Homework_2

Hamed

2/12/2020

1. Load the required packages and the cpus dataset from the MASS package

```
## -- Attaching packages --------------------------------------- tidyverse
1.3.0 --

## <U+2713> ggplot2 3.2.1     <U+2713> purrr   0.3.3
## <U+2713> tibble  2.1.3     <U+2713> dplyr   0.8.3
## <U+2713> tidyr   1.0.0     <U+2713> stringr 1.4.0
## <U+2713> readr   1.3.1     <U+2713> forcats 0.4.0

## -- Conflicts -----------------------------------------
tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x dplyr::select() masks MASS::select()

## Loading required package: lattice

##
## Attaching package: 'caret'

## The following object is masked from 'package:purrr':
##
##     lift

##             name syct mmin  mmax cach chmin chmax perf estperf
## 1  ADVISOR 32/60  125  256  6000  256    16   128  198     199
## 2  AMDAHL 470V/7   29 8000 32000   32     8    32  269     253
## 3  AMDAHL 470/7A   29 8000 32000   32     8    32  220     253
## 4 AMDAHL 470V/7B   29 8000 32000   32     8    32  172     253
## 5 AMDAHL 470V/7C   29 8000 16000   32     8    16  132     132
## 6  AMDAHL 470V/8   26 8000 32000   64     8    32  318     290
```

2. Use syct, mmin , mmax , cach , chmin, chmax as the predictors (independent variables) to predict performance (perf).

From the model output we use the p-value to check the best predictors in the model, condition: (p-value < 0.05).

```
my_model<-lm(perf~syct + mmin + mmax + cach + chmin + chmax, data = cpus,
nvmax = 6)
summary(my_model)
```

```
##
## Call:
## lm(formula = perf ~ syct + mmin + mmax + cach + chmin + chmax,
##     data = cpus, nvmax = 6)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -195.84  -25.17    5.41   26.53  385.75
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.590e+01  8.045e+00  -6.948 4.99e-11 ***
## syct         4.886e-02  1.752e-02   2.789  0.00579 **
## mmin         1.529e-02  1.827e-03   8.371 9.42e-15 ***
## mmax         5.571e-03  6.418e-04   8.680 1.33e-15 ***
## cach         6.412e-01  1.396e-01   4.594 7.64e-06 ***
## chmin       -2.701e-01  8.557e-01  -0.316  0.75263
## chmax        1.483e+00  2.201e-01   6.738 1.64e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 59.99 on 202 degrees of freedom
## Multiple R-squared:  0.8649, Adjusted R-squared:  0.8609
## F-statistic: 215.5 on 6 and 202 DF,  p-value: < 2.2e-16
```

3. Perform best subset selection in order to choose the best predictors from the above predictors. What is the best model obtained according to Cp, BIC, and adjusted R2?

```
models <- regsubsets(perf~syct + mmin + mmax + cach + chmin + chmax,
                     data = cpus, nvmax = 6)
summary(models)
```

```
## Subset selection object
## Call: regsubsets.formula(perf ~ syct + mmin + mmax + cach + chmin +
##     chmax, data = cpus, nvmax = 6)
## 6 Variables  (and intercept)
##       Forced in Forced out
## syct      FALSE      FALSE
## mmin      FALSE      FALSE
## mmax      FALSE      FALSE
## cach      FALSE      FALSE
## chmin     FALSE      FALSE
## chmax     FALSE      FALSE
## 1 subsets of each size up to 6
## Selection Algorithm: exhaustive
##          syct mmin mmax cach chmin chmax
## 1  ( 1 ) " "  " "  "*"  " "  " "   " "
## 2  ( 1 ) " "  " "  "*"  "*"  " "   " "
## 3  ( 1 ) " "  "*"  "*"  " "  " "   "*"
## 4  ( 1 ) " "  "*"  "*"  "*"  " "   "*"
```

```
## 5  ( 1 ) "*"  "*"  "*"  "*"  " "   "*"
## 6  ( 1 ) "*"  "*"  "*"  "*"  "*"   "*"

res.sum <- summary(models)
```

Displays the CP values at each predictor number from 1-predictor to 6-predictor

```
res.sum$cp

## [1] 176.563616  95.808585  28.225948  10.977588   5.099604   7.000000
```

Displays the BIC values at each predictor number from 1-predictor to 6-predictor

```
res.sum$bic

## [1] -274.7146 -320.4675 -370.5300 -383.5185 -386.1684 -380.9290
```

This shows the number of predictors that BIC, CP, ADJ.R2 support as best subset is the one with lowest BIC, CP, ADJ.R2 values.

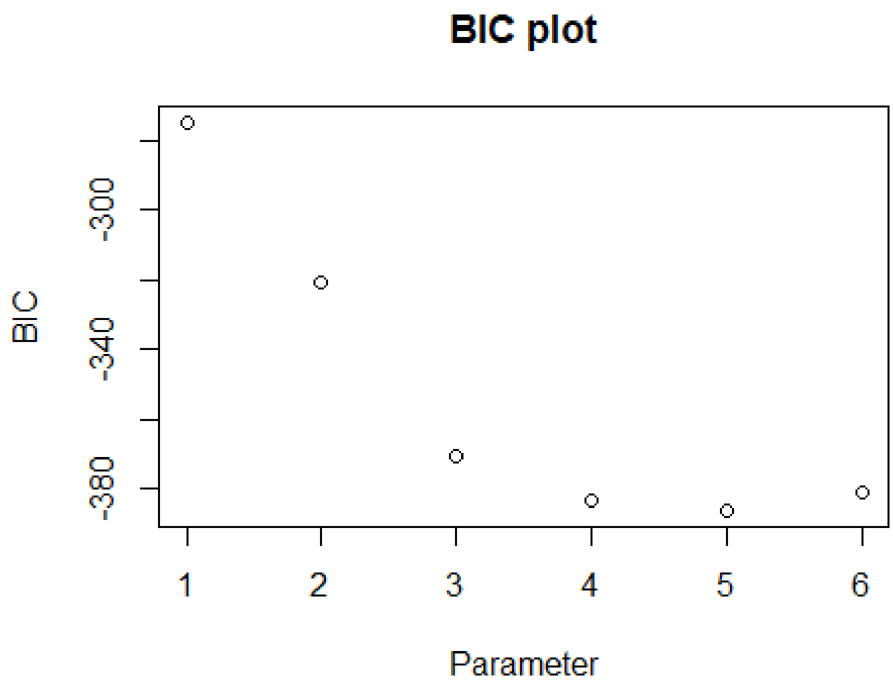
```
data.frame(
  Adj.R2 = which.max(res.sum$adjr2),
  CP = which.min(res.sum$cp),
  BIC = which.min(res.sum$bic)
)

##   Adj.R2 CP BIC
## 1      5  5   5
```

- The function summary() reports the best set of variables for each model size. From the output, an asterisk specifies that a given variable is included in the corresponding model.
- For example, it can be seen that the best 2-variables model contains only mmax and cach variables (perf ~ mmax + cach). The best three-variable model is (perf ~ mmax + cach + mmin), and so forth.
- As shown above, adjusted R2,BIC and Cp criteria, tells us that the best model is the one with 5 predictor variables.
- A natural question is: which of these best models should we finally choose for our predictive analytics?

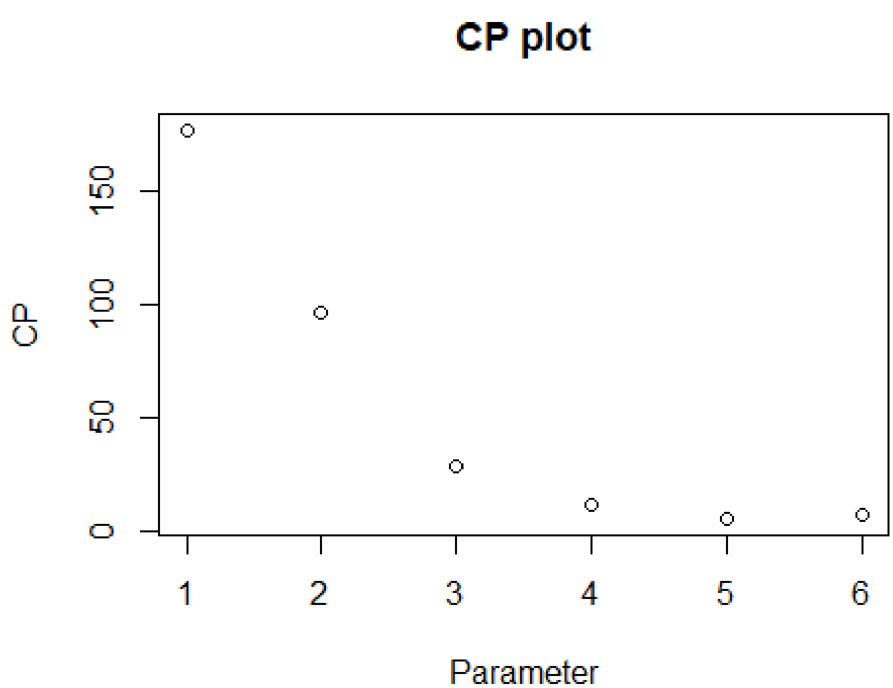
4. Show some plots to provide evidence for your answer, and report the coefficients of the best model obtained for each criteria.

- The plots show that the BIC reduces as the number of parameters increase upto 5 then it becomes constant. The same goes for CP.

```
plot(res.sum$bic, xlab="Parameter", ylab="BIC",main="BIC plot")
```




```r
plot(res.sum$cp, xlab="Parameter", ylab="CP",main = "CP plot")
```



5. Repeat using forward stepwise selection and also using backwards stepwise selection.How does your answer compare to the best subset results?

- nvmax: the number of variable in the model. For example nvmax = 2, specify the best 2-variables model
- RMSE and MAE are two different metrics measuring the prediction error of each model. The lower the RMSE and MAE, the better the model.
- Rsquared indicates the correlation between the observed outcome values and the values predicted by the model. The higher the R squared, the better the model.

Fit the full model to show the performance before subset selection

```
library(MASS)
full.model <- lm(perf ~syct + mmin + mmax + cach + chmin + chmax, data =
cpus)
full.model

##
## Call:
## lm(formula = perf ~ syct + mmin + mmax + cach + chmin + chmax,
##     data = cpus)
##
## Coefficients:
## (Intercept)         syct         mmin         mmax         cach
chmin
##  -55.900116     0.048863     0.015294     0.005571     0.641207    -
0.270065
##       chmax
##    1.482694
```

Model information

- Specify the tuning parameter nvmax, which corresponds to the maximum number of predictors to be incorporated in the model.
- For example, you can vary nvmax from 1 to 5. In this case, the function starts by searching different best models of different size, up to the best 5-variables model.
- That is, it searches the best 1-variable model, the best 2-variables model, ..., the best 5-variables models.
- As the data set contains only 6 predictors, we'll vary nvmax from 1 to 6 resulting to the identification of the 6 best models with different sizes: the best 1-variable model, the best 2-variables model, ..., the best 6-variables model.
- We'll use 10-fold cross-validation to estimate the average prediction error (RMSE) of each of the 6 models
- The output of the final model has predictors selected based on the number of asterics , the more the better.

Forward selection

```
# Set seed for reproducibility
set.seed(123)
```

```r
# Set up repeated k-fold cross-validation
train.control <- trainControl(method = "cv", number = 10)
# Train the model

step.model <- train(perf ~syct + mmin + mmax + cach + chmin + chmax, data =
cpus,
                    method = "leapForward",
                    tuneGrid = data.frame(nvmax = 1:6),
                    trControl = train.control
)
step.model$results

##   nvmax     RMSE  Rsquared      MAE   RMSESD RsquaredSD    MAESD
## 1     1 93.46400 0.6488552 56.59877 60.38499  0.1779760 22.54073
## 2     2 89.08646 0.7168064 53.65864 59.70345  0.1501266 21.52435
## 3     3 83.01186 0.7670026 48.96523 59.63821  0.1347547 21.51615
## 4     4 74.41985 0.8004094 45.31268 49.53468  0.1247854 17.23585
## 5     5 72.35948 0.8086084 44.53950 48.65457  0.1051621 17.19711
## 6     6 72.78049 0.8063649 44.82444 47.12075  0.1081818 16.66283
```

Here we show the number of predictors the best subset will have after forward selection

```r
step.model$bestTune

##   nvmax
## 5     5
```

The final model after forward selection showing the importance of each predictor.

```r
summary(step.model$finalModel)

## Subset selection object
## 6 Variables  (and intercept)
##       Forced in Forced out
## syct      FALSE      FALSE
## mmin      FALSE      FALSE
## mmax      FALSE      FALSE
## cach      FALSE      FALSE
## chmin     FALSE      FALSE
## chmax     FALSE      FALSE
## 1 subsets of each size up to 5
## Selection Algorithm: forward
##          syct mmin mmax cach chmin chmax
## 1  ( 1 ) " "  " "  "*"  " "  " "   " "
## 2  ( 1 ) " "  " "  "*"  "*"  " "   " "
## 3  ( 1 ) " "  "*"  "*"  "*"  " "   " "
## 4  ( 1 ) " "  "*"  "*"  "*"  " "   "*"
## 5  ( 1 ) "*"  "*"  "*"  "*"  " "   "*"
```

Backward selection

```
set.seed(123)
# Set up repeated k-fold cross-validation
train.control <- trainControl(method = "cv", number = 10)
# Train the model

# Train the model
step.model2 <- train(perf ~syct + mmin + mmax + cach + chmin + chmax, data =
cpus,
                    method = "leapBackward",
                    tuneGrid = data.frame(nvmax = 1:6),
                    trControl = train.control)
step.model2$results

##   nvmax     RMSE  Rsquared      MAE   RMSESD RsquaredSD    MAESD
## 1     1 90.67875 0.6109622 53.41392 62.55296  0.2355191 24.00383
## 2     2 88.89009 0.7007681 52.00267 62.55203  0.1235651 23.38322
## 3     3 76.09071 0.7765757 46.44309 60.69159  0.1381791 23.17784
## 4     4 74.41985 0.8004094 45.31268 49.53468  0.1247854 17.23585
## 5     5 72.35948 0.8086084 44.53950 48.65457  0.1051621 17.19711
## 6     6 72.78049 0.8063649 44.82444 47.12075  0.1081818 16.66283
```

Here we show the number of predictors the best subset will have after Backward selection.

```
step.model2$bestTune

##   nvmax
## 5     5
```

The final model after after Backward selection showing the importance of each predictor.

```
summary(step.model2$finalModel)

## Subset selection object
## 6 Variables  (and intercept)
##       Forced in Forced out
## syct      FALSE      FALSE
## mmin      FALSE      FALSE
## mmax      FALSE      FALSE
## cach      FALSE      FALSE
## chmin     FALSE      FALSE
## chmax     FALSE      FALSE
## 1 subsets of each size up to 5
## Selection Algorithm: backward
##          syct mmin mmax cach chmin chmax
## 1  ( 1 ) " "  "*"  " "  " "  " "   " "  
## 2  ( 1 ) " "  "*"  " "  " "  " "   "*"  
## 3  ( 1 ) " "  "*"  "*"  " "  " "   "*"  
## 4  ( 1 ) " "  "*"  "*"  "*"  " "   "*"  
## 5  ( 1 ) "*"  "*"  "*"  "*"  " "   "*"  
```

Stepwise selection

```r
set.seed(123)
# Set up repeated k-fold cross-validation
train.control <- trainControl(method = "cv", number = 10)
# Train the model

step.model3 <- train(perf ~syct + mmin + mmax + cach + chmin + chmax, data =
cpus,
                    method = "leapSeq",
                    tuneGrid = data.frame(nvmax = 1:6),
                    trControl = train.control)
step.model3$results
```

```
##   nvmax     RMSE  Rsquared      MAE   RMSESD RsquaredSD    MAESD
## 1     1 93.46400 0.6488552 56.59877 60.38499 0.17797603 22.54073
## 2     2 88.10355 0.7235298 52.90480 60.02006 0.14462287 21.58357
## 3     3 76.85737 0.7660263 46.67915 60.69824 0.15230524 23.38628
## 4     4 73.30065 0.8123798 45.88578 48.41967 0.08509873 16.35415
## 5     5 72.00203 0.8281705 44.90645 47.86131 0.08199148 16.33195
## 6     6 72.78049 0.8063649 44.82444 47.12075 0.10818179 16.66283
```

Here we show the number of predictors the best subset will have

```r
step.model3$bestTune
```

```
##   nvmax
## 5     5
```

The final model after stepwise selection showing the importance of each predictor.

```r
summary(step.model3$finalModel)
```

```
## Subset selection object
## 6 Variables  (and intercept)
##       Forced in Forced out
## syct      FALSE      FALSE
## mmin      FALSE      FALSE
## mmax      FALSE      FALSE
## cach      FALSE      FALSE
## chmin     FALSE      FALSE
## chmax     FALSE      FALSE
## 1 subsets of each size up to 5
## Selection Algorithm: 'sequential replacement'
##          syct mmin mmax cach chmin chmax
## 1  ( 1 ) " "  " "  "*"  " "  " "   " "  
## 2  ( 1 ) " "  " "  "*"  "*"  " "   " "  
## 3  ( 1 ) " "  "*"  "*"  " "  " "   "*"  
## 4  ( 1 ) "*"  "*"  "*"  "*"  " "   " "  
## 5  ( 1 ) "*"  "*"  "*"  "*"  "*"   " "  
```

REGRESSION METHODS: OLS, RIDGE, LASSO, PCR, & PLS USING "College" dataset

Predict the number of applications received using the other variables in the College data set in library ISLR

(a) Split the data set into a training set and a test set using caret library and fit each of the following models using caret and ten fold cross validation.

```
library(ISLR)
library(glmnet)
attach(College)
head(College)

##                              Private Apps Accept Enroll Top10perc
Top25perc
## Abilene Christian University     Yes 1660   1232    721        23
52
## Adelphi University               Yes 2186   1924    512        16
29
## Adrian College                   Yes 1428   1097    336        22
50
## Agnes Scott College              Yes  417    349    137        60
89
## Alaska Pacific University        Yes  193    146     55        16
44
## Albertson College                Yes  587    479    158        38
62
##                              F.Undergrad P.Undergrad Outstate Room.Board
Books
## Abilene Christian University        2885         537     7440       3300
450
## Adelphi University                  2683        1227    12280       6450
750
## Adrian College                      1036          99    11250       3750
400
## Agnes Scott College                  510          63    12960       5450
450
## Alaska Pacific University            249         869     7560       4120
800
## Albertson College                    678          41    13500       3335
500
##                              Personal PhD Terminal S.F.Ratio perc.alumni
Expend
## Abilene Christian University     2200  70       78      18.1          12
7041
## Adelphi University               1500  29       30      12.2          16
10527
## Adrian College                   1165  53       66      12.9          30
8735
## Agnes Scott College               875  92       97       7.7          37
19016
```

```
## Alaska Pacific University        1500  76        72       11.9           2
10922
## Albertson College                 675  67        73        9.4          11
9727
##                              Grad.Rate
## Abilene Christian University        60
## Adelphi University                  56
## Adrian College                      54
## Agnes Scott College                 59
## Alaska Pacific University           15
## Albertson College                   55
```

```
x <- model.matrix(Apps~., College)[,-1]
y <- College$Apps
lambda <- 10^seq(10, -2, length = 100)


# Train test split
set.seed(489)
train = sample(1:nrow(x), nrow(x)/2)
test = (-train)
ytest = y[test]
```

(b) Fit a linear model using ordinary least squares on the training set, and report the test mean squared error obtained.

```
OLS_lm <- lm(Apps~., data = College, subset = train)
OLS_lm

## 
## Call:
## lm(formula = Apps ~ ., data = College, subset = train)
## 
## Coefficients:
## (Intercept)   PrivateYes       Accept       Enroll     Top10perc
Top25perc
##  -544.41744   -170.52279      1.74160     -1.41087      38.28257      -
6.06587
## F.Undergrad  P.Undergrad     Outstate   Room.Board         Books
Personal
##     0.07306      0.08748     -0.08632      0.16650       0.06319
0.09351
##         PhD     Terminal    S.F.Ratio  perc.alumni        Expend
Grad.Rate
##   -11.10782      2.19668      4.12585      3.56206       0.05095
1.92934

#Find the best lambda from our list via cross-validation
cv.out <- cv.glmnet(x[train,], y[train], alpha = 0)
cv.out
```

```
##
## Call:  cv.glmnet(x = x[train, ], y = y[train], alpha = 0)
##
## Measure: Mean-Squared Error
##
##     Lambda Measure      SE Nonzero
## min  397.4 2103455 1270039      17
## 1se 2554.6 3360297 2169940      17
```

```
#Best lambda
bestlam <- cv.out$lambda.min
bestlam
```

```
## [1] 397.4201
```

```
#Make predictions
OLS.pred <- predict(OLS_lm, newdata = College[test,])
head(OLS.pred)
```

```
##        Adelphi University            Adrian College         Albertson
College
##                3350.61158                1397.93516
608.67123
##   Albertus Magnus College Alderson-Broaddus College         Allegheny
College
##                  54.98646                 686.22811
2922.74735
```

- MEAN SQUARED ERROR FOR OLS

```
#check Mean Squared Error
mean((OLS.pred-ytest)^2)
```

```
## [1] 1403054
```

(c) Fit a ridge regression model on the training set, with λ chosen by cross-validation. Report the test mean squared error obtained. Report the value of λ used in the model

```
ridge.mod <- glmnet(x[train,], y[train], alpha = 0, lambda = lambda)
summary(ridge.mod)
```

```
##           Length Class     Mode
## a0         100   -none-    numeric
## beta      1700   dgCMatrix S4
## df         100   -none-    numeric
## dim          2   -none-    numeric
## lambda     100   -none-    numeric
## dev.ratio  100   -none-    numeric
## nulldev      1   -none-    numeric
## npasses      1   -none-    numeric
## jerr         1   -none-    numeric
## offset       1   -none-    logical
```

```
## call         5   -none-    call
## nobs         1   -none-    numeric
```

```
#Find the best lambda from our list via cross-validation
cv.out <- cv.glmnet(x[train,], y[train], alpha = 0)
cv.out
```

```
##
## Call:  cv.glmnet(x = x[train, ], y = y[train], alpha = 0)
##
## Measure: Mean-Squared Error
##
##     Lambda Measure      SE Nonzero
## min  397.4 2352967 1646036      17
## 1se 3077.1 3903384 2937840      17
```

```
#Best lambda
bestlam <- cv.out$lambda.min
bestlam
```

```
## [1] 397.4201
```

```
#make predictions
ridge.pred <- predict(ridge.mod, s = bestlam, newx = x[test,])
head(ridge.pred)
```

```
##                                  1
## Adelphi University       3000.9738
## Adrian College           1164.0138
## Albertson College         595.0114
## Albertus Magnus College   317.8752
## Alderson-Broaddus College  549.4096
## Allegheny College        2677.7668
```

- MEAN SQUARED ERROR FOR RIDGE REGSRESSION

```
#Mean squared error
mean((ridge.pred-ytest)^2)
```

```
## [1] 1298095
```

(d) Fit a lasso model on the training set, with fraction chosen by cross validation. Report the test mean squared error obtained, along with the number of non-zero coefficient estimates and the fraction.

```
lasso.mod <- glmnet(x[train,], y[train], alpha = 1, lambda = lambda)
summary(lasso.mod)
```

```
##           Length Class     Mode
## a0         100   -none-    numeric
## beta      1700   dgCMatrix S4
## df         100   -none-    numeric
## dim          2   -none-    numeric
```

```
## lambda     100   -none-    numeric
## dev.ratio  100   -none-    numeric
## nulldev      1   -none-    numeric
## npasses      1   -none-    numeric
## jerr         1   -none-    numeric
## offset       1   -none-    logical
## call         5   -none-    call
## nobs         1   -none-    numeric
```

```
lasso.pred <- predict(lasso.mod, s = bestlam, newx = x[test,])
head(lasso.pred)
```

```
##                                     1
## Adelphi University        2741.3266
## Adrian College            1686.0656
## Albertson College          998.9299
## Albertus Magnus College    629.1303
## Alderson-Broaddus College  875.6115
## Allegheny College         2954.1927
```

- MEAN SQUARED ERROR FOR LASSO REGRESSION

```
mean((lasso.pred-ytest)^2)
```

```
## [1] 1798354
```

(e) Fit a PCR model on the training set, with no. of principal components M chosen by cross-validation. Report the test mean squared error obtained, along with the value of M selected by cross-validation.

```
set.seed(123)
smp_size <- floor(0.75 * nrow(mtcars))
train_ind <- sample(seq_len(nrow(College)), size = smp_size)
train_p <- College[train_ind, ]
test_p <- College[-train_ind,c(1,3:18) ]
y_test=College[-train_ind,2]

require(pls)
pcr_model <- pcr(Apps~., data = train_p,scale =TRUE, validation = "CV")
summary(pcr_model)
```

```
## Data:    X dimension: 24 17
##  Y dimension: 24 1
## Fit method: svdpc
## Number of components considered: 17
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV            2426     2779     1375     1389     1371     1509     1612
## adjCV         2426     2749     1351     1365     1342     1477     1568
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
```

```
## CV         1605      1625      1842       1786       1664       1373       1346
## adjCV      1559      1574      1779       1714       1606       1305       1282
##        14 comps  15 comps  16 comps  17 comps
## CV          1180     936.9      1293      2503
## adjCV       1126     890.5      1224      2386
##
## TRAINING: % variance explained
##       1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8
comps
## X       37.40    62.89    73.63    81.16    87.87    91.82    93.96
95.65
## Apps    23.12    81.60    82.53    84.39    86.72    89.36    90.93
91.92
##       9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
## X       97.14     97.93     98.65     99.09     99.50     99.79     99.96
## Apps    92.87     95.00     95.49     98.20     98.27     98.44     98.99
##       16 comps  17 comps
## X        99.98    100.00
## Apps     98.99     99.03
```

```
pcr_pred <- predict(pcr_model, test_p, ncomp = 3)
head(pcr_pred)
```

```
## [1] 1930.6961 1451.1950  704.8014 2322.1893  815.2842 1231.9749
```

- MEAN SQUARED ERROR FOR Principal component regression (PCR)

```
mean((pcr_pred - y_test)^2)
```

```
## [1] 3664827
```

(f) Fit a PLS model on the training set, with M chosen by cross validation. Report the test error obtained, along with the value of M selected by cross-validation.

```
library(caret)
# Compile cross-validation settings
set.seed(100)
myfolds <- createMultiFolds(train_p$Apps, k = 5, times = 10)
control <- trainControl("repeatedcv", index = myfolds, selectionFunction =
"oneSE")

# Train PLS model
mod1 <- train(Apps ~ ., data = train_p,
              method = "pls",
              metric = "RMSE",
              tuneLength = 20,
              trControl = control,
              preProc = c("zv","center","scale"))

summary(mod1)
```

```
## Data:    X dimension: 24 17
##  Y dimension: 24 1
## Fit method: oscorespls
## Number of components considered: 8
## TRAINING: % variance explained
##           1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X           30.84    61.11    69.40    75.50    81.94    85.84    89.89
## .outcome    82.50    87.84    92.86    95.44    96.89    97.95    98.46
##           8 comps
## X           94.22
## .outcome    98.71
```

The model results display the metrics in the model including: ncom (number of predictors in a subset,also is the value of M), root mean squared error (RMSE), R-squared, mean absolute error (MAE) etc. The lowest RMSE indicates the best subset size and inturn the best model.

```
mod1$results
```

```
##    ncomp      RMSE  Rsquared       MAE    RMSESD RsquaredSD     MAESD
## 1      1 1084.9854 0.7133798  861.7848  521.4875  0.3403221  332.2876
## 2      2 1180.7783 0.7798500  875.6709  564.5886  0.3116584  354.9473
## 3      3 1230.5371 0.7154510  897.1544  561.0249  0.3305893  389.9395
## 4      4 1191.8641 0.7283764  905.3304  475.5401  0.2813624  375.5198
## 5      5 1132.9057 0.7626434  881.3255  443.3958  0.2641401  354.1532
## 6      6 1074.9639 0.7841007  846.2498  384.8527  0.2348362  305.2852
## 7      7 1030.9962 0.8037494  823.7655  344.5840  0.2158111  280.5208
## 8      8  977.8485 0.8270842  799.8415  320.7414  0.1946075  256.7419
## 9      9  961.4021 0.8474581  796.9018  369.8159  0.1860991  279.0550
## 10    10 1003.3505 0.8466150  833.7763  403.9196  0.1928643  310.0666
## 11    11 1066.8502 0.8373313  887.4662  421.3553  0.1989530  320.6783
## 12    12 1137.5497 0.8239245  944.5730  463.5097  0.2021272  351.6782
## 13    13 1229.7565 0.7731685 1007.7498  525.0526  0.2575168  399.2797
## 14    14 1472.4609 0.7342725 1172.4776  702.0939  0.2797465  505.0709
## 15    15 1936.8054 0.6771353 1474.1501 1218.4259  0.3113862  797.1762
## 16    16 2309.6684 0.6659932 1770.6816 1720.6389  0.3131435 1165.2617
```

(g) Comment on the results obtained. Is there much difference among the test errors resulting from these five approaches?

- There is a noticeable difference between OLS, Ridge, PCR and PLS regression in terms of mean squared error whereby Ridge regression had the lowest mean squared error followed by PLS, OLS,Lasso and then Principal Component Regression had the highest mean squared error.