$$
\begin{bmatrix} F & B^{\mathrm{T}} & C^{\mathrm{T}} & 0 \\ B & 0 & 0 & 0 \\ -C & 0 & M & D^{\mathrm{T}} \\ 0 & 0 & D & 0 \end{bmatrix} \begin{bmatrix} u \\ p \\ b \\ r \end{bmatrix} = \lambda \begin{bmatrix} F & B^{\mathrm{T}} & C^{\mathrm{T}} & 0 \\ 0 & -M_s & 0 & 0 \\ -C & 0 & M + D^{\mathrm{T}} L^{-1} D & 0 \\ 0 & 0 & 0 & L \end{bmatrix} \begin{bmatrix} u \\ p \\ b \\ r \end{bmatrix}
$$

Writing this as four equations gives:

$$
(\lambda - 1)(Fu + B^{\mathrm{T}} p + C^{\mathrm{T}} b) = 0 \tag{1a}
$$

$$
Bu = -\lambda M_s p \tag{1b}
$$

$$
(\lambda - 1)Cu + (1 - \lambda)Mb + D^{\mathrm{T}} r - \lambda D^{\mathrm{T}} L^{-1} D b = 0 \tag{1c}
$$

$$
Db = \lambda L r \tag{1d}
$$

Substituting (1d) into (1c) we obtain

$$
(\lambda - 1)Cu + (1 - \lambda)Mb + (D^{\mathrm{T}} - \lambda^2 D^{\mathrm{T}})r = 0. \tag{2}
$$

From (1a) we can see that $\lambda = 1$ satisfies the equation. Substituting $\lambda = 1$ into the other three equations gives:

$$
Bu = -M_s p \tag{3a}
$$

$$
D^{\mathrm{T}} r - D^{\mathrm{T}} L^{-1} D b = 0 \tag{3b}
$$

$$
Db = Lr \tag{3c}
$$

From (2) we then obtain the following eigenvector $(u, -M_s^{-1} B u, b, L^{-1} D b)$. I can't see why $u \neq 0$ or $b \neq 0$ to get the exact statement which you asked me to show. I coded up the eigenvalue problem with random matrices and it seemed to show that there where $n + \hat{n} + 4$ eigenvalues of $\lambda = 1$.

Consider $\lambda \neq 1$ then (1) is

$$
\begin{aligned}
Fu + B^{\mathrm{T}}p + C^{\mathrm{T}}b &= 0 && \text{(4a)}\\
Bu &= -\lambda M_s p && \text{(4b)}\\
(\lambda - 1)Cu + (1-\lambda)Mb + D^{\mathrm{T}}r - \lambda D^{\mathrm{T}}L^{-1}Db &= 0 && \text{(4c)}\\
Db &= \lambda Lr && \text{(4d)}
\end{aligned}
$$

Substitute (4d) into (4c) and with some simplification gives

$$-\lambda Cu + (\lambda Mu + (1+\lambda)D^{\mathrm{T}}L^{-1}D)b = 0. \tag{5}$$

Let $A = \lambda Mu + (1+\lambda)D^{\mathrm{T}}L^{-1}D$, then since M is positive semi-definite and $D^{\mathrm{T}}L^{-1}D$ is positive definite then A is non-singular, hence, $b = \lambda A^{-1}Cu$. Using this expression for b and $p = -\frac{1}{\lambda}M_s^{-1}Bu$ with (4a) to eliminate b and p gives

$$(F - \frac{1}{\lambda}B^{\mathrm{T}}M_s^{-1}B + \lambda C^{\mathrm{T}}A^{-1}C)u = 0 \implies \mathcal{A}u = 0.$$

Since $\mathcal{A}$ is invertible then $u = 0$, hence $p = 0$. Looking at the case when $\lambda = -1$ then (5)

$$Cu = Mb,$$

since M is singular then $u = 0$, hence $p = 0$. Therefore, the eigenvector associated with $\lambda = -1$ is $(0, 0, b, -L^{-1}Db)$.

When considering $\lambda \neq 1$ or -1 then (5) reduces to

$$(\lambda Mu + (1+\lambda)D^{\mathrm{T}}L^{-1}D)b = 0$$

which means that $b \in \ker(\lambda Mu + (1+\lambda)D^{\mathrm{T}}L^{-1}D)$ since $u = p = 0$.

I tried to download and install deall.II but if said that I needed to install loads of other packages first which I am currently installing now.