CS517 - Project proposal

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In this project we will be considering the solution of a divergence free finite element approximation to the steady state Stokes equations:

$$\begin{aligned} -\nu \Delta \vec{u} - \nabla p &= \vec{f}, \\ \nabla \cdot \vec{u} &= 0. \end{aligned} \tag{0.1}$$

First we will go through what makes a good preconditioner and then look at a good approximation to ideal Schur complement preconditioner. Using the preconditioner proposed in [3, 5] given below

$$\mathcal{P} = \begin{bmatrix} A & 0 \\ 0 & M \end{bmatrix},$$

where M is the mass matrix we will show that M is spectrally equivalent to the Schur complement.

Using a multigrid V-cycle (see [1, 4] for description) allied to the (1,1) block and Conjugate Gradient [2] with a Jacobi preconditioner works extremely well for standard Taylor-Hood ($\mathbb{P}_2/\mathbb{P}_1$) elements. However, applying a standard V-cycle to the (1,1) block when using the divergence free elements ($\mathbb{BDM}_2/\mathbb{DG}_1$) will cause the number of outer iterations of your iterative scheme to increase. Therefore, we will be looking at an algebraic multigrid methods (AMG) approach where one can change the parameters within the the V-cycle.

References

[1] William L Briggs, V Emden Henson, and SF McCormick, *A multigrid tutorial siam*, Philadelphia, PA (2000).

[2] Magnus Rudolph Hestenes and Eduard Stiefel, *Methods of conjugate gradients for solving linear systems*, 1952.

[3] David Silvester and Andrew Wathen, *Fast iterative solution of stabilised stokes systems part ii: using general block preconditioners*, SIAM Journal on Numerical Analysis **31** (1994), no. 5, 1352–1367.

[4] U Trottenberg, C Oosterlee, and A Schüuller, *Multigrid methods: Basics*, Parallelism, and Adaptivity (forthcoming) (2000).

[5] Andrew Wathen and David Silvester, *Fast iterative solution of stabilised stokes systems. part i: Using simple diagonal preconditioners*, SIAM Journal on Numerical Analysis **30** (1993), no. 3, 630–649.