Question 5

5a.

$\mathbf{P} = \begin{vmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{2}{5} & \frac{3}{5} \end{vmatrix}$

We can use $\pi_j = \sum_{i \epsilon S} \pi_{ij} p_{ij}$ to find the stationary distibution.

$\pi_0 = \frac{1}{4}\pi_0 + \frac{2}{5}\pi_1 \rightarrow \pi_0 = \frac{8}{15}\pi_1$

Then using the fact that $\pi_0 + \pi_1 = 1$ we can write $\pi_0 = \frac{8}{15}(1 - \pi_0)$

$\rightarrow \frac{23}{15}\pi_0 = \frac{8}{15} \rightarrow \pi_0 = \frac{8}{23}$

Therefore $\pi_0 = \frac{8}{15}$ and $\pi_1 = 1 - \frac{8}{15} = \frac{15}{23}$

5b.

```r
sim <- function(nrep, start){
  x <- rep(NA, nrep)
  ran <- runif(nrep)
  x[1] <- start

  for (i in 2:nrep){
      if ((x[i-1] == 0 & ran[i] <= 1/4) | (x[i-1] == 1 & ran[i] <= 2/5)){
        val <- 0
      }
      else{
        val <- 1
     }
     x[i] <- val
    }
  return(x)
}
```

Here I define the function to simulate many paths of the chain. The input parameters are the number of repetitions and the starting point of the chain.

```r
n <- 1000000
start <- 0
x <- sim(n, start)
print(paste(c("The proportion of time ending in state 0 is "),
            round(1 - mean(x), 3), " and 1 is ", round(mean(x), 3), sep = ""))
```

```
## [1] "The proportion of time ending in state 0 is 0.348 and 1 is 0.652"
```

These proportions match the values of $\pi_0 = \frac{8}{23}$ and $\pi_1 = \frac{15}{23}$ found in part (a).

5c.

```r
n <- 100000
z <- rep(NA, n)

for (i in 1:n){
  start <- rbinom(1, 1, 15/23)
```

```r
  y <- sim(2, start)
  z[i] <- y[2]
}

print(paste(c("The proportion of time ending in state 0 is "),
            round(1 - mean(z), 3), " and 1 is ", round(mean(z), 3), sep = ""))
```

```
## [1] "The proportion of time ending in state 0 is 0.347 and 1 is 0.653"
```

Here start is randomly chosen as 0 or 1 with the proportions found in part (b) and run for one time step. After running this many times we see that the proportions match the distributions in part (a).