Numerical Integration

Basic Idea:

- approx. $f(x)$ by polynomial $p(x)$
- then: $I(f) = \int_a^b f(x)dx \approx \int_a^b p(x)dx$

That, the quadrature rule is

$$R(f) = \int_a^b p(x)dx$$

- using Lagrange form of $p(x)$, we get:

$$R(f) = \sum_{i=0}^{n} w_i f(x_i), \quad w_i = \int_a^b L_{n,i}(x)dx$$

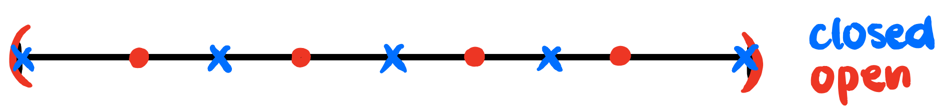
Note

$$\int_a^b p(x)dx = \int_a^b L_{n,i}(x) f(x_i)\, dx$$
$$= \sum_{i=0}^{n} \underbrace{\int_a^b L_{n,i}(x)dx}_{w_i} \cdot f(x_i)$$

When nodes are equally spaced, we obtain the Newton-Cotes (NC) formulae:

Closed NC rule: include boundary points $[a, b]$

Open NC rule: include boundary points (a, b)



Ex. $n=2$