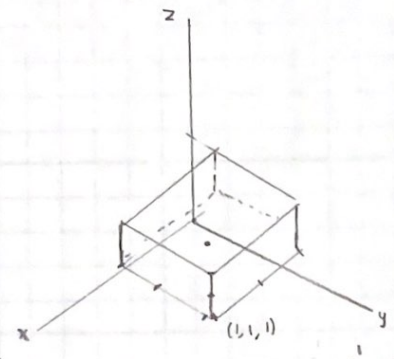
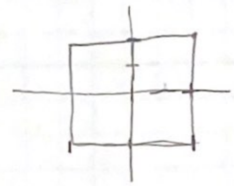
1) Find the net charge enclosed in a cube 2m on an edge, parallel to the axes and centered at the origin, if the charge density is given.

$$\rho = 50x^2\cos\left(\frac{\pi}{2}y\right) \quad (\mu C/m^3)$$

total charge enclosed $= Q = \int_v \rho_v \, dv$





$$Q_{enc} = \int_v \rho_v \, dv \Rightarrow \int_{-1}^{1}\int_{-1}^{1} 50x^2\cos\left(\frac{\pi}{2}y\right)dx\,dy$$

$$50\int_{-1}^{1} x^2\,dx \cdot \int_{-1}^{1}\cos\left(\frac{\pi}{2}y\right)dy$$

$$\left.\frac{x^3}{3}\right|_{-1}^{1} = \left(\frac{1}{3} - \frac{(-1)^3}{3}\right) \cdot \frac{4}{\pi}$$

$$\frac{\pi}{\pi}\frac{2}{3} - \frac{4}{\pi}\frac{3}{3} \Rightarrow \frac{2\pi}{3\pi} - \frac{12}{3\pi} = \frac{2\pi - 12}{3\pi}$$

$$\cancel{\boxed{= \frac{2\pi - 12}{3\pi} \quad \mu C/m^3}}$$

$u = \frac{\pi}{2}y$

$du = \frac{\pi}{2}dy$

$dy = \frac{2du}{\pi}$

$\frac{2}{\pi}\int \cos(u)du$

$\frac{2}{\pi}\sin(u)$

$\left.\frac{2}{\pi}\sin\left(\frac{\pi}{2}y\right)\right|_{-1}^{1}$

$= \frac{2}{\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{2}\right)\right]$

$\quad 1 \quad + \; -1$

$= \frac{2}{\pi}(2) = \frac{4}{\pi}$

$$\frac{2}{3} \cdot \frac{4}{\pi} = \frac{8}{3\pi}$$

$$\frac{50 \cdot 8}{3\pi} = 2\left(\frac{400}{3\pi}\ \mu C/m^3\right) = \boxed{\frac{800}{3\pi}\ \frac{\mu C}{m^3}}$$

2) Charge is distributed in the spherical region $r \leq 2m$ with density.

$$\rho_\ell = -\frac{200}{r^2} \ (\mu C/m^3)$$

What net flux crosses the surfaces $r = 1m$, $r = 4m$, $r = 500m$?

$r \leq a$
$r = 1m$

$$Q_{enc} = \frac{-200}{r^2}\left(\frac{4}{3}\right)\pi r^3 \Rightarrow \frac{-200}{(1)^2}\left(\frac{4}{3}\right)\pi (1)^3 =$$

$$-\cos\Big|_0^{\pi} = -\cos(\pi) - -\cos(0)$$
$$1 + 1 = 2$$

$$Q_{enc} = \rho_\ell \int_0^{2\pi} d\phi \cdot \int_0^{\pi} \sin\theta \, d\theta \cdot \int_0^{r} r^2 dr$$

$$= \frac{-200}{\cancel{r^2}}(2\pi)(2)\int_0^{r} \cancel{r^2} dr \Rightarrow \frac{r^3}{3}\Big|_0^1 = \left(\frac{1}{3} - 0\right) = \frac{1}{3}$$

$$\frac{-200}{1}(4\pi)\left(\frac{1}{3}\right) = = \boxed{-800\pi \ \mu C}$$

$r = 4m$

$$Q_{enc} = \rho_\ell \int_0^{2\pi} d\phi \cdot \int_0^{\pi} \sin\theta \, d\theta \cdot \int_2^{4} \cancel{r^2} dr$$

$$Q_{enc} = \frac{-200}{\cancel{r^2}}(2\pi)(2)\left(\cancel{\frac{r^3}{3}\Big|_2^4}\right) \Rightarrow \left(\frac{4^3}{3} - \frac{2^3}{3}\right) = 18.66 = \frac{56}{3}$$

$$-200(2\pi)(2)(2) = \boxed{-1600\pi \ \mu C}$$

$$Q_{enc} = -200(2\pi)(2)\left[r\Big|_2^{500}\right]$$

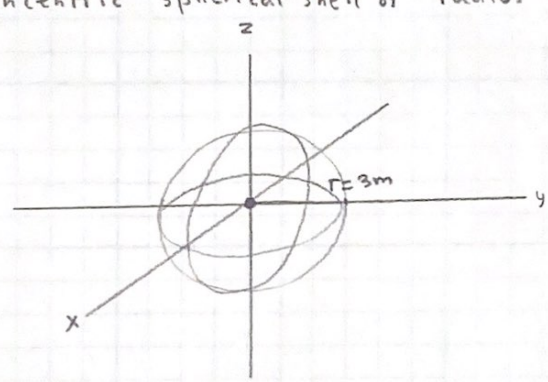
$$\boxed{= -1600\pi \ \mu C}$$

* the flux outside the surface is zero so it's always going to be $-1600\pi \mu C$ for anything above $r \leq 2m$.

3) If a point charge Q is at the origin, find an expression for the flux which crosses the portion of a shere centered at the origin, described by $\alpha \le \phi \le \beta$

$$\frac{\beta - \alpha}{2\pi} Q$$

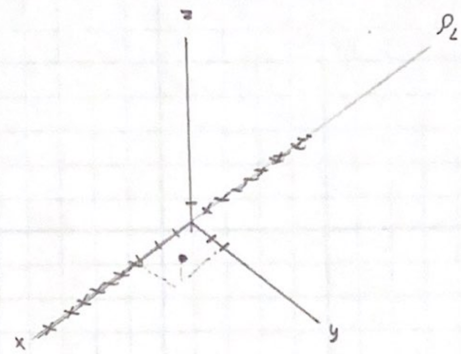
4) A point charge of Q(C) is at the origin of a spherical coordinate system. Find the flux Ψ which crosses an area of $4\pi\ m^2$ on a concentric spherical shell of radius r=3m

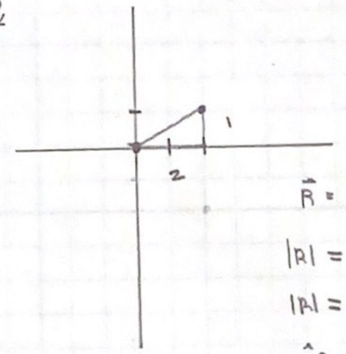


$$D = \frac{QA}{4\pi r^2} \approx \frac{Q(4\pi)}{4\pi r^2} = \frac{Q}{r^2} = \frac{Q}{9}\ c$$

5) A uniform line charge with $\rho = 5\mu C/m$ lies along the x-axis

Find $\vec{D}$ = at P(3,2,1)m





$\vec{R} = \begin{matrix} 2\,a_x \\ 1\,a_y \end{matrix}$

$|R| = \sqrt{2^2+1^2}$

$|R| = \sqrt{5}$

$\hat{a}_R = <2,1>$

$\oiint \tilde{D}\cdot d\tilde{s} = Q_{enc}$

$\Delta \vec{E} = \frac{1}{4\pi\varepsilon}\frac{\Delta q}{R^2}\hat{a}_R$

$E = \frac{\rho_L}{2\pi\varepsilon_0 r} a_r$

$\vec{D} = \varepsilon\vec{E}$ $\quad \vec{D} = \frac{\rho_L}{2\pi r} a_r = \frac{5\mu C}{2\pi}\frac{1}{\sqrt{5}m}\hat{a}_R$

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6) Given that $\vec{A} = <30e^{-r}, 0, -2z>$ in cylindrical coordinates, eval both sides of the divergence theorem for the volume enclosed by

$r = 2 \qquad 0 < z < 5$

$\oiint \bar{A}\cdot d\bar{s} = \iiint(\nabla\cdot\bar{A})dv$

$\nabla\cdot A = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \cancel{\frac{1}{r}\frac{\partial A_\phi}{\partial\phi}} + \frac{\partial A_z}{\partial z}$

$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\,30e^{-r}\right) + \frac{\partial}{\partial z}(-2z)$

$= \frac{30}{r}\frac{\partial}{\partial r}(re^{-r}) + (-2)$

$= \frac{30}{r}\left[e^{-r} + re^{-r}\right] - 2$

$= 30e^{-r}\left(\frac{1}{r} - 1\right) - 2$

$\iiint \tilde{\nabla}\cdot\tilde{A}\,dv = 30\int_0^5\int_0^{2\pi}\int_0^2 e^{-r}\left(\frac{1}{r}-1\right) - 2(r\,dr\,d\phi\,dz)$

$= 30\int_0^5 dz\int_0^{2\pi} d\phi\int_0^2 e^{-r}(1-r) - 2r\,dr$

$30\cdot 5\cdot 2\pi \cdot \quad = -3.729$

$=$ ~~5520.56~~

7) Given that $\bar{D} = 2.5 r^3 \hat{a}_r$ (C/m²) in cylindrical coordinates evaluate both sides of the divergence theorm for the volume enclosed by $1 \le r \le 2$m and $0 \le z \le 10$m

$$\oint\!\!\oint \bar{D} \cdot d\bar{s} = \iiint (\nabla \cdot \bar{D})\, dv$$

$$\vec{D} = \langle 2.5r^3, 0, 0 \rangle$$

$$\nabla \cdot D = \frac{1}{r}\frac{\partial}{\partial r}(r D_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial G_z}{\partial z}$$

$$\nabla \cdot D = \frac{1}{r}\frac{\partial}{\partial r}(r\, 2.5r^3) + \cancel{\frac{1}{r}}^{\,0} + \cancel{\frac{\partial G_z}{\partial z}}^{\,0}$$

$$\iiint \frac{1}{r}(10r^3)\, dr \Rightarrow \iiint 10r^2\, dr\, d\phi\, dz$$

$$\int_1^2 10r^2 \cdot \int_0^{2\pi} d\phi \cdot \int_0^{10} dz \Rightarrow 23.33 \cdot 2\pi \cdot 10 = 1466.07\ \text{c}$$

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8) Given that $D = \langle 10 \sin\phi\, \hat{a}_r, + 2\cos\theta\, \hat{a}_\theta \rangle$ evaluate both sides of the divergence theorem for the volume enclosed by the shell $r=2$

$$\nabla \cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 G_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(G_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial G_\phi}{\partial \phi}$$

$$\bar{D} = \langle 10 \sin\phi, 2\cos\theta, 0 \rangle$$

$r = 2$

$$\nabla \cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \sin\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(10\sin^2\phi) + \cancel{\frac{1}{r\sin\theta}\frac{\partial 0}{\partial\phi}}^{\,0}$$

* I'm not sure if I'm doing this correctly or not: I'm not understanding How to correctly evaluate both sides of the integral of the divergence ~~therem~~