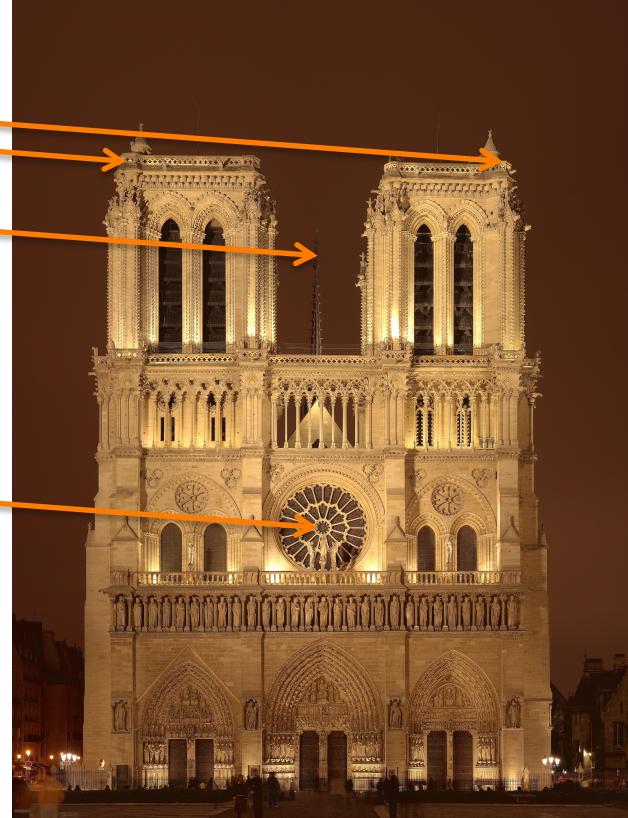
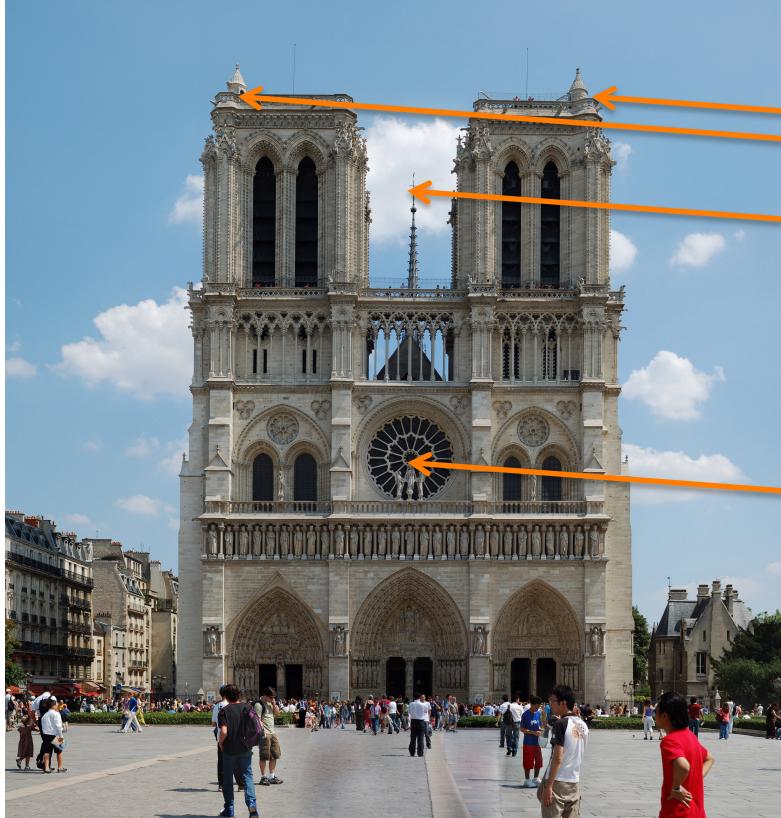


FEATURE REPRESENTATIONS

Why do we care about representation?



Discussion

- What information can we use to represent features?
- How do we make it robust to scale, rotation?

Proposed by David Lowe (2004)

SIFT: SCALE INVARIANT FEATURE TRANSFORM

Main parts of SIFT

1. Keypoint (i.e., Blobs) detection
 - a. Scale-space extrema detection
 - b. Keypoint localization
 - c. Orientation assignment
2. Descriptor computation
 - a. Keypoint descriptor

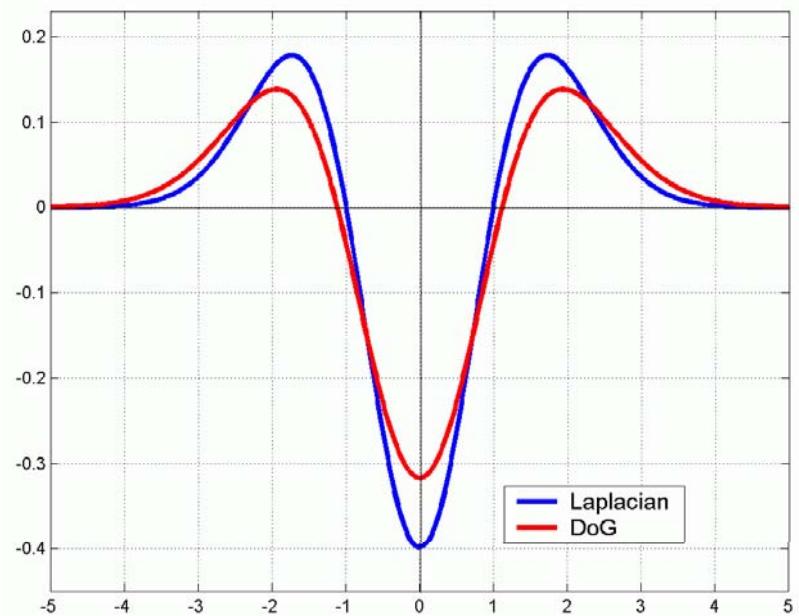
Approximating LoG with DoG

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

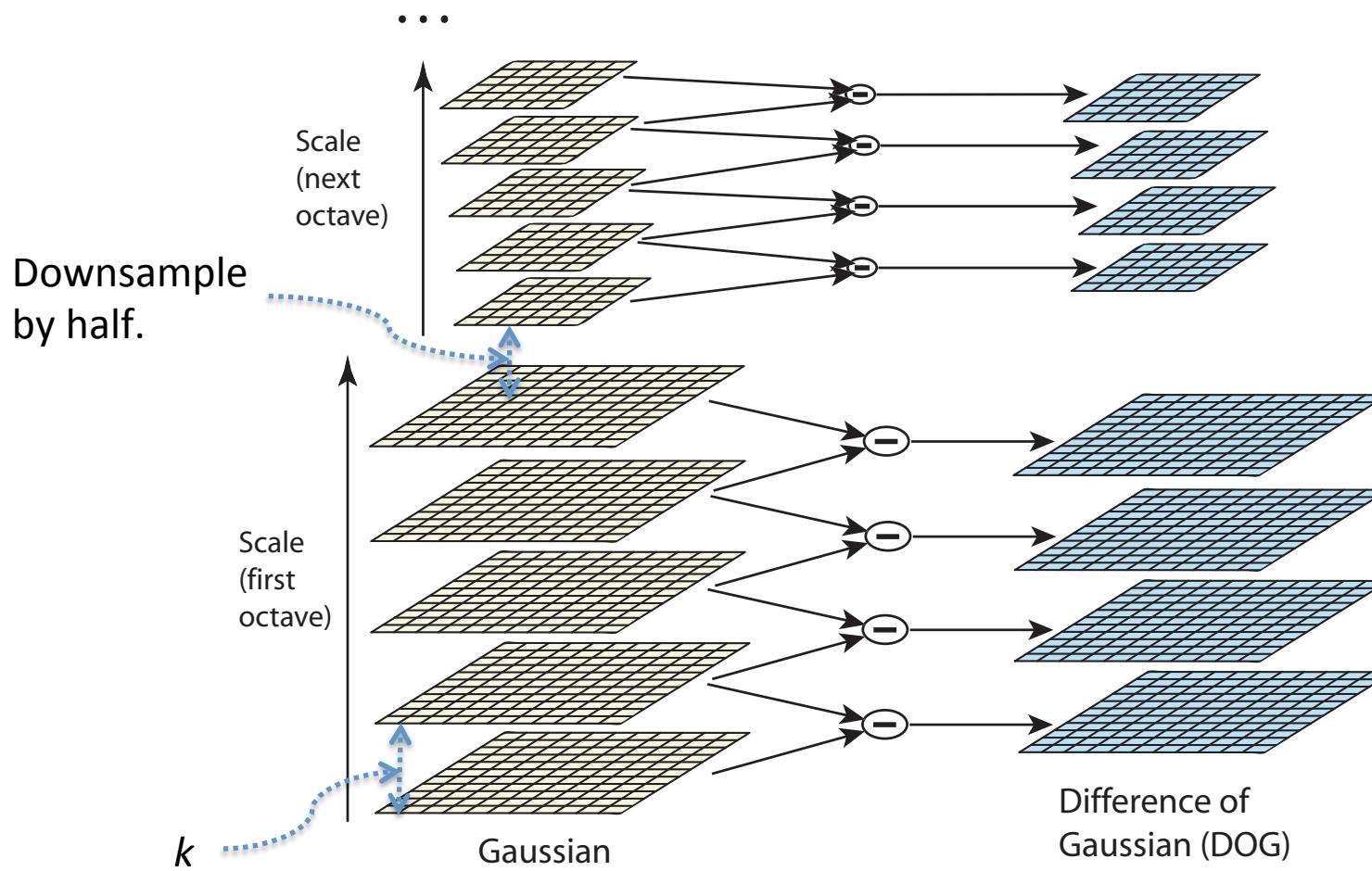
(Difference of Gaussians)



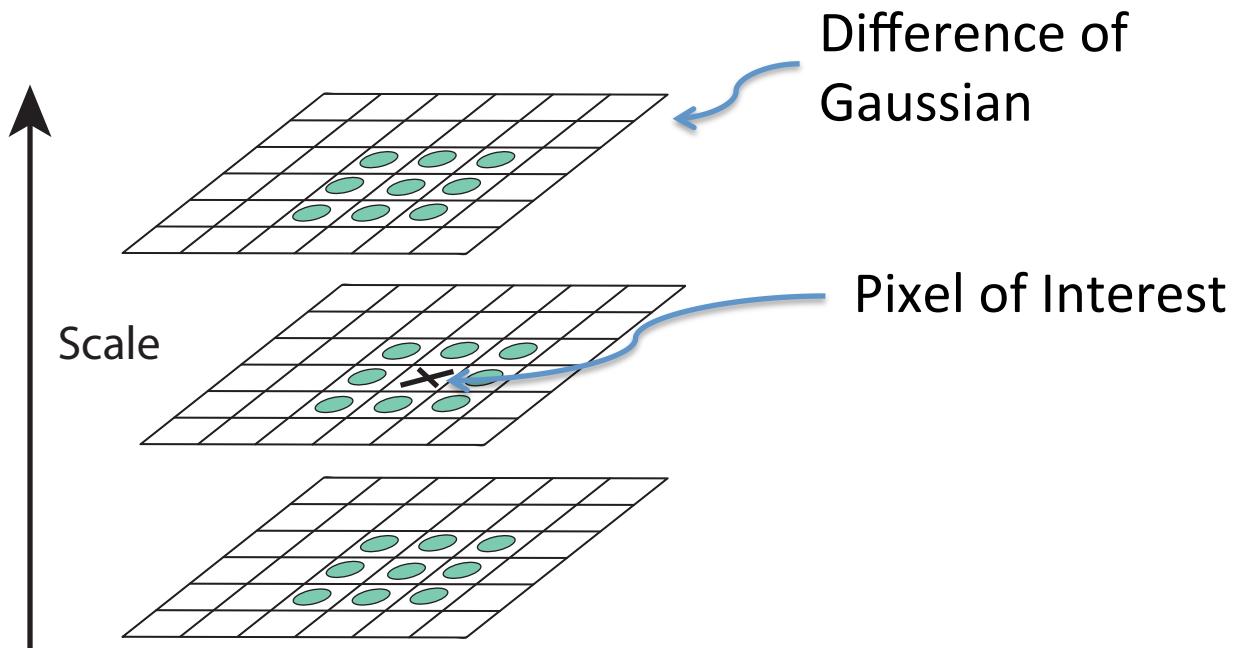
$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k - 1)\sigma^2 \nabla^2 G$$

Remember that blobs are detected using the Laplacian of Gaussian (LoG).

Efficient construction of the Difference of Gaussian

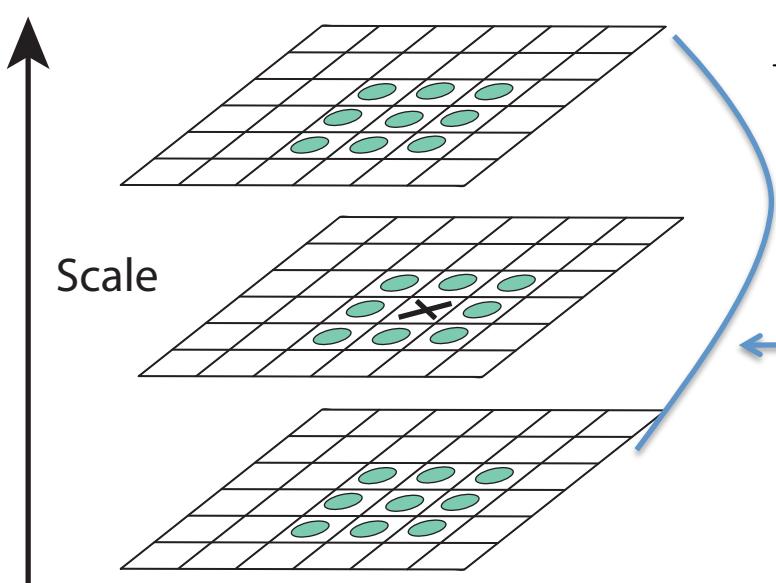


Local Extrema Detection



Maxima and minima of the difference-of-Gaussian images are detected by comparing a pixel (marked with X) to its 26 neighbors in 3x3 regions at the current and adjacent scales (marked with circles).

Accurate keypoint localization



$$D(\mathbf{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \mathbf{x}$$

Quadratic interpolation of $D(x, y, \sigma)$

$$\hat{\mathbf{x}} = -\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2} \frac{\partial D}{\partial \mathbf{x}}$$

Keypoint refinement

Filter Unstable Extrema

$$D(\hat{\mathbf{x}}) = D + \frac{1}{2} \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}}.$$



Refined DoG response

Filter points with a low DoG response

Eliminating Edges

- Calculate the principal curvature from the Hessian matrix.
- Similar to Harris corner detector, we analyze the eigenvalues of H .

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Hessian of the
Difference of Gaussian

Eliminating Edges (cont'd)

Eigenvalues

$$\frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} = \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(r\beta + \beta)^2}{r\beta^2} = \frac{(r + 1)^2}{r}$$

Largest Eigenvalue  $\alpha = r\beta$

Accept a keypoint if

$$\frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} < \frac{(r + 1)^2}{r}$$

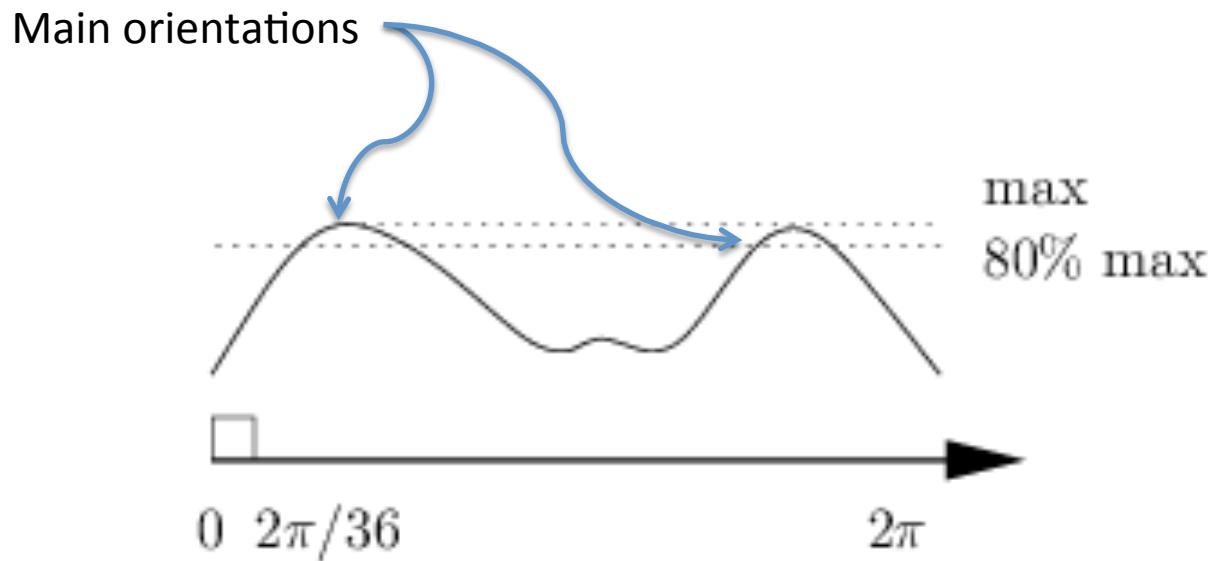
Discussion about invariance and covariance properties

- Laplacian of Gausian (i.e., blobs) response is invariant w.r.t scale and rotation.
- Blob location and scale are covariant w.r.t. Rotation and scaling.

Orientation Assignment

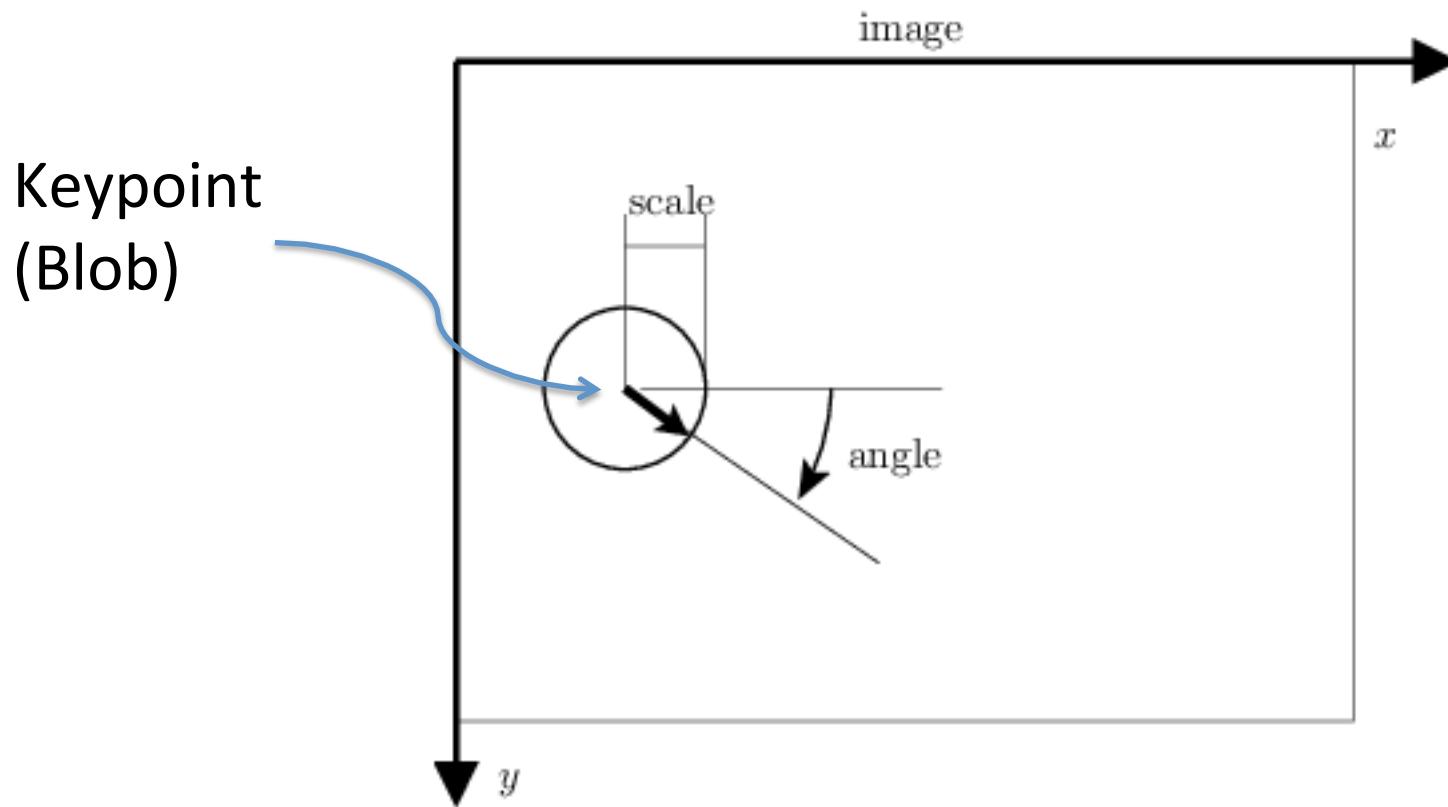
- For every keypoint (blob) do:
 1. Find the smoothed image (L) with the closest σ to the scale of the keypoint.
 2. Compute gradient magnitude and orientation of the selected image.
 3. Form an orientation histogram with 36 bins considering a region around the keypoint.
 - I. Each sample added is weighted by its gradient magnitude and by a Gaussian window with $\sigma=1.5 \sigma_{\text{scale}}$.
 4. Highest peak is detected.
 - I. Add any other peaks if their height exceeds 80% of the highest peak.
 - II. NOTE: A blob/keypoint might have multiple orientations assigned.
 5. Fit a parabola to each peak and interpolate to estimate the maximum orientation.

Orientation Assignment (cont'd)



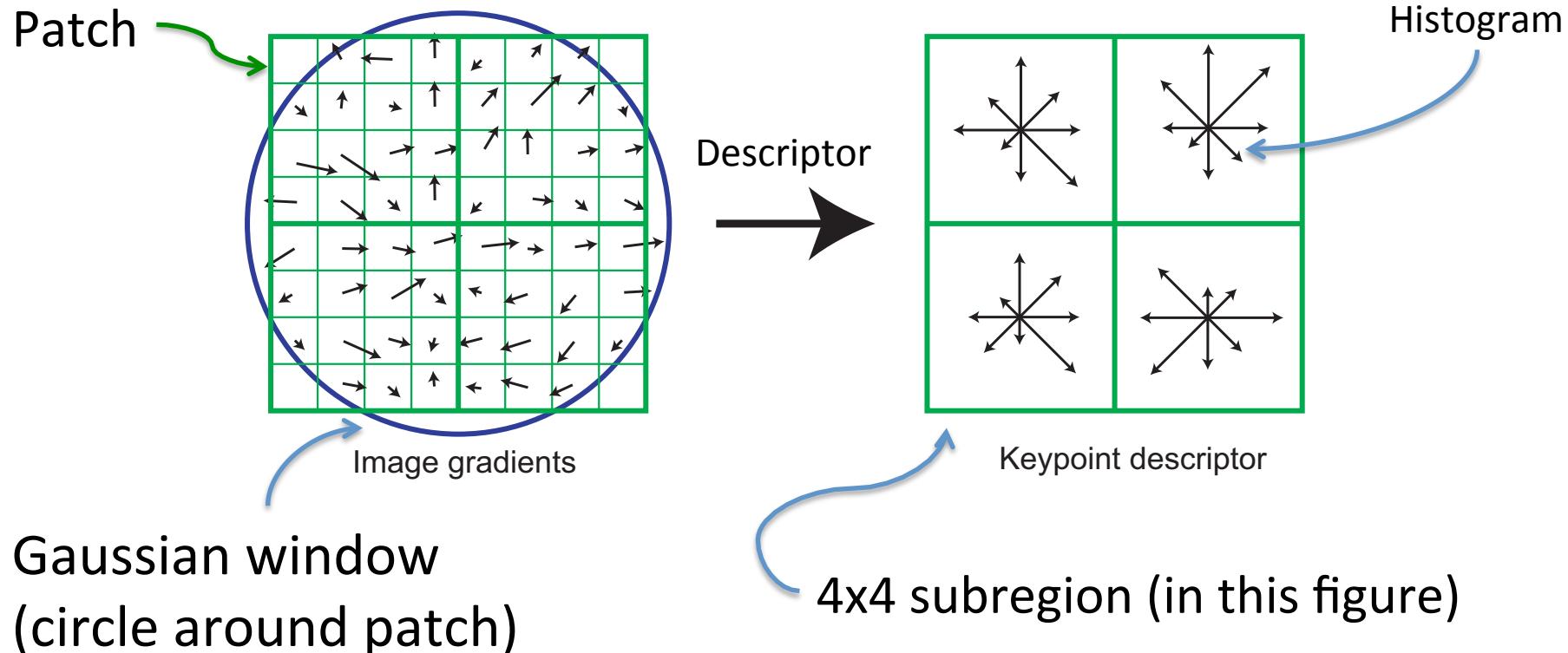
Blobs can have multiple assigned orientations!

Orientation Assignment (cont'd)



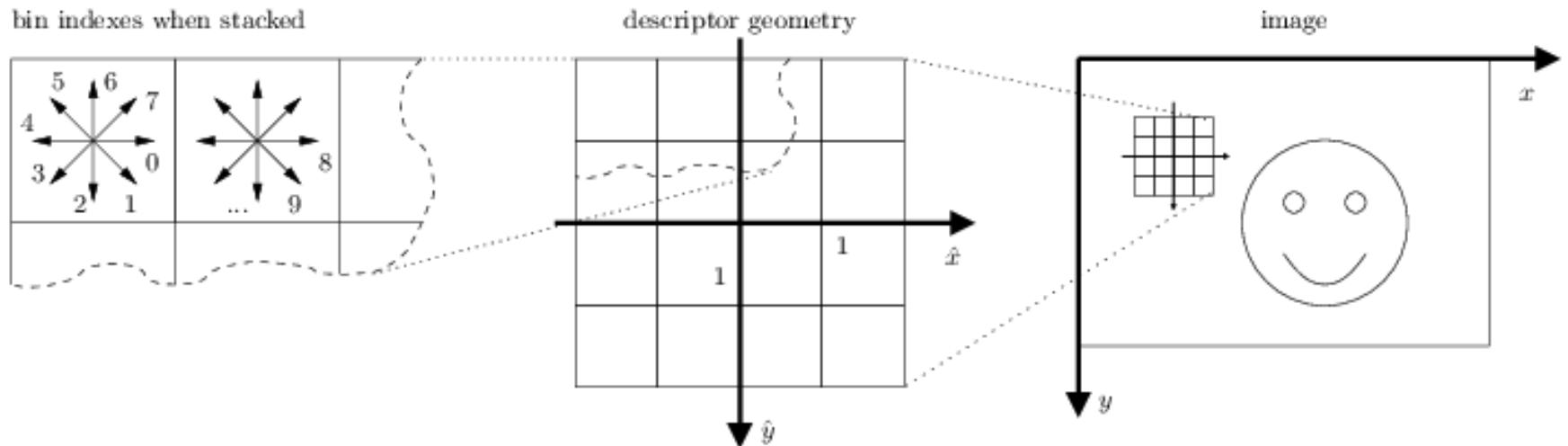
Descriptor computation

Rotating first so that the orientation of the patch looks upright.



A descriptor is computed per orientation.

SIFT descriptor

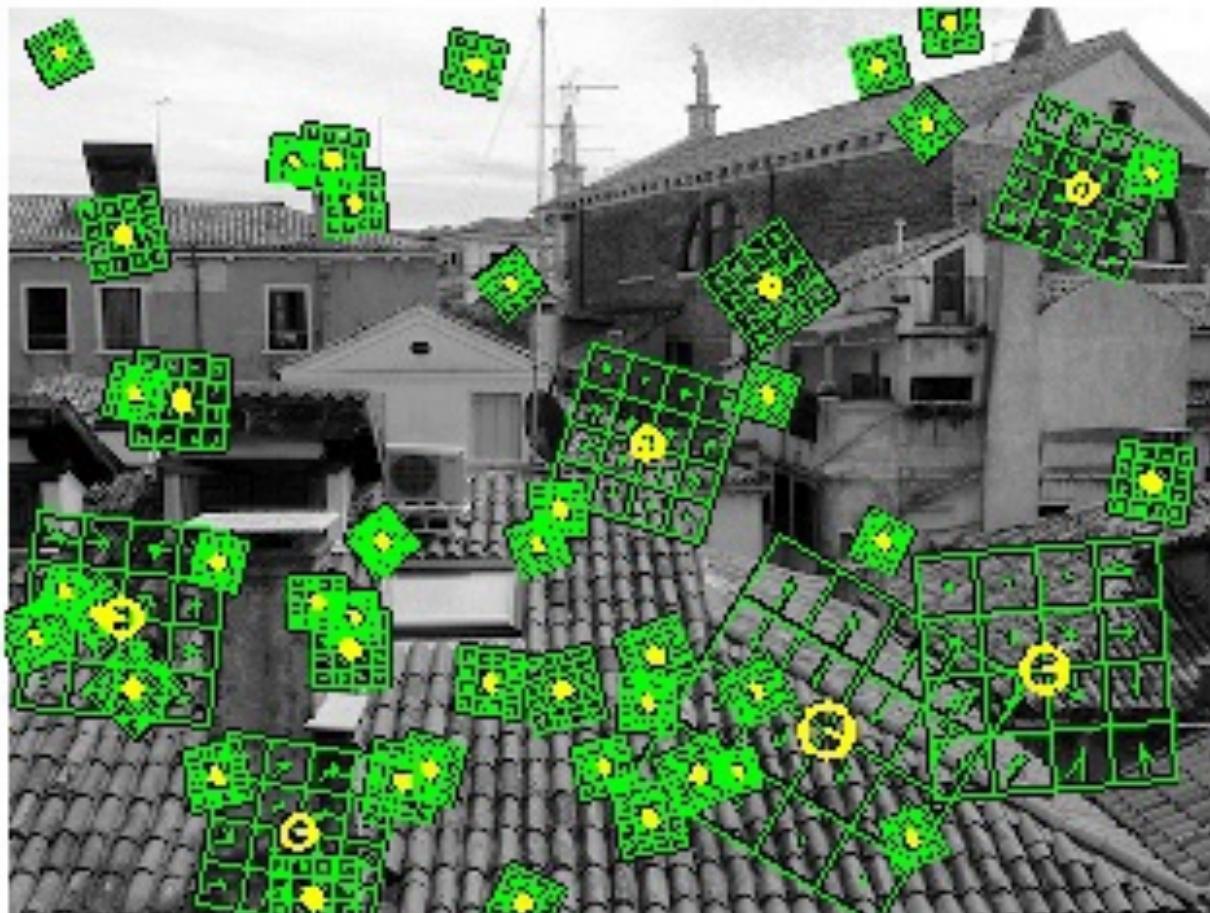


- 4x4 bins/cells
- 8 orientation per bin
- The dimension of the descriptor thus is 128

SIFT descriptor

- A weighting scheme (Gaussian window) is used to form the histograms.
- To reduce illumination effects in the descriptor, the feature vector is normalized.
- Threshold the normalized vector (threshold =0.2).
- Keep samples higher than the threshold and renormalize.

SIFT descriptors visualization



SIFT implementations

- VLFeat: <http://www.vlfeat.org/overview/sift.html>
 - Matlab and C/C++
- OpenCV:
[http://docs.opencv.org/modules/nonfree/doc/
feature detection.html?highlight=sift](http://docs.opencv.org/modules/nonfree/doc/feature_detection.html?highlight=sift)
 - C/C++ and Python
- David Lowe:
<http://www.cs.ubc.ca/~lowe/keypoints/>
 - Binary (SIFT is patented)

Discussion

- How do we match features?
 - Brute force Nearest-Neighbor search: Given a query feature find the reference feature that produces the smallest distance.
 - Approximate Nearest-Neighbor (ANN).

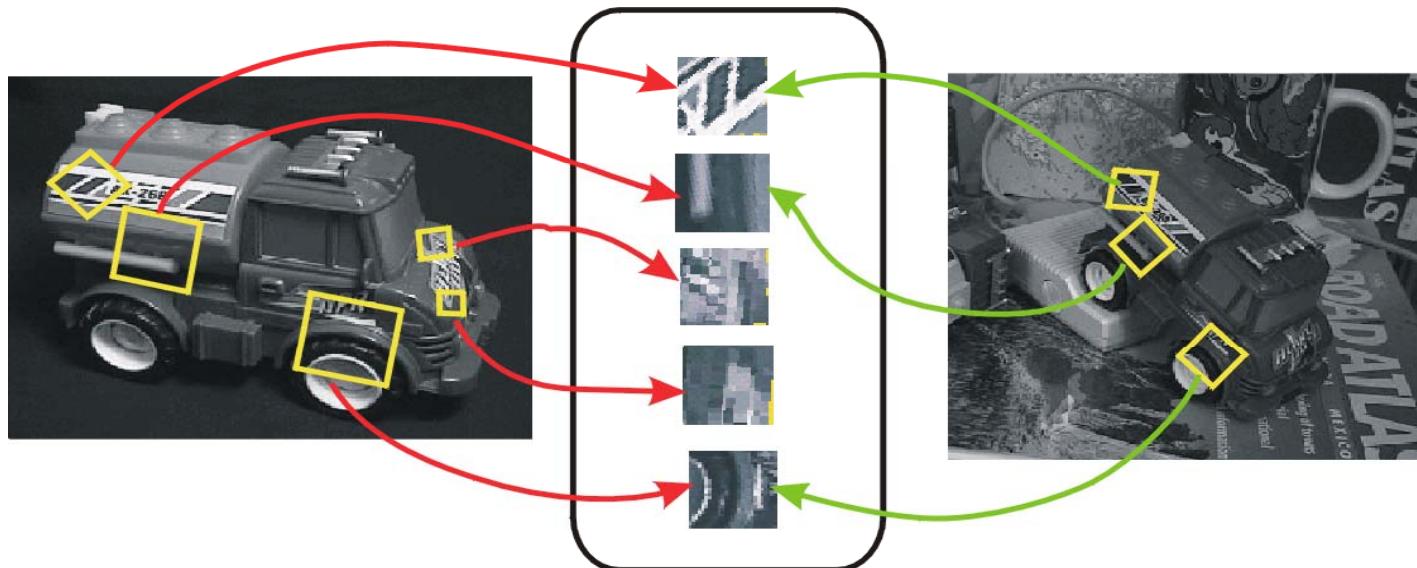
Invariance vs. Covariance

Invariance:

- $\text{features}(\text{transform}(\text{image})) = \text{features}(\text{image})$

Covariance:

- $\text{features}(\text{transform}(\text{image})) = \text{transform}(\text{features}(\text{image}))$



Covariant detection => invariant description

Proposed by Bay et al. (2006)

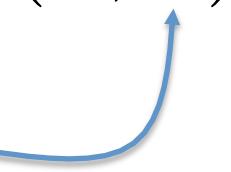
SURF: SPEEDED UP ROBUST FEATURES

Blob detection via Hessian

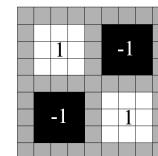
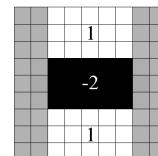
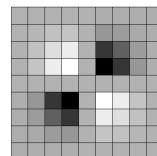
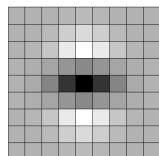
- Hessian matrix using LoG:

$$H = \begin{bmatrix} L_{xx}(\mathbf{x}, \sigma) & L_{xy}(\mathbf{x}, \sigma) \\ L_{xy}(\mathbf{x}, \sigma) & L_{yy}(\mathbf{x}, \sigma) \end{bmatrix}$$

Point 

Scale 

- Problem: LoG is expensive to compute, why?
- Solution: Approximate with box filters!



Blob detection via Hessian (cont'd)

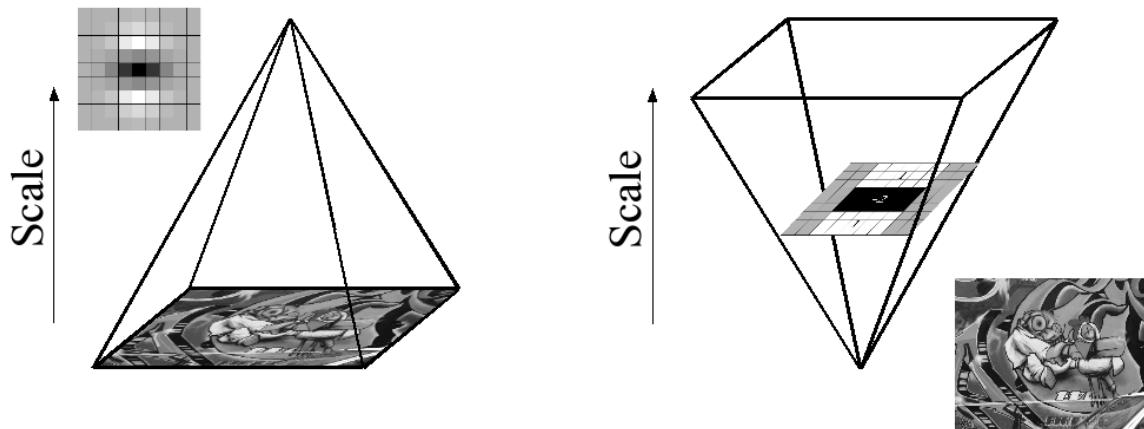
$$\det H = D_{xx}D_{yy} - (wD_{xy})^2$$

LoG approximations Weight

w = 0.9

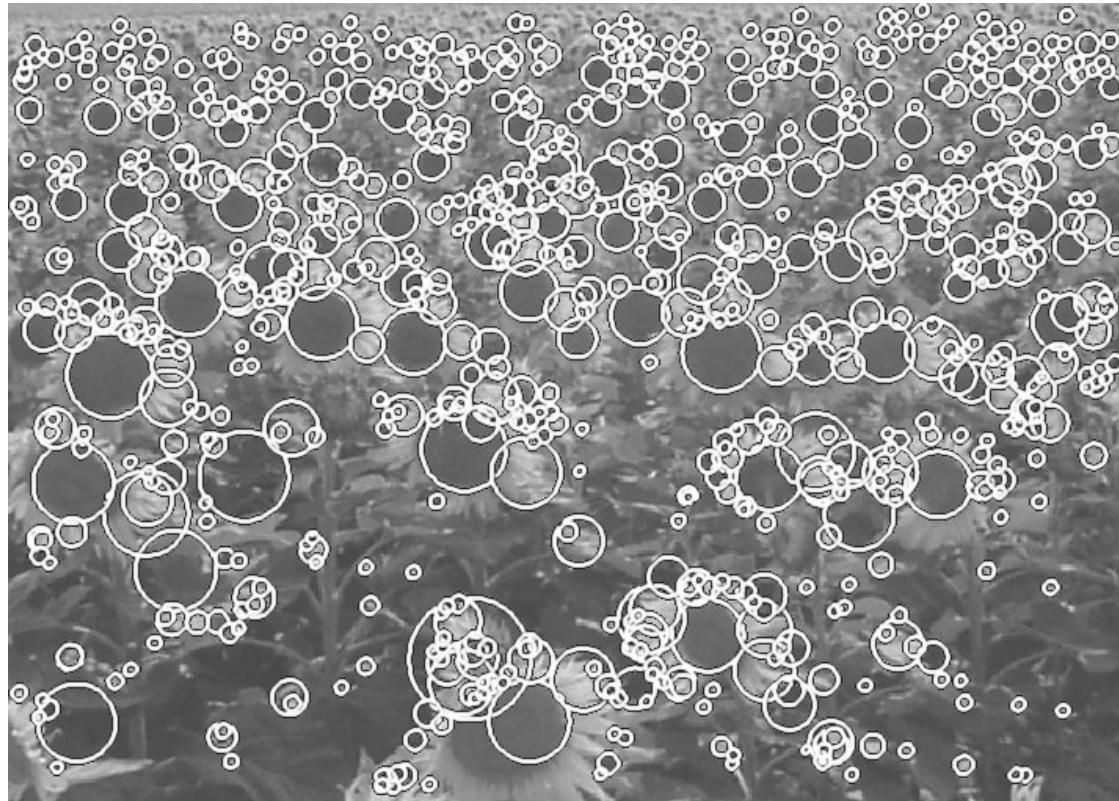
The determinant is used as the blob response.

Scale-space blob detection



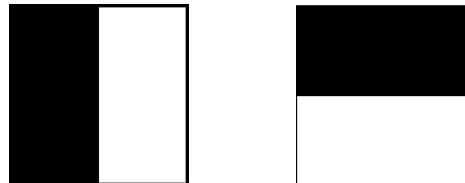
- Instead of reducing the image size (left), Surf increase the filter size exploiting box filters.

Blob detection via approximation of the determinant of the Hessian

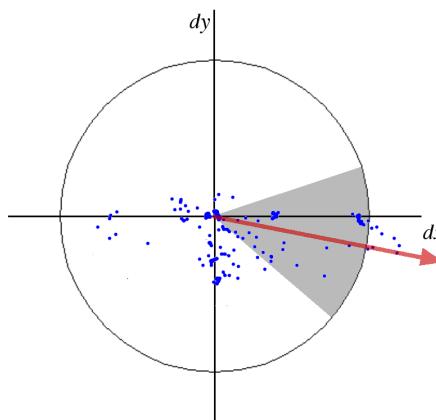


Orientation Assignment

- Convolve image with Haar-like filters; estimate some sort of gradient.



- Represent the “gradients” as points, and calculate the orientation from a window.
 - Sum points over the angular window
 - The longest vector is kept as the main orientation

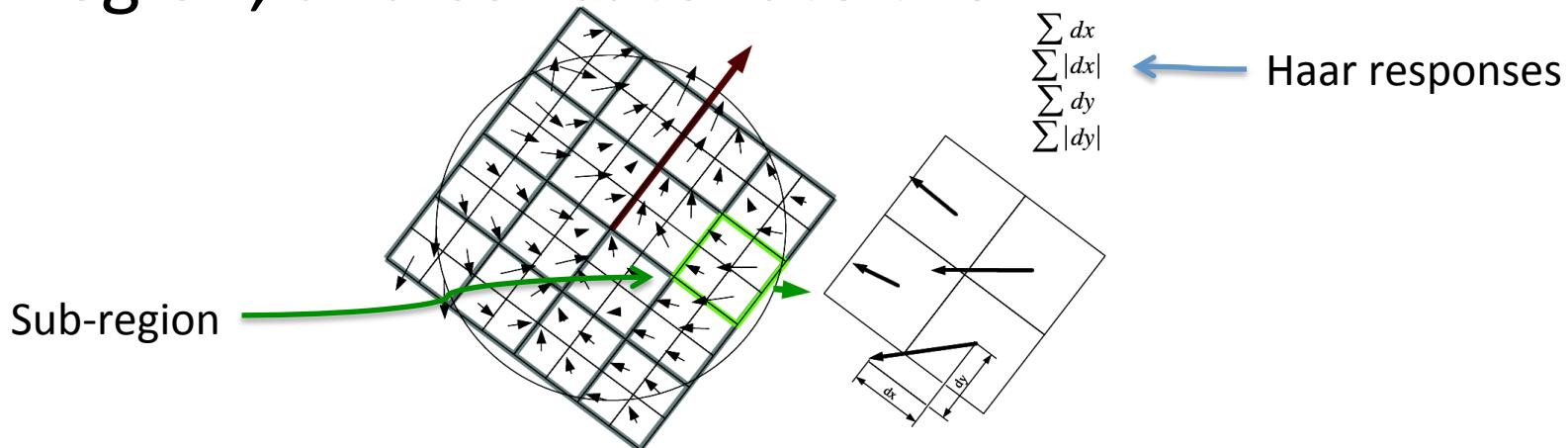


Descriptor computation

- Grab an oriented patch around the blob.



- Compute the following sums at each sub-region, and concatenate them.



BINARY DESCRIPTORS

BRIEF: Binary Robust Independent Elementary Features

- Proposed by Calonder et al. in ECCV 2010.
- SIFT and SURF work great, but the descriptor can be expensive to compute.
- Comparing SIFT or SURF descriptors involve Euclidean norms, which also add computational cost.

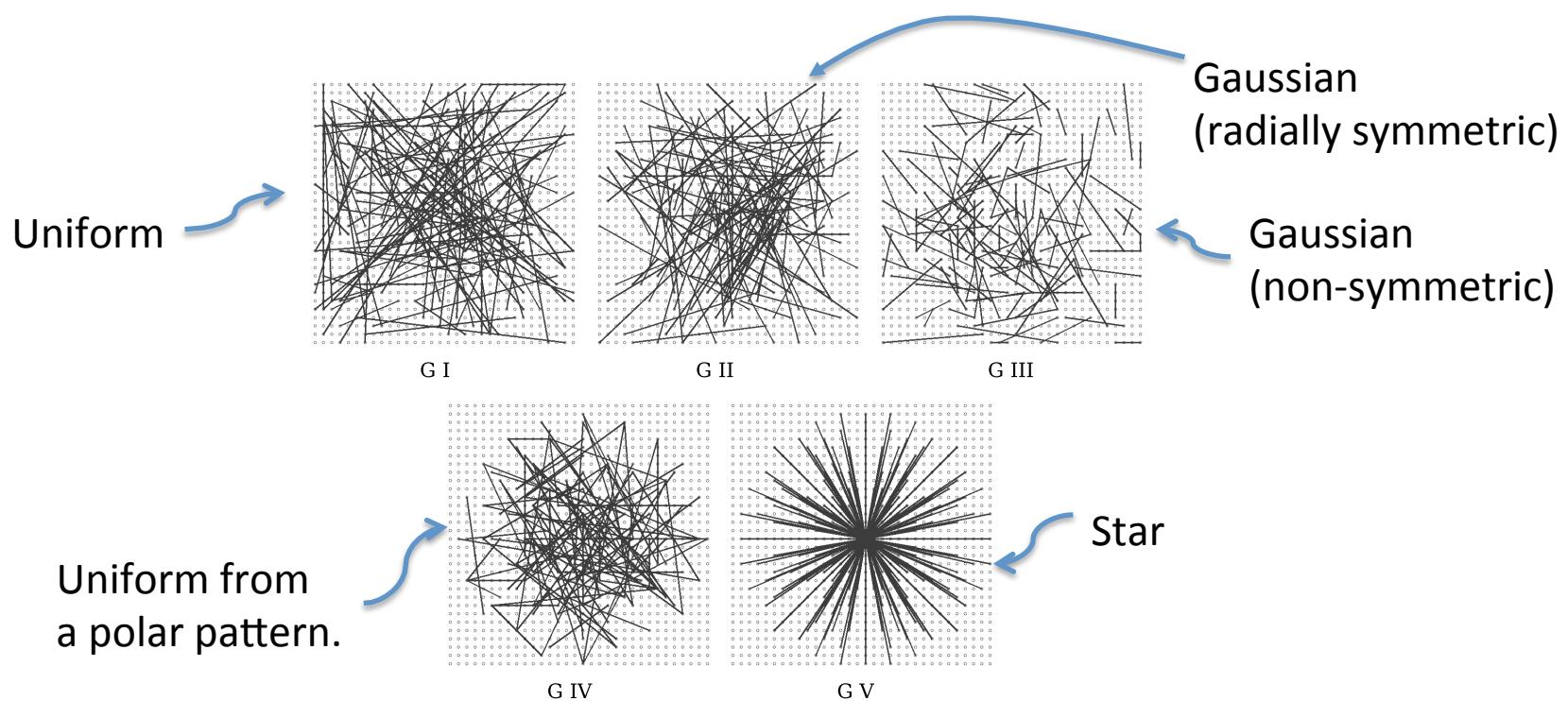
BRIEF Descriptor

- Idea: Compute cheaply a descriptor that can be compared efficiently.
- Binary Feature:

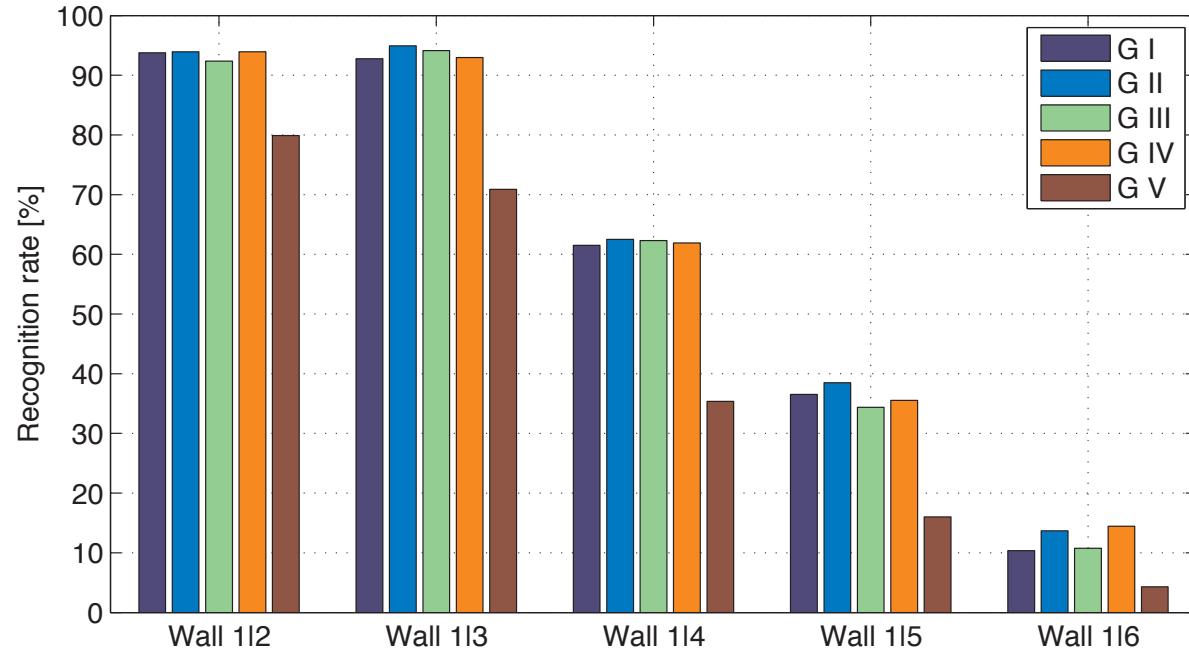
$$T(p; \mathbf{x}, \mathbf{y}) = \begin{cases} 1 & \text{if } p(\mathbf{x}) < p(\mathbf{y}) \\ 0 & \text{otherwise} \end{cases}$$

BRIEF Descriptor (cont'd)

- Concatenate the binary features to form the descriptor.
- Patterns used to form several descriptors:



What pattern works the best?



The random patterns work the best, according to experiments.

Number of bits for BRIEF

- Each bit is a binary feature.
- The sizes range from 16-512 bits.
- BRIEF is a descriptor, it needs keypoints to describe
 - FAST corners are used because they can be detected quickly.
 - CenSurE (a fast BLOB detector) is another good choice.

FEATURE MATCHING

Matching descriptors

- SIFT & SURF usually are matched with a nearest-neighbor (NN) classifier
 - Approximate NN methods also exist
- Binary descriptors are matched using a Hamming distance and a NN classifier.

Feature Matching Example



How do we fix those bad matches?

- We need geometric models to check consistency of the correspondences!