CS517 - Project proposal

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In this project we will be considering the solution of a divergence free finite element approximation to the steady state Stokes equations:

$$
\begin{aligned}
-\nu\Delta\vec{u} - \nabla p &= \vec{f}, \\
\nabla\cdot\,\vec{u} &= 0,
\end{aligned}
\tag{0.1}
$$

where $\vec{u}$ is the velocity vector, p is the pressure and ν is the dynamic viscosity. Using a mixed discretisation we get the following saddle point system

$$
\begin{bmatrix} A & B^{\mathrm{T}} \\ B & 0 \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix},
$$

where A, B^{T} and B are the discrete laplacian, gradient and divergence matrices. First we will go through what makes a good preconditioner and then look at a good approximation to ideal Schur complement preconditioner. Using the preconditioner proposed in [3, 5] namely

$$
\mathcal{P} = \begin{bmatrix} A & 0 \\ 0 & M \end{bmatrix},
$$

where M is the mass matrix, we will show the important property that M is spectrally equivalent to the Schur complement.

Using a multigrid V-cycle (see [1, 4] for description) to the (1,1) block and Conjugate Gradient [2] with a Jacobi preconditioner with the mass works extremely well for standard Taylor-Hood ($\mathbb{P}_2/\mathbb{P}_1$) elements. However, applying a standard V-cycle to the (1,1) block when using the divergence free elements ($\mathbb{BDM}_2/\mathbb{DG}_1$) will cause the number of outer iterations of the iterative scheme to increase. Therefore, we will be looking at an algebraic multigrid methods (AMG) approach where one can change the parameters (which maybe more suitable for these elements) within the the V-cycle.

References

[1] William L Briggs, V Emden Henson, and SF McCormick, *A multigrid tutorial siam*, Philadelphia, PA (2000).

[2] Magnus Rudolph Hestenes and Eduard Stiefel, *Methods of conjugate gradients for solving linear systems*, National Bureau of Standards (1952).

[3] David Silvester and Andrew Wathen, *Fast iterative solution of stabilised stokes systems part ii: using general block preconditioners*, SIAM Journal on Numerical Analysis **31** (1994), no. 5, 1352–1367.

[4] U Trottenberg, C Oosterlee, and A Schüller, *Multigrid methods: Basics*, SIAM, 2000.

[5] Andrew Wathen and David Silvester, *Fast iterative solution of stabilised stokes systems. part i: Using simple diagonal preconditioners*, SIAM Journal on Numerical Analysis **30** (1993), no. 3, 630–649.