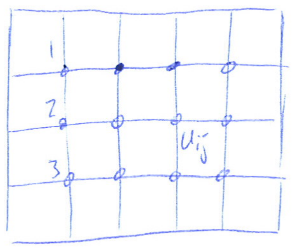
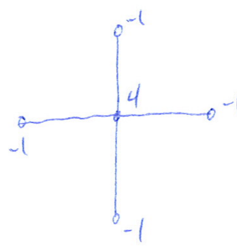
October 22, 2013

$-\Delta u = q$ in Ω

$u = 0$ in $\partial\Omega$





$$\frac{1}{h^2}\left(4u_{ij} - u_{i+1j} - u_{i-1j} - u_{ij+1} - u_{ij-1}\right) = q_{ij}$$

$$Au = q$$

if we refine the mesh $\Rightarrow$ A gets bigger

+ more ill-conditioned

$Ax = b$

$(M-N)x = b$

$Mx_{k+1} = b + Nx_k$

convergence of the method: $M^{-1}N$

$e = x^* - x_k$ error

$r = b - Ax_k$ residual

Symmetric Gauss Seidel smooths the residual

$x_{SMT} \leftarrow SGS$

Pre-smoothing $r_{SMT} = b - Ax_{SMT}$ $\xleftarrow{\text{post}}$ smooth $x \leftarrow x_{SMT} + e_h$ update

$\xrightarrow{P}$ r_H, A_H $\xrightarrow{P}$ e_h

coarse grid correction $e_H = A_H^{-1} r_H$

$$\begin{pmatrix}0&1&0&0&0\\0&0&0&1&0\end{pmatrix}$$ injection

$$\frac{1}{4}\begin{pmatrix}1&2&1&0&0\\0&0&1&2&1\end{pmatrix}$$

transposed $\rightarrow 2\times\frac{1}{4}\begin{pmatrix}1&0\\2&0\\1&1\\0&2\\0&1\end{pmatrix}$ interpolation

$Pr = r_H$

define

$$\underbrace{PAP^T}_{\text{Gallerking coarsening}} x_H = r_H$$

Start w/ $Ax = b$

1) smooth r, x

2) Pr, $\underbrace{PAP^T}_{A_H}$

$A_H e_H = r_H$

3) $e_H = P^T e_H$

$x \leftarrow x + e_h$

$r = b - Ax$

e-mail eldad for code