GTS210 : Math 3 Final. Summary

- Lecture 7 : Projection & Cross Product.
  - $v = proj_y x \Rightarrow \frac{y}{\|y\|} \times \frac{(x, y)}{\|y\|}$ - if $x \perp y \rightarrow proj_y x = 0$
  - distance b/w point to line * 2D *
    - $d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$; line : $ax + by + c = 0$, point = (x_0, y_0)
    - another way by construct line perpendicular to l and pass through point p. and find distance b/w 2 points.
  - distance b/w 2 points
    - $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
  - distance b/w point to line * 3D *
    - given line as parametric equation ; $l(t) = p_0 + dt$ and point p
    - find vector u as $u = p - p_0 - dt$ and $(u, d) = 0$
    - find point and distance b/w such point and point p.
  - distance b/w point to plane * 3D, obviously *
    - given plane equation and point p.
    - find *perpendicular* line (using normal vector as directional vector, p as p_0) and find intersection point , then distance is distance b/w point p and intersection point.
  - Cross product.
    - $W = v_1 \times v_2 \Rightarrow v_1, v_2$ is orthogonal to W
    - $W = \theta$ iff only if $v_1 \parallel v_2$
    - $\|w\| = \|v_1\| \|v_2\| \sin \theta$; area of parallelogram.
  - Shortest distance b/w skew line.
    - given line $l_1(t)$, $l_2(s)$
    - different b/w must be proportional to $n = d_1 \times d_2$; set as $l_1(t) - l_2(s) = \lambda n$
    - distance is $\|\lambda n\| = |\lambda| \|n\|$

- Lecture 8 : l.c. / l.d. / span

$R^n \to$ vector space $\quad P_n \to$ polynomial space with n as highest degree. $\quad M_{m\times n} \to$ matrix space with m×n matrix

- linear combination (l.c.)
  - $w = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n \quad \to$ if $\alpha_1, \ldots \alpha_n \begin{cases} \text{has solution.} \to \text{l.c.} \\ \text{no solution} \to \text{not. l.c.} \end{cases}$

- span.
  - must have at least 1 solution for $\forall w$
  - might check w/ det if $\det(A) \neq 0 \to$ unique solution $\to$ span
    $\det(A) = 0 \to$ infinite sol$^n \to$ span
    $\to$ no solution for some $w \to$ not span!

- linear independence.
  - $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \ldots + \alpha_n v_n = \theta$
    if only get trivial solution $(\alpha_1 = \alpha_2 = \ldots = \alpha_n = 0) \to$ linear independent.
  - observation point : if v_n is multiple of v_m or linear combination of 12 vectors is said to be linear dependent vector,
  - R^n ; if r (# of vector) > n $\to$ l.d.
    P_n ; if r > n+1 $\to$ l.d.
    $M_{m\times n}$; if r > m×n $\to$ l.d.

- Lecture 9 : Basis, Dimension, Fundamental Matrix
  - Basis
    - must be l.i. and span !
    - R^n ; n l.i. vectors $\to$ span $\to$ basis $\quad M_{m\times n}$; mn l.i. vectors $\to$ span $\to$ basis
      P^n ; n+1 l.i. vectors $\to$ span $\to$ basis
    - dimension = number of basis vector.
    - $\dim\{\theta\} = 0$
    - find basis of span by removing l.d. vectors.

No.
Date :

Fundamental Subspace.

- Rowspace (R_A) • Columnspace (C_A) • Nullspace (N_A)
- if $\det(A) \neq 0 \rightarrow N_A = \{\theta\}$
- $\dim(N_A)$ = Nullity A / $\dim(R_A) = \dim C_A = \dim R_{A^T} = \dim C_{A^T} =$ rank. A

rank A + nullity A = n (# of col.)

rank A^T + nullity A^T = m (# of row).

$R_A = C_{A^T}$, $C_A = R_{A^T}$, rank A = rank A^T

To find N_A, use R_A • To find N_{A^T}, use C_A

- Basis on line and plane
  - transform into vector equation then you will get the basis vector.

Lecture 10: Change of basis.

A_{B,B_s} ⇒ transformation matrix from basis B to B_s

- A_{B,B_s} can be obtained directly from set of vectors.

formular.

$v_{B_s} = A_{B,B_s} v_B$ ▶ $A_{B_s,B} = A^{-1}_{B,B_s}$

▶ $A_{B_1,B_2} = A^{-1}_{B_2,B_s} A_{B_1,B_s}$

Lecture 11: Orthogonal / Orthonormal, GSO

- $(v_1, v_2) = 0 \rightarrow v_1 \perp v_2$
- $\|v\| = \sqrt{(v,v)}$
- $\|v_1 + v_2\| \leq \|v_1\| + \|v_2\|$
- $\|\alpha v\| = |\alpha| \|v\|$

if all vectors in set are orthogonal/orthonormal → l.i.

$\text{proj}_H v = (v, v_1)v_1 + (v, v_2)v_2 + \dots + (v, v_n)v_n$ only when H is orthonormal basis

otherwise, use this: $\text{proj}_H v = v - \text{proj}_{H^\perp} v$

GSO; l.i. system → orthonormal basis.

① $u_1 = \frac{v_1}{\|v_1\|}$, ② $u_2' = v_2 - (v_2, u_1)u_1$, $u_2 = \frac{u_2'}{\|u_2'\|}$ ③ ... ⓝ $u_n' = v_n - (v_n, u_1)u_1 - (v_n, u_2)u_2 - \dots - (v_n, u_{n-1})u_{n-1}$

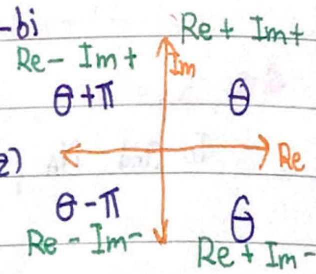
$u_n = \frac{u_n'}{\|u_n'\|}$ orthonormal basis $= \{u_1, u_2, \dots u_n\}$

- orthogonal complement of H is $H^{\perp}$
  - $N_A^{\perp} = R_A = C_{A^T}$ - $N_{A^T}^{\perp} = C_A = R_{A^T}$

  ↳ find by dot product of $(x, y, \dots)$ with every vector in $H = 0$ then solve system.

- Lecture 12: Complex Number.
  - $z = a+bi$ / $Re(z) = a$ / $Im(z) = b$ / $\bar{z}$ (conjugate) $= a-bi$

    $|z|$ = magnitude $= \sqrt{a^2+b^2}$



  - $a+bi \leftrightarrow r\,cis(\theta)$ by $r = |z|$, $\theta = \tan^{-1}\left(\frac{b}{a}\right) = arg(z)$

    ☆ $arg(z) \in [-\pi, \pi]$

  - $z^n = r^n cis(n\theta)$
  - $\sqrt[n]{z} = \sqrt[n]{r}\, cis\left(\frac{\theta + 2k\pi}{n}\right)$ where $k \in \{0, 1, \dots, n-1\}$

    if $\sqrt[4]{z}$ → have 4 answer, vice versa.

    ☆☆ x in quadratic equation ; $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

- Lecture 13: Eigenvalue, Eigenvectors.

  ☆☆ $Av = \lambda v$

  $Av - \lambda v = 0$

  $Av - \lambda I v = 0$

  $(A - \lambda I)v = 0 \rightarrow \overbrace{\det(A - \lambda I)}^{p(\lambda)} = 0 \rightarrow$ get λ (eigen value)!

  - multiplicity → algebraic (m_a) → # of root answer in $p(\lambda)$
    ↳ geometric (m_g) → # of vector get from E_λ for each λ $(\dim E_\lambda)$

  ☆ $m_g \leq m_a$ always!

  ☆ C^n = complex vector space (like vector space all did the same)

  ↳ $(u, v) = \sum u_i \bar{v}_i$ ☆ use conjugate ☆