# Derivative

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Spring Term 2024

## Introduction

The assignment focuses on implementing function to derivate mathematical expressions. Since, the language is Elixir, it is not so convenient reading the expressions in Elixir syntax. Therefore, they need to be simplified.

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## Derivatives

A list of derivative rules should be implemented and simplified. First a number of four rules, and then some additional rules will be added to the implementation. To implement ln(x), the following deriv/2 function was created.

```
def deriv({:ln, {:num, n}}, _) do {:div, 1, {:num, n}} end
def deriv({:ln, e1}, x) do {:mul, {:div, 1, e1}, deriv(e1, x)} end
```

ln(x) is an expression that can receive an expression as an argument, :ln, expr(). In order to represent the derivative of this expression a division function was needed which takes to arguments, :div, expr(), expr(). By having access to this function the derivative of ln(x) became easy to represent. The first case is when the there is a logarithm of a number, where one then divided by that number. The second case is when and the logarithm of an expression is to be performed. In that case, one is divided by the expression and the division is then multiplied with the inner derivative, meaning the derivative of the expression.

To derivate expression resembling 1/x the following implementation was done.

```
def deriv({:div, {:num, 1}, {:var, x}}, x) do
    {:div, {:num, -1}, {:expo, {:var, x}, {:num, 2}}}
  end
```

The division function comes to help in both representing the expression and derivating the expression here.

Next expression to be derivated is square root of x. Since there is an exponential function available ,which has earlier been implemented, it can be used to represent x to the power of 1/2.

The first case in the code below handles the simple case of square root of x and the second case handles a more general case where the x can be an expression and therefore has to be derivated too and be multiplied to the answer.

```
def deriv({:expo, {:var, x}, {:div, {:num, 1},
{:num, 2}}}, x) do
     {:div, {:num, 1}, {:mul, {:num, 2},
     {:expo, {:var, x}, {:div, {:num, 1}, {:num, 2}}}}}
   end

def deriv({:expo, e1, {:div, {:num, 1}, {:num, 2}}}, x) do
     {:mul,
     deriv(e1, x),{:div, {:num, 1},
     {:mul, {:num, 2}, {:expo, {:var, x},
     {:div, {:num, 1}, {:num, 2}}}}}
     }
   end
```

There are three cases represented in the code below that handles the derivative of sin(x). The first one is a simple expression being sin(x) which is derivated to cos(x). The other case when x is an expression instead. In that case, it is an inner derivative and should be handled.

```
def deriv({:sin, {:var, x}}, x) do {:cos, {:var, x}} end
def deriv({:sin, e1}, x) do {:mul, deriv(e1, x), {:cos, e1}} end
```

In cases where there is a division to be derivated, the general rule must be implemented. So derivating 1/sin(2x) the function below and the derivate function for sin(x) is applied.

```
 def deriv({:div, e1, e2}, x) do
       {:div,
         {:sub,
         {:mul, deriv(e1, x), e2},
         {:mul, e1, deriv(e2,x)}
         },
         {:expo, e2, {:num, 2}}
       }
   end
```

## Results

The outcome derivation expression for an expression including 1/sin(x) would be:

Expression:

$$1/sin(2*x)+x^3+ln(5)$$

Derivative:

$$(0*sin((2*x)))-(1*((0*x)+(2*1)*cos((2*x))))/sin((2*x))^2+((3*x^2)*1)+1/5$$

Simple_version:

$$(0*sin((2*x)))-(1*((0*x)+(2*1)*cos((2*x))))/sin((2*x))^2+(3*x^2)+1/5$$

The simplification didn't fully work on this expression but the outcome is correct. Other expressions give a more simplified result as shown down below.

Expression:

$$ln((4*x^6))$$

Derivative:

$$(1/(4*x^6)*(0*x^6)+(4*((6*x^5)*1)))$$

Simple_version:

$$(1/(4*x^6)*(4*(6*x^5)))$$

This is still not fully simplified but better.

## Discussion

Considering the simplification, there are different ways an expression can be counted as simplified. In the last example in the results section the expression

$$(4*(6*x^5))$$

could also be more simplified to this expression instead:

$$(24*x^5)$$

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