OpenThrust Documentation

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2 Injector Model

The injector model implemented in OpenThrust is a weighted average of the Homogenous Equilibrium Model and the Single Phase Equilibrium models, as proposed by Dyer et al. with corrections by Brian J. Solomon.

2.1 Discharge Coefficient

The discharge coefficient C_d accounts for frictional and vena contracta effects and features prominently in the injector models used herein. The coefficient is normally determined experimentally; however it is sometimes useful to estimate the value of C_d based on the injector geometry. An equation to do so is therefore derived below.

For simple single-phase flow through an injector the flow rate can be determined using the following equation:

$$\dot{m} = C_d A_2 \sqrt{2\rho\Delta P} \quad (1)$$

Bernoulli's equation with losses, disregarding the effects of elevation difference (equation 2) can then be combined with mass flow relations pre- and post-injector (equation 3) to express the post-injector flow velocity (equation 4).

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2}\left(1 + k + f\frac{L}{D}\right) \tag{2}$$

$$v_1 A_1 = v_2 A_2 \tag{3}$$

$$v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left(1 + k + f\frac{L}{D} + \left(\frac{A_2}{A_1}\right)^2\right)}} \tag{4}$$

If the L/D ratio is sufficiently small, a vena contracta effect will make the effective cross-sectional area less than A_2, resulting in an additional flow loss.

$$C_{vc} = \frac{A_{vc}}{A_2} \tag{5}$$

This combined with equation 4 and equation 6, below, results in equation 7, expressing the actual mass flow rate (with losses taken into account).

$$\dot{m}_i = \rho v_2 A_2 \tag{6}$$

$$\dot{m}_i = C_{vc} A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{1 + k + f\frac{L}{D} + \left(\frac{A_2}{A_1}\right)^2}} \tag{7}$$

C_d is defined as the ratio of the actual mass flow rate to the mass flow rate assuming zero losses, allowing it to be expressed by the equation below:

$$\mathrm{C_d} = \frac{C_{vc} A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{1 + k + f\frac{L}{D} + \left(\frac{A_2}{A_1}\right)^2}}}{\mathrm{A_2}\sqrt{2\rho\Delta P}} = \frac{\mathrm{C_{vc}}}{\sqrt{1 + \mathrm{k} + \mathrm{f}\frac{\mathrm{L}}{\mathrm{D}} + \left(\frac{\mathrm{A_2}}{\mathrm{A_1}}\right)^2}} \tag{8}$$

For an injector with multiple orifices, this equation can be extended to yield the following expression:

$$C_d = \frac{\mathrm{C_{vc}}}{\sqrt{1 + \mathrm{n}\left(\mathrm{k} + \mathrm{f}\frac{\mathrm{L}}{\mathrm{D}}\right) + \left(\frac{\mathrm{A_2}}{\mathrm{A_1}}\right)^2}} \tag{9}$$

Where n is the number of injector holes and A is the sum of all the hole areas. This can be further simplified depending on the injector design. For smaller L/D ratios ($L/D \cong 1$), the friction factor will be insignificant, resulting in equation 10. For larger L/D ratios, $C_{vc} = 1$, resulting in equation 11.

$$C_d = \frac{C_{vc}}{\sqrt{1 + nk + \left(\frac{A_2}{A_1}\right)^2}} \tag{10}$$

$$C_d = \frac{1}{\sqrt{1 + n\left(k + f\frac{L}{D}\right) + \left(\frac{A_2}{A_1}\right)^2}} \tag{11}$$

Note that while these equations provide a good reference point for the discharge coefficient, they neglect some other factors such as the compressibility of liquid NOS. For simplicity it has been assumed that the flow is fully turbulent. Because the length of the orifices tends to be relatively small, this assumption does not have a large impact.

2.2 Homogeneous Equilibrium Model (HEM) [1]

The Homogeneous Equilibrium Model is the simplest model for predicting the two-phase critical flow rate through an injector. Predictions made using this model assume the following conditions:

- The liquid and gas phases have the same velocity
- The temperatures of the liquid and gas are the same
- Pressure and temperature are related due to saturation
- Flow is isentropic

Based on these assumptions, the following equations apply:

$$\dot{m} = \rho_2 v_2 A_2 \tag{12}$$

$$h_1 = h_2 + \frac{1}{2}v_2^2 \tag{13}$$

These equations are combined along with the discharge coefficient to solve for the mass flow rate through the injector in the case of thermodynamic equilibrium:

$$\dot{m}_{HEM} = C_d A \rho_2 \sqrt{2(h_1 - h_2)} \tag{14}$$

2.3 Single Phase Equilibrium Model (SPI) [1]

The Single Phase Equilibrium Model predicts the mass flow rate through an injector by modeling the rocket propellant as an incompressible liquid, meaning the density of the propellant remains constant. As a result, an expression can be constructed relating the velocity of the flow pre- and post-injector:

$$\dot{m} = \rho v_1 A_1 = \rho v_2 A_2 \tag{15}$$

$$v_1 = \frac{A_2}{A_1} v_2 \tag{16}$$

So as to be able to substitute the above relationship into the steady-state Bernoulli equation, steady-state operation is assumed. The expression can be further simplified by assuming that the gravitational potential energy pre- and post-injector is effectively equivalent, resulting in the equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \tag{17}$$

Substituting v_1 with the equivalent expression from equation 5 and expressing v_2 as a function of $\dot{m}$ according to the relationship in equation 4, equation 6 can be rearranged to give the critical mass flow rate.

$$\dot{m}_{SPI_{crit}} = A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}} \tag{18}$$

Assuming that $A_1 \gg A_2$ and including the discharge coefficient results in the final expression:

$$\dot{m}_{SPI} = C_d A_2 \sqrt{2\rho(P_1 - P_2)}$$

2.4 Dyer Model [1]

The Dyer et al. model combines the predictions from the HEM and SPI models using a weighted average according to a non-equilibrium parameter, κ, shown in the equation below. The purpose of this factor is to account for the finite rate of vapor bubble growth and provides a smooth transition between the two models.

$$\kappa = \sqrt{\frac{P_1 - P_2}{P_v - P_2}} \propto \frac{\tau_b}{\tau_r} \tag{19}$$

When the residence time of bubbles $\tau_r \ll \tau_b$, the characteristic time of bubble growth, the single-phase assumption of the SPI model is more likely to be valid and so its mass flow rate prediction is given more weight. When the opposite is true ($\tau_r \gg \tau_b$) the mass flow rate is expected to approach that predicted by the HEM model for injectors with a large L/D ratio, such as the current injector design for the Kismet engine. As a result, the HEM model's prediction is given more weight. This relationship is expressed by the following equation:

$$\dot{m}_{DYER} = \left(\frac{\kappa}{1 + \kappa}\right) \dot{m}_{SPI} + \left(\frac{1}{1 + \kappa}\right) \dot{m}_{HEM} \tag{20}$$

2.5 Class Implementation

The Dyer model is implemented as a Python class in OpenThrust, with a total of 5 members and 1 method.

2.5.1 Members

`Cd`	The discharge coefficient (see section 2.1)
`Ac`	The total cross-sectional area of all injector orifices
`mdot_HEM`	The mass flow rate as predicted by the Homogeneous Equilibrium Model (section 2.2)
`mdot_SPI`	The mass flow rate as predicted by the Single Phase Equilibrium Model (section 2.3)
`parent`	The overall model that created and is calling this injector model. Use to pass pressure and enthalpy values required for mass flow rate calculations.

2.5.2 Constructor

```
def __init__(self, callingModel, Ac, Cd=0.8):
    self.Cd = Cd
    self.Ac = Ac
    self.mdot_HEM = 0
    self.mdot_SPI = 0
    self.parent = callingModel
```

The constructor for creating a new instance of the DyerModel class takes a minimum of 2 arguments and a maximum of 3. The first argument indicates the overarching model which the injector model should pull values such as the pre- and post-injector pressure from. The object specified here must be an instance of a class with defined values for the following members:

`P1`	Pre-injector pressure (PSI)
`P2`	Post-injector pressure (PSI)
`Pv1`	Pre-injector vapour pressure of NOS (PSI)
`rho1`	Pre-injector NOS density
`rho2`	Post-injector NOS density
`h1`	Pre-injector specific enthalpy
`h2`	Post-injector specific enthalpy

The second required argument represents the total cross-sectional area of all the injector orifices and should be a floating-point value. The discharge coefficient is included as an optional third argument when instantiating the injector model object and takes a floating point value such that $0 < C_d \leq 1$. If no value is specified, the discharge coefficient is assigned a default value of 0.8, a standard rough value in the case of an injector with exactly one orifice.

2.5.3 Methods

The DyerModel class contains only one method, getMassFlowRate(), which takes no arguments and returns a floating-point value corresponding to the mass flow rate through the injector as predicted by the Dyer model.

```python
def getMassFlowRate(self):

    if min(self.parent.P1, self.parent.Pv1) < self.parent.P2 or
self.parent.P2 == self.parent.Pv1:
        raise ValueError("Pressure values are invalid")

    # Dyer et. al factor. Always 1 for self-pressurizing fluid
    k = ((self.parent.P1 - self.parent.P2) / (self.parent.Pv1 -
self.parent.P2)) ** 0.5
    W = (1 / (k + 1))

    self.mdot_SPI = self.Cd * self.Ac * (
            (2 * self.parent.rho1 * ((self.parent.P1 - self.parent.P2) *
PSItoPASCAL)) ** 0.5)
    self.mdot_HEM = self.Cd * self.parent.rho2 * self.Ac * (
            (2 * (self.parent.h1 - self.parent.h2)) ** 0.5)
    if isinstance(self.mdot_HEM, complex):
        self.mdot_HEM = 0  # Case where h1<h2

    return (1 - W) * self.mdot_SPI + W * self.mdot_HEM
```

This method includes an implementation of the injector model first proposed by Dyer et al. (section 2.4) as well as some error-checking measures.

If either the pre-injector actual pressure or vapour pressure is less than the post-injector pressure, or if the pre-injector vapour pressure is equivalent to the post-injector pressure, a value error is returned. This is because the model being used does not handle these cases, and such a case would indicate an issue with the values fed in by the parent model as it should not occur during normal rocket operation.

In instances where the pre-injector specific enthalpy is less than the post-injector specific enthalpy, the mass flow rate according to the HEM model is set to 0. This allows situations where the mass flow rate is a complex value and therefore has no physical meaning to be avoided.

Nomenclature

References

[1] Waxman, Injectors for Use with High Vapour Pressure