Project proposal

This proposal concerns the development of fully scalable and efficient solution techniques for a magnetohydrodynamics (MHD) problem. MHD models describes a coupling between electromagnetic effects and fluid flow governed by the Maxwells equations and Navier-Stokes equations respectively. These problems appear frequently in many applications in science and engineering, and any progress in the development of solution methods has the potential of making a high impact.

Using my Master's degree work (at UBC) as a starting point, we will aim to develop a fully scalable iterative solution method, that is, a solver with optimal complexity. To that end, specialised multigrid methods will be implemented, which are tailored to the specific differential operators arising in the problem. One option is to employ and extend the multigrid solver developed in [1] which is known to be very robust. The implementation of scalable preconditioner solves has the potential of significantly improving the efficiency of the solution procedure, especially for large scale three-dimensional problems.

Currently, there is a very limited set of available iterative solution techniques for the MHD model, in part due to enormous computational effort that is required. The availability of increased computing resources and parallel computational platforms has been instrumental in generating new possibilities and horizons in this regard.

Our proposed discretization and linearization, results in a 4×4 non-symmetric block-structured linear system needs to be (repetitively) solved. One of the principal challenges is the presence of a skew-symmetric term that couples the fluid velocity with the electric field. The proposed techniques exploit the block structure of the underlying linear system, utilizing and combining effective preconditioners for the mixed Maxwell and Navier-Stokes subproblems. The preconditioner is based on dual and primal Schur complement approximations to yield a scalable solution method.

An aim of this work is to provide engineers, geophysicists and others with the capability of performing computations on specific large scale applications they work on. As part of the project, we will generate a customisable and flexible, publicly available software package.

As an integral part of the research project, we will also consider iterative solution techniques for the Helmholtz equations. These equations are a subclass of electromagnetics problems, which give rise to indefinite linear systems. These are notoriously difficult problems, which are typically ill-posed and hence cause ill-conditioning of the discrete systems. Iterative solution methods are typically limited in their ability to deal with high frequencies, and often require the design of complicated solvers with fine meshes (which generally lead to large linear systems that are hard to solve). Any development of efficient solvers for Helmholtz equations will help the advancement of many diverse applications, such as radar and sonar technologies, noise scattering and seismology. Our plan is to design and implement a preconditioner based on hierarchical semi-separable (HSS) matrices. This is a recursive technique based on block decompositions and repeated rank-revealing factorisations of the off-diagonal blocks [2], which has been used primarily as a direct solution technique, but to date has not been fully explored as a paradigm for preconditioning.

By the end of project the plan is to release all code onto a public repository such as GitHub `https://github.com/`. We believe that making the code public will help increase the development of MHD and Helmholtz solvers for real physical applications. It will allow for combining together several prominent software packages (namely `FEniCS`, `PETSc`, `Trilinos`, and `hypre`), to solve significant physical problems described by partial differential equations (PDEs).

[1] R. Hiptmair and J. Xu, *Nodal auxiliary space preconditioning in H(curl) and H(div) spaces*, **45** (2007), no. 6, 2483–2509.

[2] S. Wang, J. Xia, Y. Situ, H. V. de Hoop, and X. Li, *Efficient parallel algorithms for hierarchically semiseparable (HSS) matrices: kernel of a massively parallel structured direct Helmholtz solver*, 2011 SEG Annual Meeting, Society of Exploration Geophysicists, 2011.