Degree of Position (DOP)

If a quadrature rule $R(f) = \sum_{i=0}^{n} w_i f(x_i)$

Rmk. For interpolating quadrature,

$R(f) = \sum_{i=0}^{n} w_i f(x_i)$ is constructed

Ex. Find DOP for midpoint rule

$$R(f) = (b-a) f\left(\frac{a+b}{2}\right)$$

- $f(x) = 1 \rightarrow f\left(\frac{a+b}{2}\right) = 1$

$$R(f) = (b-a) \cdot 1 = b-a$$

$$\int_a^b 1\,dx = b-a$$ same!

- $f(x) = x \rightarrow f\left(\frac{a+b}{2}\right) = \frac{a+b}{2}$

$$R(f) = (b-a)\left(\frac{a+b}{2}\right) = \frac{b^2-a^2}{2}$$

$$\int_a^b x\,dx = \frac{1}{2}x^2\Big|_a^b = \frac{b^2-a^2}{2}$$ same!

$\cdot f(x)=x^2 \to f\left(\frac{a+b}{2}\right)=\frac{(a+b)^2}{4}$

$R(f)=(b-a)\frac{(a+b)^2}{4}$ NOT same!

$\int_a^b x^2 dx = \frac{1}{3}x^3 \Big|_a^b = \frac{b^3-a^3}{3}$

$\therefore$ midpt rule has DOP $=1$.

Note Midpoint rule is interpolatory quad. rule using degree $m=0$ poly.
BUT DOP $= m+1 = 1$.

Ex. Find DOP for trapezoidal rule

$R(f)=\frac{b-a}{2}(f(a)+f(b))$

We know: $R(1)=\int_a^b 1\,dx$

$R(x)=\int_a^b x\,dx$

$f(x)=x^2$: $R(f)=\frac{b-a}{2}(a^2+b^2)$

$\int_a^b x^2 dx = \frac{1}{3}(b^3-a^3)$

$\therefore$ trapezoidal rule has DOP $=1$.

Ex. Find DOP for Simpson's ⅓ rule

$R(f)=\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$

We know: $R(1)=\int_a^b 1\,dx$

$R(x)=\int_a^b x\,dx$

$R(x^2)=\int_a^b x^2 dx$

Check: $R(x^3)$

$=\frac{b-a}{8}\left(a^3+\frac{4(a+b)^3}{8}+b^3\right)$

$= \frac{1}{4}(b^4 - a^4)$

$\int_a^b x^3 dx = \frac{1}{4}(b^4 - a^4)$ same!

Check: $R(x^4)$

can show: $R(x^4) \neq \int_a^b x^4 dx$

Note Simpson's rule is interpolatory quad. rule using degree $m=2$ poly.

BUT DOP $= m+1 = 3$.

Rmk. This pattern is true in general

That is, for interpolatory quad. with equally-spaced points

- If degree of $p(x) = m$ is odd, then $R(f)$ has DOP $=$.
- If degree of $p(x) = m$ is even, then $R(f)$ has DOP $= m+1$.