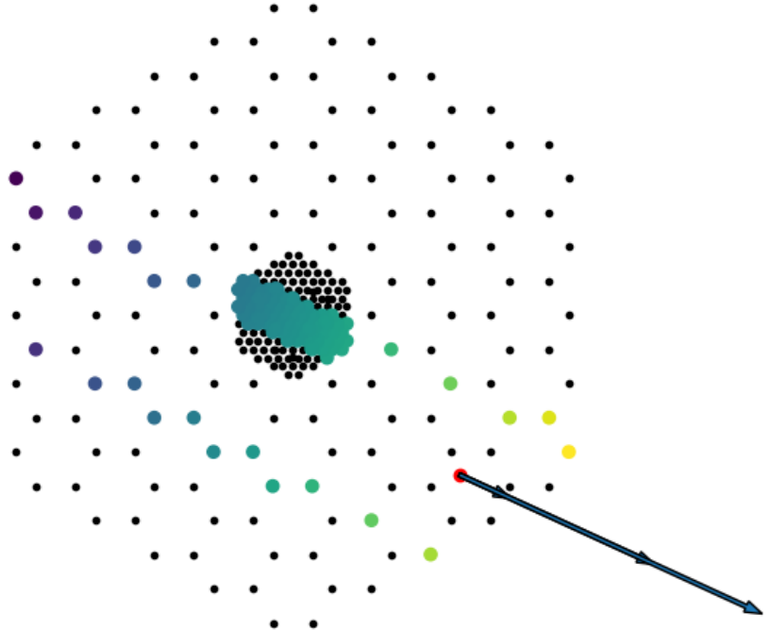
PWF reconstruction with DC2, theoretical uncertainties, and layout considerations

More detailed results/plots on the DC2 and layout Forge wikis



TL; DR:

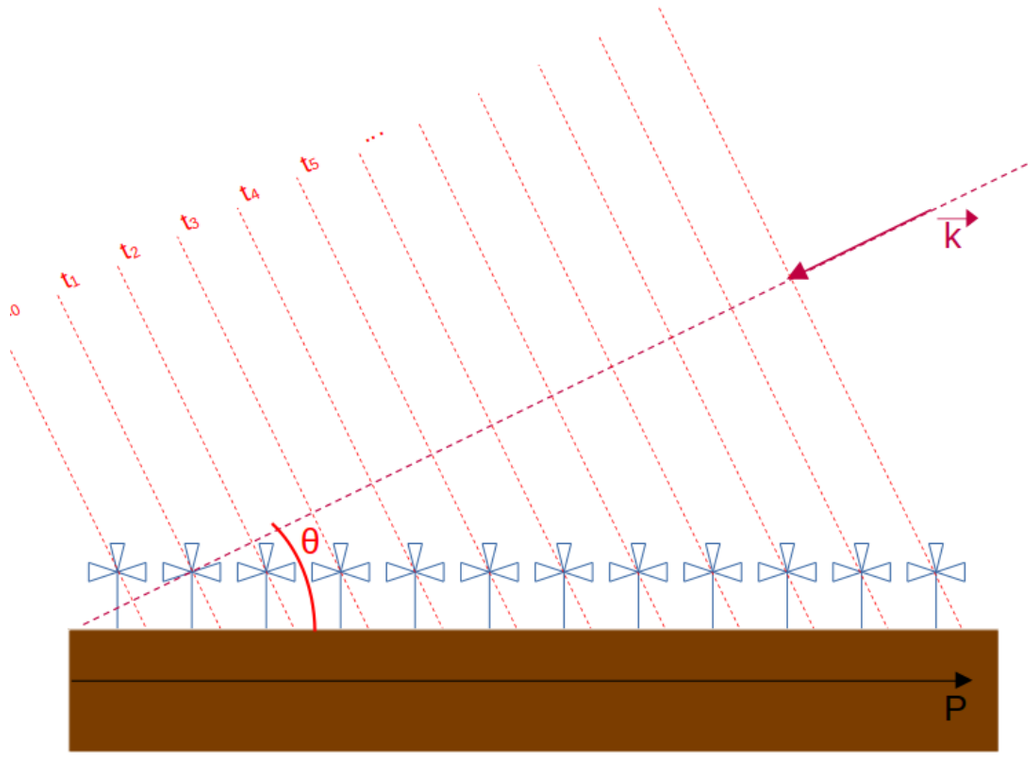
- reconstruction of zenith works fine
- reconstruction of azimuth is ok for zenith <75
- Things get interesting for azimuth reconstruction for zenith>75

A. Benoit-Lévy, GRAND analysis meeting, June 6th, 2024

PWF reconstruction and theoretical uncertainties

Wave Front Approximations: Planar

Slide from Arsène's talk @Nanjing



In planar approximation, and known Gaussian noise we have :

$$\sim \mathcal{N}\left(0,\ \sigma^2\right)$$

$$t_i = \frac{1}{c} P_i^T k + t_0 + \epsilon_i$$

Antenna position

To reconstruct **k**, we minimise :

$$\frac{1}{2}(c\mathbf{T} - \mathbf{P}\mathbf{k})^T (c\mathbf{T} - \mathbf{P}\mathbf{k}) \quad \text{with} \quad \|\mathbf{k}\| = 1$$

PWF reconstruction and theoretical uncertainties

Uncertainty estimation:
Small error approximation

Slide from Arsène's talk @Nanjing

- **With no constraint:**

$$k \sim \mathcal{N}(k^*,\ \Sigma) \ \text{ with: } \begin{cases} k^* = c(P^T P)^{-1} P^T T \\ \Sigma = (c\sigma)^2 (P^T P)^{-1} \end{cases}$$

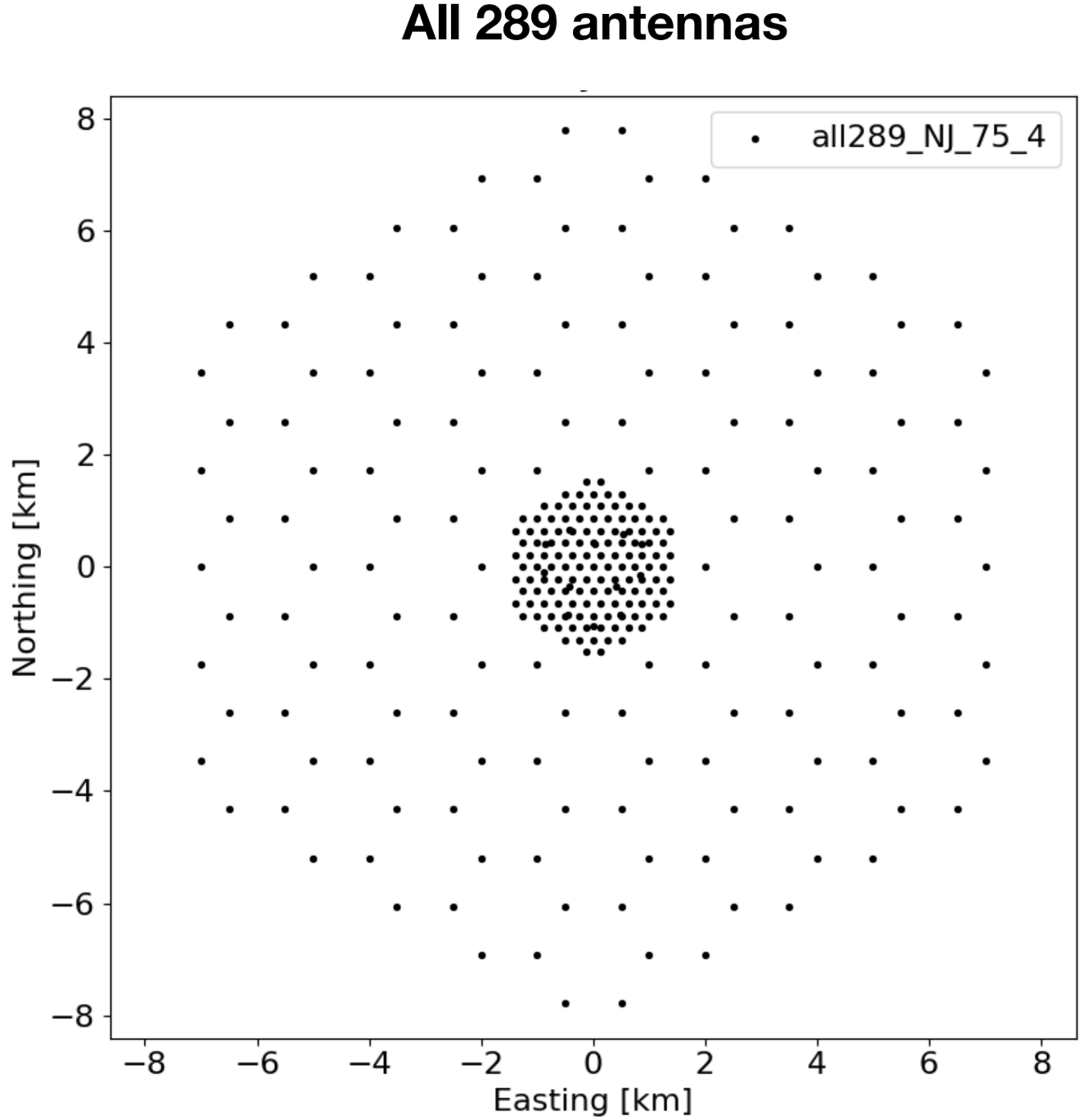
- **But we want the distribution of :** $\mathbf{k} \mid \|\mathbf{k}\| = 1$
  - Graphically : It is intersection of white envelope with the sphere

If error small: The sphere is equal to its tangent plane.

- With this approximation : $\begin{pmatrix} \theta \\ \phi \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \theta_s \\ \phi_s \end{pmatrix}, \bar{\mathbf{\Sigma}}\right)$ nd $\bar{\mathbf{\Sigma}} = \left[\mathbf{R_a}^T \mathbf{\Sigma}^{-1} \mathbf{R_a}\right]^{-1}$ $\mathbf{R_a} = \begin{pmatrix} -\cos(\theta_s)\cos(\phi_s) & \sin(\theta_s)\sin(\phi_s) \\ -\cos(\theta_s)\sin(\phi_s) & -\sin(\theta_s)\cos(\phi_s) \\ \sin(\theta_s) & 0 \end{pmatrix}$
- Same than with error propagation. Analytic formula for layout optimisation.

We have an estimation of the prediction uncertainties
We must know the timing measurement noise.

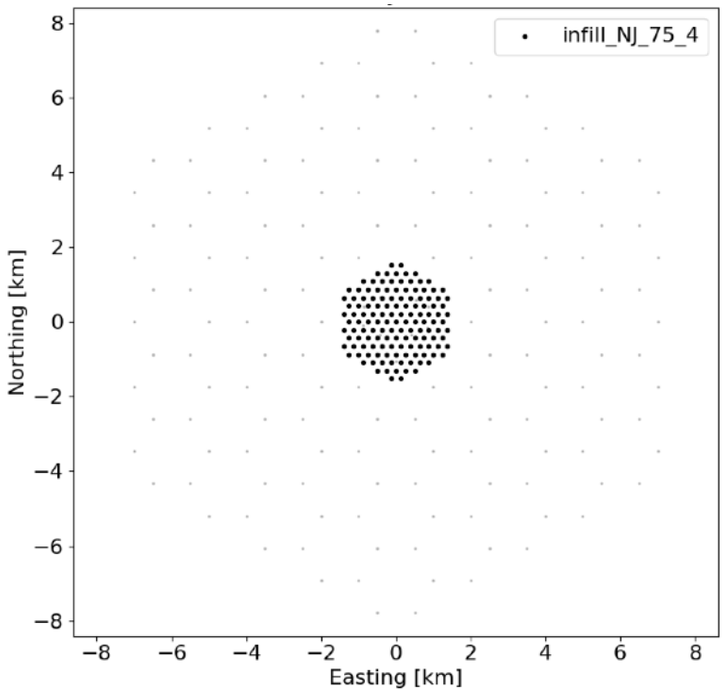
DC2 Layouts



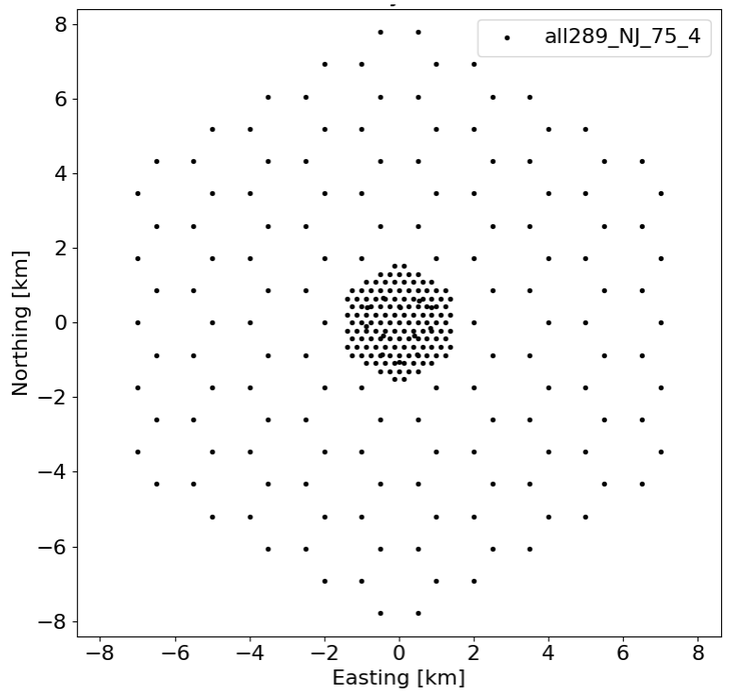
DC2 allows for nice layout studies: event rate (of course),
but more importantly on reconstruction!

DC2 Layouts

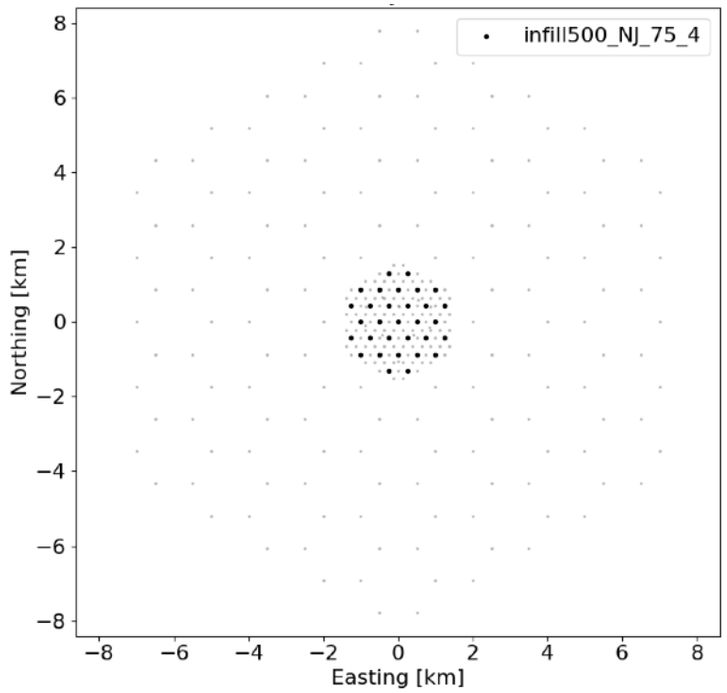
infill 250m



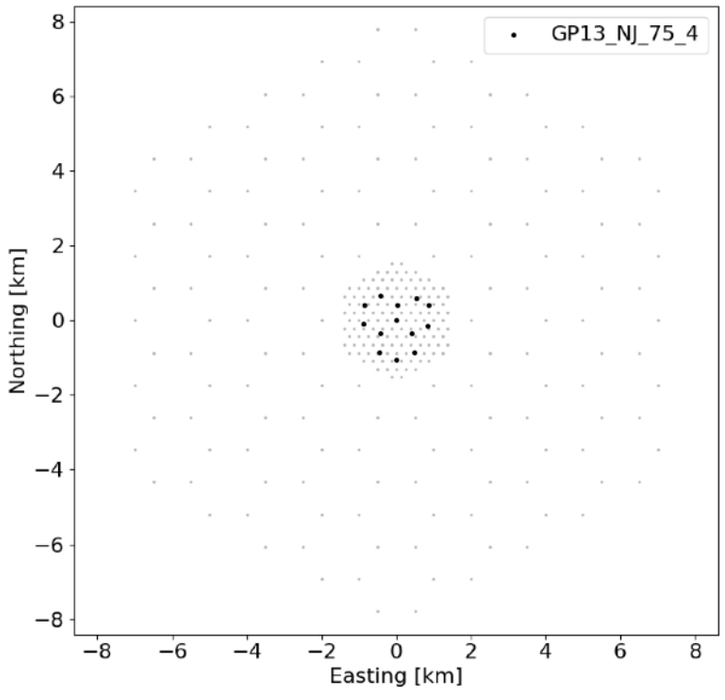
All 289 antennas



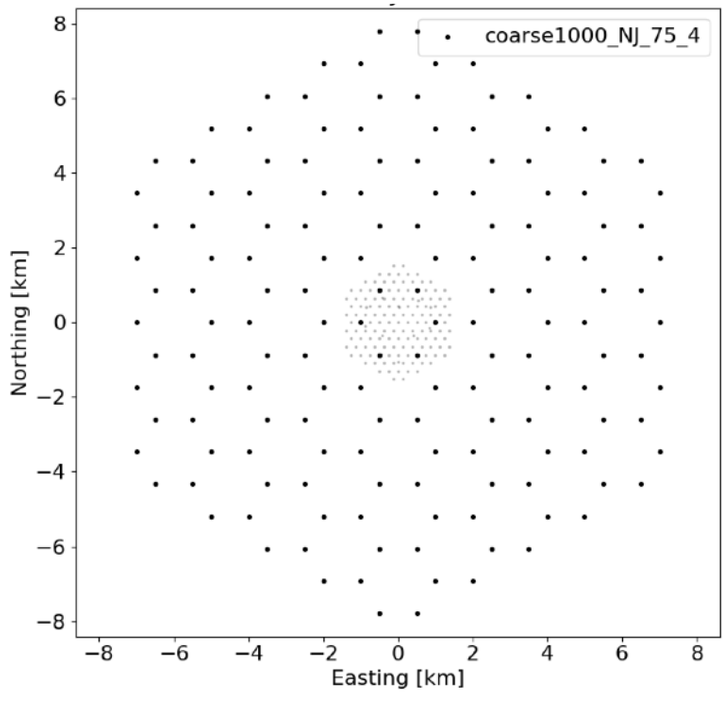
infill 500m



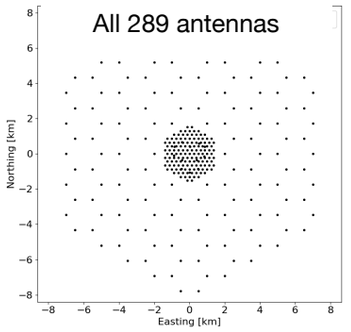
GP13



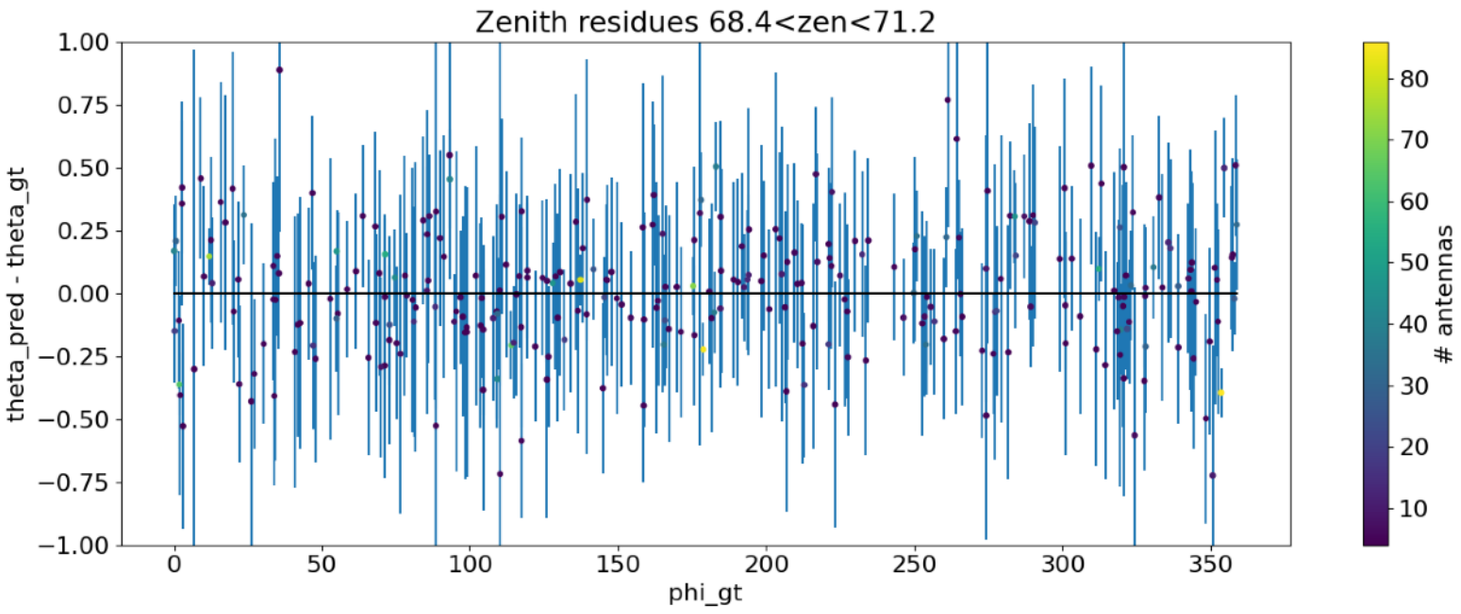
coarse 1000m

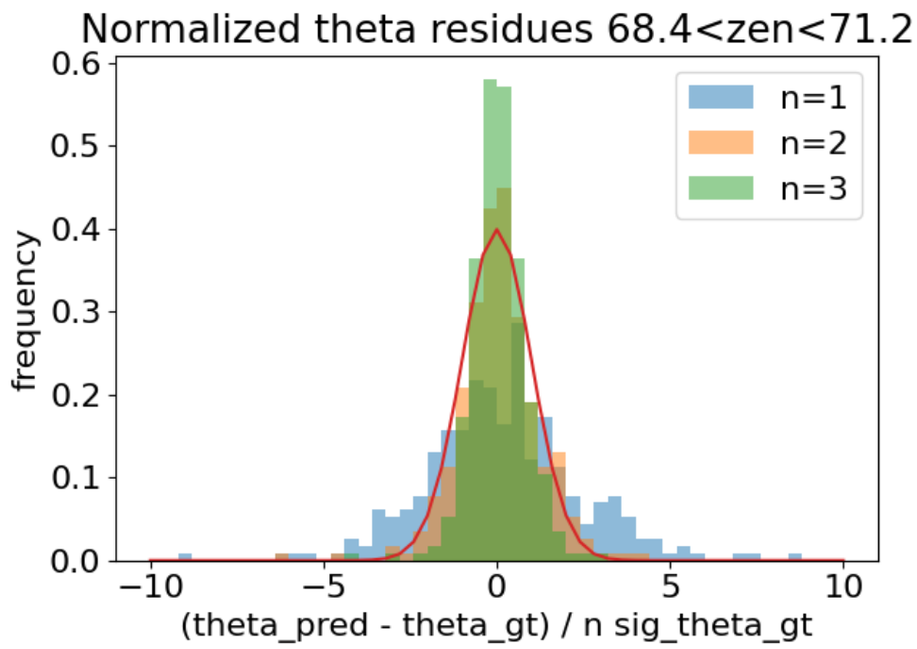


I. Zenith reconstruction



Error recalibration

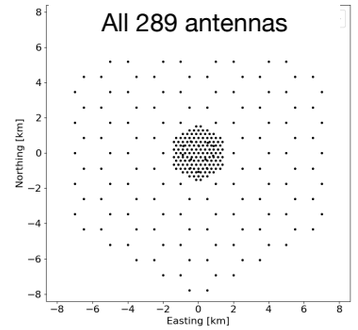


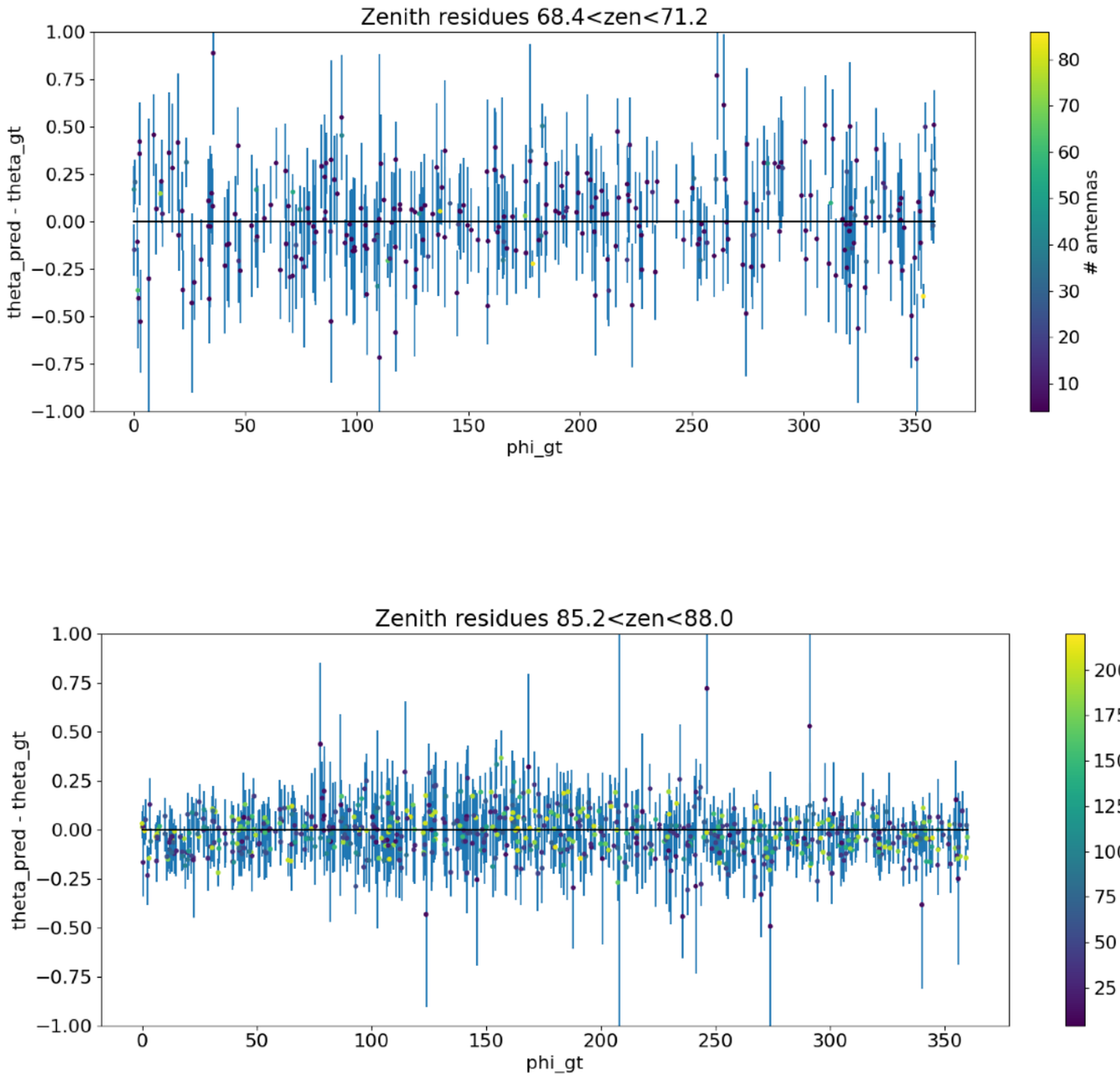


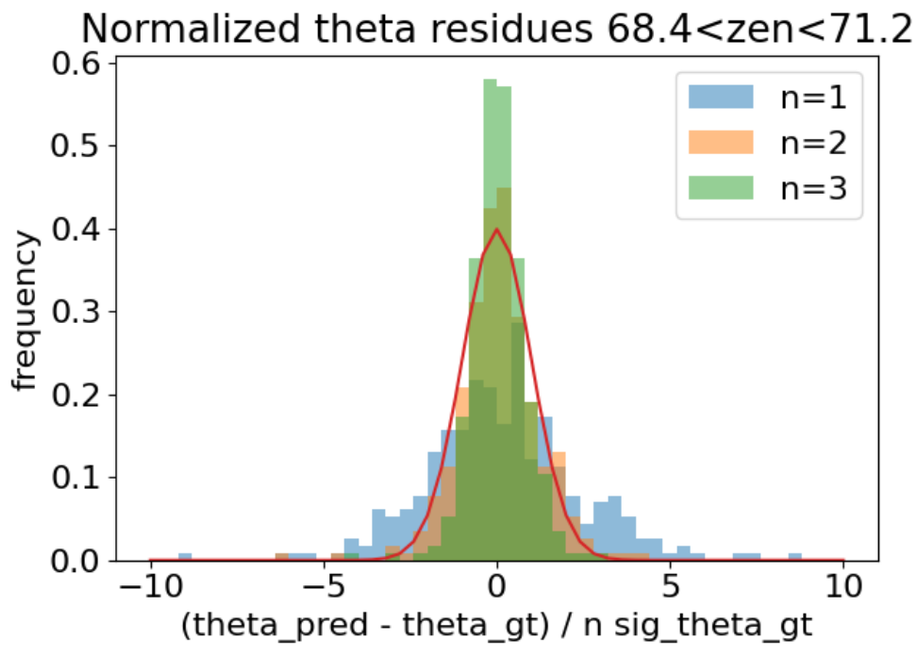
- Theoretical errors from PWF formalism are too small, model is overconfident.
- We can apply a corrective factor (typically ~2 for zenith, and ~3 for azimuth)
- For calibrated errors, we expect the histogram of normalized residues to be normal

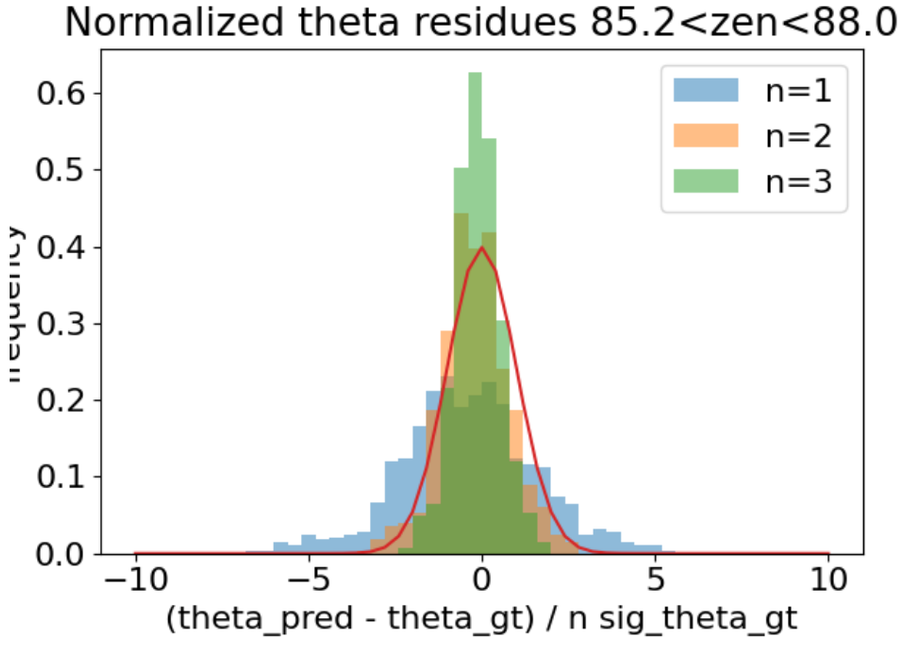
I. Zenith reconstruction

No significant issues for zenith reconstruction, errors can be nicely recalibrated



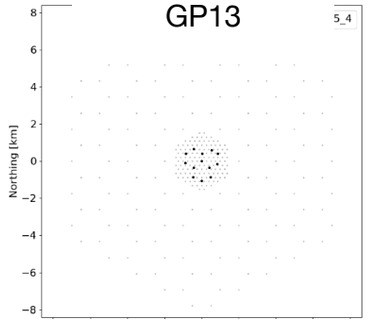


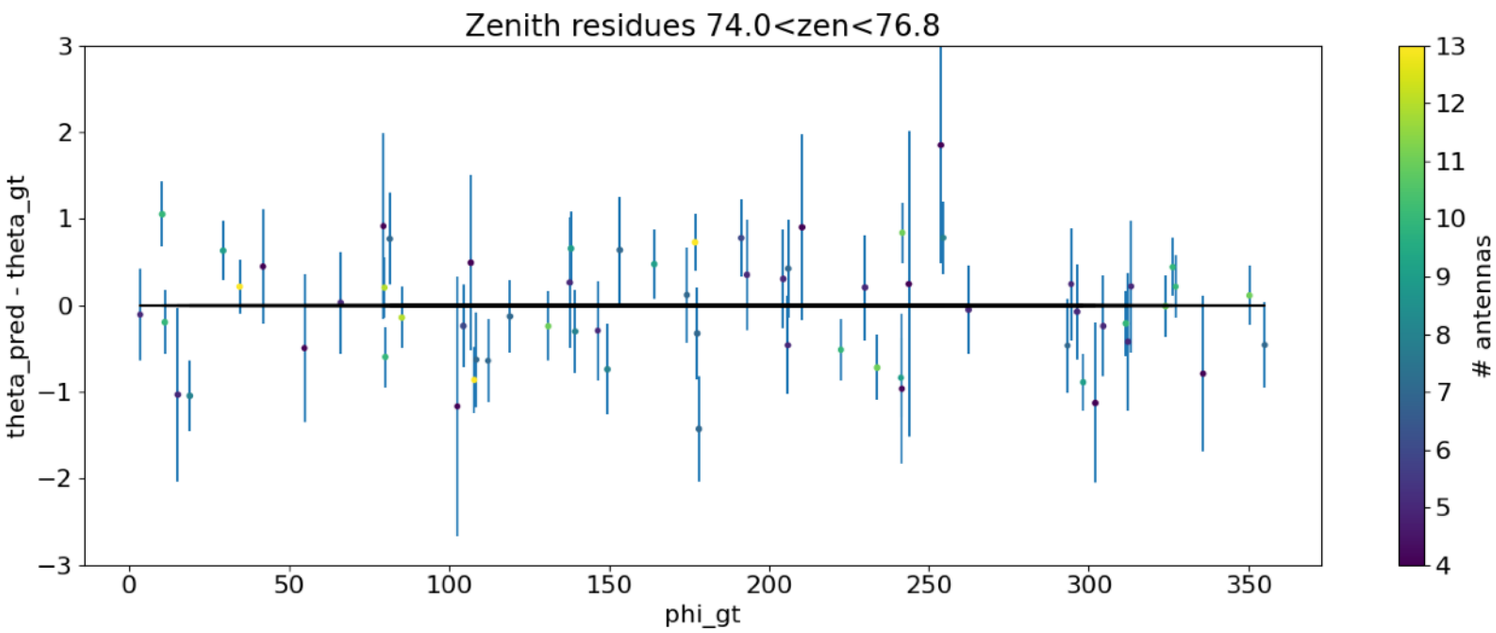


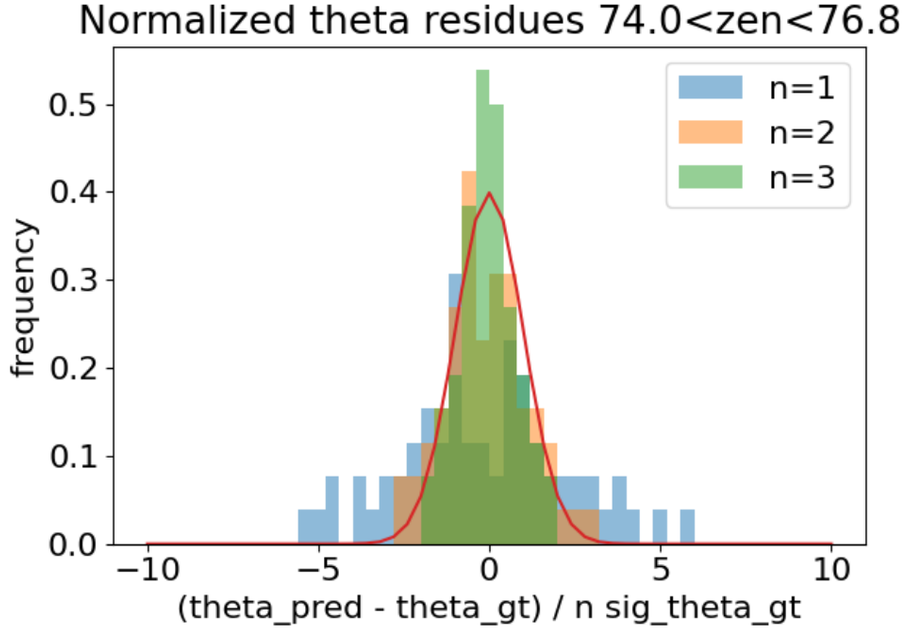


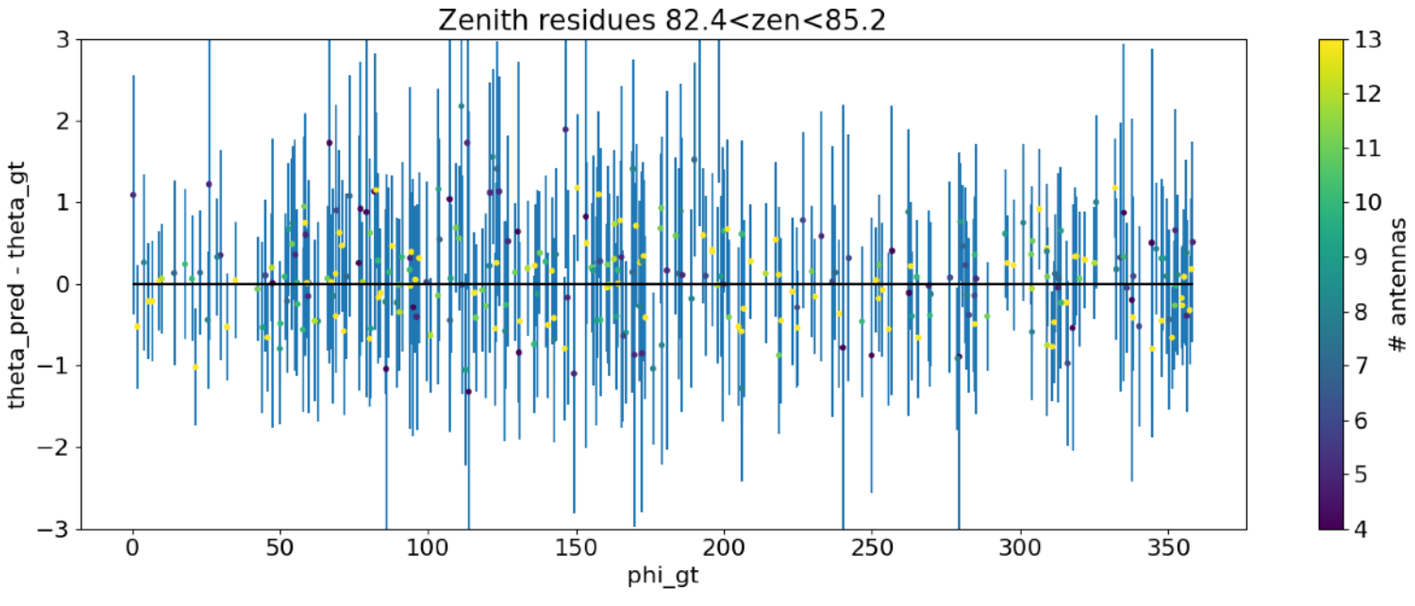
I. Zenith reconstruction

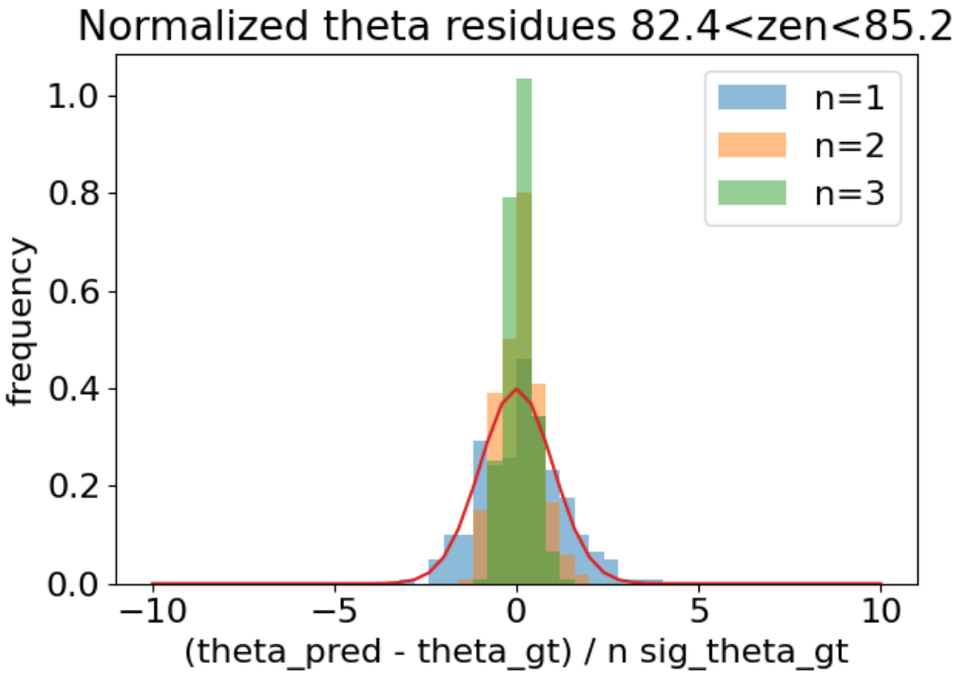
No significant issues for zenith reconstruction, errors can be nicely recalibrated





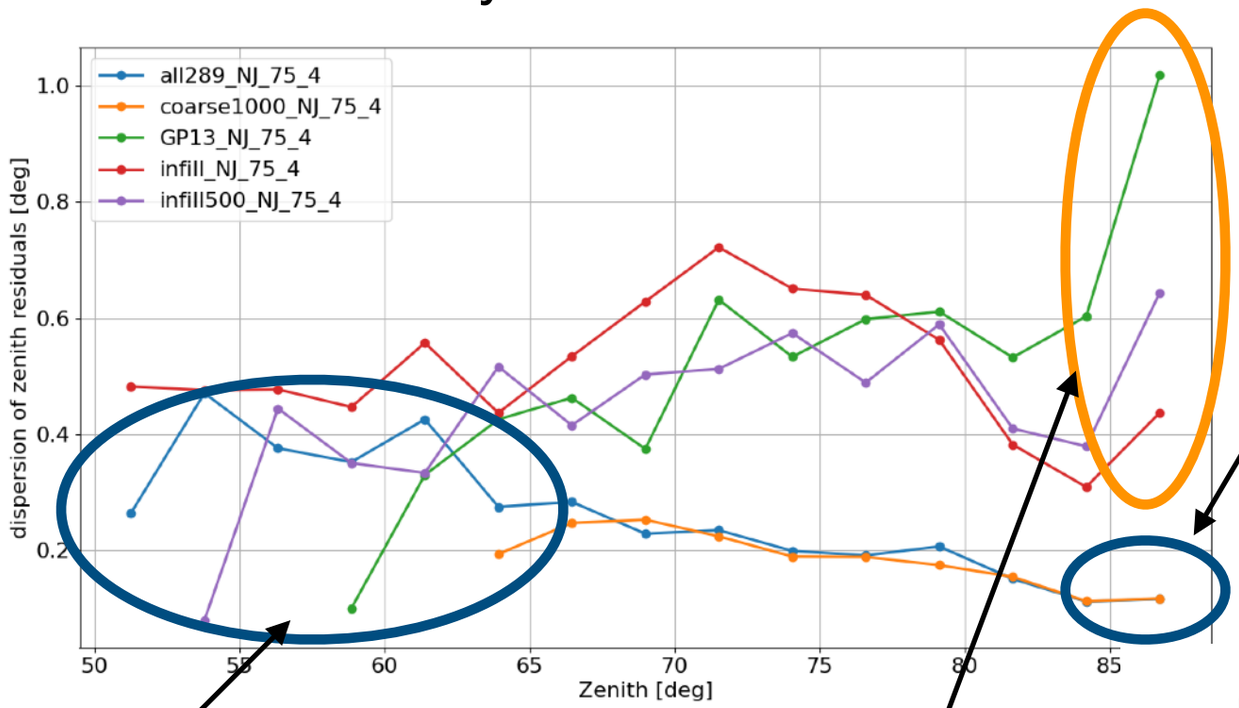






I. Zenith reconstruction

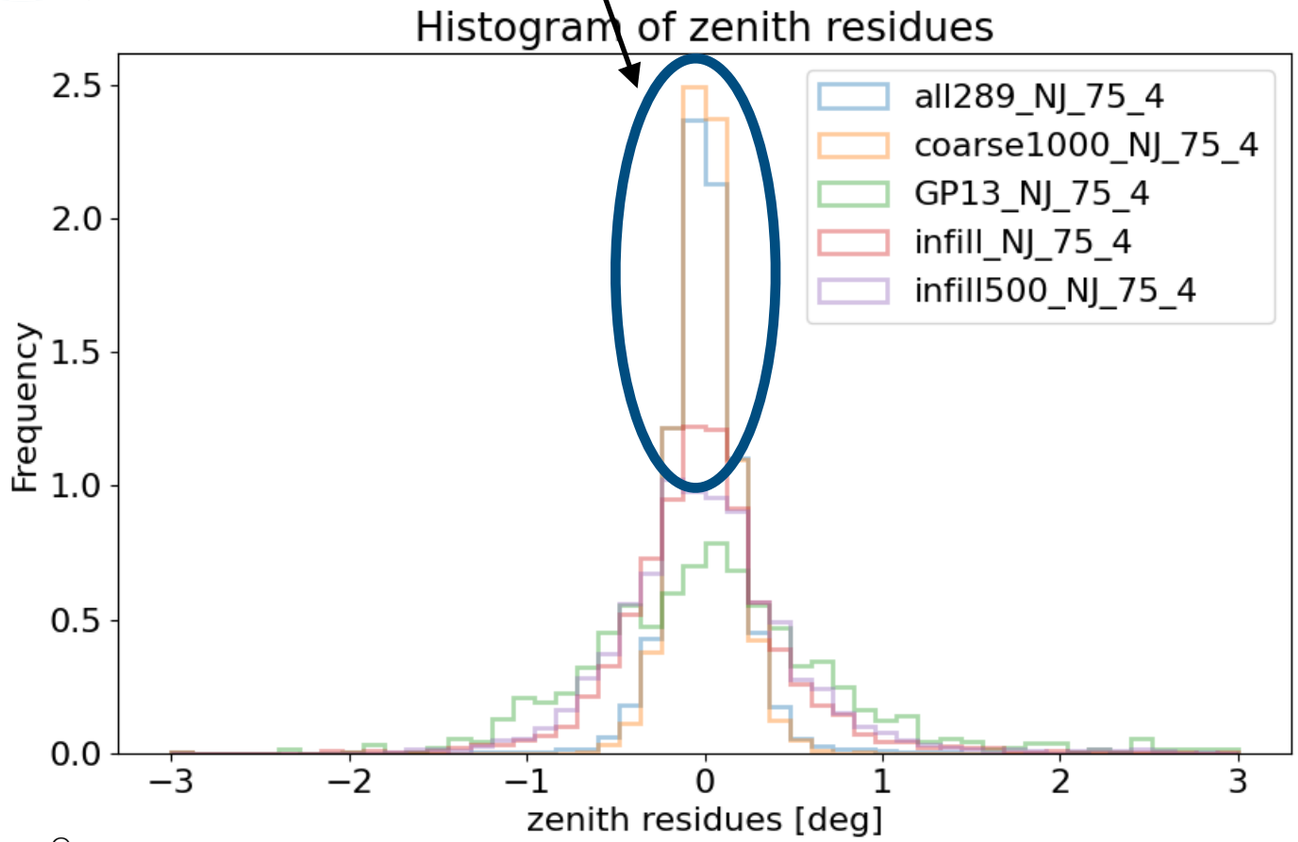
No significant issues for zenith reconstruction, errors can be nicely recalibrated



Infill allows for reconstruction of vertical showers

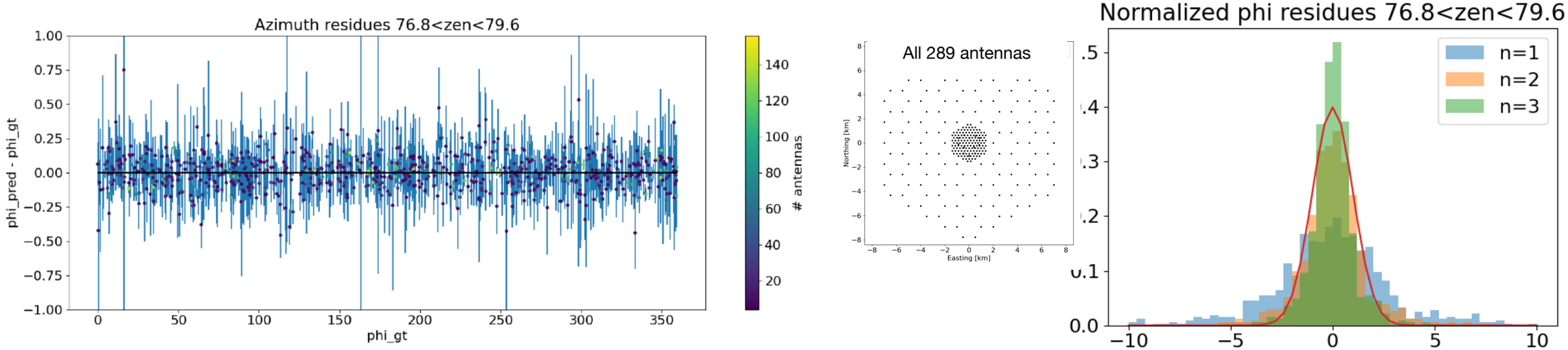
Having only small infill does not permit great zenith reconstruction…

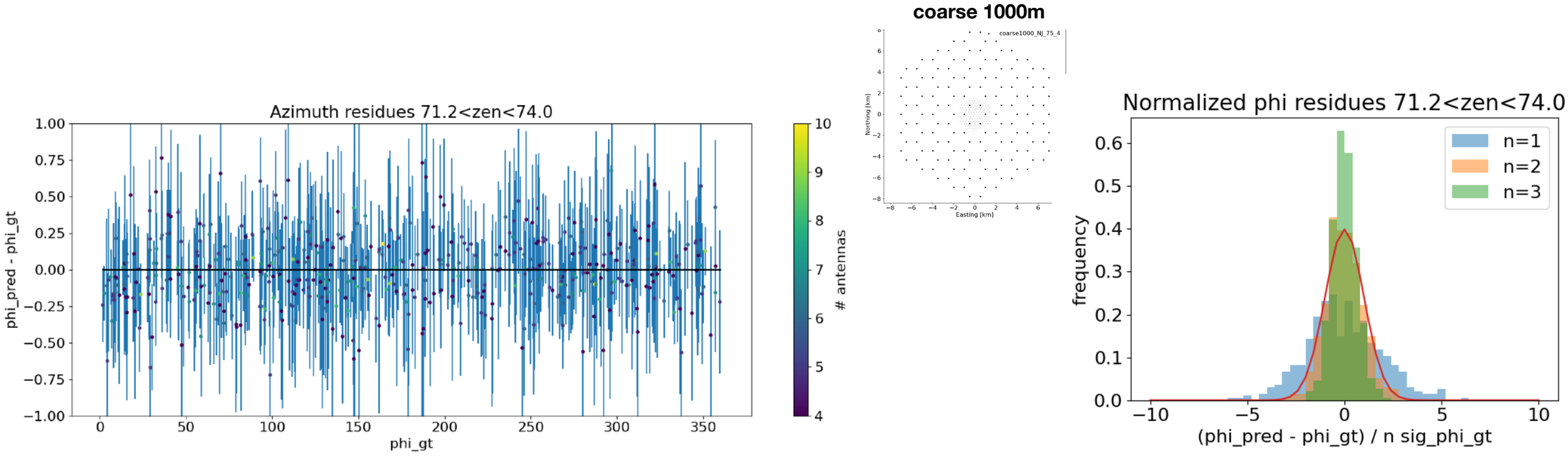
For inclined event, don't need the infill to reconstruct zenith!



II. Azimuth reconstruction

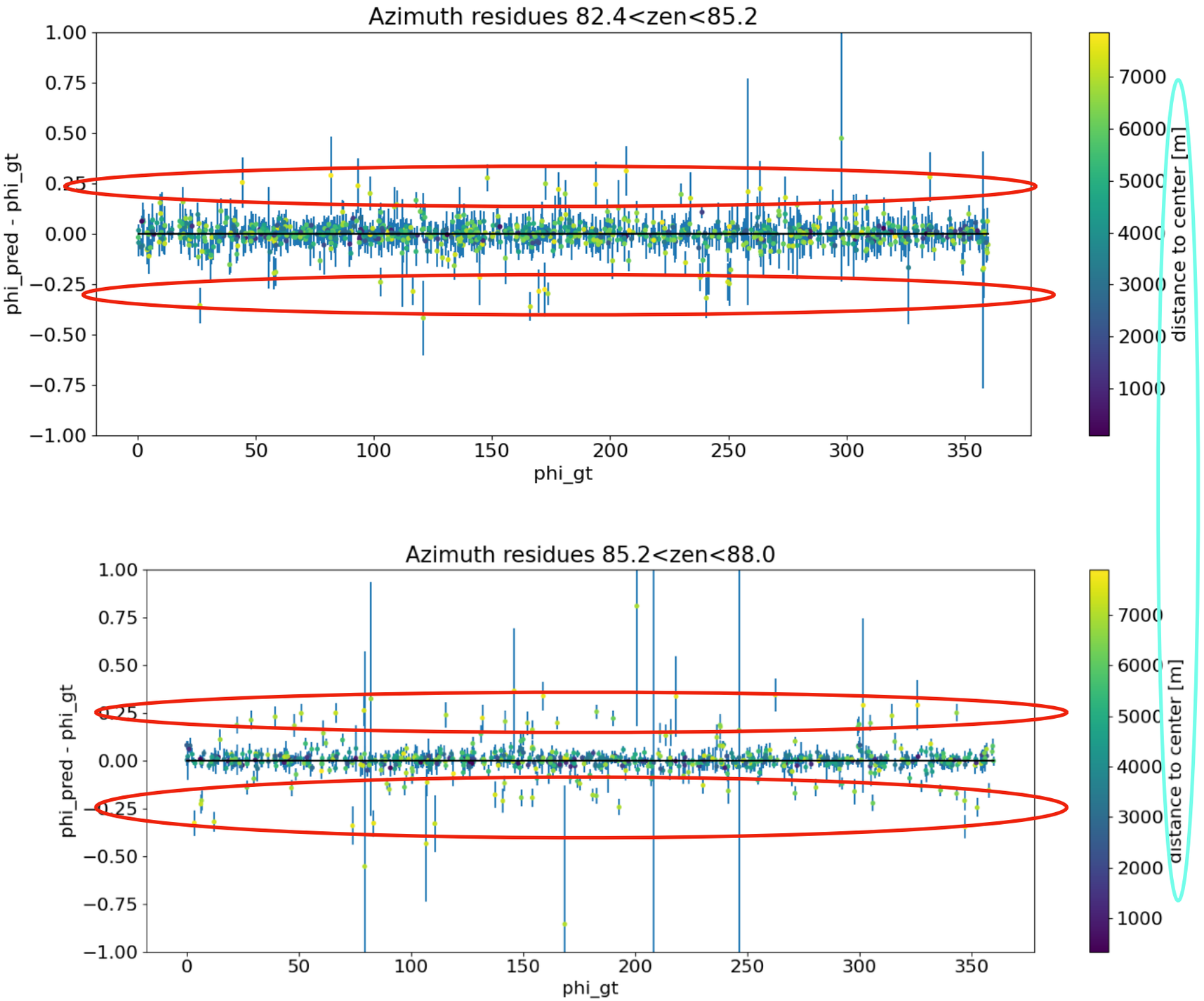
Things are more complicated for azimuth.
Below 75-80 deg, no significant issues

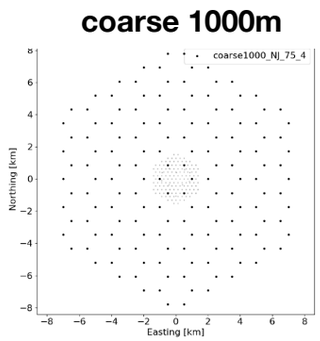




II. Azimuth reconstruction

Things are more complicated for azimuth.
For more inclined showers (the ones we're after), there are some interesting features…

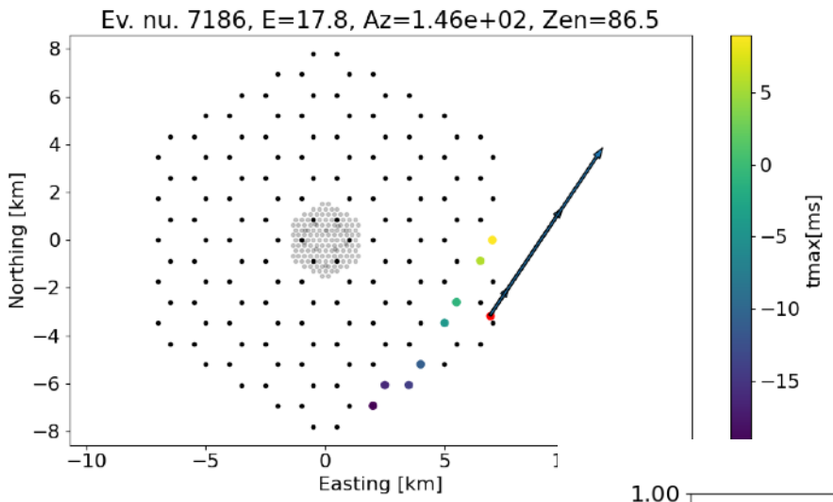


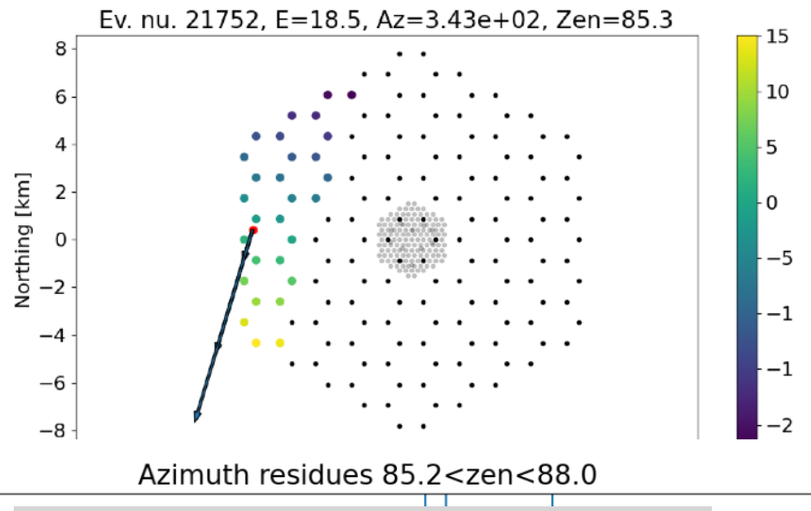


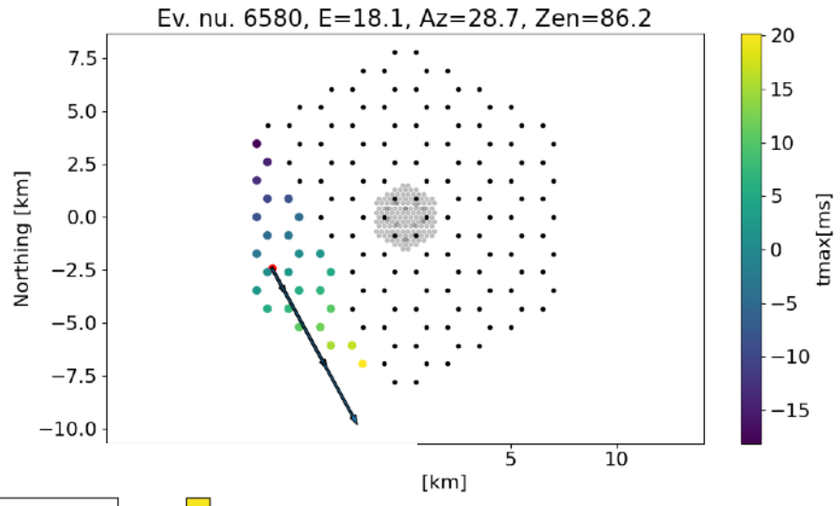
These are event close to the border of the array, for which we only probe one side of the footprint (at best…)

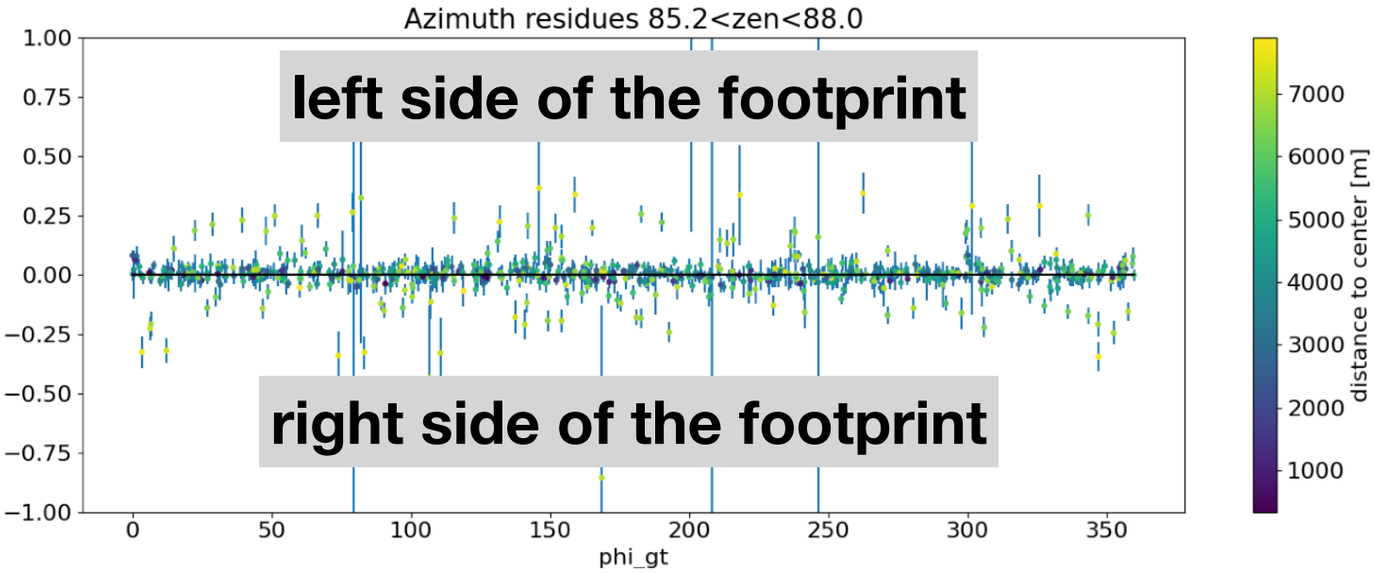
II. Azimuth reconstruction

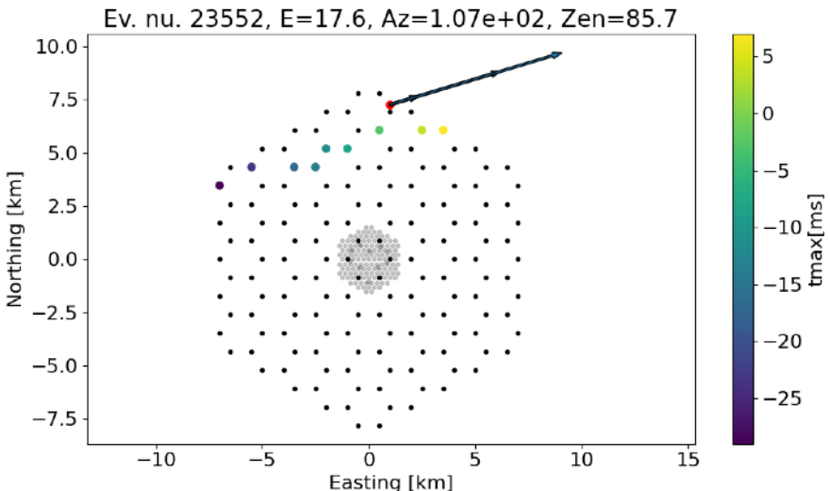
Things are more complicated for azimuth.
For more inclined showers (the ones we're after), there are some interesting features…

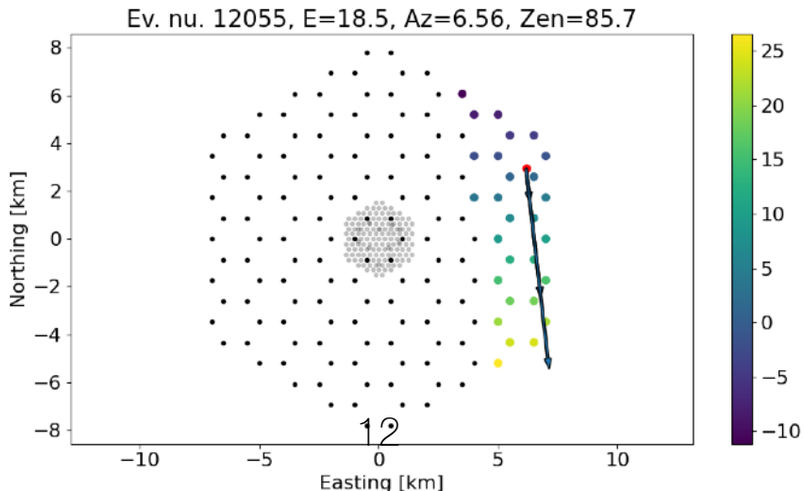


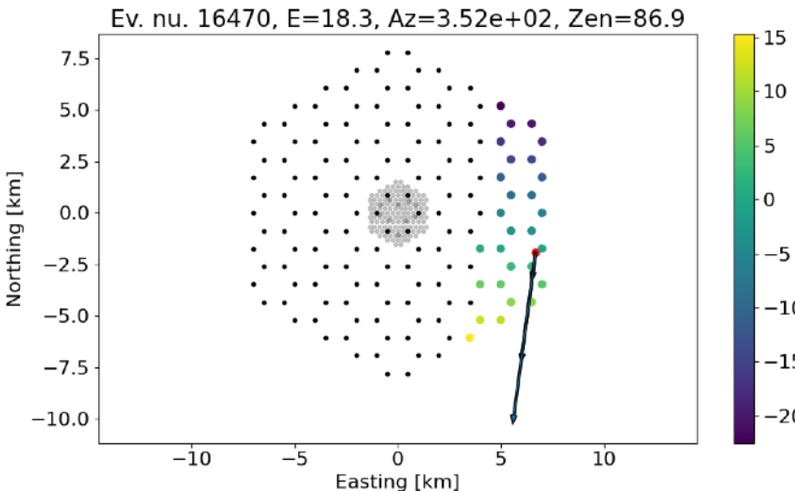






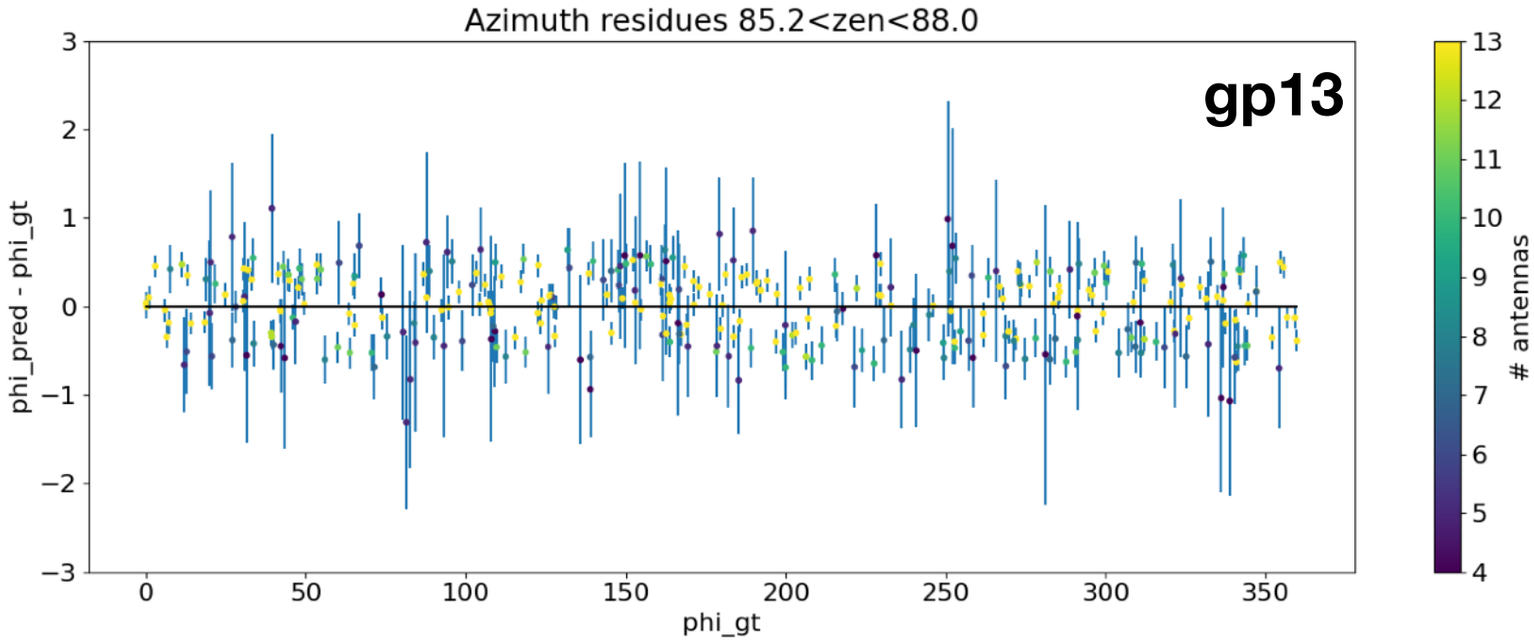


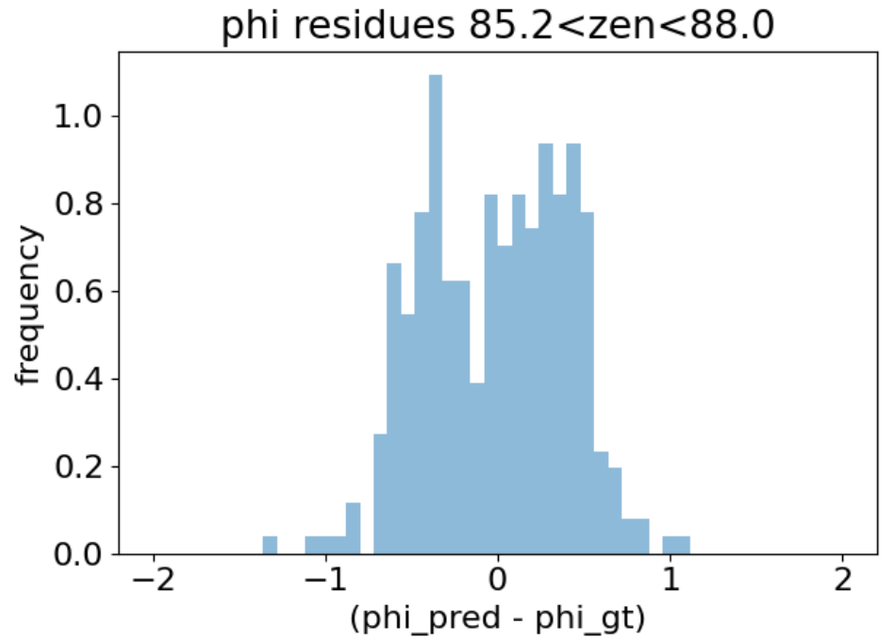


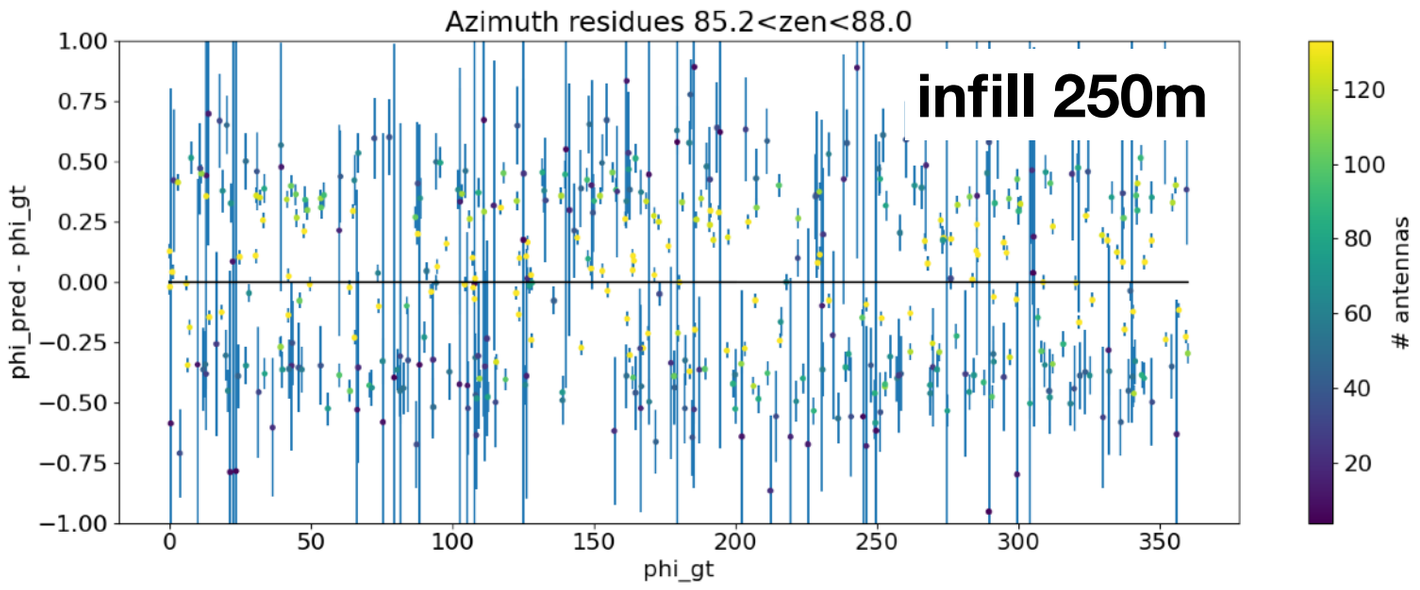


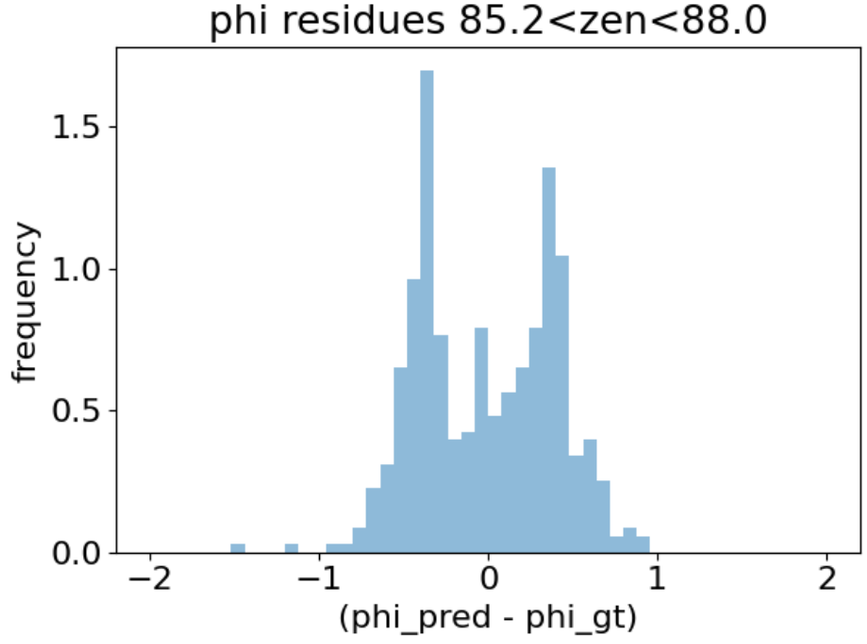
II. Azimuth reconstruction

Things are more complicated for azimuth.
Similar behaviour is seen with small arrays for events that hit the infill on the side





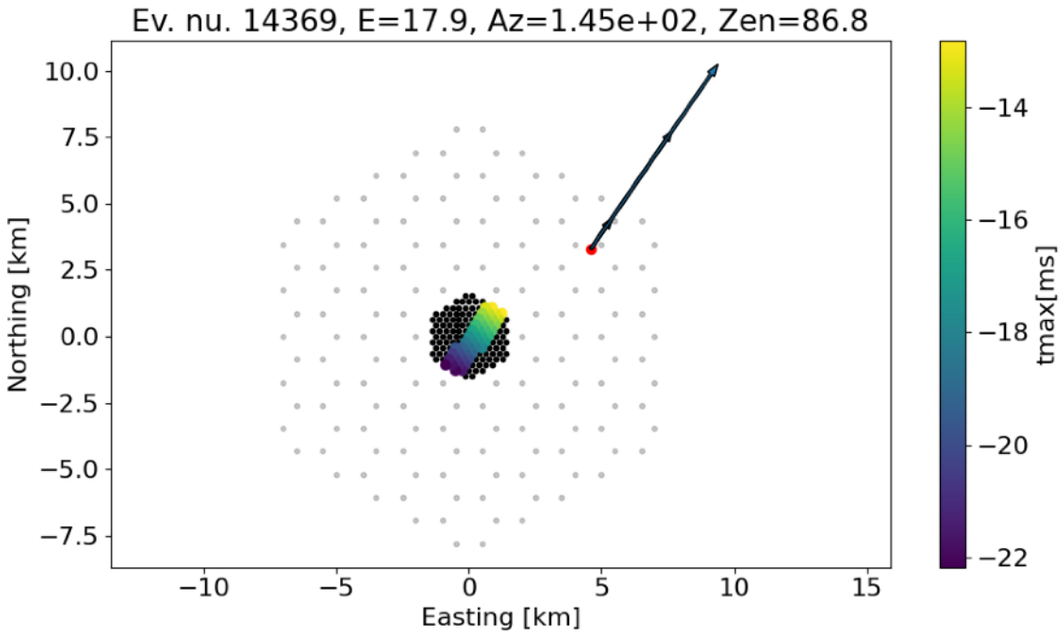


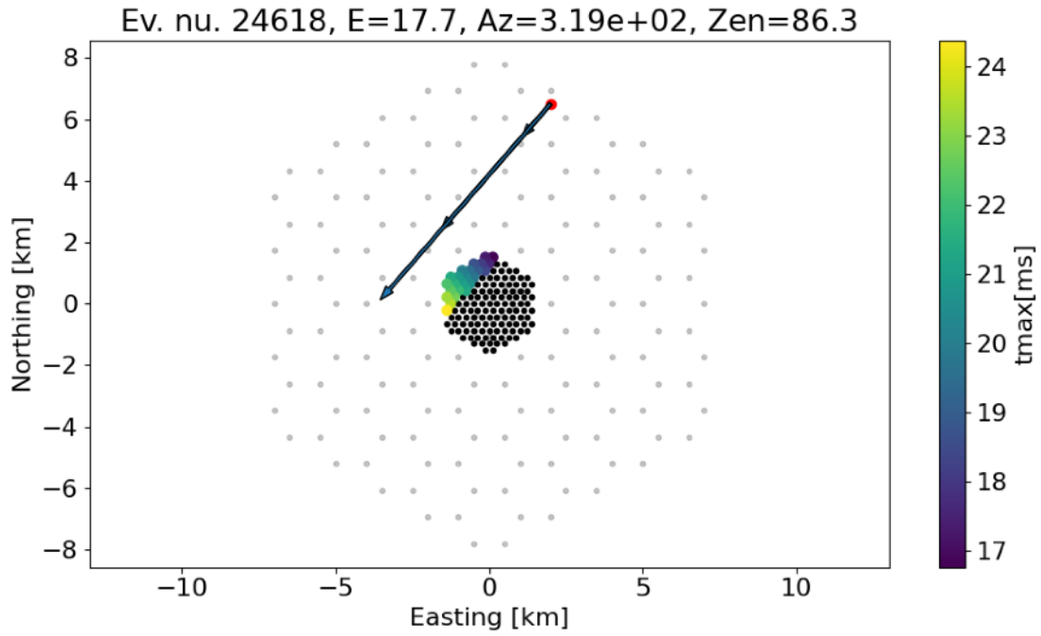


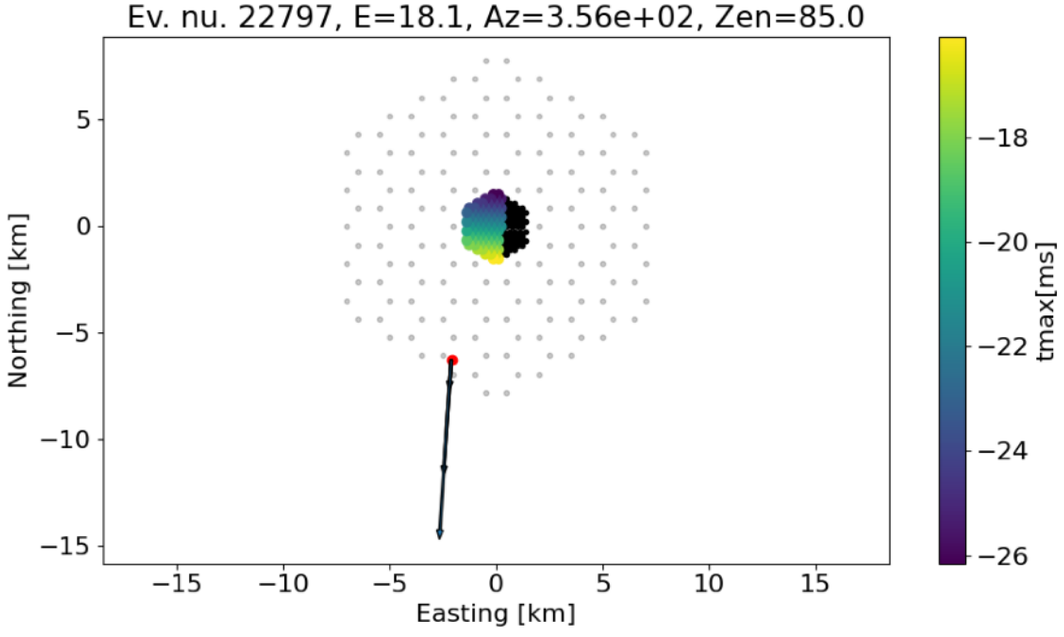
II. Azimuth reconstruction

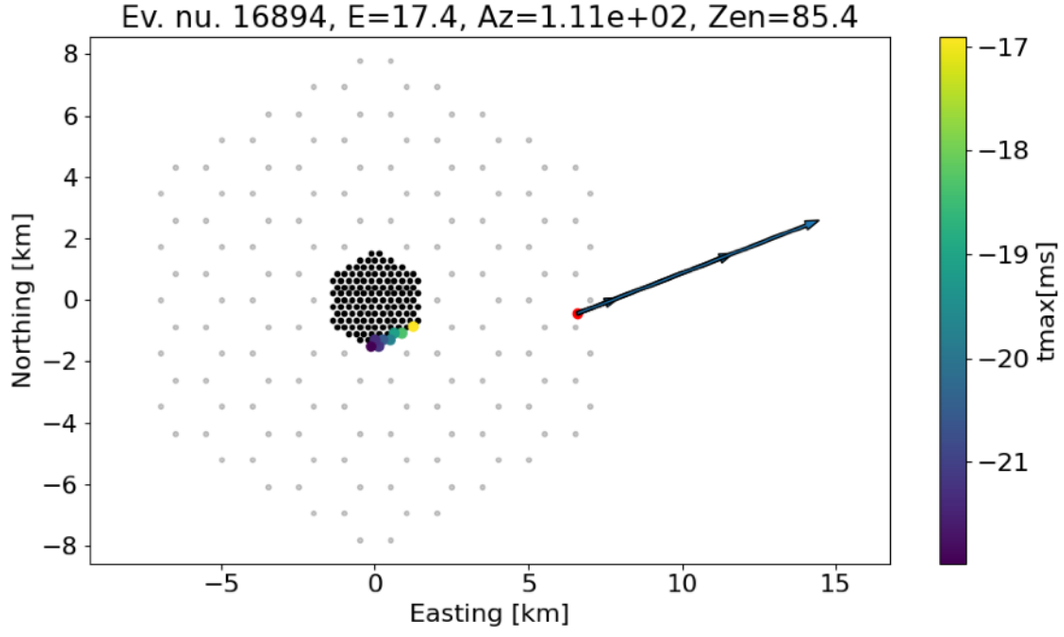
Things are more complicated for azimuth.
Similar behaviour is seen with small arrays for events that hit the infill on the side

infill 250, zen > 85deg, res_theta> 0.3deg: left side of the footprint





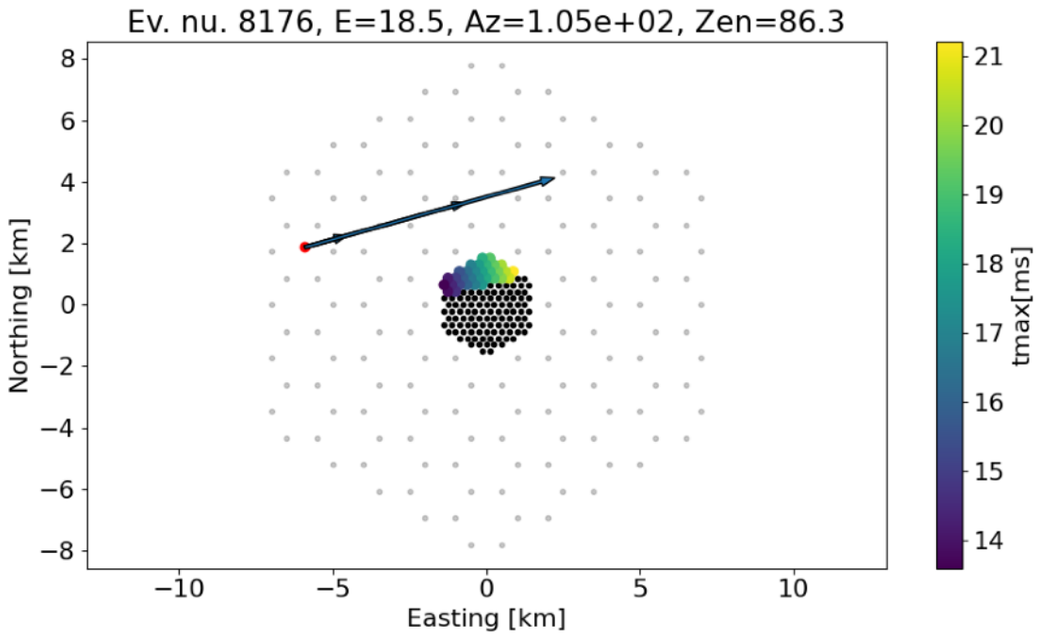


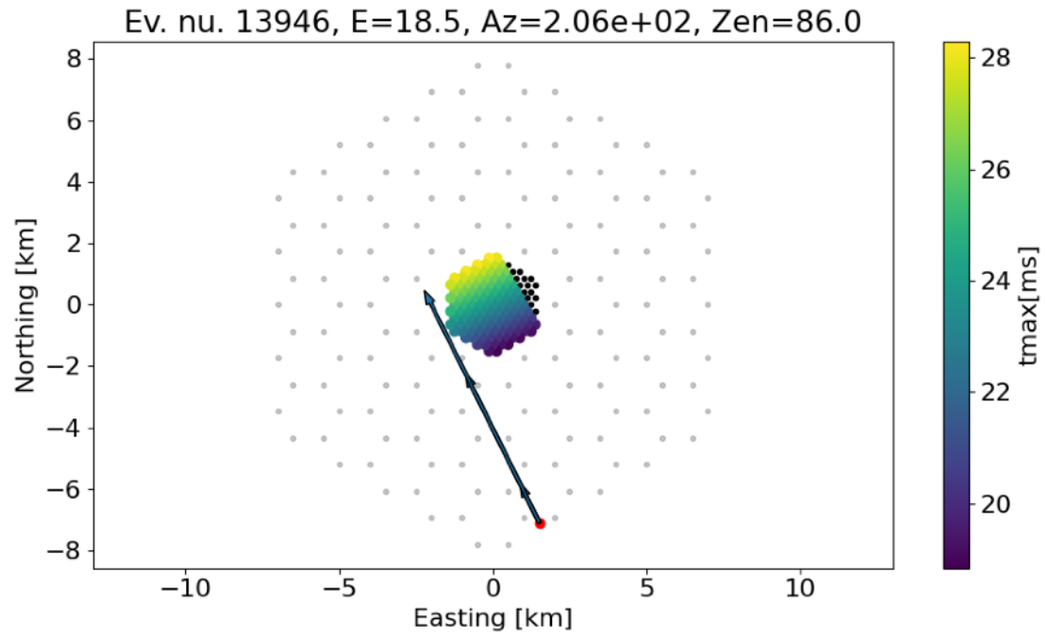


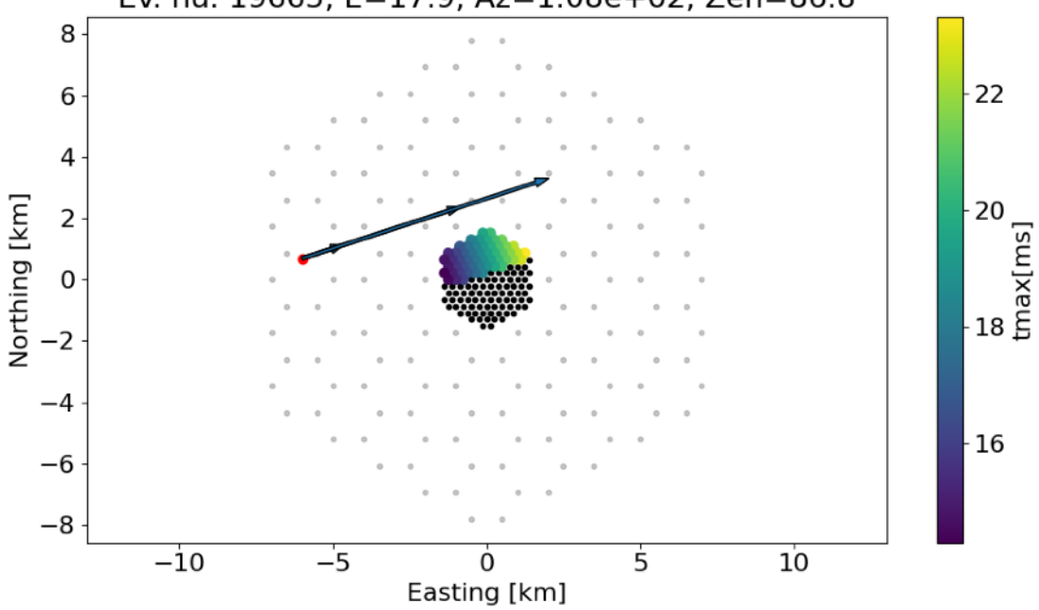
II. Azimuth reconstruction

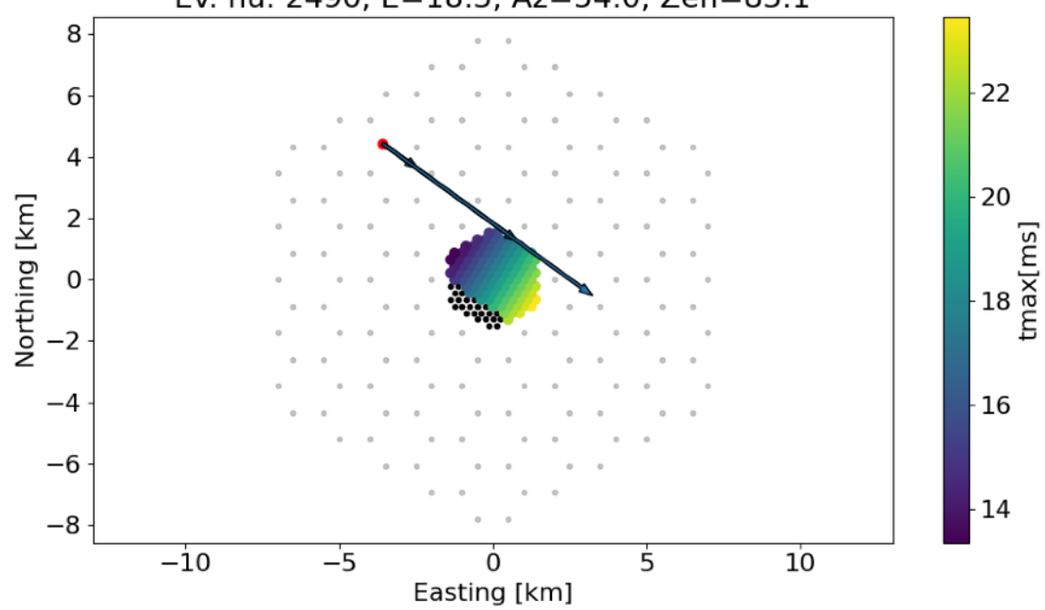
Things are more complicated for azimuth.
Similar behaviour is seen with small arrays for events that hit the infill on the side

infill 250, zen > 85deg, res_theta< -0.3deg: right side of the footprint





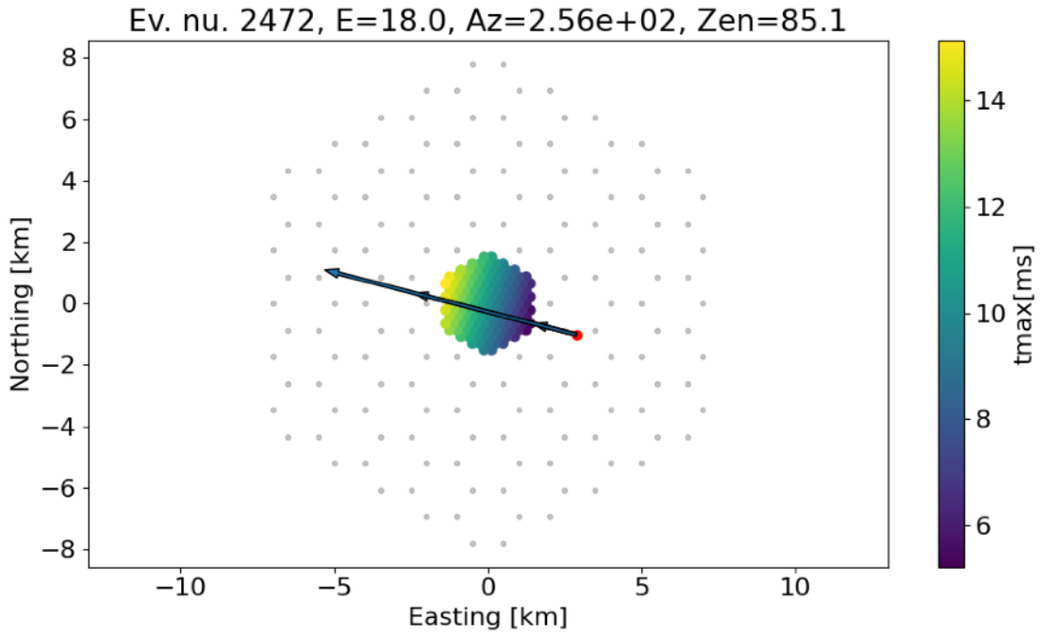


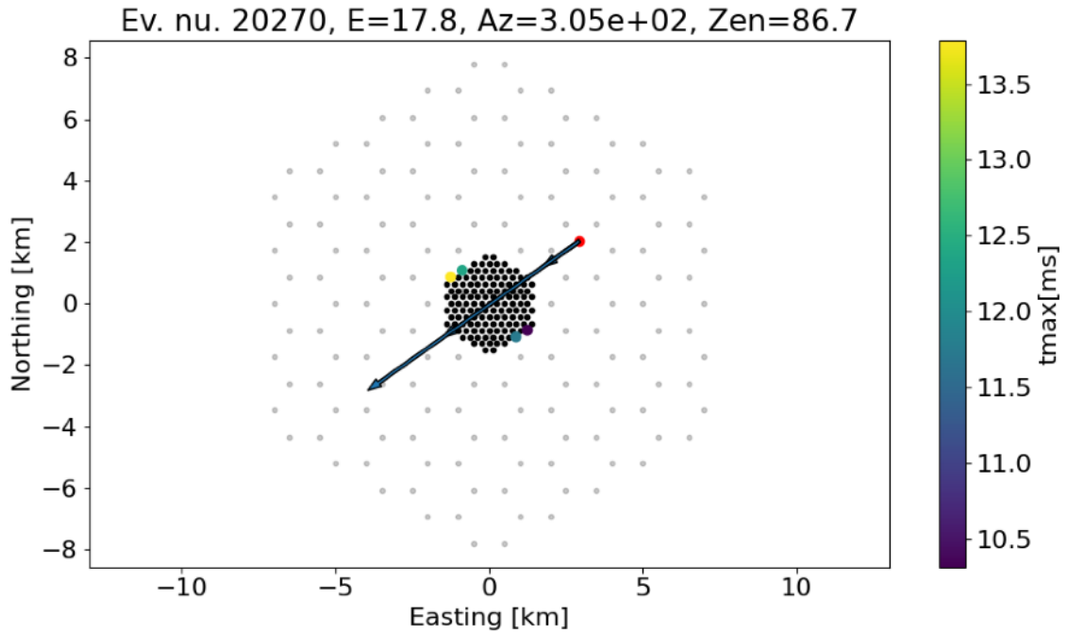


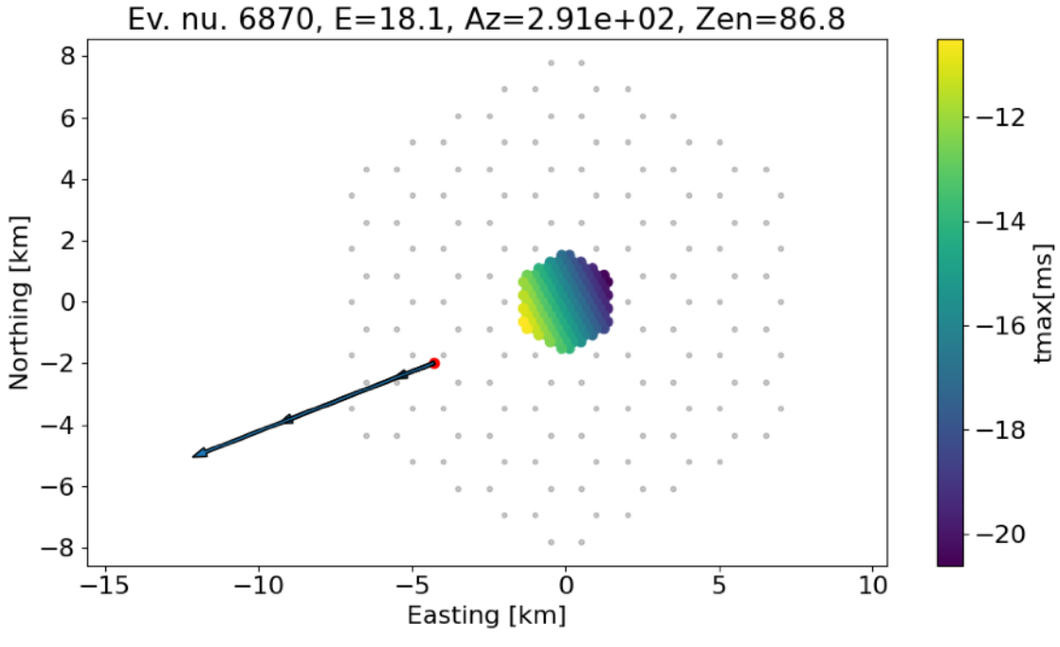
II. Azimuth reconstruction

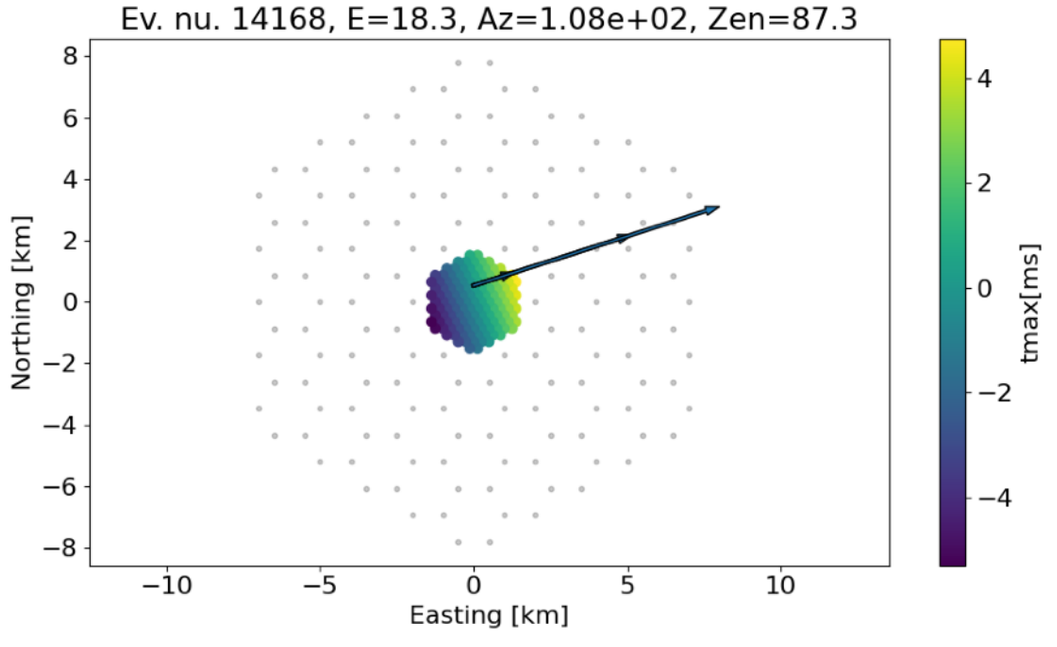
Things are more complicated for azimuth.
Similar behaviour is seen with small arrays for events that hit the infill on the side

infill 250, zen > 85deg, |res_theta|< 0.1deg: right in the middle!



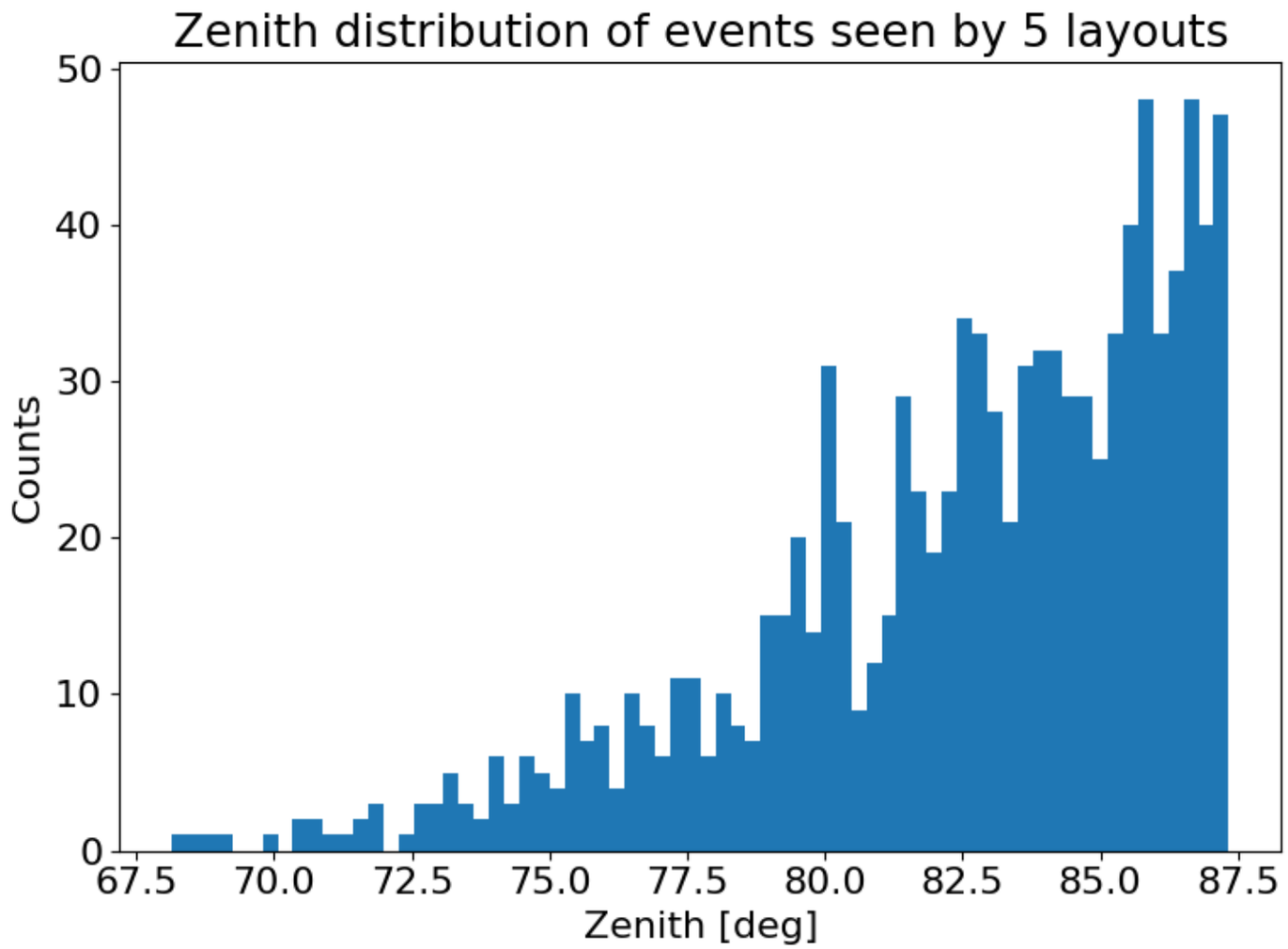






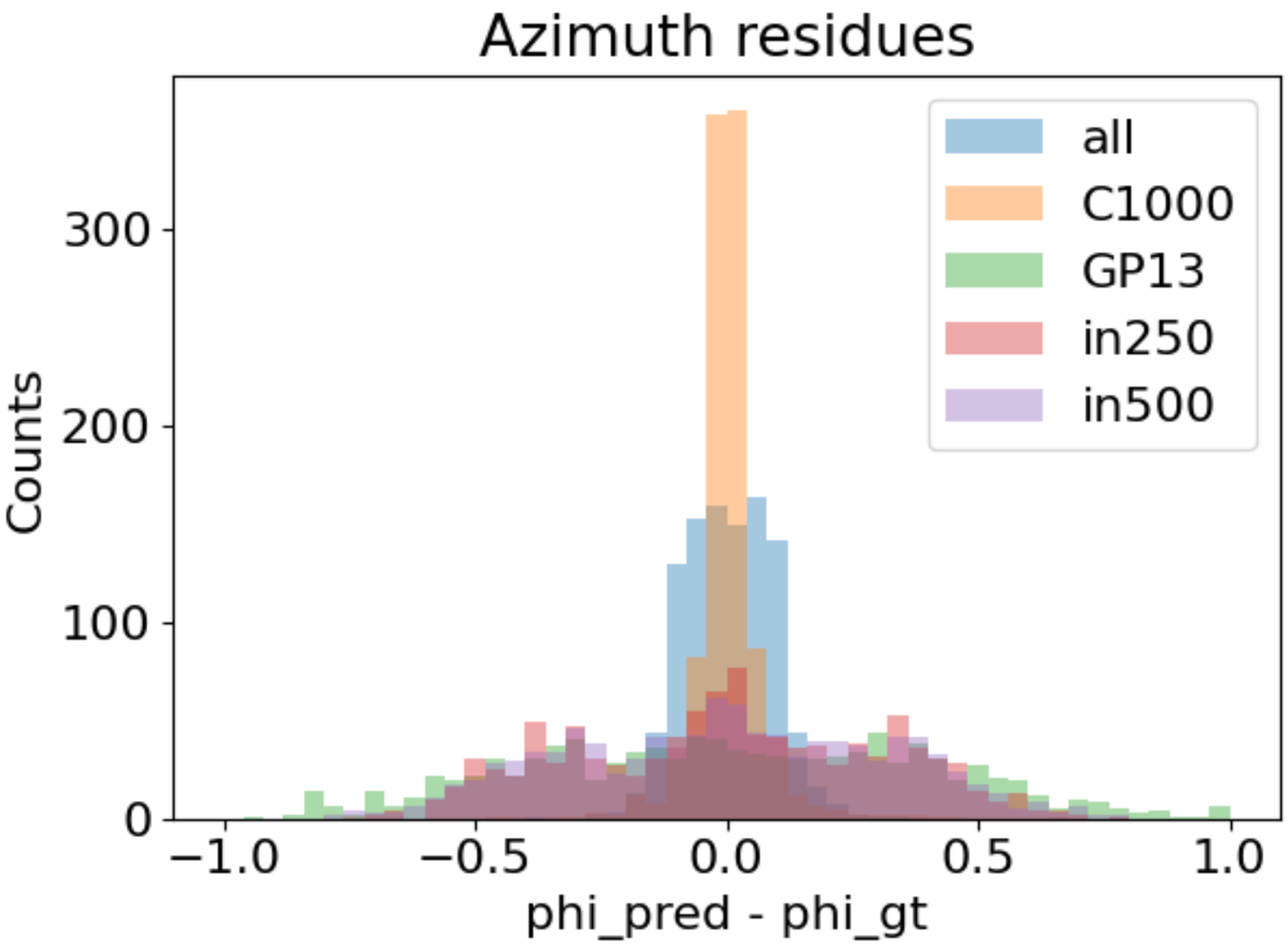
III. Events which are seen by the 5 layouts (1029)

Things are more complicated for azimuth.
Interesting result?



III. Events which are seen by the 5 layouts (1029)

Things are more complicated for azimuth.
Interesting result?



- Small layout have « bad » residues (green, red, purple)
- They get significantly improved by adding the 1000m coarse antennas (blue)
- Residues further improved by removing the infill (orange!)

Tentative conclusion

- Very first study with DC2 and PWF modeling
- Zenith angle corretly reconstructed
- Indication that PWF need full footprint to recover azimuth at high zenith
- Does ADF have the same issue?
- Need to think this through for GP80 layout studies