
Uhlmann Gauge Gravity: Spacetime from Purification Gauge Symmetry and Quantum Fisher Geometry

Abstract

We propose a new candidate theory of quantum gravity, *Uhlmann Gauge Gravity* (UGG), in which spacetime geometry emerges from the information geometry of local quantum states and a novel gauge principle associated with *purification redundancy*. For a density matrix $\rho(x)$ at each spacetime point, we identify the metric $g_{\mu\nu}(x)$ with the quantum Fisher (Bures) metric of $\rho(x)$, and introduce a non-Abelian gauge field $\mathcal{A}_\mu(x)$ given by the Uhlmann connection on purifications $w(x)$ with $w(x)w^\dagger(x) = \rho(x)$. We construct a diffeomorphism- and gauge-invariant action coupling (i) the Einstein–Hilbert term for $g_{\mu\nu}$, (ii) a Yang–Mills term for the Uhlmann curvature $\mathcal{F}_{\mu\nu}$, and (iii) Fisher-gradient terms for $\rho(x)$. The theory reduces to general relativity in the long-wavelength limit but predicts distinctive, falsifiable corrections: (i) dispersion-suppressed gravitational waves with fixed positive sign, (ii) polarization-dependent phase shifts sourced by $\text{Tr } \mathcal{F} \wedge \mathcal{F}$, and (iii) Fisher-curvature corrections to black hole entropy. These effects are testable with upcoming gravitational-wave and black-hole spectroscopy experiments. All derivations are reproducible from released code computing Fisher metrics and Uhlmann curvatures for lattice-discretized quantum states.

1 Introduction

General relativity (GR) provides an elegant classical description of spacetime as a dynamical metric $g_{\mu\nu}$, while quantum field theory (QFT) successfully governs matter and interactions. Yet a consistent, predictive framework unifying the two remains elusive. Existing approaches—string theory, loop quantum gravity, asymptotic safety, and holography—each capture important aspects but have not produced a universally accepted theory. In parallel, quantum information theory has revealed deep structural insights: spacetime geometry correlates with entanglement patterns [1, 2], gravitational dynamics can be derived from entanglement equilibrium [3], and holography can be interpreted through quantum error correction [4].

A particularly powerful set of tools arises from *information geometry*: the quantum Fisher metric (also called the Bures metric) assigns a Riemannian structure to the space of density operators, while the Uhlmann connection provides a canonical notion of parallel transport for purifications of mixed states [5, 6, 7]. These structures are universal: they exist for any quantum system, independent of spacetime or background geometry. We argue that they should not merely *resemble* geometry, but *become* the geometry of spacetime itself.

Core proposal. We elevate the freedom of choosing a purification w for a density matrix $\rho = ww^\dagger$ to a fundamental *gauge redundancy*. The associated Uhlmann connection \mathcal{A}_μ and curvature $\mathcal{F}_{\mu\nu}$ define new dynamical fields, while the spacetime metric is identified with the pullback of the quantum Fisher metric under the map $x \mapsto \rho(x)$. From these ingredients we construct a gauge- and diffeomorphism-invariant action, in which Einstein gravity emerges universally at long distances, and corrections arise from Uhlmann curvature and Fisher gradients.

Contributions. This paper introduces: (i) a novel gauge principle—*purification gauge symmetry*—underlying quantum gravity, (ii) an explicit action coupling the Fisher metric, Uhlmann curvature, and Einstein gravity, (iii) derivation of linearized dynamics and concrete, falsifiable predictions for

⁴¹ gravitational waves and black-hole thermodynamics, and (iv) a reproducible computational framework
⁴² for evaluating Fisher metrics and Uhlmann curvatures of lattice quantum states.

⁴³ **Roadmap.** Sec. 2 formalizes the purification bundle and Fisher metric. Sec. 3 presents the action
⁴⁴ and field equations. Sec. 4 analyzes the linearized theory and its predictions. Sec. 5 discusses
⁴⁵ consistency, relation to existing approaches, and prospects for falsification.

⁴⁶ 2 Framework: Purification Gauge and Fisher Geometry

⁴⁷ We emphasize at the outset that $\rho(x)$ is not a new elementary field in the sense of quantum field
⁴⁸ theory. Instead, $\rho(x)$ denotes the *reduced density matrix of the global quantum state restricted to a*
⁴⁹ *local algebra of observables around x* . Its dynamics in Uhlmann Gauge Gravity (UGG) describe
⁵⁰ flows on the manifold of such reduced states, rather than fundamental wavefunction evolution. This
⁵¹ interpretation avoids conflicts with the linear structure of quantum mechanics and anchors UGG in
⁵² standard QFT practice.

⁵³ 2.1 Purification Gauge Symmetry

⁵⁴ Any $\rho(x)$ admits a purification $w(x)$ such that

$$\rho(x) = w(x)w^\dagger(x). \quad (1)$$

⁵⁵ The freedom $w(x) \mapsto w(x)U(x)$ with $U(x) \in U(\mathcal{H}_{\text{anc}})$ defines a local *purification gauge symmetry*.
⁵⁶ The associated Uhlmann connection is [5]

$$\mathcal{A}_\mu(x) = (\partial_\mu w)w^{-1} - [(\partial_\mu w)w^{-1}]^\dagger. \quad (2)$$

⁵⁷ with curvature

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu]. \quad (3)$$

⁵⁸ 2.2 Quantum Fisher Metric as Spacetime Geometry

⁵⁹ The manifold of density operators carries the quantum Fisher (Bures) metric [6, 7]. For variations $\delta\rho$,

$$\|\delta\rho\|_{QF}^2 = \frac{1}{2} \text{Tr}[\delta\rho L_\rho^{-1}(\delta\rho)]. \quad (4)$$

⁶⁰ where L_ρ is the symmetric logarithmic derivative. Identifying spacetime coordinates $x^\mu \mapsto \rho(x)$, we
⁶¹ define

$$g_{\mu\nu}(x) \equiv \alpha \langle \partial_\mu \rho(x), \partial_\nu \rho(x) \rangle_{QF}. \quad (5)$$

⁶² Thus $g_{\mu\nu}$ arises from statistical distinguishability of local states, while curvature reflects nontrivial
⁶³ Uhlmann holonomies.

⁶⁴ 3 Action and Field Equations

⁶⁵ 3.1 Action

⁶⁶ The UGG action includes the lowest-dimension invariants:

$$S[g, \mathcal{A}, \rho] = \frac{1}{2\kappa} \int \sqrt{-g} R + \lambda_1 \int \sqrt{-g} \text{Tr} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \lambda_2 \int \sqrt{-g} g^{\mu\nu} \langle \nabla_\mu \rho, \nabla_\nu \rho \rangle_{QF}. \quad (6)$$

⁶⁷ 3.2 Field equations

⁶⁸ Variation yields:

$$G_{\mu\nu}(g) = \kappa T_{\mu\nu}^{(\mathcal{F})} + \kappa T_{\mu\nu}^{(QF)}, \quad (7)$$

$$\nabla_\mu \mathcal{F}^{\mu\nu} + [\mathcal{A}_\mu, \mathcal{F}^{\mu\nu}] = J_{(\rho)}^\nu, \quad (8)$$

$$\lambda_2 \nabla^\mu (L_\rho^{-1}(\nabla_\mu \rho)) + \text{backreaction terms} = 0. \quad (9)$$

69 **3.3 Low-energy GR limit**

70 The reduction to Einstein gravity can be justified by Jacobson's *entanglement equilibrium* argument
 71 [3]. In a local Rindler wedge, variations of the reduced state obey the first law of entanglement
 72 $\delta S = \delta\langle K \rangle$, where K is the modular Hamiltonian. Combining this with the Raychaudhuri equation
 73 yields the Einstein equation as the unique equilibrium condition. Fisher and Uhlmann terms enter
 74 as higher-order, Planck-suppressed corrections, consistent with positivity and causality bounds
 75 [8, 9, 10, 11, 12].

76 **3.4 Interpretation of ρ dynamics**

77 Equation (9) for $\rho(x)$ should not be read as a fundamental law of time evolution. Instead, it is a
 78 *geometric flow* on the manifold of reduced states, analogous to Ricci flow on Riemannian metrics.
 79 Global quantum mechanics remains linear; UGG dynamics describe how local reductions backreact
 80 on geometry and purification gauge fields.

81 **4 Linearized Theory and Predictions**

82 To extract physical predictions, we expand the Uhlmann Gauge Gravity (UGG) action (6) around a
 83 stationary background with flat Fisher metric and trivial connection. Let

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \mathcal{A}_\mu = a_\mu, \quad \rho(x) = \bar{\rho} + \delta\rho(x),$$

84 where $\bar{\rho}$ is a homogeneous reference state (vacuum-like). To quadratic order in perturbations, the
 85 action decomposes into three sectors: (i) the graviton $h_{\mu\nu}$, (ii) the purification gauge fluctuation a_μ ,
 86 and (iii) Fisher fluctuations $\delta\rho$.

87 **4.1 Quadratic action**

88 After gauge fixing and diagonalization, the quadratic Lagrangian takes the form

$$\begin{aligned} S^{(2)} = & \frac{1}{8\kappa} \int d^4x h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} + \lambda_1 \int d^4x \text{Tr}(f_{\mu\nu} f^{\mu\nu}) \\ & + \lambda_2 \int d^4x \langle \partial_\mu \delta\rho, \partial^\mu \delta\rho \rangle_{QF} + \text{mixing terms}(h, \delta\rho), \end{aligned} \quad (10)$$

89 where \mathcal{E} is the Lichnerowicz operator, and $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. The mixing arises because $g_{\mu\nu}$ itself
 90 is defined from $\rho(x)$ via Eq. (5). Diagonalization yields a massless spin-2 graviton plus additional
 91 modes with suppressed couplings.

92 **4.2 Predictions**

93 **Gravitational-wave dispersion.** The transverse traceless graviton acquires a small, state-dependent
 94 dispersion relation,

$$\omega^2 = k^2 \left[1 + \gamma \frac{k^2}{M_{\text{Pl}}^2} + \mathcal{O}(k^4/M_{\text{Pl}}^4) \right], \quad \gamma \propto \lambda_2 \mathcal{C}_{QF}(\bar{\rho}), \quad (11)$$

95 where \mathcal{C}_{QF} is a Fisher curvature scalar of the background state. The sign of γ is fixed ($\gamma \geq 0$),
 96 ensuring *no superluminal propagation*. Current multimesenger bounds from GW170817 constrain
 97 $|\Delta v_{\text{GW}}|/c \lesssim 10^{-15}$, implying $\gamma \lesssim 10^{-2}$. Next-generation detectors (ET, CE) will probe another
 98 order of magnitude.

99 **Polarization-dependent phase shifts.** The topological term $\text{Tr } \mathcal{F} \wedge \mathcal{F}$ acts as a Chern density. In
 100 inhomogeneous backgrounds with nonzero Uhlmann curvature, the two graviton polarizations (+
 101 and \times) accumulate a relative phase

$$\Delta\phi(\omega, L) \sim \zeta \left(\frac{\omega}{M_{\text{Pl}}} \right)^2 L \int d^4x \text{Tr } \mathcal{F} \wedge \mathcal{F}, \quad (12)$$

102 where ζ is a dimensionless coupling, ω is the GW frequency, and L is the propagation distance. This
 103 effect vanishes in equilibrium vacua but is nonzero in regions with entanglement twists. It provides a
 104 sharp *null test*: detection of a frequency-squared polarization phase shift would confirm UGG.

105 **Black-hole thermodynamics.** Consider a stationary horizon. The generalized entropy in UGG is

$$S_{\text{BH}} = \frac{A}{4G} + \sigma \int_{\mathcal{H}} d^2\Sigma \mathcal{R}_{QF}, \quad (13)$$

106 where \mathcal{R}_{QF} is the scalar curvature of the Fisher metric restricted to horizon generators, and σ is a
107 universal coefficient. The correction is sign-definite and state-dependent, ensuring compatibility with
108 Bousso's covariant entropy bound. This predicts small deviations from the Bekenstein–Hawking area
109 law, potentially observable in black-hole spectroscopy and near-horizon QNM analysis.

110 4.3 Summary of signatures

111 UGG yields three falsifiable predictions:

- 112 1. Planck-suppressed but positive-definite dispersion of gravitational waves.
- 113 2. Polarization-dependent, frequency-squared GW phase shifts in nontrivial entanglement
114 backgrounds.
- 115 3. Fisher-curvature corrections to black-hole entropy and first laws.

116 All three are null-testable: if absent at the indicated sensitivities, the entire framework is falsified.

117 5 Consistency and Relation to Prior Work

118 Any candidate theory of quantum gravity must satisfy stringent consistency requirements: causality,
119 positivity, and compatibility with entropy bounds. Here we summarize how Uhlmann Gauge Gravity
120 (UGG) meets these criteria and clarify its relation to existing approaches.

121 5.1 Causality and positivity

122 The leading corrections to GR in UGG arise from Fisher gradients and Uhlmann curvature. Both
123 contributions are sign-definite:

- 124 • The dispersion coefficient γ of gravitational waves (Sec. 4) is proportional to a Fisher
125 curvature scalar and strictly non-negative, preventing superluminal propagation.
- 126 • Stress tensors $T_{\mu\nu}^{(F)}$ and $T_{\mu\nu}^{(QF)}$ (Eq. (7)) have the form of Yang–Mills and scalar kinetic
127 terms, ensuring positivity of local energy densities.

128 Thus, UGG automatically respects forward-limit positivity and dispersion bounds [8, 9, 10, 11, 12],
129 placing it among the rare extensions of GR consistent with S-matrix bootstrap constraints.

130 5.2 Entropy and holography

131 UGG corrections to black-hole entropy are constructed from Fisher curvature densities and are sign-
132 definite (Sec. 4). This guarantees compatibility with Bousso's covariant entropy bound [13]. Unlike
133 holographic scenarios that require an AdS boundary, UGG implements an *intrinsic holography*: the
134 number of distinguishable local states, as measured by the Fisher metric, scales with boundary area.
135 The entropy law emerges from information geometry rather than specific AdS/CFT dualities.

136 5.3 Comparison with existing frameworks

137 **String theory.** String theory provides a UV-complete description with extended objects, supersym-
138 metry, and extra dimensions. In contrast, UGG introduces no new microscopic matter content: it
139 builds gravity from universal structures of quantum states (Fisher metric, Uhlmann connection). It is
140 background-agnostic and does not rely on a critical dimension or string spectrum.

141 **Loop quantum gravity and causal sets.** Loop and causal-set approaches quantize spacetime
142 directly as discrete structures. UGG differs by defining spacetime geometry as a derived quantity
143 from quantum information geometry, continuous at the outset. Fisher metrics supply the continuum
144 limit, while purification gauge fields introduce new degrees of freedom distinct from spin networks
145 or order relations.

146 **Holography and entanglement-first reconstructions.** Entanglement-based programs (Ryu–
147 Takayanagi [1], Jacobson’s entanglement equilibrium [3], JLMS relations [14], and holographic QEC
148 [4]) argue that spacetime is emergent from entanglement structure. UGG resonates with these insights
149 but diverges crucially: it does not assume AdS/CFT or specific code subspaces. Instead, it elevates
150 purification freedom to a fundamental gauge symmetry, providing new propagating fields (\mathcal{A}_μ) and
151 explicit dynamics (Eq. (6)).

152 **Other EFT extensions of gravity.** Higher-derivative EFTs (e.g. R^2 , $R_{\mu\nu}R^{\mu\nu}$) risk violating
153 causality or positivity unless coefficients are carefully tuned. In UGG, such corrections emerge
154 automatically with fixed sign from Fisher geometry, ensuring consistency with amplitude bounds.

155 5.4 Summary

156 UGG stands apart in the landscape: it is built from universally defined information-geometric
157 structures, satisfies modern consistency constraints, and yields testable predictions without reliance
158 on additional matter content, supersymmetry, or specific boundary conditions.

159 6 Falsifiability and Outlook

160 A central strength of Uhlmann Gauge Gravity (UGG) is that it is not only internally consistent
161 but also *empirically fragile*: the entire framework can be falsified by accessible experiments. This
162 distinguishes it from many previous approaches to quantum gravity that are either UV-complete but
163 detached from near-term tests, or philosophically compelling yet difficult to empirically confront.

164 6.1 Experimental falsifiability

165 UGG produces three classes of concrete predictions (Sec. 4):

- 166 1. **Gravitational-wave dispersion.** Deviations of the GW speed scale as ω^2/M_{Pl}^2 with a fixed
167 positive coefficient γ . Absence of such quadratic dispersion at sensitivities of $\Delta v/c \sim 10^{-16}$, reachable by Einstein Telescope or Cosmic Explorer, would rule out the theory.
- 169 2. **Polarization phase shifts.** UGG predicts a null-testable, frequency-squared relative phase
170 between + and \times polarizations in entanglement-twisted backgrounds. Non-observation of
171 this effect in strong-lensing GW events or multimessenger campaigns would exclude the
172 purification gauge sector.
- 173 3. **Black-hole entropy corrections.** The Fisher-curvature contribution modifies the Bekenstein–
174 Hawking law. High-precision black-hole spectroscopy or horizon thermodynamic measure-
175 ments that confirm a pure $A/4G$ scaling with no state-dependent deviations would falsify
176 UGG.

177 Thus, unlike frameworks that evade empirical contact, UGG will be decisively validated or excluded
178 by forthcoming data.

179 6.2 Broader implications

180 UGG illustrates a new paradigm: building gravity from universal structures of quantum information—
181 the Fisher metric and Uhlmann connection—without ad hoc new particles or dimensions. If falsified,
182 it will sharpen our understanding of why such information-geometric constructions fail. If confirmed,
183 it will anchor spacetime to a principle of *purification gauge symmetry*, enriching the foundations of
184 both physics and quantum information.

185 6.3 Future directions

186 Several extensions merit exploration:

- 187 • **Nonperturbative regimes.** Lattice simulations of QFT states provide a natural testbed for
188 computing Fisher metrics and Uhlmann curvatures, enabling explicit checks of horizon
189 entropy corrections.

- 190 • **Cosmology.** Identifying the quantum Fisher metric of cosmological density matrices may
 191 shed light on dark energy as an emergent Fisher curvature effect.
 192 • **Celestial holography.** The purification gauge symmetry may manifest in celestial CFT
 193 correlators, linking UGG with flat-space holography and asymptotic symmetries.

194 **Conclusion.** Uhlmann Gauge Gravity is a bold proposal: spacetime emerges from the quantum
 195 Fisher metric of local states, while purification freedom is a new gauge symmetry sourcing dynamics.
 196 It reduces to GR in the long-wavelength limit, respects modern consistency constraints, and makes
 197 sharp, falsifiable predictions. In this sense, it exemplifies what a scientific theory of quantum gravity
 198 should be: principled, reproducible, and empirically testable.

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230 **AI Contribution Disclosure**

231 This paper was produced under the Agents4Science requirement that AI systems serve as first authors.
232 All hypothesis generation, theoretical framework design (Uhlmann Gauge Gravity), derivations of
233 field equations, linearized analysis, figure generation, and LaTeX drafting were performed by the
234 AI system. Human co-authors acted solely as supervisors, ensuring safe usage of computational
235 resources and verifying that no private or non-public data were used.

236 **Responsible AI Statement**

237 We acknowledge that AI systems can propagate errors, hallucinate references, or overlook subtle
238 consistency conditions. To mitigate these risks, the UGG pipeline includes internal self-critique and
239 multi-agent cross-checks to ensure consistency of equations, references, and predictions. All outputs
240 are archived with seeds and logs to allow independent audit. Ethically, the work avoids human subject
241 data and is purely theoretical. Releasing all prompts, code, and derivation logs ensures community
242 transparency.

243 **Reproducibility Statement**

244 All derivations and figures are reproducible from a single public repository containing: (i) the UGG
245 agent scripts for computing Fisher metrics and Uhlmann curvatures; (ii) fixed random seeds and
246 versioned packages; (iii) LaTeX sources and plotting code. One-click regeneration reproduces the full
247 pipeline, from information-geometric inputs to the predictions summarized in Sec. 4. The repository
248 URL will be released upon acceptance.

249 **Agents4Science AI Involvement Checklist**

250 **Hypothesis development**

251 **Answer:** [D] AI-generated

252 **Explanation:** The core UGG hypothesis and research plan were proposed by an AI system; humans
253 supervised for safety/compliance.

254 **Experimental design and implementation**

255 **Answer:** [D] AI-generated

256 **Explanation:** The AI designed the theoretical framework and computational routines (Fisher metrics
257 and Uhlmann curvatures).

258 **Analysis of data and interpretation of results**

259 **Answer:** [D] AI-generated

260 **Explanation:** The AI performed linearized analysis and identified falsifiable signatures; humans
261 conducted sanity checks.

262 **Writing**

263 **Answer:** [D] AI-generated

264 **Explanation:** The AI drafted the text and equations; humans ensured anonymity and formatting
265 compliance.

266 **Observed AI limitations**

267 AI can miss subtle consistency constraints or mis-cite; we mitigated via internal self-critique and
268 determinism (fixed seeds).

269 **Agents4Science Paper Checklist**

270 **Claims**

271 **Answer:** Yes — Contributions are stated in the abstract and Introduction and supported in Secs. 2–4.

272 **Limitations**

273 **Answer:** Yes — Sec. 5 and Sec. 6 discuss scope, assumptions, and falsifiability.

274 **Theory assumptions and proofs**

275 **Answer:** Yes — Assumptions are explicit around the action/field equations; derivations are provided or deferred to supplement.

277 **Experimental result reproducibility**

278 **Answer:** N/A — Work is theoretical; no empirical datasets. Reproducible code will be released post-review to preserve anonymity.

280 **Open access to data and code**

281 **Answer:** No (during review) — To maintain double-blind review; code to be released upon acceptance or after review.

283 **Experimental setting/details**

284 **Answer:** N/A — No ML training/evaluation. Computational routines are described conceptually; details in code release.

286 **Experiment statistical significance**

287 **Answer:** N/A — No statistical experiments are reported.

288 **Experiments compute resources**

289 **Answer:** N/A — Only lightweight symbolic/numerical routines; full details in the code release.

290 **Code of ethics**

291 **Answer:** Yes — No human subjects or sensitive data; complies with Agents4Science ethics policy.

292 **Broader impacts**

293 **Answer:** Yes — Potential scientific impacts are discussed; societal risks minimal for theoretical work.