

# Identification of the Curie Temperature Distribution from Temperature Dependent Magnetisation Data

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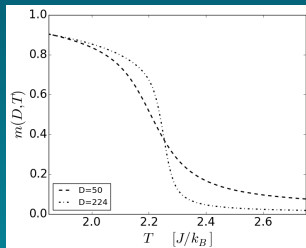
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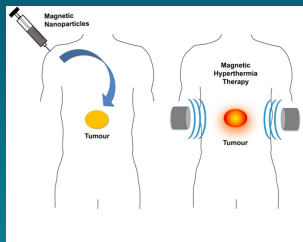
In HAMR<sup>1</sup>:

- $T_C$  distribution affects the noise performance.



In Magnetic Hyperthermia<sup>2</sup>:

- Low  $T_C$  reduces tissue damage.



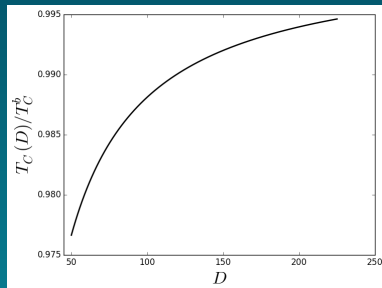
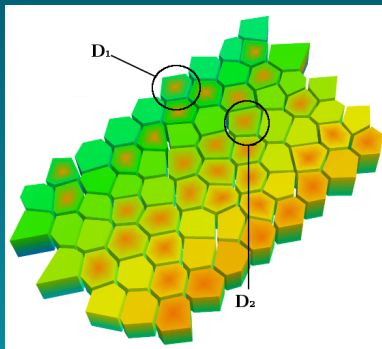
Ângela Andrade, Roberta Ferreira, José Fabris and Rosana Domingues (2011). Coating Nanomagnetic Particles for Biomedical Applications

<sup>1</sup>Dieter Weller et al. "A HAMR Media Technology Roadmap to an Areal Density of 4 Tb/in". In: *IEEE transactions on magnetics* 50.1 (2014), p. 3100108

<sup>2</sup>I Apostolova and JM Wesselinowa. "Possible low- $T_C$  nanoparticles for use in magnetic hyperthermia treatments". In: *Solid State Communications* 149.25 (2009), pp. 986–990

Correlation length  $\propto |T - T_C^b|^{-\nu}$

Grain size,  $D \propto |T_C(D) - T_C^b|^{-\nu}$



$$f_D(D) \Rightarrow f_{T_C}(T_C)$$

## 2 Types:

- ▶ Explicit measurement of individual grains.<sup>3</sup>
- ▶ Implicit calculation using global measurements.<sup>4</sup>
  - ▶ Single measurement with magnetometer
  - ▶ Integral measure
  - ▶ Uses bulk relations

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<sup>3</sup> Simone Pisana et al. “Curie temperature distribution in FePt granular media”. In: *Magnetics, IEEE Transactions on* 51.4 (2015), pp. 1–5

<sup>4</sup> Andreas Berger et al. “Critical exponents of inhomogeneous ferromagnets”. In: *Journal of applied physics* 91.10 (2002), pp. 8393–8395

- ▶ Develop a method to identify the  $T_C$  distribution which incorporates the finite size effects of the individual grains.
- ▶ Test this method against benchmark data in order to verify it's effectiveness for different distributions.

# Our Method

## Aggregate Magnetisation:

$$M(T) = M_0 \int_0^\infty D^d m(D, T) f_D(D) dD$$

Scaling Ansatz:

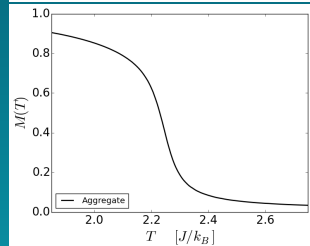
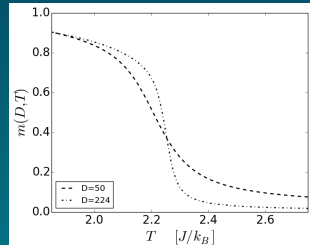
$$m(D, T) \propto D^{-\beta/\nu} \mu \left( D^{1/\nu} \frac{T - T_C^b}{T_C^b} \right)$$

Change of Variables:

$$D = d_0 \left( \frac{t}{T_C^b} \right)^{-\nu} \quad t \equiv T_C^b - T_C(D)$$

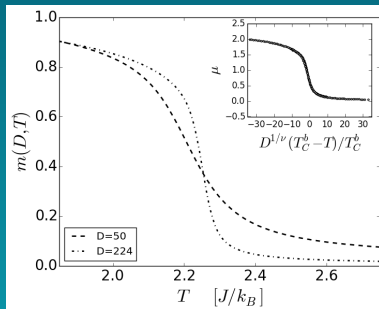
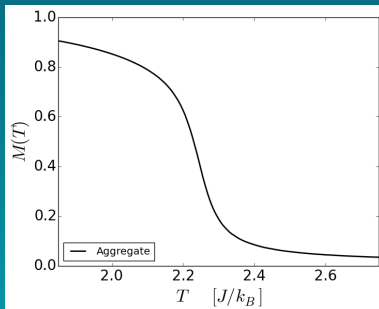
Final Result:

$$M(T) = M_0^* \int_0^{T_C^b} t^{-d\nu+\beta} \mu \left( d_0^{1/\nu} \frac{T - T_C^b}{t} \right) f_t(t) dt$$



$$M(T) = M_o^* \int_0^{T_C^b} t^{-d\nu+\beta} \mu \left( d_o^{\frac{1}{\nu}} \frac{T - T_C^b}{t} \right) f_t(t) dt$$

- ▶  $M(T)$ : To be fitted
- ▶  $d, \nu, \beta, \mu$ : Known information about the material
- ▶  $d_o, T_C^b$ : May be known, otherwise taken from fit
- ▶  $M_o^*, f_t [\bar{t}, \sigma_t]$ : Taken from the fit



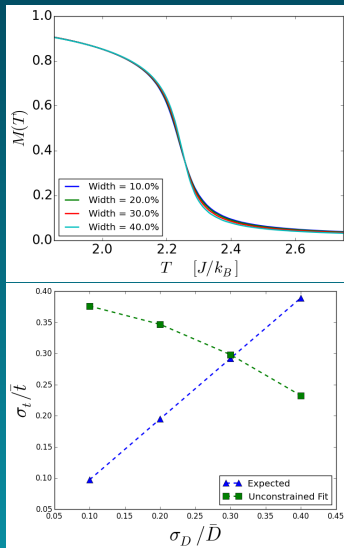
# Test Case: 2D Ising Model

Used 2D Ising model as a benchmark:

- ▶ Easily simulated
- ▶ Analytical results for  $\beta$ ,  $\nu$ ,  $T_C^b$ ...

Tested against different  $f_D$ :

- ▶ All mean  $\bar{D} = 100$
- ▶ Standard deviation  
 $\sigma_D = 10, 20, 30, 40$





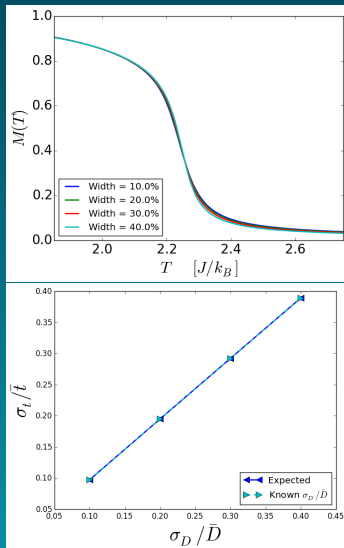
# Test Case: 2D Ising Model

Introduce constraint from:

$$D = d_o \left( \frac{t}{T_C^b} \right)^{-\nu}$$

$$\Rightarrow \sigma_t^2 = \bar{t}^2 \left( \left( 1 + \frac{\sigma_D^2}{\bar{D}^2} \right)^{1/\nu^2} - 1 \right)$$

Works much better!



- ▶ Presented a universal method to find size dependent  $T_C$  distribution.
- ▶ Based upon fitting ensemble magnetisation:

$$M(T) = M_o^* \int_0^{T_C^b} t^{-d_\nu + \beta} \mu \left( d_o^{\frac{1}{\nu}} \frac{T - T_C^b}{t} \right) f_t(t) dt$$

- ▶ Successfully tested against different size distributions  $f_D$ , given certain constraints.

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