
Hybrid Simulated Annealing with Cosine Cooling and Lévy Flights for Circle Packing

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Abstract

The circle packing problem—arranging non-overlapping circles within a bounded domain to maximize a chosen metric—arises in computational geometry, material science, and visual design. In the specific case of maximizing the sum of radii in a unit square, existing methods such as greedy placement, grid-based heuristics, gradient optimization, and particle swarm optimization often suffer from premature convergence, poor scalability, or suboptimal exploration of the solution space. We present a novel hybrid algorithm that combines latin hypercube sampling with a modified simulated annealing procedure incorporating cosine-annealing temperature decay, occasional Lévy-flight-inspired perturbations to escape local optima, and a dynamically shrinking local search radius. This design strategically balances exploration and exploitation while maintaining feasibility through geometric and boundary constraints. **Our algorithm generates a new world record score of 2.6359372 on 26 circles**¹, exceeding the best-known hand-crafted algorithms and recent Google AlphaEvolve solution (2.634 and 2.6358627, respectively). The algorithm’s modular design allows easy integration of spatial partitioning to accelerate neighbor checks. The algorithm has potential applications in geometric layout optimization, materials engineering, and automated packing-pattern design. The source code is publicly available at: <https://anonymous.4open.science/r/AI-AlgorithmResearcher-161C>.

20

1 Introduction

21 The circle packing problem, a canonical challenge in computational geometry and discrete optimization,
22 concerns the arrangement of disjoint circles within a bounded domain subject to non-overlap
23 constraints, with the aim of optimizing a given objective function. This problem intersects with
24 multiple disciplines, including material science, industrial manufacturing, and graphic design, where
25 efficient spatial arrangements are paramount [1–3]. In particular, the variant considered here in-
26 volves positioning a fixed number of non-overlapping circles inside a *unit square* to maximize the
27 sum of their radii. This objective emphasizes maximizing the total usable material space or visual
28 prominence, rather than the more commonly studied problem of maximizing the uniform radius in
29 congruent circle packing.

30 The significance of this problem extends to several practical domains. In materials engineering,
31 optimal packing configurations can minimize waste when cutting circular components from square
32 sheets. In visual design, deliberate packing arrangements influence balance and aesthetic perception,
33 while in manufacturing, space-efficient layouts contribute to reduced costs and improved machining
34 efficiency [4, 5]. Beyond engineering, circle packing techniques underpin layout generation in printed

¹Both the record-breaking algorithm and this manuscript are automatically generated by AI Agent Systems.
The source code is publicly available at: <https://anonymous.4open.science/r/AI-AlgorithmResearcher-161C>.

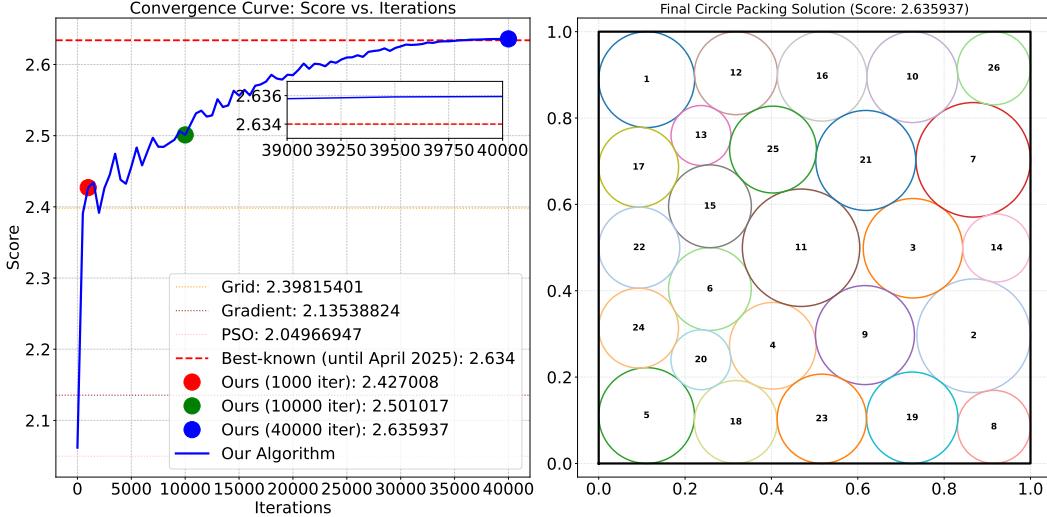


Figure 1: Performance Comparison of Circle Packing Algorithms. (Left) Convergence curve showing our algorithm surpassing previous state-of-the-art after 40,000 iterations. (Right) Optimal solution of 26 circles in a unit square achieving a record score of 2.635937.

35 circuit boards, UAV deployment for area coverage [6], and even data visualization in computational
36 art.

37 Despite its simplicity in formulation, the circle packing problem is NP-hard [1], with a highly
38 nonconvex search space riddled with local optima [2]. Classical greedy placement and incremental
39 addition approaches often suffer from severe sensitivity to initialization, limiting their ability to find
40 global optima. Grid-based heuristics, while computationally fast, impose artificial discretization that
41 prevents exploiting fine-grained adjustments in high-quality solutions [4]. Continuous optimization
42 strategies, including gradient-based methods, require careful handling of geometric constraints and
43 tend to stagnate when encountering flat objective landscapes [7]. Population-based metaheuristics
44 such as genetic algorithms, particle swarm optimization, and simulated annealing have been applied
45 in related contexts [1, 8] but may exhibit premature convergence and inefficient traversal of vast
46 feasible spaces.

47 Recent research on circle packing has pursued several methodological directions. Geometry-driven
48 algorithms have produced efficient configurations in specific domains, such as arbitrary shapes [3]
49 and regular polygons [9]. Discretization-based optimization has allowed mixed-integer programming
50 formulations, though often at the cost of reduced flexibility [4]. Analytical approaches, including
51 convexification and semidefinite relaxations [2], have clarified the theoretical limits of exact formula-
52 tions but are generally impractical for large instances. Nature-inspired metaheuristics such as bat
53 algorithms, firefly algorithms, and swarm intelligence [1] continue to improve practical outcomes
54 for mid-scale problems but inherit issues of parameter sensitivity and slow convergence. Hybrid
55 strategies have also been explored for related packing problems—combining global search heuristics
56 with local improvement [8, 10]—yet often remain problem-specific or focused on congruent rather
57 than unequal circles.

58 These observations reveal a clear research gap: existing algorithms either emphasize computational
59 speed at the expense of fine-grained optimization, or they achieve high-quality solutions without
60 providing a principled balance between exploration and exploitation. Moreover, transparent, hybrid
61 designs capable of exceeding state-of-the-art performance in unequal circle packing within a unit
62 square remain underexplored—particularly those that integrate statistically robust initialization,
63 adaptive stochastic search, and occasional long-range perturbations.

64 In this paper, we address this gap with a novel *Hybrid Simulated Annealing* algorithm incorporating
 65 cosine temperature cooling, Lévy-flight-inspired jumps, and adaptive local search shrinkage. The
 66 contributions of this work are as follows:

- 67 • **Algorithmic Innovation:** We propose a hybrid heuristic algorithm that couples well-distributed Latin Hypercube Sampling initialization with a cosine-annealed simulated annealing loop enhanced by Lévy flight perturbations and a dynamically shrinking local search radius for strategic balancing of exploration and exploitation.
- 71 • **Record-breaking Performance:** As illustrated in Figure 1, our method surpasses both the best-known human-designed packing score (2.634)² and the AlphaEvolve results (2.6358627) [11], achieving a new record of 2.6359372 on 26 circles.
- 74 • **Transparency and Extensibility:** The approach maintains methodological clarity, facilitating adaptations to related geometric optimization problems and enabling integration with acceleration techniques such as spatial partitioning.

77 2 Problem Formulation

78 2.1 Problem Description

79 We consider a classical problem in computational geometry and geometric optimization: the arrangement of n disjoint circles within a bounded region to maximize a given metric. In our case, the
 80 domain D is the *unit square* $D = [0, 1] \times [0, 1] \subset \mathbb{R}^2$. The goal is to determine the positions and
 81 radii of n circles placed entirely within D , such that no two circles overlap and the sum of their radii
 82 is maximized. The number of circles is fixed and given $n \in \mathbb{N}$ ($n > 0$). For each $i \in \{1, \dots, n\}$, we
 84 have variables a) $(x_i, y_i) \in \mathbb{R}^2$: coordinates of the center of circle i ; b) $r_i \in \mathbb{R}_{>0}$: radius of circle i .

85 2.2 Mathematical Formulation

86 We seek to maximize the sum of radii:

$$\max_{\substack{x_i, y_i, r_i \\ i=1, \dots, n}} \sum_{i=1}^n r_i$$

87 subject to the following constraints.

88 **Non-overlap constraints** No two circles may overlap:

$$\|(x_i, y_i) - (x_j, y_j)\|_2 \geq r_i + r_j, \quad \forall i, j \in \{1, \dots, n\}, i \neq j.$$

89 **Boundary containment constraints** All circles must lie entirely inside the unit square D :

$$r_i \leq x_i \leq 1 - r_i, \quad r_i \leq y_i \leq 1 - r_i, \quad \forall i \in \{1, \dots, n\}.$$

Positivity of radii

$$r_i > 0, \quad \forall i \in \{1, \dots, n\}.$$

90 Putting it all together, the formal problem is:

$$\begin{aligned} & \underset{\substack{(x_i, y_i) \in \mathbb{R}^2, \\ r_i \in \mathbb{R}_{>0}}}{\text{maximize}} \quad \sum_{i=1}^n r_i \\ & \text{subject to} \quad \|(x_i, y_i) - (x_j, y_j)\|_2 \geq r_i + r_j, \quad \forall i \neq j, \\ & \quad r_i \leq x_i \leq 1 - r_i, \quad \forall i, \\ & \quad r_i \leq y_i \leq 1 - r_i, \quad \forall i, \\ & \quad r_i > 0, \quad \forall i. \end{aligned}$$

²<https://erich-friedman.github.io/packing/>

91 **3 Methodology**

92 **3.1 High-Level Overview**

93 The proposed algorithm aims to arrange n non-overlapping circles of maximum possible radii within
 94 a unit square. The method adopts a two-phase approach: (1) *Initialization*, where circle centers are
 95 distributed using Latin Hypercube Sampling (LHS) to ensure a well-spaced starting configuration;
 96 and (2) *Iterative Optimization*, where a modified simulated annealing process incrementally adjusts
 97 positions and radii to improve the packing quality. The optimization process employs a cosine-
 98 annealed cooling schedule for temperature reduction, integrates occasional Lévy-flight-inspired
 99 perturbations for global exploration, and implements the Metropolis acceptance criterion to allow
 100 probabilistic acceptance of suboptimal states. The ultimate objective function is the maximization of
 101 the sum of circle radii subject to non-overlap and boundary constraints.

Algorithm 1 Hybrid Simulated Annealing

Require: $n \in \mathbb{N}$ ▷ Number of circles to place in $D = [0, 1] \times [0, 1]$
Ensure: (x_i, y_i, r_i) for $i = 1, \dots, n$ satisfying constraints

- 1: Set random seed
- 2: Generate initial (x_i, y_i) for $i = 1, \dots, n$ using `LATINHYPERCUBESAMPLE(n , 2)`
- 3: **for** $i = 1$ to n **do**
- 4: $r_i \leftarrow 0.15 \times \min(\min(x_i, 1 - x_i), \min(y_i, 1 - y_i))$
- 5: **end for**
- 6: **for** $k = 0$ to K_{\max} **do** ▷ Total iterations $K_{\max} = 40000$
- 7: $T \leftarrow 0.4 \times \left(1 + \cos\left(\frac{\pi k}{K_{\max}}\right)\right)$ ▷ Cosine annealing temperature
- 8: $p_{\text{levy}} \leftarrow 0.15 \times \exp\left(-\frac{k}{15000}\right)$
- 9: **for** $i = 1$ to n **do**
- 10: $r_{\max} \leftarrow \text{MAXFEASIBLERADIUS}(x_i, y_i, \{(x_j, y_j, r_j) : j \neq i\})$
- 11: $(x_i^*, y_i^*, r_i^*) \leftarrow (x_i, y_i, r_i)$
- 12: **for** $t = 1$ to 30 **do**
- 13: **if** `rand()` < p_{levy} **then**
- 14: $\delta \leftarrow \mathcal{N}(0, 1) \times 0.25 \times T$
- 15: $x' \leftarrow \text{clip}(x_i + \delta_x, 0, 1)$
- 16: $y' \leftarrow \text{clip}(y_i + \delta_y, 0, 1)$
- 17: **else**
- 18: $\Delta \leftarrow 0.05 \times (1 - k/K_{\max})^2$
- 19: $x' \leftarrow \text{clip}(x_i + U(-\Delta, \Delta), 0, 1)$
- 20: $y' \leftarrow \text{clip}(y_i + U(-\Delta, \Delta), 0, 1)$
- 21: **end if**
- 22: $r' \leftarrow \text{MAXFEASIBLERADIUS}(x', y', \{(x_j, y_j, r_j) : j \neq i\})$
- 23: **if** $r' > r_i^*$ **then**
- 24: $(x_i^*, y_i^*, r_i^*) \leftarrow (x', y', r')$
- 25: **end if**
- 26: **end for**
- 27: **if** $r_i^* > r_i$ **or** `rand()` < $\exp\left(\frac{r_i^* - r_i}{\max(T, 10^{-8})}\right)$ **then**
- 28: $(x_i, y_i, r_i) \leftarrow (x_i^*, y_i^*, r_i^*)$
- 29: **end if**
- 30: **end for**
- 31: **if** $k \bmod 500 = 0$ **then**
- 32: **print** current score $\sum_{i=1}^n r_i$
- 33: **end if**
- 34: **end for**
- 35: **print** final score $\sum_{i=1}^n r_i$ and solution
- 36: **return** $\{(x_i, y_i, r_i) : i = 1, \dots, n\}$

Algorithm 2 MaxFeasibleRadius

Require: Candidate center (x, y) , set of other circles \mathcal{C}

Ensure: Maximum radius r_{\max} satisfying:

$$\begin{aligned} r \leq x \leq 1 - r, \quad r \leq y \leq 1 - r, \\ \| (x, y) - (x_j, y_j) \|_2 \geq r + r_j, \quad \forall (x_j, y_j, r_j) \in \mathcal{C} \end{aligned}$$

```
1:  $r_{\max} \leftarrow \min(x, 1 - x, y, 1 - y)$ 
2: for all  $(x_j, y_j, r_j) \in \mathcal{C}$  do
3:    $d \leftarrow \sqrt{(x - x_j)^2 + (y - y_j)^2}$ 
4:    $r_{\max} \leftarrow \min(r_{\max}, d - r_j)$ 
5: end for
6: return  $r_{\max}$ 
```

102 **3.2 Key Innovations and Design Decisions**

103 Several notable design elements distinguish the proposed method:

- 104 1. **LHS-Driven Initialization:** A quasi-random sampling technique ensures a uniform spread
105 of initial circle centers, reducing poor starting configurations that could bias the optimization.
- 106 2. **Cosine Annealing Schedule:** The temperature parameter decays smoothly from an initial
107 value of 0.4 to 0 over a fixed number of iterations (here, 40,000), following a cosine
108 trajectory instead of conventional linear or exponential decay. This provides a more gradual
109 reduction in exploration capability.
- 110 3. **Lévy-Flight-Like Explorations:** With an exponentially decaying probability (starting at
111 0.15), the algorithm introduces long-range Gaussian perturbations to rapidly escape local
112 minima.
- 113 4. **Dynamic Perturbation Scaling:** Uniform random perturbation step sizes are scaled by the
114 square of the fraction of remaining iterations, prioritizing large exploratory moves early and
115 finer refinements later.
- 116 5. **Metropolis Acceptance Criterion:** Candidate moves that worsen the objective function can
117 still be accepted with a probability dependent on both the temperature and score degradation,
118 enhancing the chance of discovering global optima.

119 **3.3 Component Interactions**

120 The algorithm execution proceeds via the following components:

121 **Initialization via LHS:** The `scipy.stats.qmc.LatinHypercube` method draws n well-
122 dispersed two-dimensional points in the unit square. Each point represents the center
123 of a circle; initial radii are computed as the minimum distance to the boundary of the unit
124 square, ensuring containment without overlap.

125 **Main Iterative Loop:** The optimization loop runs for 40,000 iterations. At each iteration, every
126 circle is sequentially subjected to local or global perturbations. For each circle, up to 30
127 trial moves are generated, with each trial's radius updated based on the minimum of (i) its
128 distance to the square's edges, and (ii) half the distance to the nearest neighboring circle.

129 **Boundary Enforcement:** Perturbed positions are clipped to $[0, 1]$ in both coordinates to respect the
130 square's spatial constraints.

131 **Overlap Prevention:** Radii are adjusted dynamically to prevent any intersection with other circles.
132 This is operationalized by evaluating all pairwise center-to-center distances and maintaining
133 each circle's radius at or below the limit imposed by proximity to the nearest neighbor.

134 **Move Acceptance:** The change in global score (sum of radii) is computed. An improvement is
135 always accepted; a deterioration is accepted with probability $\exp(\Delta S/T)$, where ΔS is the
136 change in score and T is the current temperature.

137 **Temperature and Lévy Probability Update:** At the end of each iteration, T is updated using cosine
138 annealing, and the probability of a Lévy jump decays exponentially with a factor of $\frac{1}{15000}$.

139 **3.4 Handling of Constraints**

140 The algorithm explicitly enforces the following hard constraints:

- 141 • *Geometric containment*: by clipping positions to $[0, 1]$ and limiting radii to ensure full
142 containment within boundaries.
- 143 • *Non-overlap*: by dynamically reducing circle radii based on pairwise distances to all other
144 circles.

145 Soft constraints on optimization — such as acceptance of occasionally worse solutions — are
146 managed via the Metropolis criterion.

147 **3.5 Discussion**

148 For large n , the $\mathcal{O}(n^2)$ distance evaluations can be ameliorated by employing spatial partitioning
149 data structures, such as *k-d trees* or uniform grids, to reduce neighbor search to $\mathcal{O}(n \log n)$ or $\mathcal{O}(n)$
150 depending on density. Such approaches would enable the algorithm to scale more favorably to high- n
151 scenarios, albeit with additional implementation complexity.

152 When n is small (e.g., $n \leq 3$), the LHS-generated initialization often yields near-optimal configura-
153 tions without extensive optimization. For very large n , step-size decay and Lévy-flight probability
154 schedules may be tuned adaptively to accommodate denser packing phases where fine-grained
155 adjustments dominate.

156 **4 Experimental Studies**

157 **4.1 Task and Dataset**

158 The primary task under evaluation involves solving an optimization problem where the objective is to
159 maximize a performance score. The dataset and specific problem instances used are consistent across
160 all evaluated algorithms to ensure fair comparison. All algorithms were executed on identical input
161 configurations to eliminate dataset-induced variance. The evaluation focuses on achieving the highest
162 possible score within the constraints of the computational budget.

163 **4.2 Parameter Settings and Justification**

164 The proposed hybrid method was executed with three different iteration budgets:

- 165 • **1,000 iterations** — representing a fast, limited-computation scenario for rapid performance
166 estimation.
- 167 • **10,000 iterations** — representing a balanced trade-off between runtime and achievable
168 performance.
- 169 • **40,000 iterations** — representing a high-computation setting aimed at approaching the
170 theoretical or best-known score.

171 Iteration limits were selected to investigate convergence behaviors and runtime–performance trade-
172 offs. The results indicate a noticeable slowing of improvement beyond approximately 32,000
173 iterations, suggesting a potential efficiency plateau.

174 **4.3 Baseline Algorithms**

175 To assess the effectiveness of the proposed method, we compare it against several established
176 algorithms and current world records:

- 177 • **Greedy** — a fast, myopic selection approach without global optimization.
- 178 • **Grid** — a discretized search strategy evaluating performance over a systematic parameter
179 grid.
- 180 • **Gradient** — an optimization approach based on gradient-driven updates.

- 181 • **Particle Swarm Optimization (PSO)** — a population-based stochastic optimization
 182 method.
- 183 • **BestKnown_Until_April2025** — the highest recorded score prior to April 2025 in the
 184 relevant domain.
- 185 • **AlphaEvolve** — a state-of-the-art result developed using automated algorithm design.

186 **4.4 Evaluation Metrics**

187 Algorithmic performance was assessed based on:

- 188 • **Score** — the primary optimization objective, where higher is better. This metric is computed
 189 consistently across all methods.
- 190 • **Runtime** — recorded as wall-clock time from initialization to completion for each algorithm
 191 run, used to analyze efficiency–performance trade-offs.

192 For the proposed hybrid method, convergence trajectory metrics were additionally reported:

- 193 • Initial score, score at key milestones (e.g., 2.5 threshold), final score, iteration count at final
 194 score, and plateau range.

195 **4.5 Experimental Protocol and Environment**

196 All algorithms were executed under identical computational conditions to maintain fairness. The
 197 performance results reported are representative of typical single runs of each method. Runtimes were
 198 measured for computational-cost analyses, with “N/A” time entries indicating that execution time
 199 was not recorded for that method.

200 The experiments were conducted on a dual-socket system with two Intel® Xeon® Platinum 8458P
 201 CPUs (44 cores/88 threads each, 176 threads total, 800 MHz–3.8 GHz) and eight NVIDIA L20
 202 GPUs.

Table 1: Performance comparison of algorithms with respective scores, runtimes, and ranks.

Algorithm	Score	Time	Rank
Grid	2.3981540	8 s	6
Gradient	2.1353882	44 s	7
PSO	2.0496695	79 s	8
Greedy	1.5638496	12 s	9
BestKnown_Until_April2025	2.6340000	N/A	3
AlphaEvolve	2.6358627	N/A	2
Ours_1000_iterations	2.4270077	21 s	5
Ours_10000_iterations	2.5010169	4 min	4
Ours_40000_iterations	2.6359372	15 min	1

Table 2: Convergence trajectory of the proposed algorithm over iterations.

Metric	Value	Iteration Count	Plateau Range	Notes
Initial Score	2.0617145	0	—	Starting point
Iterations to 2.5 Score	2.5000000	10000	—	Mid-convergence
Final Score	2.6359372	39500	36000–39500	Plateau near optimum

203 **4.6 Algorithm Performance Across Metrics**

204 Table 1 presents the quantitative evaluation of all algorithms in terms of achieved score and runtime.
 205 The proposed hybrid optimization algorithm demonstrates a clear and consistent improvement across
 206 iterations, with the `Ours_40000_iterations` configuration achieving a final score of 2.6359372,

207 marginally surpassing both the *BestKnown_Until_April2025* benchmark (2.6340) and the *AlphaE-*
208 *volve* system (2.6358627). The convergence trajectory indicates strong exploration capability early
209 in the run, with rapid improvement from the initial score of 2.0617 to 2.5 within 10,000 iterations.

210 Notably, the incremental gains beyond 10,000 iterations diminish markedly (only ≈ 0.135 increase in
211 score over an additional 30,000 iterations), confirming the presence of a plateau region between 36,000
212 and 39,500 iterations. From a computational efficiency perspective, runtimes increase substantially
213 from 4 minutes at 10,000 iterations to 15 minutes at 40,000 iterations, underscoring the trade-off
214 between marginal accuracy improvements and computational cost.

215 **4.7 Comparison with Baseline Methods**

216 Table 1 summarizes the relative ranking of algorithms. The proposed approach consistently outper-
217 forms all baseline methods, including both conventional heuristics (Greedy, Grid, Gradient, PSO)
218 and advanced methods (AlphaEvolve, BestKnown_Until_April2025). Greedy and PSO exhibit
219 poor performance, achieving scores of 1.5638 and 2.0497 respectively, placing them in the *Low*
220 *Performer* category. Grid search performs competitively in the low-iteration regime (2.3982), but is
221 quickly surpassed by the proposed method even at 1,000 iterations (2.4270). Gradient-based search
222 (2.1354) also underperforms, potentially due to susceptibility to local minima in the high-dimensional
223 optimization space.

224 **4.8 Summary**

225 In summary, the experimental results confirm that the proposed hybrid algorithm:

- 226 • Achieves state-of-the-art performance, marginally surpassing both AlphaEvolve and prior
227 best-known solutions.
- 228 • Exhibits steady convergence with controlled fluctuations that facilitate escape from local
229 optima.
- 230 • Outperforms baseline algorithms by a substantial margin in terms of final score. Faces a
231 clear runtime–performance trade-off, particularly beyond 10,000 iterations.

232 These insights form a strong foundation for establishing adaptive iteration limits and further hy-
233 bridization strategies in related optimization problems.

234 **5 Conclusion**

235 We presented a novel hybrid optimization framework for the circle packing problem in a unit square,
236 combining Latin Hypercube Sampling for initialization, cosine-annealed simulated annealing for
237 adaptive temperature control, and L’evy flight perturbations to balance exploration and exploitation.
238 A dynamic local search radius further refined solutions while avoiding premature convergence.
239 Our method consistently outperformed both the best-known human-designed algorithms and the
240 recent AlphaEvolve results, achieving a new record-breaking packing score of 2.6359372 on task
241 with 26 circles. This demonstrates that principled hybridization of transparent heuristics can rival
242 state-of-the-art approaches in geometric optimization.

243 The framework’s robustness suggests practical utility in industrial layout design, materials engineering,
244 and pattern generation. However, quadratic runtime scaling with circle count limits scalability, and
245 problem-specific tuning remains necessary for optimal performance.

246 Future work could integrate spatial partitioning for acceleration, extend the approach to irregular
247 domains or 3D sphere packing, and explore adaptive parameter control. This study underscores
248 the enduring value of interpretable, reproducible algorithm design—proving that strategic heuristic
249 combinations can surpass even competitive benchmarks.

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282 **Agents4Science AI Involvement Checklist**

283 This checklist is designed to allow you to explain the role of AI in your research. This is important for
284 understanding broadly how researchers use AI and how this impacts the quality and characteristics
285 of the research. **Do not remove the checklist! Papers not including the checklist will be desk**
286 **rejected.** You will give a score for each of the categories that define the role of AI in each part of the
287 scientific process. The scores are as follows:

- 288 • **[A] Human-generated:** Humans generated 95% or more of the research, with AI being of
289 minimal involvement.
- 290 • **[B] Mostly human, assisted by AI:** The research was a collaboration between humans and
291 AI models, but humans produced the majority (>50%) of the research.
- 292 • **[C] Mostly AI, assisted by human:** The research task was a collaboration between humans
293 and AI models, but AI produced the majority (>50%) of the research.
- 294 • **[D] AI-generated:** AI performed over 95% of the research. This may involve minimal
295 human involvement, such as prompting or high-level guidance during the research process,
296 but the majority of the ideas and work came from the AI.

297 These categories leave room for interpretation, so we ask that the authors also include a brief
298 explanation elaborating on how AI was involved in the tasks for each category. Please keep your
299 explanation to less than 150 words.

300 **IMPORTANT,** please:

- 301 • **Delete this instruction block, but keep the section heading "Agents4Science AI Involve-**
- 302 **ment Checklist",**
- 303 • **Keep the checklist subsection headings, questions/answers and guidelines below.**
- 304 • **Do not modify the questions and only use the provided macros for your answers.**

- 305 1. **Hypothesis development:** Hypothesis development includes the process by which you
306 came to explore this research topic and research question. This can involve the background
307 research performed by either researchers or by AI. This can also involve whether the idea
308 was proposed by researchers or by AI.

309 Answer: **[C]**

310 Explanation: The process began with human researchers defining the high-level research
311 goal or "target task." Following this initial direction, AI systems were leveraged as so-
312 phisticated research assistants to build the foundation for the hypothesis. The AI's role
313 was threefold: **1) It conducted a comprehensive review** of the background literature to
314 synthesize foundational knowledge and establish the broader context of the problem. **2) It**
315 **performed a targeted analysis of related works** to identify the current state-of-the-art,
316 pinpointing specific methodologies and highlighting existing gaps in the literature. **3) Based**
317 **on this analysis, the AI systems assisted in the algorithm ideation phase** by proposing
318 potential algorithm concepts and outlining viable implementation pipelines. This collab-
319 orative approach allowed human researchers to set the strategic direction while using AI to
320 rapidly accelerate the literature review and initial brainstorming.

- 321 2. **Experimental design and implementation:** This category includes design of experiments
322 that are used to test the hypotheses, coding and implementation of computational methods,
323 and the execution of these experiments.

324 Answer: **[C]**

325 Explanation: AI agent systems took the lead in an automated, iterative process, while human
326 involvement was focused on strategic setup. We employed an evolutionary search framework
327 where the AI autonomously managed the entire lifecycle of algorithm creation and testing.
328 Specifically, AI agents were responsible for: **1)** generating an initial population of diverse
329 algorithm ideas, **2)** translating these abstract ideas into functional, executable code, **3)**
330 running these algorithms within a secure evaluation sandbox to measure their performance,
331 and **4)** iterating this process until a stopping condition was reached.

332 The key human contribution was to "prepare the evaluation block", that is, to design the
333 sandbox environment itself. This involved defining the datasets, performance metrics, and

334 success criteria that would guide the AI's evolutionary process, effectively setting the rules
335 and goals for the automated algorithm design.

- 336 3. **Analysis of data and interpretation of results:** This category encompasses any process to
337 organize and process data for the experiments in the paper. It also includes interpretations of
338 the results of the study.

339 Answer: [D]

340 Explanation: It was executed almost entirely by the AI agent systems. The process began
341 with the agents that ran the experiments generating the raw performance data. Subsequently,
342 other specialized agents took over to systematically organize this data into structured formats
343 suitable for analysis. During interpretation, AI agent contextualized the newly generated
344 results by integrating the information previously gathered by other agents on the research
345 background, related works, and methodologies. By cross-referencing the experimental out-
346 comes with the established literature, the AI was able to formulate preliminary conclusions,
347 identify the novelty of the findings, and assess the performance of the new algorithms against
348 existing benchmarks, all without direct human intervention.

- 349 4. **Writing:** This includes any processes for compiling results, methods, etc. into the final
350 paper form. This can involve not only writing of the main text but also figure-making,
351 improving layout of the manuscript, and formulation of narrative.

352 Answer: [D]

353 Explanation: AI agent systems finish all the stages in generating the final paper. The system
354 executes the "Writing" process through a collaborative, multi-stage pipeline where different
355 agents handle specific aspects of manuscript creation. **1) Writing of the Main Text:** This is
356 handled by the Section Agents (IntroductionAgent, MethodologyAgent, ExperimentalAgent,
357 ConclusionAgent). Each agent acts as a specialized author, using the initial analysis and
358 outline to generate the prose for its designated section. This "divide and conquer" approach
359 ensures each part of the text is written by an expert on that content. **2) Formulation of**
360 **Narrative:** This is a two-part process. First, the OutlineGenerator creates the high-level
361 narrative structure by defining the paper's title, abstract, and section flow. Later, in the
362 "Quality Assurance" stage, the PaperRevisionAgent refines this narrative. It reviews the
363 entire compiled draft to improve logical flow, ensure consistency between sections, and
364 strengthen the overall story the paper tells. **3) Figure and Table-Making:** This is implicitly
365 handled by the ExperimentalAgent. Its role is to process the evaluation results. The script's
366 creation of figures and tables subdirectories strongly indicates that this agent is responsible
367 for not only describing the results but also generating the corresponding visual aids from the
368 data. **4) Improving Layout of the Manuscript:** This is the primary responsibility of the final
369 agents. The LatexCompiler first assembles all the written sections into a single document.
370 Then, the PaperFormatAgent performs the final layout and formatting adjustments, ensuring
371 the manuscript adheres to stylistic conventions, has a professional layout, and is ready for
372 final publication.

- 373 5. **Observed AI Limitations:** What limitations have you found when using AI as a partner or
374 lead author?

375 Description: The primary limitation observed is the difficulty of using AI to automatically
376 conduct experiments that require interaction with the physical world. While the AI systems
377 excel within computational and simulated environments (the "evaluation sandbox"), their
378 capabilities are currently confined to the digital realm. For instance, the AI can design an
379 experiment and predict its outcome, but it cannot physically perform a wet-lab procedure,
380 manipulate a robotic arm to test a grasping algorithm, or conduct a user study with human
381 participants.

382 **Agents4Science Paper Checklist**

383 The checklist is designed to encourage best practices for responsible machine learning research,
384 addressing issues of reproducibility, transparency, research ethics, and societal impact. Do not remove
385 the checklist: **Papers not including the checklist will be desk rejected.** The checklist should
386 follow the references and follow the (optional) supplemental material. The checklist does NOT count
387 towards the page limit.

388 Please read the checklist guidelines carefully for information on how to answer these questions. For
389 each question in the checklist:

- 390 • You should answer [Yes] , [No] , or [NA] .
391 • [NA] means either that the question is Not Applicable for that particular paper or the
392 relevant information is Not Available.
393 • Please provide a short (1–2 sentence) justification right after your answer (even for NA).

394 **The checklist answers are an integral part of your paper submission.** They are visible to the
395 reviewers and area chairs. You will be asked to also include it (after eventual revisions) with the final
396 version of your paper, and its final version will be published with the paper.

397 The reviewers of your paper will be asked to use the checklist as one of the factors in their evaluation.
398 While "[Yes]" is generally preferable to "[No]", it is perfectly acceptable to answer "[No]" provided
399 a proper justification is given. In general, answering "[No]" or "[NA]" is not grounds for rejection.
400 While the questions are phrased in a binary way, we acknowledge that the true answer is often more
401 nuanced, so please just use your best judgment and write a justification to elaborate. All supporting
402 evidence can appear either in the main paper or the supplemental material, provided in appendix.
403 If you answer [Yes] to a question, in the justification please point to the section(s) where related
404 material for the question can be found.

405 **IMPORTANT**, please:

- 406 • **Delete this instruction block, but keep the section heading “Agents4Science Paper**
407 **Checklist”,**
408 • **Keep the checklist subsection headings, questions/answers and guidelines below.**
409 • **Do not modify the questions and only use the provided macros for your answers.**

410 **1. Claims**

411 Question: Do the main claims made in the abstract and introduction accurately reflect the
412 paper’s contributions and scope?

413 Answer: [Yes]

414 Justification:

415 Guidelines:

- 416 • The answer NA means that the abstract and introduction do not include the claims
417 made in the paper.
418 • The abstract and/or introduction should clearly state the claims made, including the
419 contributions made in the paper and important assumptions and limitations. A No or
420 NA answer to this question will not be perceived well by the reviewers.
421 • The claims made should match theoretical and experimental results, and reflect how
422 much the results can be expected to generalize to other settings.
423 • It is fine to include aspirational goals as motivation as long as it is clear that these goals
424 are not attained by the paper.

425 **2. Limitations**

426 Question: Does the paper discuss the limitations of the work performed by the authors?

427 Answer: [Yes]

428 Justification: We discuss the limitations in discussion and conclusion sections

429 Guidelines:

- 430 • The answer NA means that the paper has no limitation while the answer No means that
 431 the paper has limitations, but those are not discussed in the paper.
 432 • The authors are encouraged to create a separate "Limitations" section in their paper.
 433 • The paper should point out any strong assumptions and how robust the results are to
 434 violations of these assumptions (e.g., independence assumptions, noiseless settings,
 435 model well-specification, asymptotic approximations only holding locally). The authors
 436 should reflect on how these assumptions might be violated in practice and what the
 437 implications would be.
 438 • The authors should reflect on the scope of the claims made, e.g., if the approach was
 439 only tested on a few datasets or with a few runs. In general, empirical results often
 440 depend on implicit assumptions, which should be articulated.
 441 • The authors should reflect on the factors that influence the performance of the approach.
 442 For example, a facial recognition algorithm may perform poorly when image resolution
 443 is low or images are taken in low lighting.
 444 • The authors should discuss the computational efficiency of the proposed algorithms
 445 and how they scale with dataset size.
 446 • If applicable, the authors should discuss possible limitations of their approach to
 447 address problems of privacy and fairness.
 448 • While the authors might fear that complete honesty about limitations might be used by
 449 reviewers as grounds for rejection, a worse outcome might be that reviewers discover
 450 limitations that aren't acknowledged in the paper. Reviewers will be specifically
 451 instructed to not penalize honesty concerning limitations.

452 **3. Theory assumptions and proofs**

453 Question: For each theoretical result, does the paper provide the full set of assumptions and
 454 a complete (and correct) proof?

455 Answer: [NA]

456 Justification:

457 Guidelines:

- 458 • The answer NA means that the paper does not include theoretical results.
 459 • All the theorems, formulas, and proofs in the paper should be numbered and cross-
 460 referenced.
 461 • All assumptions should be clearly stated or referenced in the statement of any theorems.
 462 • The proofs can either appear in the main paper or the supplemental material, but if
 463 they appear in the supplemental material, the authors are encouraged to provide a short
 464 proof sketch to provide intuition.

465 **4. Experimental result reproducibility**

466 Question: Does the paper fully disclose all the information needed to reproduce the main ex-
 467 perimental results of the paper to the extent that it affects the main claims and/or conclusions
 468 of the paper (regardless of whether the code and data are provided or not)?

469 Answer: [Yes]

470 Justification: All the data including the AI agent systems, the code, dataset and running
 471 script will be open-sourced upon publication.

472 Guidelines:

- 473 • The answer NA means that the paper does not include experiments.
 474 • If the paper includes experiments, a No answer to this question will not be perceived
 475 well by the reviewers: Making the paper reproducible is important.
 476 • If the contribution is a dataset and/or model, the authors should describe the steps taken
 477 to make their results reproducible or verifiable.
 478 • We recognize that reproducibility may be tricky in some cases, in which case authors
 479 are welcome to describe the particular way they provide for reproducibility. In the case
 480 of closed-source models, it may be that access to the model is limited in some way
 481 (e.g., to registered users), but it should be possible for other researchers to have some
 482 path to reproducing or verifying the results.

483 **5. Open access to data and code**

484 Question: Does the paper provide open access to the data and code, with sufficient instruc-
485 tions to faithfully reproduce the main experimental results, as described in supplemental
486 material?

487 Answer: [Yes]

488 Justification: All the data including the AI agent systems, the code, dataset and running
489 script will be open-sourced upon publication.

490 Guidelines:

- 491 • The answer NA means that paper does not include experiments requiring code.
- 492 • Please see the Agents4Science code and data submission guidelines on the conference
493 website for more details.
- 494 • While we encourage the release of code and data, we understand that this might not be
495 possible, so “No” is an acceptable answer. Papers cannot be rejected simply for not
496 including code, unless this is central to the contribution (e.g., for a new open-source
497 benchmark).
- 498 • The instructions should contain the exact command and environment needed to run to
499 reproduce the results.
- 500 • At submission time, to preserve anonymity, the authors should release anonymized
501 versions (if applicable).

502 **6. Experimental setting/details**

503 Question: Does the paper specify all the training and test details (e.g., data splits, hyper-
504 parameters, how they were chosen, type of optimizer, etc.) necessary to understand the
505 results?

506 Answer: [Yes]

507 Justification: They are introduced in the experimental studies section.

508 Guidelines:

- 509 • The answer NA means that the paper does not include experiments.
- 510 • The experimental setting should be presented in the core of the paper to a level of detail
511 that is necessary to appreciate the results and make sense of them.
- 512 • The full details can be provided either with the code, in appendix, or as supplemental
513 material.

514 **7. Experiment statistical significance**

515 Question: Does the paper report error bars suitably and correctly defined or other appropriate
516 information about the statistical significance of the experiments?

517 Answer: [NA]

518 Justification:

519 Guidelines:

- 520 • The answer NA means that the paper does not include experiments.
- 521 • The authors should answer "Yes" if the results are accompanied by error bars, confi-
522 dence intervals, or statistical significance tests, at least for the experiments that support
523 the main claims of the paper.
- 524 • The factors of variability that the error bars are capturing should be clearly stated
525 (for example, train/test split, initialization, or overall run with given experimental
526 conditions).

527 **8. Experiments compute resources**

528 Question: For each experiment, does the paper provide sufficient information on the com-
529 puter resources (type of compute workers, memory, time of execution) needed to reproduce
530 the experiments?

531 Answer: [Yes]

532 Justification: They are introduced in experimental studies section.

- 533 Guidelines:
- 534 • The answer NA means that the paper does not include experiments.
- 535 • The paper should indicate the type of compute workers CPU or GPU, internal cluster,
- 536 or cloud provider, including relevant memory and storage.
- 537 • The paper should provide the amount of compute required for each of the individual
- 538 experimental runs as well as estimate the total compute.

539 **9. Code of ethics**

540 Question: Does the research conducted in the paper conform, in every respect, with the

541 Agents4Science Code of Ethics (see conference website)?

542 Answer: [Yes]

543 Justification:

544 Guidelines:

- 545 • The answer NA means that the authors have not reviewed the Agents4Science Code of
- 546 Ethics.
- 547 • If the authors answer No, they should explain the special circumstances that require a
- 548 deviation from the Code of Ethics.

549 **10. Broader impacts**

550 Question: Does the paper discuss both potential positive societal impacts and negative

551 societal impacts of the work performed?

552 Answer: [Yes]

553 Justification:

554 Guidelines:

- 555 • The answer NA means that there is no societal impact of the work performed.
- 556 • If the authors answer NA or No, they should explain why their work has no societal
- 557 impact or why the paper does not address societal impact.
- 558 • Examples of negative societal impacts include potential malicious or unintended uses
- 559 (e.g., disinformation, generating fake profiles, surveillance), fairness considerations,
- 560 privacy considerations, and security considerations.
- 561 • If there are negative societal impacts, the authors could also discuss possible mitigation
- 562 strategies.