Comments and differences of Philips et al.

The main difference between our approach and theirs is the formation of the preconditioner. Consider the block matrix:

$$\mathcal{K} = \begin{pmatrix} \mathcal{K}_{\mathrm{NS}} & \mathcal{K}_{\mathrm{C}}^T \\ -\mathcal{K}_{\mathrm{C}} & \mathcal{K}_{\mathrm{M}} \end{pmatrix},$$

where $\mathcal{K}_{\mathrm{NS}}$, $\mathcal{K}_{\mathrm{C}}$ and $\mathcal{K}_{\mathrm{M}}$ are the Navier-Stokes, coupling and Maxwell block systems, respectively. We took the approach to consider combining well-known preconditioners where as they went for an "all-at-once" block approach. The approaches are as follows:

$$\mathcal{P}_{\mathrm{WGS}} = \begin{pmatrix} \mathcal{P}_{\mathrm{NS}} & \mathcal{K}_{\mathrm{C}}^T \\ -\mathcal{K}_{\mathrm{C}} & \mathcal{P}_{\mathrm{M}} \end{pmatrix},$$

we then combine the coupling terms in the Navier-Stokes preconditioner to form a block triangular preconditioner and

$$\mathcal{P}_{\mathrm{PSCEP}} = \begin{pmatrix} \mathcal{X}_{\mathrm{NS}} & \mathcal{K}_{\mathrm{C}}^T \\ 0 & \mathcal{K}_{\mathrm{M}} \end{pmatrix} \quad \text{where} \quad \mathcal{X}_{\mathrm{NS}} = \mathcal{K}_{\mathrm{NS}} + \mathcal{K}_{\mathrm{C}}^T \mathcal{K}_{\mathrm{M}}^{-1} \mathcal{K}_{\mathrm{C}}.$$

The resulting preconditioners have a similar block structure, however the approximations for $\mathcal{K}_{\mathrm{M}}^{-1}$ and $\mathcal{X}_{\mathrm{NS}}^{-1}$ are slightly different.