Fact: If n has n elements, then there are $n!$ permutations of X.

$$n! = n(n-1)(n-2)\ldots 1$$

Q: How many ways are there to order 5 students in a single file line out of 30 students?

$$30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$$

$$30! = 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot (25!)$$

$$= 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot (30-5)!$$

$$\text{Answer} = \frac{30!}{(30-5)!}$$

Def. An r-permutation of an n-element set is the same as a permutation.

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$= n(n-1)(n-2)\ldots(n-r+1)$$

Combinations

Def. An r-combination of an n-element set is an r-element subset.

Q: How many ways are there to choose 5 students out of 30 to go on a field trip?

- in order: $\frac{30!}{25!}$

overcounted each group of 5 by 5! times

$\rightarrow \frac{30!}{25!\,5!}$

Notation

$C(n,r)$ or $\binom{n}{r}$ } # of r-elements of an n-element set

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!\,r!}$$

Q: How many ways are there to choose 25 students out of 30 to go on a field trip?

$$C(30,25) = \frac{30!}{5!\,25!} = C(30,5)$$

in general: $C(n,r) = C(n, n-r)$

$\bar{P}(X)$ = power set of X
= set of subsets of X
$= \{ Y \mid Y \subseteq X \}$

if $|X| = n$, then $|P(X)| = 2^n$.

$X = \{x_1, x_2, \dots x_n\}$

By multiplication principle, $|P(X)| = 2^n$.

- Cool fact:

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

- The RHS is number of subsets of an n-element set, since if $|X| = n$, $|P(X)| = 2^n$.

- The LHS is $\sum_{i}^{n}$ of # of subsets of X of size i
  = # of subsets of X.

Ex. Reorder 'MISSOURI'

$$\frac{8!}{2!\,2!} \quad \text{(I repeats , S repeats)}$$

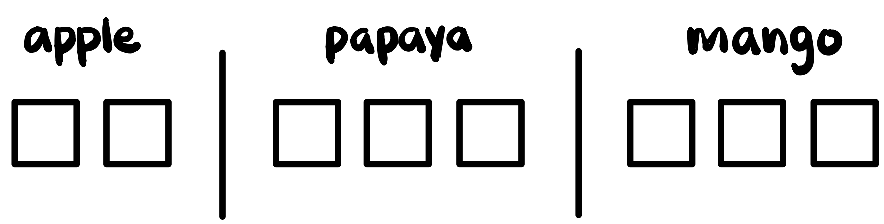
$$\frac{8!}{1!\,2!\,2!\,1!\,1!\,1!}$$

Thm. 6.3.2

Suppose a sequence S of length n consists of n_1 identical objects of type 1, n_2 identical objects of type 2, … n_t identical objects of type t. Then the number of orderings of S is $\frac{n!}{n_1!\, n_2! \cdots n_k!}$.

Ex. Pick 8 fruits

"Stars and bars"



There are C(10 2) = C(10 8) ways

10 = 8 boxes + 2 bars

Thm. 6.3.5

If a set X contains t elements, the number of unordered selections from X of size k, allowing repetitions, is $C(k+t-1, t-1) = C(k+t-1, k)$.