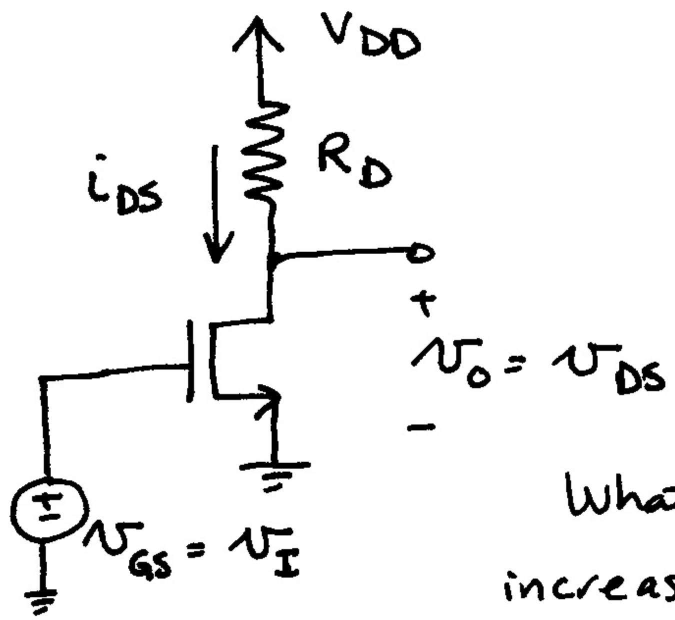
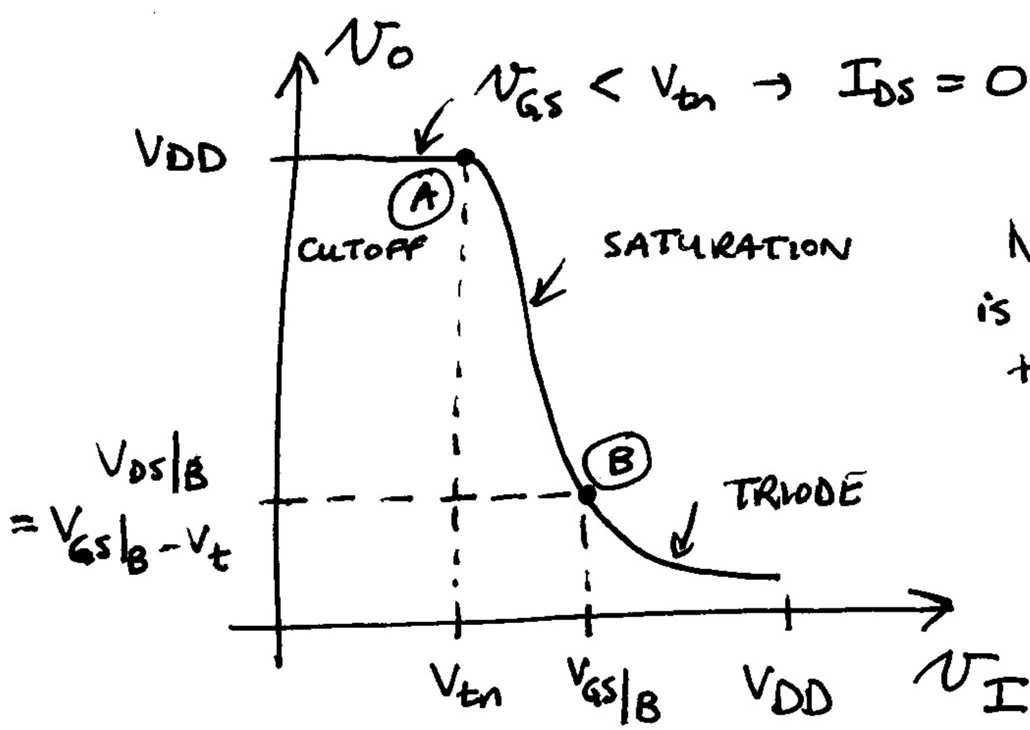
MOSFET in an Amplifier §7.1

Consider the following circuit (known as a common-source amplifier), where we apply an input signal to the gate of a MOS transistor, and where the transistor's drain current, I_{DS}, flows across a load resistor R_D, to produce an output voltage V_O:



Clearly, $v_O = V_{DD} - R_D\, i_{DS}$, since the transistor's current must flow onto R_D.

What happens as v_I is increased from 0V to V_{DD}?



NOTE: the slope is steepest as the transistor is in the saturation region

$V_I < V_{tn} \rightarrow$ transistor CUTOFF

$I_{DS} = 0 \rightarrow V_O = V_{DD}$

$V_{tn} \le V_I \le V_{GS}|_B \rightarrow V_{DS} = V_O > V_I - V_{tn}$

$\rightarrow$ transistor in SATURATION

$$V_O = V_{DD} - R_D \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_I - V_{tn})^2$$

$V_I > V_{GS}|_B \rightarrow V_{DS} = V_O < V_I - V_{tn}$

$\rightarrow$ transistor in TRIODE

$$V_O = V_{DD} - R_D \mu_n C_{ox} \frac{W}{L} \left((V_I - V_{tn}) V_O - \frac{V_O^2}{2} \right)$$

(quadratic equation)

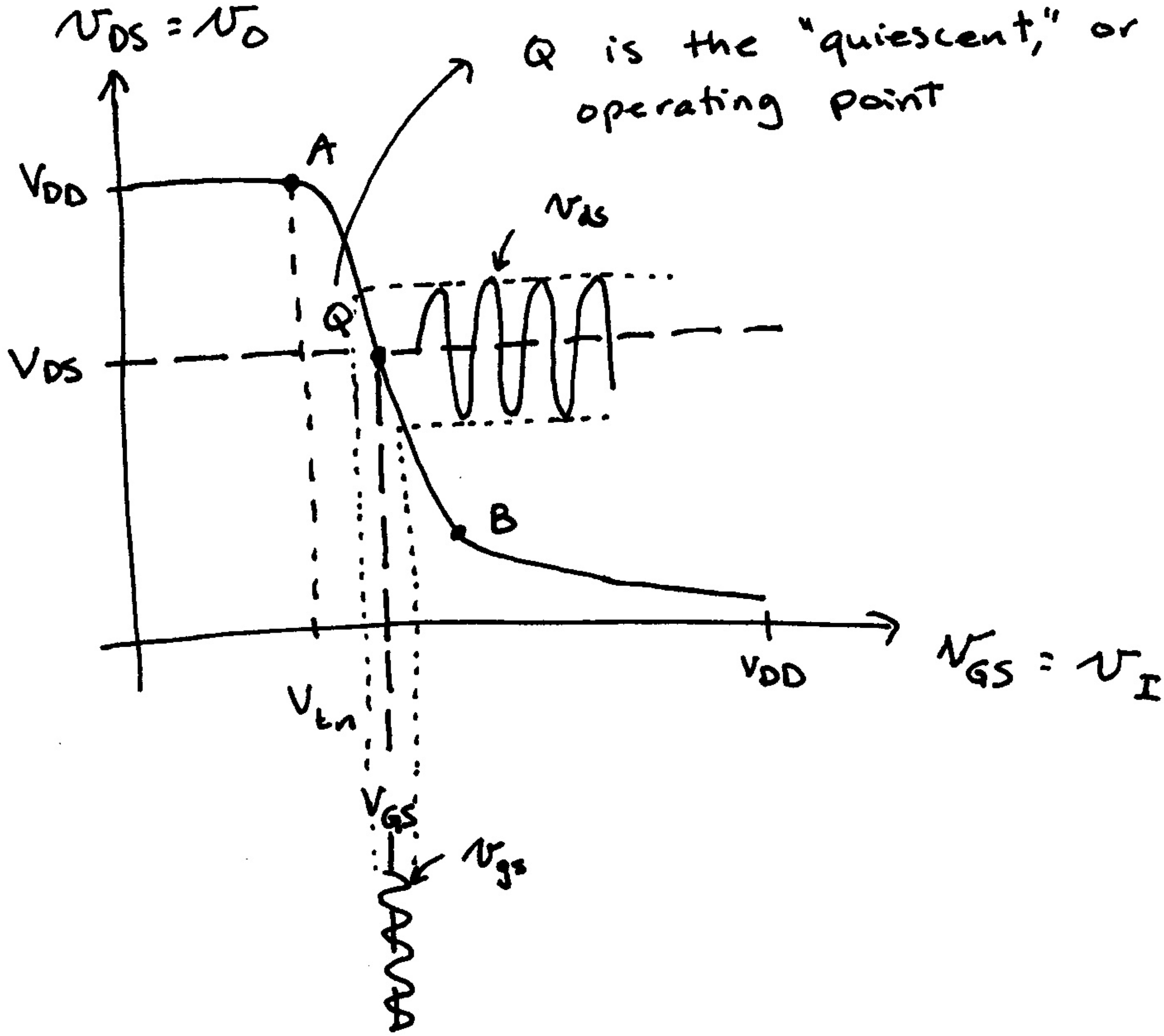
Question: where is point B ?

$V_{DS}|_B = V_{GS}|_B - V_{tn}$, i.e. transition between SAT. & TRIODE

$$\underbrace{V_{DS}|_B}_{V_{GS}|_B - V_{tn}} = V_{DD} - R_D \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS}|_B - V_{tn})^2$$

Solve for $V_{GS}|_B = V_{tn} + \dfrac{\sqrt{2 k_n R_D V_{DD} + 1} - 1}{k_n R_D}$

Let's use the circuit as an amplifier



At the input, we have a signal

$$v_{GS}(t) = V_{GS} + v_{gs}(t)$$

whole signal — DC "biasing" component — small-signal to be amplified

Question: how much "bigger" is $v_{ds}(t)$ than $v_{gs}(t)$?

We could solve for $v_{DS}(t)$ exactly,

i.e. $v_{DS}(t) = V_{DD} - R_D \frac{\mu_n C_{ox}}{2}\frac{W}{L}\left(V_{GS} + v_{gs}(t) - V_{tn}\right)^2$

$$= V_{DD} - R_D \underbrace{\frac{\mu_n C_{ox}}{2}\frac{W}{L}}_{k_n/2}\left[(V_{GS}-V_{tn})^2 + 2(V_{GS}-V_{tn})v_{gs}(t) + v_{gs}^2(t)\right]$$

$v_{gs}^2(t) \to 0$

constant, doesn't vary with t $\to V_{DS}$

varies with t $\to v_{ds}(t)$

puny. $v_{gs} \ll V_{GS} - V_{tn}$ $\to$ ignore!

$$v_{DS}(t) = \left[V_{DD} - R_D \frac{k_n}{2}(V_{GS}-V_{tn})^2\right]$$

$$\left[- R_D \times k_n (V_{GS}-V_{tn}) v_{gs}(t)\right]$$

$= V_{DS}$

$+ \; v_{ds}(t)$

Note:

the slope, $\frac{\partial v_{DS}}{\partial v_{GS}}$ at point Q is:

$$A_v = -R_D \times k_n (V_{GS} - V_{tn})$$

so $v_{ds}(t) = (A_v)\, v_{gs}(t)$

the voltage gain of the circuit.

We can rewrite this as:

$$A_v = -k_n V_{ov} R_D$$

$$= -\underbrace{\frac{2I_D}{V_{ov}}}\, R_D$$

note: there is always one term dependent on the transistor, times R_D.

Summary:

<u>Step 1</u> Find Q, i.e. the DC operating point → find V_{GS} and V_{DS}.

<u>Step 2</u> Find the gain, by finding the slope at point Q.

$$v_{DS}(t) \approx V_{DS} + (A_v) \times v_{gs}(t)$$

↳ $v_{gs}(t)$ small so we ignore square term.

Notation:

small v

$v_{DS}(t)$: total signal

caps

cap

V_{DS} : dc component

caps

small v

$v_{ds}(t)$: small-signal (ac) component.

small ds

Example part (a) (Based on Example 7.1)

In our circuit, $V_{tn} = 0.4V$, $k_n' = 0.4mA/V^2$, $W/L = 10$, $\lambda = 0$, $V_{DD} = 1.8V$, $R_D = 17.5k\Omega$, $V_{GS} = 0.6V$.

What are: $\underbrace{V_{ov}, I_D, V_{DS}}_{\text{dc values.}}$, A_v

① $V_{ov} = V_{GS} - V_{tn} = 0.6V - 0.4V = 0.2V$

② $I_D = \frac{k_n'}{2}\frac{W}{L} V_{ov}^2 = 0.08mA$

③ $V_{DS} = V_{DD} - I_D R_D = 0.4V$

④ $A_v = -k_n V_{ov} R_D = \left(\frac{0.4mA}{V^2}\right)(10)(0.2V)(17.5k\Omega)$

$= -14 \; V/V$

to clearly indicate that this is voltage gain

Small-Signal Operation and Models §7.2

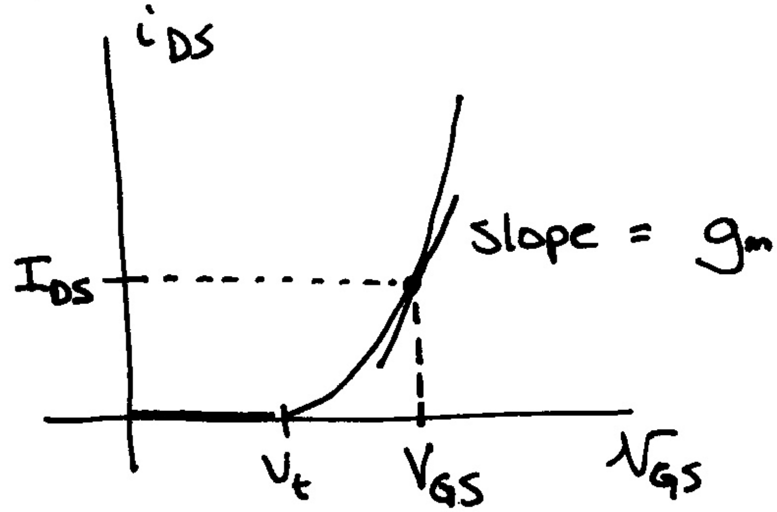
Recall that $A_v = - \underbrace{k_n V_{ov}}_{\text{dependent on transistor}} \times R_D \quad V/V$

$$= \left. \frac{\partial v_{DS}}{\partial v_{GS}} \right|_{v_{GS} = V_{GS}}$$

$$k_n V_{ov} = \mu_n C_{ox} (V_{GS} - V_t) = \left. \frac{\partial i_{DS}}{\partial v_{GS}} \right|_{V_{GS}},$$

a parameter with units of conductance (Ω^{-1}), that relates how much the output current changes in relation to a change in the gate voltage.

We call this parameter the <u>transconductance</u> g_m of the transistor:

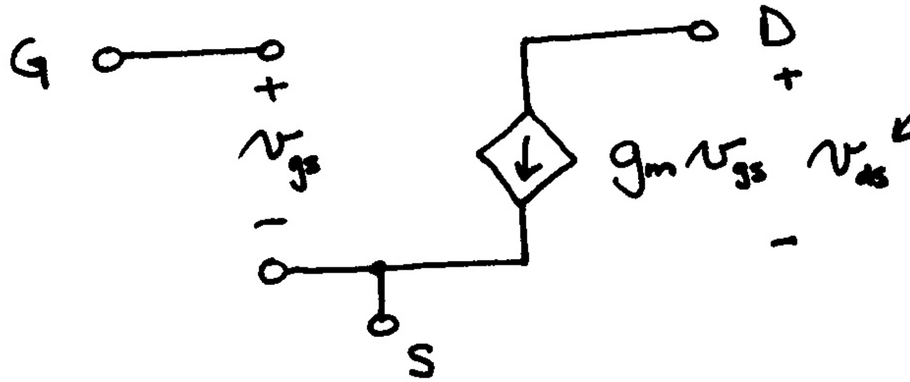


We can therefore write

$$A_v = -g_m R_D,$$ the small-signal

voltage gain of the common-source amplifier.

In small-signal analysis, we can replace the transistor with its small-signal equivalent circuit:



note small-signal notation – NO DC COMPONENT!

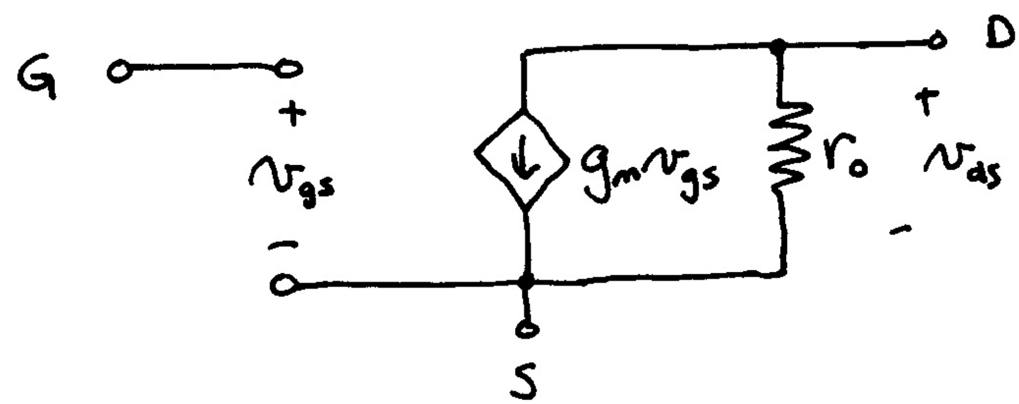
So far, we have implicity assumed that $\lambda = 0$. In a real transistor, i_{DS} also varies with v_{DS}. Recall,

$$r_o = \left[\left. \frac{\partial i_{DS}}{\partial v_{DS}} \right|_{V_{DS}, V_{GS}} \right]^{-1} = \frac{|V_A|}{I_{DS}} = \frac{1}{\lambda I_{DS}}$$

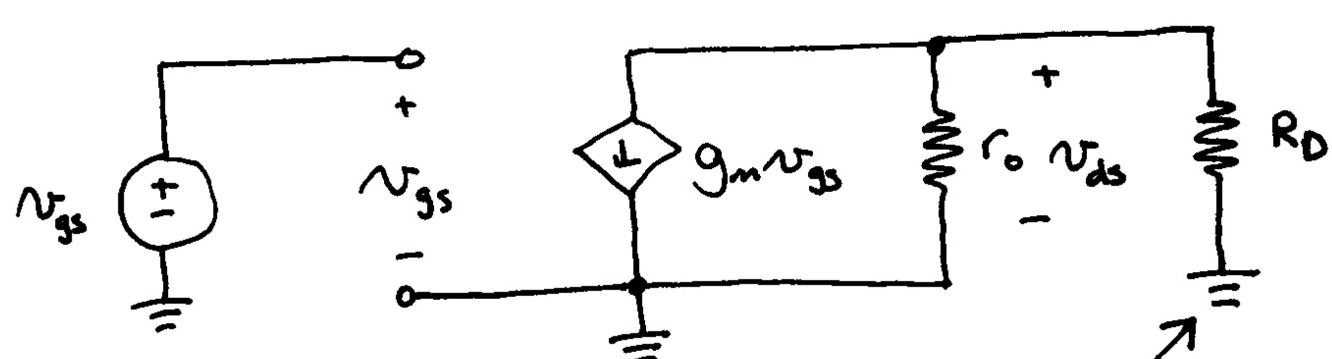
DC bias point

Typically, r_o is in the range of $10\,k\Omega$ to $1\,M\Omega$.

In a transistor's small-signal model, we add this as a resistor r_o between D and S:



In the common-source amplifier, we replace the transistor with its small-signal equivalent:



$$\frac{\partial V_{DD}}{\partial v_{gs}} = 0$$

small-signal ground

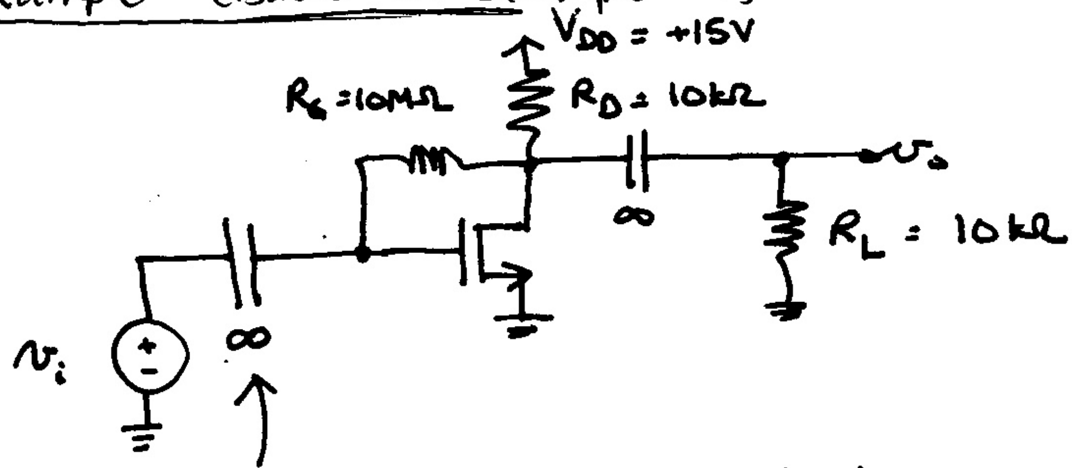
Analyzing this linear circuit, we get $\frac{v_{ds}}{v_{gs}} = A_v = -g_m (R_D \parallel r_o)$

r_o acts as if it's in parallel with R_D.

Problems 7.1, 3, 8, 24, 26, 30, 31, 33

Example 7.1, 3

Example (Based on Example 7.3)



means open circuit at dc
short circuit at ac frequencies

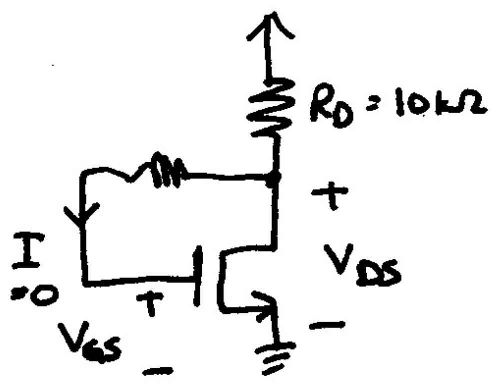
$V_t = 1.5V$

$k_n = 0.25 mA/V^2$

$V_A = 50V$

What is $A_v = v_o / v_i$?

Step 1 Find the dc operating point. (V_{GS}, V_{DS}, I_{DS})



Since $I = 0$,

$V_{DS} = V_{GS}$ (saturation!)

$= V_{DD} - I_{DS} R_D$

$I_{DS} = \frac{k_n}{2} (V_{GS} - V_t)^2$ (We usually ignore λ at this stage)

$\rightarrow V_{GS} = V_{DD} - \frac{R_D k_n}{2} (V_{GS} - V_t)^2$

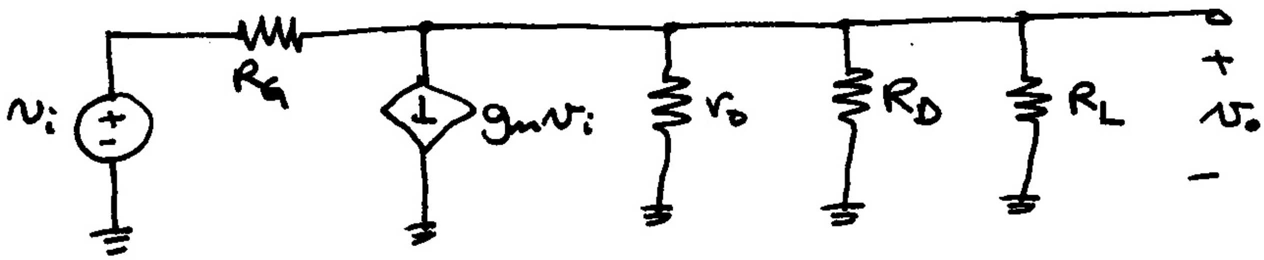
$\rightarrow$ quadratic equation, with meaningful solution $V_{GS} = V_{DS} = 4.4V$

$I_{DS} = 1.06mA$

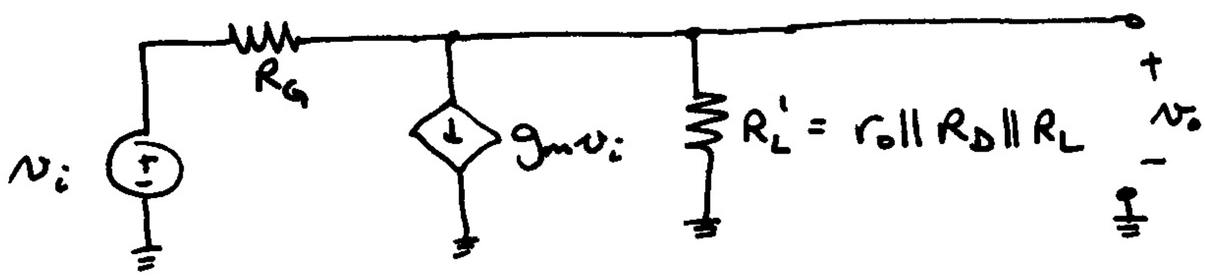
$\rightarrow V_{ov} = V_{GS} - V_t = 2.9V$

Step 2 Small-Signal Analysis

Small-signal model:



simplifies to:



$$g_m = k_n V_{ov} = \left(\frac{0.25\text{mA}}{V^2}\right)(2.9V) = 0.725\text{mA}/V$$

$$r_o = \frac{V_A}{I_D} = \frac{50V}{1.06\text{mA}} = 47\text{k}\Omega$$

$$R_L' = (47\text{k}\Omega \,\|\, 10\text{k}\Omega \,\|\, 10\text{k}\Omega) = 4.52\text{k}\Omega$$

→ Doing all the math we get

$$A_v = -g_m R_L' \left(\frac{1 - 1/g_m R_G}{1 + R_L'/R_G}\right)$$

but notice R_G is big (it was only used to set the dc operating point...)

so $A_v \approx -g_m R_L' = -0.725\frac{\text{mA}}{V} \times 4.52\text{k}\Omega = -3.3\frac{V}{V}$

what we had before