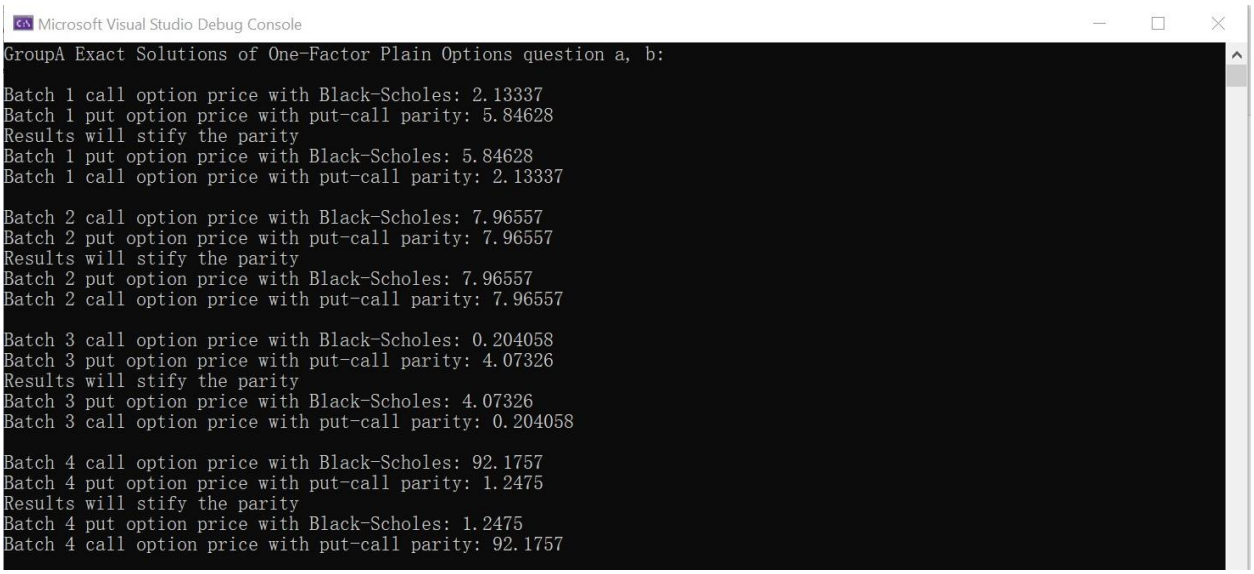
Group A

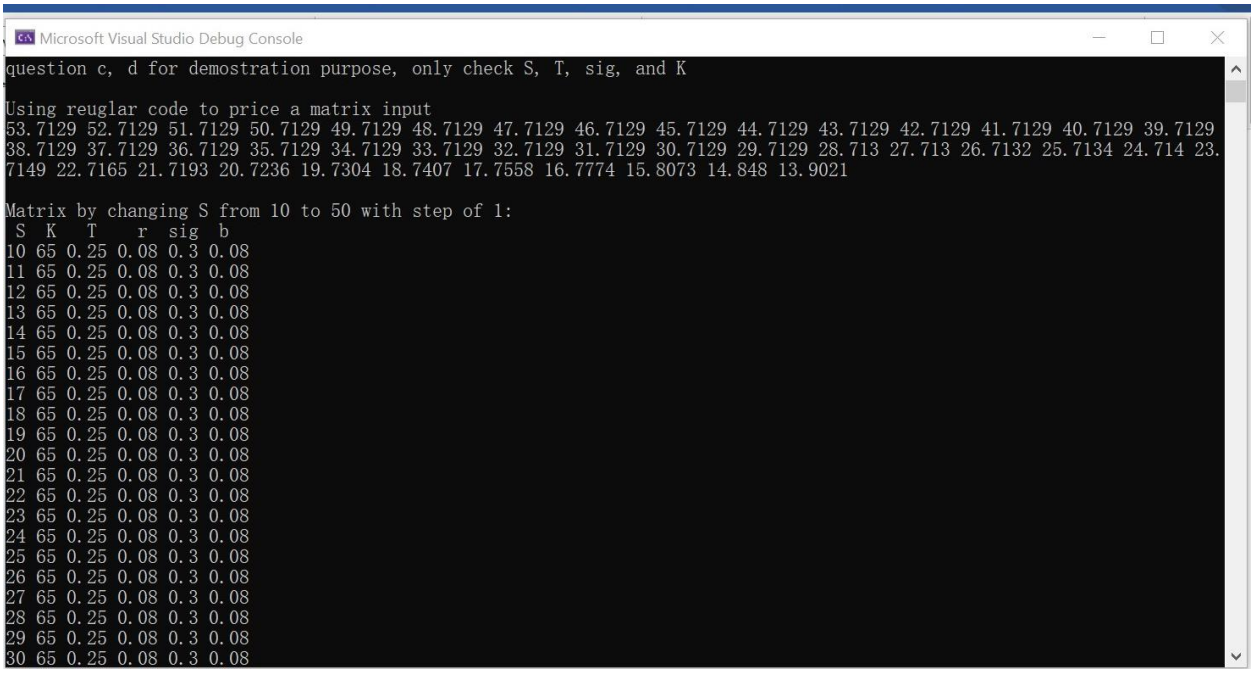
Exact Solutions of One-Factor Plain Options

Question a, b



Results of 4 batches. Put-Call parity was applied to either check if the results match its results from put-call parity, or calculate its opposite result.

Question c, d



Question c was answered by regular code. The results are pricing by passing different underlying asset price S.

Question d was answered by class MatrixCal. The class accepts an option engine (could be either European or American option, and will support other type option if needed), input matrix, and an option type to control engine's type. The test sets are changing S, T, sig, K with different steps. The matrices are from a global function mersh().

Matrix by changing S from 10 to 50 with step of 1:

S K T r sig b

10 65 0.25 0.08 0.3 0.08

11 65 0.25 0.08 0.3 0.08

12 65 0.25 0.08 0.3 0.08

13 65 0.25 0.08 0.3 0.08

14 65 0.25 0.08 0.3 0.08

15 65 0.25 0.08 0.3 0.08

16 65 0.25 0.08 0.3 0.08

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36 65 0.25 0.08 0.3 0.08

37 65 0.25 0.08 0.3 0.08

38 65 0.25 0.08 0.3 0.08

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45 65 0.25 0.08 0.3 0.08

46 65 0.25 0.08 0.3 0.08

47 65 0.25 0.08 0.3 0.08

48 65 0.25 0.08 0.3 0.08

49 65 0.25 0.08 0.3 0.08

50 65 0.25 0.08 0.3 0.08

The matrix price results with put option are:

53.7129 52.7129 51.7129 50.7129 49.7129 48.7129 47.7129 46.7129 45.7129 44.7129 43.7129 42.7129 41.7129 40.7129 39.7129 38.7129 37.7129 36.7129 35.7129 34.7129 33.7129 32.7129 31.7129 30.7129 29.7129 28.713 27.713 26.7132 25.7134 24.714 23.7149 22.7165 21.7193 20.7236 19.7304 18.7407 17.7558 16.7774 15.8073 14.848 13.9021

Matrix by changing T from 0.25 to 3 with step of 0.25:

S K T r sig b

60 65 0.25 0.08 0.3 0.08

60 65 0.5 0.08 0.3 0.08

60 65 0.75 0.08 0.3 0.08

60 65 1 0.08 0.3 0.08

60 65 1.25 0.08 0.3 0.08

60 65 1.5 0.08 0.3 0.08

60 65 1.75 0.08 0.3 0.08

60 65 2 0.08 0.3 0.08

60 65 2.25 0.08 0.3 0.08

60 65 2.5 0.08 0.3 0.08

60 65 2.75 0.08 0.3 0.08

The matrix price results with call option are:

2.13337 4.03758 5.6754 7.15299 8.51879 9.79951 11.0118 12.1667 13.2724 14.3348 15.3584

Matrix by changing sig from 0.1 to 0.56 with step of 0.05:

S K T r sig b

60 65 0.25 0.08 0.1 0.08

60 65 0.25 0.08 0.15 0.08

60 65 0.25 0.08 0.2 0.08

60 65 0.25 0.08 0.25 0.08

60 65 0.25 0.08 0.3 0.08

60 65 0.25 0.08 0.35 0.08

60 65 0.25 0.08 0.4 0.08

60 65 0.25 0.08 0.45 0.08

60 65 0.25 0.08 0.5 0.08

The matrix price results put option are:

3.88601 4.26954 4.75437 5.28745 5.84628 6.42012 7.00325 7.59235 8.18535

Matrix by changing K from 10 to 71 with step of 5:

S K T r sig b

60 10 0.25 0.08 0.3 0.08

60 15 0.25 0.08 0.3 0.08

60 20 0.25 0.08 0.3 0.08

60 25 0.25 0.08 0.3 0.08

60 30 0.25 0.08 0.3 0.08

60 35 0.25 0.08 0.3 0.08

60 40 0.25 0.08 0.3 0.08

60 45 0.25 0.08 0.3 0.08

60 50 0.25 0.08 0.3 0.08

60 55 0.25 0.08 0.3 0.08

60 60 0.25 0.08 0.3 0.08

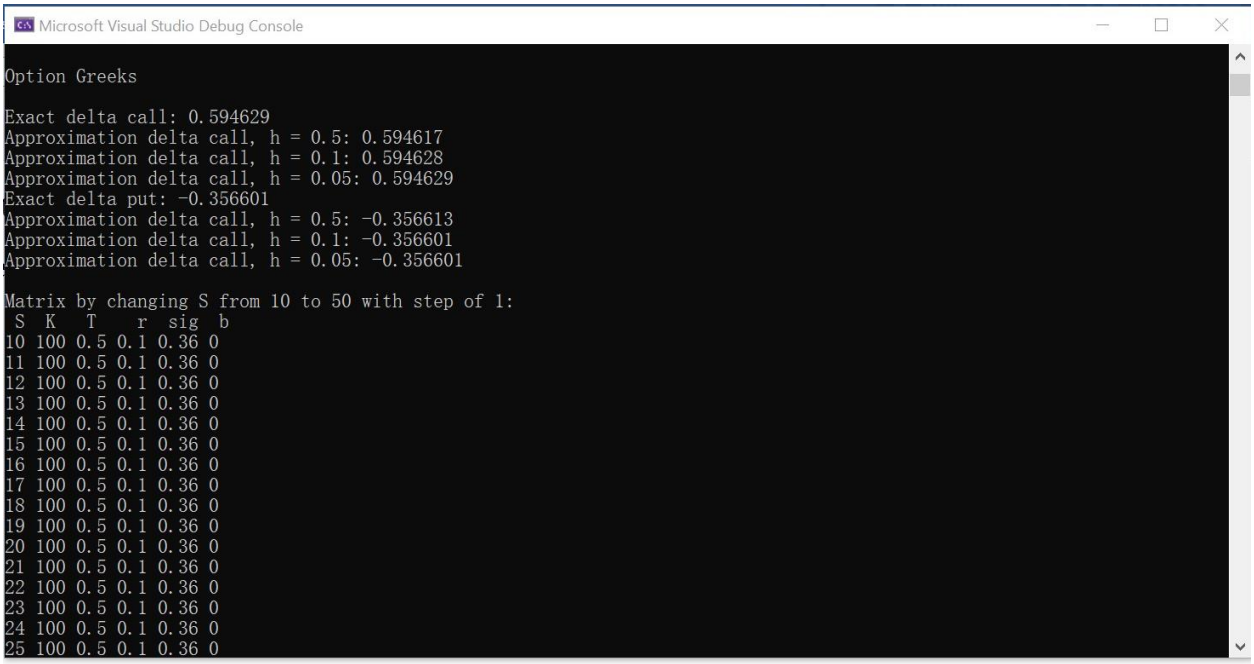
60 65 0.25 0.08 0.3 0.08

The matrix price results with call option are:

50.198 45.297 40.396 35.495 30.594 25.6932 20.7969 15.948 11.3229 7.27865 4.1771 2.13337

Option Sensitivities, aka the Greeks

Question a, d



Question a and d are answered by calling different methods. Exact Greeks support Delta, Gamma, Vega, Theta, and Rho. Approximate version needs argument h, and only support Delta and Gamma.

Question b is answered by regular code:

Matrix by changing S from 10 to 50 with step of 1:

S K T r sig b

10 100 0.5 0.1 0.36 0

11 100 0.5 0.1 0.36 0

12 100 0.5 0.1 0.36 0

13 100 0.5 0.1 0.36 0

14 100 0.5 0.1 0.36 0

15 100 0.5 0.1 0.36 0

16 100 0.5 0.1 0.36 0

17 100 0.5 0.1 0.36 0

18 100 0.5 0.1 0.36 0

19 100 0.5 0.1 0.36 0

20 100 0.5 0.1 0.36 0

21 100 0.5 0.1 0.36 0

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27 100 0.5 0.1 0.36 0

28 100 0.5 0.1 0.36 0

29 100 0.5 0.1 0.36 0

30 100 0.5 0.1 0.36 0

31 100 0.5 0.1 0.36 0

32 100 0.5 0.1 0.36 0

33 100 0.5 0.1 0.36 0

34 100 0.5 0.1 0.36 0

35 100 0.5 0.1 0.36 0

36 100 0.5 0.1 0.36 0

37 100 0.5 0.1 0.36 0

38 100 0.5 0.1 0.36 0

39 100 0.5 0.1 0.36 0

40 100 0.5 0.1 0.36 0

41 100 0.5 0.1 0.36 0

42 100 0.5 0.1 0.36 0

43 100 0.5 0.1 0.36 0

44 100 0.5 0.1 0.36 0

45 100 0.5 0.1 0.36 0

46 100 0.5 0.1 0.36 0

47 100 0.5 0.1 0.36 0

48 100 0.5 0.1 0.36 0

49 100 0.5 0.1 0.36 0

50 100 0.5 0.1 0.36 0

Using regular code to calculate exact delta for call option with a matrix input

2.25551e-19 6.18174e-18 1.12536e-16 1.46673e-15 1.44882e-14 1.1336e-13 7.27491e-13 3.93787e-12 1.83921e-11 7.55191e-11 2.76878e-10 9.18317e-10 2.78591e-09 7.80377e-09 2.0348e-08 4.97346e-08 1.14647e-07 2.50577e-07 5.21714e-07 1.03903e-06 1.98667e-06 3.65877e-06 6.5092e-06 1.1216e-05 1.87624e-05 3.05351e-05 4.84406e-05 7.50367e-05 0.00011368 0.000168682 0.000245471 0.000350761 0.000492699 0.000681019 0.000927157 0.00124435 0.00164771 0.00215423 0.00278276 0.00355398 0.00449025

Question c is answered by MatrixCal and it calculates the Greeks (Delta, Gamma) by passing different argument. And h for MatrixCal is adjustable, the argument only called to calculate approximate approaches.

Delta: 2.25551e-19 Gamma: 8.409e-19

Delta: 6.18174e-18 Gamma: 2.00933e-17

Delta: 1.12536e-16 Gamma: 3.22244e-16

Delta: 1.46673e-15 Gamma: 3.73243e-15

Delta: 1.44882e-14 Gamma: 3.30098e-14

Delta: 1.1336e-13 Gamma: 2.32738e-13

Delta: 7.27491e-13 Gamma: 1.35348e-12

Delta: 3.93787e-12 Gamma: 6.67174e-12

Delta: 1.83921e-11 Gamma: 2.85008e-11

Delta: 7.55191e-11 Gamma: 1.07453e-10

Delta: 2.76878e-10 Gamma: 3.62994e-10

Delta: 9.18317e-10 Gamma: 1.1128e-09

Delta: 2.78591e-09 Gamma: 3.12924e-09

Delta: 7.80377e-09 Gamma: 8.14606e-09

Delta: 2.0348e-08 Gamma: 1.97861e-08

Delta: 4.97346e-08 Gamma: 4.51471e-08

Delta: 1.14647e-07 Gamma: 9.7348e-08

Delta: 2.50577e-07 Gamma: 1.99386e-07

Delta: 5.21714e-07 Gamma: 3.89679e-07

Delta: 1.03903e-06 Gamma: 7.2963e-07

Delta: 1.98667e-06 Gamma: 1.31348e-06

Delta: 3.65877e-06 Gamma: 2.28058e-06

Delta: 6.5092e-06 Gamma: 3.82994e-06

Delta: 1.1216e-05 Gamma: 6.23682e-06

Delta: 1.87624e-05 Gamma: 9.87068e-06

Delta: 3.05351e-05 Gamma: 1.52137e-05

Delta: 4.84406e-05 Gamma: 2.28788e-05

Delta: 7.50367e-05 Gamma: 3.3626e-05

Delta: 0.00011368 Gamma: 4.83754e-05

Delta: 0.000168682 Gamma: 6.82164e-05

Delta: 0.000245471 Gamma: 9.44107e-05

Delta: 0.000350761 Gamma: 0.00012839

Delta: 0.000492699 Gamma: 0.000171745

Delta: 0.000681019 Gamma: 0.000226207

Delta: 0.000927157 Gamma: 0.000293627

Delta: 0.00124435 Gamma: 0.000375938

Delta: 0.00164771 Gamma: 0.000475119

Delta: 0.00215423 Gamma: 0.000593155

Delta: 0.00278276 Gamma: 0.000731986

Delta: 0.00355398 Gamma: 0.000893459

Delta: 0.00449025 Gamma: 0.00107928

Detla and Gamma approximation approach

Apprx with h = 0.5

ApprxDelta: 3.54074e-19 ApprxGamma: 9.99315e-19

ApprxDelta: 8.77158e-18 ApprxGamma: 2.26399e-17

ApprxDelta: 1.4825e-16 ApprxGamma: 3.49505e-16

ApprxDelta: 1.8277e-15 ApprxGamma: 3.93676e-15

ApprxDelta: 1.73062e-14 ApprxGamma: 3.40984e-14

ApprxDelta: 1.31062e-13 ApprxGamma: 2.36633e-13

ApprxDelta: 8.19913e-13 ApprxGamma: 1.35939e-12

ApprxDelta: 4.34949e-12 ApprxGamma: 6.63698e-12

ApprxDelta: 1.99895e-11 ApprxGamma: 2.8138e-11

ApprxDelta: 8.10156e-11 ApprxGamma: 1.05442e-10

ApprxDelta: 2.93893e-10 ApprxGamma: 3.54456e-10

ApprxDelta: 9.66289e-10 ApprxGamma: 1.08228e-09

ApprxDelta: 2.91036e-09 ApprxGamma: 3.03341e-09

ApprxDelta: 8.10351e-09 ApprxGamma: 7.87505e-09

ApprxDelta: 2.10232e-08 ApprxGamma: 1.90843e-08

ApprxDelta: 5.11669e-08 ApprxGamma: 4.34628e-08

ApprxDelta: 1.17523e-07 ApprxGamma: 9.35654e-08

ApprxDelta: 2.56076e-07 ApprxGamma: 1.91377e-07

ApprxDelta: 5.3176e-07 ApprxGamma: 3.73591e-07

ApprxDelta: 1.05664e-06 ApprxGamma: 6.98812e-07

ApprxDelta: 2.01639e-06 ApprxGamma: 1.25693e-06

ApprxDelta: 3.70721e-06 ApprxGamma: 2.18078e-06

ApprxDelta: 6.58565e-06 ApprxGamma: 3.66001e-06

ApprxDelta: 1.13331e-05 ApprxGamma: 5.9568e-06

ApprxDelta: 1.89369e-05 ApprxGamma: 9.42296e-06

ApprxDelta: 3.07887e-05 ApprxGamma: 1.45175e-05

ApprxDelta: 4.88003e-05 ApprxGamma: 2.18239e-05

ApprxDelta: 7.55359e-05 ApprxGamma: 3.20651e-05

ApprxDelta: 0.000114358 ApprxGamma: 4.61168e-05

ApprxDelta: 0.000169586 ApprxGamma: 6.50151e-05

ApprxDelta: 0.000246655 ApprxGamma: 8.99603e-05

ApprxDelta: 0.000352283 ApprxGamma: 0.000122314

ApprxDelta: 0.000494627 ApprxGamma: 0.000163589

ApprxDelta: 0.000683423 ApprxGamma: 0.000215432

ApprxDelta: 0.000930112 ApprxGamma: 0.000279602

ApprxDelta: 0.00124794 ApprxGamma: 0.000357937

ApprxDelta: 0.001652 ApprxGamma: 0.000452321

ApprxDelta: 0.00215931 ApprxGamma: 0.000564639

ApprxDelta: 0.0027887 ApprxGamma: 0.000696736

ApprxDelta: 0.00356085 ApprxGamma: 0.000850367

ApprxDelta: 0.00449812 ApprxGamma: 0.00102715

Apprx with h = 0.01

ApprxDelta: 2.25596e-19 ApprxGamma: 7.99963e-19

ApprxDelta: 6.18269e-18 ApprxGamma: 1.91146e-17

ApprxDelta: 1.12549e-16 ApprxGamma: 3.06544e-16

ApprxDelta: 1.46687e-15 ApprxGamma: 3.55055e-15

ApprxDelta: 1.44893e-14 ApprxGamma: 3.14009e-14

ApprxDelta: 1.13367e-13 ApprxGamma: 2.21394e-13

ApprxDelta: 7.27527e-13 ApprxGamma: 1.2875e-12

ApprxDelta: 3.93804e-12 ApprxGamma: 6.34647e-12

ApprxDelta: 1.83927e-11 ApprxGamma: 2.71112e-11

ApprxDelta: 7.55213e-11 ApprxGamma: 1.02214e-10

ApprxDelta: 2.76885e-10 ApprxGamma: 3.45294e-10

ApprxDelta: 9.18336e-10 ApprxGamma: 1.05854e-09

ApprxDelta: 2.78596e-09 ApprxGamma: 2.97665e-09

ApprxDelta: 7.80389e-09 ApprxGamma: 7.74882e-09

ApprxDelta: 2.03483e-08 ApprxGamma: 1.88212e-08

ApprxDelta: 4.97352e-08 ApprxGamma: 4.29455e-08

ApprxDelta: 1.14648e-07 ApprxGamma: 9.26007e-08

ApprxDelta: 2.50579e-07 ApprxGamma: 1.89663e-07

ApprxDelta: 5.21718e-07 ApprxGamma: 3.70675e-07

ApprxDelta: 1.03904e-06 ApprxGamma: 6.94047e-07

ApprxDelta: 1.98668e-06 ApprxGamma: 1.24943e-06

ApprxDelta: 3.65879e-06 ApprxGamma: 2.16936e-06

ApprxDelta: 6.50923e-06 ApprxGamma: 3.64316e-06

ApprxDelta: 1.1216e-05 ApprxGamma: 5.93266e-06

ApprxDelta: 1.87624e-05 ApprxGamma: 9.3893e-06

ApprxDelta: 3.05352e-05 ApprxGamma: 1.44718e-05

ApprxDelta: 4.84407e-05 ApprxGamma: 2.17631e-05

ApprxDelta: 7.50369e-05 ApprxGamma: 3.19861e-05

ApprxDelta: 0.00011368 ApprxGamma: 4.60162e-05

ApprxDelta: 0.000168682 ApprxGamma: 6.48895e-05

ApprxDelta: 0.000245472 ApprxGamma: 8.98063e-05

ApprxDelta: 0.000350761 ApprxGamma: 0.000122128

ApprxDelta: 0.0004927 ApprxGamma: 0.000163369

ApprxDelta: 0.00068102 ApprxGamma: 0.000215175

ApprxDelta: 0.000927158 ApprxGamma: 0.000279307

ApprxDelta: 0.00124435 ApprxGamma: 0.000357603

ApprxDelta: 0.00164771 ApprxGamma: 0.000451947

ApprxDelta: 0.00215423 ApprxGamma: 0.000564227

ApprxDelta: 0.00278276 ApprxGamma: 0.000696287

ApprxDelta: 0.00355398 ApprxGamma: 0.000849885

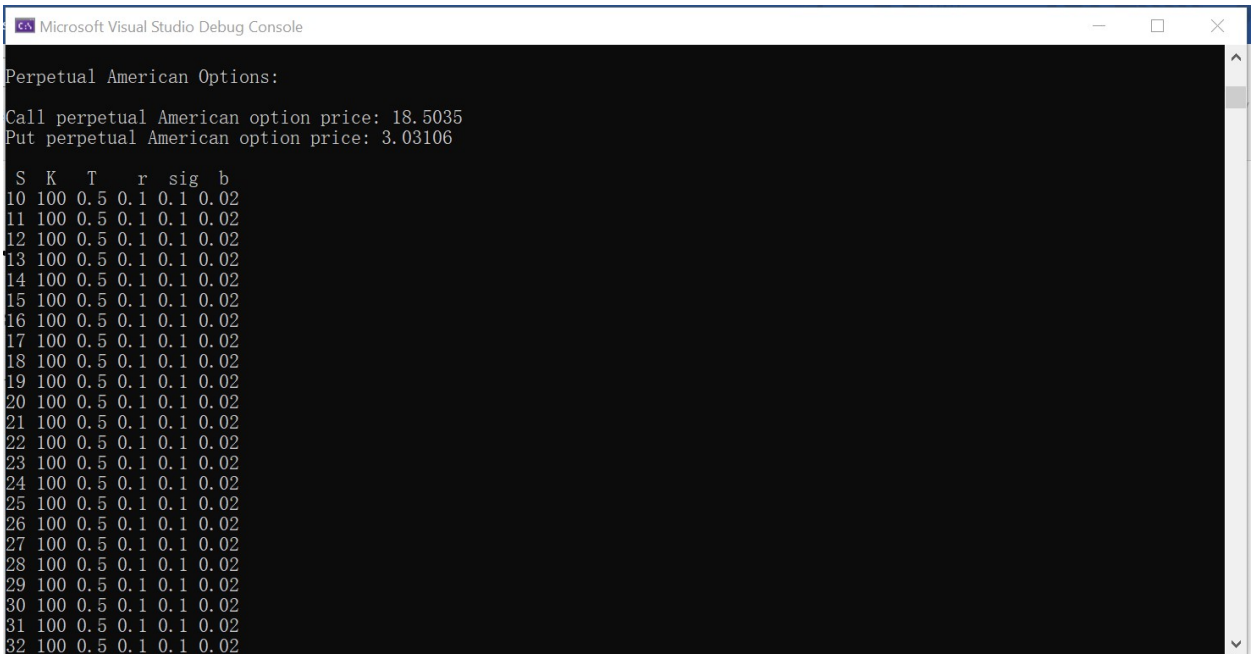
ApprxDelta: 0.00449025 ApprxGamma: 0.00102664

Group B

Perpetual American Options

Question a, b:

The American option was inherited class with base class Option (European option is also inherited). Pricing the option with given arguments:



Question c:

S K T r sig b

10 100 0.5 0.1 0.1 0.02

11 100 0.5 0.1 0.1 0.02

12 100 0.5 0.1 0.1 0.02

13 100 0.5 0.1 0.1 0.02

14 100 0.5 0.1 0.1 0.02

15 100 0.5 0.1 0.1 0.02

16 100 0.5 0.1 0.1 0.02

17 100 0.5 0.1 0.1 0.02

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25 100 0.5 0.1 0.1 0.02

26 100 0.5 0.1 0.1 0.02

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43 100 0.5 0.1 0.1 0.02

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47 100 0.5 0.1 0.1 0.02

48 100 0.5 0.1 0.1 0.02

49 100 0.5 0.1 0.1 0.02

50 100 0.5 0.1 0.1 0.02

Using regular code to price a matrix input:

0.00826235 0.011227 0.0148535 0.0192158 0.0243891 0.03045 0.0374762 0.0455465 0.054741 0.0651405 0.076827 0.0898835 0.104394 0.120442 0.138115 0.157497 0.178677 0.201742 0.226781 0.253883 0.283138 0.314637 0.348471 0.384732 0.423512 0.464906 0.509007 0.555908 0.605706 0.658495 0.714373 0.773434 0.835777 0.901499 0.970699 1.04347 1.11993 1.20015 1.28425 1.37233 1.46448

The same input with MatrixCal class to calculate:

Matrix by changing S from 10 to 50 with step of 1:

The matrix price results with call option are:

0.00826235 0.011227 0.0148535 0.0192158 0.0243891 0.03045 0.0374762 0.0455465 0.054741 0.0651405 0.076827 0.0898835 0.104394 0.120442 0.138115 0.157497 0.178677 0.201742 0.226781 0.253883 0.283138 0.314637 0.348471 0.384732 0.423512 0.464906 0.509007 0.555908 0.605706 0.658495 0.714373 0.773434 0.835777 0.901499 0.970699 1.04347 1.11993 1.20015 1.28425 1.37233 1.46448