Lightning Formation and the Monte Carlo Simulation

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Abstract

Thunderstorms are mysteries in themselves. Measuring a storm is highly difficult, and as a result, analyzing its components is also a hefty task. The focus that this paper wishes to possess lies with what people most commonly associate a storm as having: lightning. These bolts of pure electricity place people in awe at every strike, and for good reason – their breathtaking beauty lasts for a literal flash of a moment, lasting only in fluorescent illusion in your eyes until the illusion fades. It bears the question, then, *why* is lightning considered a phenomenon if not merely for its beauty? It bears the answer that lightning is a feat of electromagnetic beauty alongside its visual allure. As we will explore, the *Monte Carlo* method of simulation has allowed many interested in the phenomenon of lightning to explain parts of its mystery.

Introduction

The papers chosen for the electric anomaly of lightning are chosen for their description of either the *formation* or the *effect* of lightning. The disparity between these components lies not in their differences, but in their ability to use differing methods to describe their pieces of the puzzle. Depending on the specific subject of the paper toward storms, the measurement of phenomena varies between papers. Dwyer explicitly states the ignoring of the Earth's magnetic field and atmospheric pressure in his analysis[1], whereas Petrov and Petrova embrace pressure in their paper[2].

Despite the underlying differences between the papers, there are many similarities in the mention of lightning. There is explicit description of a preliminary breakdown *or* an 'avalanche of electrons' [1][3][4][5] that causes the birth of lightning itself. An extremely common thread is the mention of Electric Field – as there should be – when defining the storm's electrical background. Plus, the push of Electric Field thresholds being flexible under certain conditions [1][2] is a common topic. Along the theme of commonality, three papers analyze nuclear effects of lightning through gamma rays and X-rays, of which spikes occur in tandem with Electric Field increases [1][4][6].

With the differences in topics in each paper, there also comes differences in models. It was mentioned before that one paper dismisses magnetic field and atmospheric pressure; in that same paper, Compton scattering is used to describe the avalanche of runaway electrons. One paper, as well, is the only one so far to describe the cloud that lightning forms from as having 'charge deposits', or sorts of pockets of charge that push the description of excess charge being a contributor to lightning formation, plus being the *only* paper to call the cloud an electric dipole charge distribution [2]. This last distinction is important when emphasis on Electric Field is so large – one should wonder, why do the other papers not emphasize this disparity in charge distribution (i.e. more positive charges at the top of a cloud) if they also wish to describe the threshold of Electric Field?

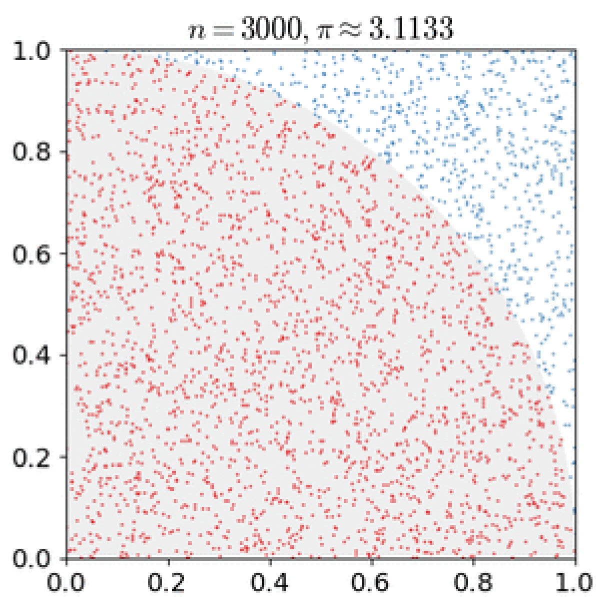
As we get deeper, we see that equations are only present if heavy assumptions are made. Krider has an excellent equation depicting current and Electric Field of a formed lightning bolt:

$$E_{\text{RAD}}(t) = -\frac{\mu_0\, v}{2\pi D} I(t - D/c),$$

Krider, however, is not the only one connecting current to lightning[3]. There is also description of the amount of current in certain strokes of lightning; “First stroke currents are typically near 30 kA, while subsequent stroke peak currents are typically 10–15 kA” says Dwyer in *The physics of lightning*[5]. However, the purpose of this paper is to roam outside of the numerical aspects of lightning. While these are interesting interpretations and calculations, there have not been many *computational* methods used to attempt to bring light to the storm. Most computational models are quite sparse, since the description of the creation of lightning is a very conceptual phenomenon.

Monte Carlo Simulation & Lightning

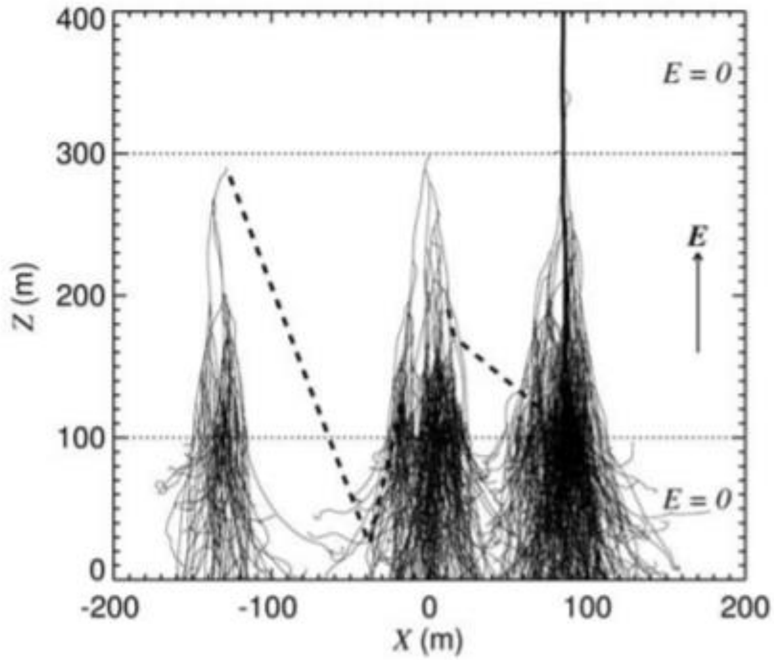
Yet, a few chosen papers have decided to use the Monte Carlo simulation to try – and succeed – to bridge the gap between concepts and computation. Those wishing to connect nuclear physics to the electrical physics use either physical instrumentation[4] or Monte Carlo simulation[1] [6] to reflect their findings. Monte Carlo simulation, in general, generates probability distributions to model some phenomena – an example of a simpler simulation is that which coders approximate pi by making a quarter of a circle and counting dots that end up in that quarter (Figure 1).



(Figure 1), approximating pi using Monte Carlo method.

Monte Carlo has been used world-wide, dating itself even prior to the 1950’s, before computation came along to simulate it. Beyond estimating pi, Monte Carlo simulation gets complicated – one must analyze their own subject, see potential statistically with using the MC method, and then apply it to the best of their ability. It is more than estimating pi: we can predict statistical values by forecasting potential values, using pseudorandom numbers, and stacking historical data en masse. As a clarification, pseudorandom numbers are that which are inserted into your data that are both ‘random’ and ‘applicable’, as in the situation with above; the dots are ‘randomly’ placed, but they are still in a range from 0 to 1 in both axes, making them not as random as they seem.

Yet, what does this mean for our evaluation of lightning? As we know, storms are very unpredictable; what we know for sure is that an ‘avalanche of electrons’ sweeps through the cloud and rapidly changes the Electric Field, changing energy just as quickly. What does this do for us?

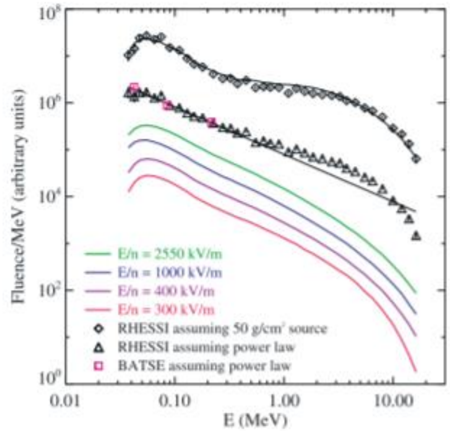


(Figure 2) Monte Carlo simulation depicting peaks of radioactivity in same peaks of Electric Field threshold breaking.

Dwyer et al. [1] give an approach toward applying Monte Carlo simulation to lightning, the first of the few gathered that is interpretable in this paper. In Figure 2, we see several lines seemingly like the ends of broomsticks, fanning out below, yet coming together to a nearly singular point atop the lines. This is one classic structure, creating a ‘bell-curve’ type shape that allows us to take the peak value as somewhat of an average for the data. To take this further, we need to delve into the research itself. This Monte Carlo simulation displays random selections of data showing runaway electrons and gamma rays produced during lightning[1].

With these selections mapped, we can see that this random data is not so random in accordance to how runaway electrons, and therefore changing Electric Field, coincides with gamma ray (and perhaps X-Ray) detection. Monte Carlo simulation here takes the seemingly random data from separate sets of data and projects them together, allowing us to compare their consistencies. Results from this simulation states that, indeed, the nuclear bursts in the clouds correspond *strikingly* with the data of Electric Field threshold being broken. This tells us the beginnings of a larger tale for lightning – with the use of the Monte Carlo method, ‘random’ phenomena like these nuclear bursts can be a valid aspect of the storm.

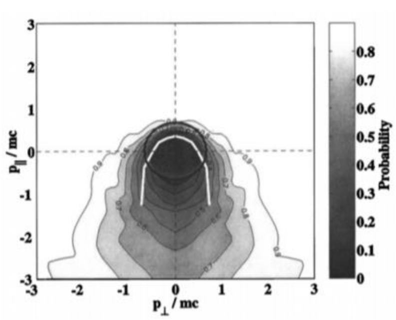
Another Monte Carlo simulation by Dwyer in a Monte Carlo-themed paper simulates a similar experiment:



(Figure 3) Monte Carlo depicting trends of runaway electrons
Affecting both E-Fields and X-Ray emission spectra.

As we can see from their results (Figure 3)[6], as the energy is lost, there is a correlation between changing Electric Field and changing emission or energies produced by colliding photons, electrons, and air particles. For this experiment, an assumption was made that the initial Electric Field was uniform, allowing the sequential assumption to be made that the runaway electrons both *propogate* the E-Field *and* spark the emission of radioactive waves. Here, we begin straying from the simpler 'nuclear bursts occur when Electric Fields break down' model and delve into the more complicated 'runaway electrons are a cause in shift of energy' model. This pushes Monte Carlo to a newer limit – using multiple datasets that vary in limitations and caps of values to try and track a pattern between these datasets. It works, and we see that the 'random' datasets have extremely similar curves.

Past this model, though, Dwyer et al. attribute their start to *Lehtinen* et al. who use Monte Carlo to plot the "probability for an electron with given initial momentum vector to become a runaway" [7] (Figure 4).



(Figure 4) Monte Carlo depicting the probability that
an electron with some momentum will become a runaway.

This evaluation is important since it gives Dwyer a basis to work his simulation off of – not simply from this above figure, but from Lehtinen's calculations themselves. There is a great quote from this work:

> "Using the Monte Carlo technique, we can estimate this probability by running the simulation many times and approximating the probability with the fraction of runs in which the electron eventually gains energy > 50 mc^2. In

> [figure 4], we plot the probability for an electron with given initial momentum vector to become a runaway, calculated in this way."

Here, we finally see the roots of the cause of lightning explained by Monte Carlo; an electron reaching or breaching the cap of energy of 50 mc^2 means that this avalanche of electrons has a *higher probability* of occurring. The boundary of the 'runaway region' was decided by the Monte Carlo calculations, making these random values a *compilation* of probabilities that coincide with the breaches of energy. What an amazingly simple analysis! One would think, "*Of course that runaway electron having a ton of energy would cause the avalanche! Of course it has a higher probability for creating that avalanche!*" However, it shows the versatility of the MC simulation toward explaining lightning from its root cause. Lightning is not only electric fields and nuclear activity, it is also *momentum* and *pure motion*, and that is truly incredible.

Discussion

These three cases of Monte Carlo simulation and lightning are profoundly different for having the same basic subject – for Figure 2 and its relation of Electric Field threshold breaking with gamma ray activity, for Figure 3 and its starting relation to runaway electrons, E-Fields, and X-Rays, and lastly Figure 4 with an electron's energy and momentum relative to probability, we know *so much* about lightning and its phenomena. And this is despite the three subjects only vaguely being connected, as lightning has so many pieces to unpack; we have only scratched the surface with what MC can offer to us. We should review a few pieces of the other non-MC-based papers before continuing to prod the simulation.

It was mentioned before that with the strike of lightning comes a current – to anyone experienced with electricity and magnetism, this may be a given. It is known that hundreds of moving charges from cloud to ground makes a current. However, it is notable that very few mention the actual action of lightning step by step; two collected papers lovingly describe lightning as a *ladder* and as having a *return stroke*[3] [5]. What does this mean for us lightning enthusiasts? Is there a way to analyze this stage of lightning formation with the Monte Carlo method?

Long story short, it would be very, very difficult if we were using Monte Carlo to reflect the path strike itself – the path that lightning takes is variable and ever-changing, much like how no two snowflakes are the same. A simulation of the path, then, is out of the picture. Thus, we stick with the traditional method of 'measure and make a conceptual analysis', as done by those interested in the 'ladder' and 'return stroke'. This leaves us with an open-ended question of, what *can* we still measure about lightning with the MC method?

Lightning is full of phenomena. Petrov & Petrova do a wonderful job excavating a few of the lesser-traversed aspects of 'pockets of charge' and 'dipole charge distribution' type analyses[2]. There is potential here; with pockets of charge as a model for the cloud, it can be proposed that the MC simulation could inspect if higher charge density increases probability for runaway electrons, or if overall charge density change in a spot of a cloud helps to cause the breakdown of Electric Field at that spot. How about the dipole charge distribution: with the disparity between charge distribution in the upper and lower levels of the clouds, one wonders if the intensity of this distribution helps increase that same probability of runaway electrons, or perhaps if the dipole moment of a cloud could map with some other phenomena of the cloud.

Conclusion

With the beauty of a storm comes the underlying beauty of discovering the storm's secrets. The method of Monte Carlo simulations in discovering those secrets is both creative and storytelling – we weave connections between already-done analyses with new possible chapters, carving our own questions out of new theories. *Do* dipole distributions in clouds have predictable properties usable in MC simulation? What a fun question, and one that would be interesting to delve into. The seemingly 'random' subsets of information can be, and already have been, compiled together to tell the tale of lightning. It is to be noted that very few equations were used in this paper, and this tells how electricity is not simply the equation for Electric Field, but also *trends* and *conceptual investigation!* Thus, it is noted: the mystery of lightning has not yet been solved.

References

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