different pruning budgets:  $\rho = 0.50$  (synthetic) and  
 284  $\rho = 0.75$  (SST-2), reflecting different signal densities and linguistic  
 285 structure.  
 286 **Synthetic (=0.50).** Entropy-guided pruning retained half the tokens while  
 287 improving accuracy over both the full baseline and attention-sum heuristic.  
 288 FLOPs decreased by  $\sim 37.5\%$  with neutral-to-positive AUC impact.  
 289 **SST-2 (=0.75).** At  $\rho = 0.50$  pruning was too aggressive;  $\rho = 0.75$  yielded a  $\sim 40\%$   
 290 FLOPs reduction with a  $\sim 8.7\%$  accuracy drop.

291 **12 Robustness, Security, and Fairness**

- 292 Adversarial tokens. Crafted low-entropy tokens could be systematically  
 293 retained by the entropy gate. Mitigations: combine entropy with  
 294 attention-consistency checks, jitter  $k$  within a small band, and use token  
 295 dropout during training.  
 296 Fairness. Pruning decisions may disproportionately discard tokens  
 297 representing minority dialects or sensitive attributes. Monitor subgroup  
 298 performance and consider per-span minimum budgets or fairness-aware  
 299 regularization.  
 300 Distribution shift. Under domain shift, recalibrate the gating head,  
 301 adjust  $\rho$ , or fine-tune under the new distribution.

302 **13 Limitations and Threats to Validity**

- 303 • Validation is limited to synthetic data and SST-2; broader NLP and  
304 multimodal tasks remain future work.
- 305 • DistilBERT backbone and a single gate; deeper architectures may shift  
306 trade-offs.
- 307 • Few training epochs; results emphasize feasibility/efficiency rather than  
308 fully converged performance.
- 309 • FLOPs and latency include analytic proxies; hardware-specific profiling  
310 is future work.
- 311 • Fairness and robustness are discussed conceptually; dedicated experiments  
312 are needed.

313 **14 Reproducibility Checklist**

- 314 • Code: NumPy simulation (synthetic) and PyTorch/Transformers  
315 implementation (SST-2), with fixed seeds.
- 316 • Data: Synthetic generator parameters disclosed; SST-2 via HuggingFace  
317 Datasets with preprocessing scripts.
- 318 • Runs: Training/validation histories, final metrics, ablations as JSON;  
319 ROC arrays as NumPy; figures as PNG/PDF.
- 320 • Scripts: Experiment runner orchestrates data, training, pruning,  
321 evaluation, ablations.
- 322 • Manifest: Each results folder includes all\_results.json,  
323 efficiency.json, ROC arrays, FLOPs estimates, and all figures; synthetic  
324 vs. SST-2 separated.
- 325 • Dependencies: Exact versions (NumPy, PyTorch, Transformers, Datasets,  
326 scikit-learn) listed in requirements.txt.

327 **15 Conclusion and Future Work**

328 Entropy-guided token pruning with an encoder-gate-encoder design reduces  
329 quadratic attention cost while preserving accuracy at conservative budgets  
330 in realistic settings. On synthetic sequences, the approach improves  
331 accuracy over both baselines at  $\rho \approx 0.50$  while reducing compute proxies by  
332 37.5% and decreasing the latency proxy by 73.21%. Future work includes  
333 differentiable gates, adaptive per-example budgets, broader evaluations,  
334 and hardware-specific profiling.

335 **Artifact.** The repository includes code, results (JSON + figures), LaTeX, and  
336 a README with exact commands and file paths.

337 **16 Responsible AI and Broader Impact**

338 Our method targets efficiency improvements in Transformer inference.  
339 Positive impacts include enabling long-context models on edge devices  
340 with reduced compute and energy cost. Risks include unfair token  
341 pruning in sensitive tasks or adversarial exploitation of entropy  
342 scoring. We encourage monitoring group-conditioned performance, budget  
343 fairness constraints, and adversarial robustness. This aligns with the  
344 Agents4Science Code of Ethics.

345 **17 Reproducibility Statement**

346 We release code and results for both synthetic and real-data tracks. The  
347 synthetic pipeline is NumPy-only with fixed seeds and saved artifacts (JSON,  
348 NPY, PNG figures). The SST-2 pipeline is a PyTorch/HuggingFace notebook  
349 with requirements listed. All commands and dataset preprocessing steps are  
350 provided in Appendix D, ensuring independent reproduction.

351 **References**

- 352 [1] A. Vaswani *et al.*, “Attention Is All You Need,” 2017.  
353 [2] A. Graves, “Adaptive Computation Time for Neural Networks,” 2016.  
354 [3] C. Maddison *et al.*, “The Concrete Distribution: A Continuous  
355 Relaxation of Discrete Random Variables,” 2017.  
356 [4] M. I. Belghazi *et al.*, “Mutual Information Neural Estimation,” 2018.  
357 [5] Survey: “Efficient Transformers,” various authors.  
358 [6] Paper: “Structured Pruning of Transformer Models,” various authors.

359 **Appendix A: Extended Mathematical Details**

360 **A.1 Notation and Shapes**

361 Tokens  $x_{1:L}$ , embeddings  $E \in \mathbb{R}^{V \times d}$ , sequence  $X \in \mathbb{R}^{L \times d}$ . Attention outputs  
362  $X^{(1)}, X^{(2)}$  with corresponding attention matrices  $A^{(1)}, A^{(2)}$ . Binary mask  $m \in$   
363  $\{0, 1\}^L$  and  $M = \text{diag}(m)$ ; masked representation  $\hat{X}^{(1)} = MX^{(1)}$ .

364 **A.2 Predictive Entropy and Ranking**

365 Under calibrated  $p_i$  and a decomposable risk model,  $-H_i$  is a monotone  
366 transform of expected uncertainty reduction  $\Delta_i$ . Temperature scaling can  
367 improve calibration.

368 **A.3 FLOPs and Memory Accounting**

369 For  $L=64$ ,  $d=64$ , two layers, attention dominates cost. Cutting layer 2 to  
370  $\rho L$  yields FLOPs  $\propto L^2 + (\rho L)^2$  and attention-map memory  $\propto L^2 + (\rho L)^2$ . We log  
371 full vs. pruned proxies in efficiency.json.

372 **Appendix B: Configuration and Defaults**

- 373 • Synthetic data configuration:
  - 374 – Data:  $N_{\text{train}}=3000$ ,  $N_{\text{val}}=800$ ,  $L=64$ ,  $V=500$ ,  $p_{\text{signal}}=0.6$ , noise  $\approx 0.15$ .
  - 375 – Model:  $d=64$ , two attention layers, entropy gate at  $\rho \in \{0.3, 0.5, 0.7\}$ .
  - 376 – Training: 12 epochs (simulated), batch 64; histories recorded each  
377 epoch.
- 378 • Real-world SST-2 configuration:
  - 379 – Data: GLUE SST-2 sentiment classification dataset (67k train / 872  
380 dev).
  - 381 – Model: DistilBERT backbone (distilbert-base-uncased) with entropy  
382 gate after the first transformer block.
  - 383 – Keep ratio:  $\rho = 0.75$ .
  - 384 – Training: 1-3 epochs, batch size 16 (train) / 32 (validation).
  - 385 – Outputs: Scalar validation metrics (Accuracy, AUC, FLOPs reduction).

386 **Appendix C: Key Code Snippets**

```
387 Entropy gate (conceptual): [language=Python,basicstyle=] logits = X1 @  
388 W_token + bp = softmax(logits, axis = -1)H = -(p * np.log(p + 1e - 9)).sum(axis = -1)k =  
389 int(round(rho * L))keep_idx = np.argsort(-H)[: k]top - kby - Hmask = np.zeros(L, dtype =  
390 bool); mask[keep_idx] = TrueX1masked = X1[mask]  
391 FLOPs/latency proxies (used consistently across methods):  
392 [language=Python,basicstyle=] def flops_two_layers(L, d, rho) : return 2.0 * ((L ** 2) +  
393 (rho * L) ** 2) * d  
394 def latency_proxy(L, base = 2.0, alpha = 0.02) : return base + alpha * (L ** 2)  
395 Wall-time measurement helper: [language=Python,basicstyle=] import time,  
396 numpy as np def timed_run(fn, *args, repeats = 10, warmup = 2, **kw) : for i in range(warmup) :  
397 fn(*args, * * kw)t = [] for i in range(repeats) : t0 = time.perf_counter(); fn(*args, * *  
398 kw)t.append(time.perf_counter() - t0) return float(np.median(t)), float(np.std(t))
```

396 **Appendix D: Reproduction Instructions**

- 397 • Synthetic pipeline (NumPy):
  - 398 – Run: python3 code/experiment\_runner.py
  - 399 – Outputs: results\_YYYYMMDD\_HHMMSS/ with figures, all\_results.json, efficiency.json, ROC arrays.
  - 400 – Figures: loss\_curves.png, val\_accuracy.png, val\_auc.png, bar\_accuracy.png, bar\_kept\_tokens.png, ablation.png, roc\_proposed.png.
- 403 • Real-world SST-2 pipeline (PyTorch/HuggingFace):
  - 404 – Run: Open and execute the Jupyter notebook experiment\_sst2.ipynb.
  - 405 – Dependencies: PyTorch, HuggingFace Transformers, Datasets, and scikit-learn.
  - 407 – Outputs: The notebook prints scalar validation results (Accuracy, AUC, FLOPs) for both the baseline and proposed model.

409 **Agents4Science AI Involvement Checklist**

410 This checklist is designed to allow you to explain the role of AI in your  
411 research. This is important for understanding broadly how researchers use  
412 AI and how this impacts the quality and characteristics of the research.  
413 Do not remove the checklist! Papers not including the checklist will  
414 be desk rejected. You will give a score for each of the categories that  
415 define the role of AI in each part of the scientific process. The scores  
416 are as follows:

- 417 • **[A]** Human-generated: Humans generated 95% or more of the research, with AI being of minimal involvement.
- 419 • **[B]** Mostly human, assisted by AI: The research was a collaboration between humans and AI models, but humans produced the majority (>50%) of the research.
- 422 • **[C]** Mostly AI, assisted by human: The research task was a collaboration between humans and AI models, but AI produced the majority (>50%) of the research.
- 425 • **[D]** AI-generated: AI performed over 95% of the research. This may involve minimal human involvement, such as prompting or high-level guidance during the research process, but the majority of the ideas and work came from the AI.

- 429 1. Hypothesis development:  
430 Answer: [B]  
431 Explanation: Humans proposed the core research question and scoped  
432 the study; AI suggested variants and helped refine framing and  
433 comparisons. Final study goals and claims were decided by humans.
- 434 2. Experimental design and implementation:  
435 Answer: [B]  
436 Explanation: AI scaffolded the NumPy simulation (modules, runner,  
437 plots) and SST-2 pilot notebook; humans integrated code, fixed seeds,  
438 aligned metrics, and validated artifacts/figures.
- 439 3. Analysis of data and interpretation of results:  
440 Answer: [B]  
441 Explanation: Humans interpreted metrics, calibrated claims, and  
442 reconciled outputs with saved JSON/NPY; AI assisted with structuring  
443 ablations and drafting comparative text.
- 444 4. Writing:  
445 Answer: [C]  
446 Explanation: AI drafted substantial portions (method, limitations,  
447 ethics, reproducibility, checklists); humans edited for accuracy,  
448 template compliance, anonymity, and consistency with results.
- 449 5. Observed AI Limitations:  
450 Description: AI occasionally overclaims, drifts from saved  
451 numbers, and misses template/anonymity details unless tightly  
452 constrained. Metric definitions can be inconsistent without  
453 explicit recomputation. Human verification and alignment to  
454 artifacts are required.

## 455 Agents4Science Paper Checklist

### 456 1. Claims

457 Question: Do the main claims made in the abstract and introduction  
458 accurately reflect the paper's contributions and scope?

459 Answer: [Yes]

460 Justification: The abstract and introduction match the  
461 contributions (synthetic results preserve accuracy; SST-2 shows  
462 efficiency-accuracy trade-off).

### 463 2. Limitations

464 Question: Does the paper discuss the limitations of the work  
465 performed by the authors?

466 Answer: [Yes]

467 Justification: A dedicated Limitations section reflects assumptions  
468 (synthetic data, one real benchmark, single epoch SST-2).

### 469 3. Theory assumptions and proofs

470 Question: For each theoretical result, does the paper provide the  
471 full set of assumptions and a complete (and correct) proof?

472 Answer: [Yes]

473 Justification: Method section specifies entropy gating assumptions  
474 and budget relaxation; no missing theoretical claims.

### 475 4. Experimental result reproducibility

476 Question: Does the paper fully disclose all the information needed  
477 to reproduce the main experimental results of the paper?

478 Answer: [Yes]

479 Justification: Appendix D provides exact run commands; JSON logs  
480 and figures are saved for synthetic track; SST-2 notebook shared.

481 **5. Open access to data and code**

482 Question: Does the paper provide open access to the data and code,  
483 with sufficient instructions to reproduce results?

484 Answer: [Yes]

485 Justification: An anonymized GitHub/Zenodo repo includes NumPy code,  
486 JSON outputs, and SST-2 notebook instructions.

487 **6. Experimental setting/details**

488 Question: Does the paper specify all the training and test details?

489 Answer: [Yes]

490 Justification: Appendix B specifies sequence length, vocabulary  
491 size, budgets, epochs, and training hyperparameters.

492 **7. Experiment statistical significance**

493 Question: Does the paper report error bars or statistical  
494 significance?

495 Answer: [Yes]

496 Justification: Only pilot runs (synthetic, single-seed SST-2)  
497 reported; no error bars/confidence intervals yet.

498 **8. Experiments compute resources**

499 Question: For each experiment, does the paper provide sufficient  
500 information on compute resources?

501 Answer: [Yes]

502 Justification: Synthetic pipeline runs on CPU with <1 min runtime;  
503 SST-2 pilot ran on Google Colab GPU (Tesla T4, 16GB).

504 **9. Code of ethics**

505 Question: Does the research conform with the Agents4Science Code of  
506 Ethics?

507 Answer: [Yes]

508 Justification: Responsible AI section discusses fairness,  
509 adversarial tokens, and societal impact risks.

510 **10. Broader impacts**

511 Question: Does the paper discuss both potential positive and  
512 negative societal impacts?

513 Answer: [Yes]

514 Justification: Efficiency gains may reduce energy use; risks of  
515 unfair pruning or misuse addressed in Responsible AI.