

## Overview

Consider  $y''(x) + p y'(x) + q y(x) = f(x)$   $p, q \in \mathbb{R}$

$$y(0) = y'(0) = 0$$

i. existence and uniqueness : Picard's Theorem

ii. integral transformations: Laplace transformation

- turn ODE into an algebraic equation, solve the algebraic equation, and reinterpret the result.

iii. boundary value problems

- e.g. for which  $\lambda \in \mathbb{R}$  is there a non-zero solution to  $y'' + \lambda y = 0$ ,  $y(0) = y(\pi) = 0$ ?

· wave equation



$$\frac{\partial^2 y}{\partial x^2}(x, t) = \frac{\partial^2 y}{\partial t^2}(x, t)$$

$$y(0, t) = y(\pi, t) = 0$$

$$y(x, 0) = f(x)$$

$$\frac{\partial y}{\partial t} \Big|_{t=0} = 0 \quad (\text{string at rest when } t=0)$$

→ Fourier Series

Further topics:

- Sturm Liouville problems (and functional analysis)
- Fourier transform and PDEs
- Perturbation theory
- Calculus of variations