Electrical Circuits Equations

I. Introduction

This document contains various equations and formulas that are related to electrical circuits. It is written using Overleaf which is an online LaTeX editor. If you see any error please email watts.jacob.samuel@gmail.com. This document contains many equations for finding different values within electrical circuits.

II. Engineering Prefixes

It is important to know how to convert between different scientific prefixes. For example when given 12×10^3 A and you would like to know what the unit is. Use the following table.

A. SI Units Chart

Prefix	Symbol	Exponential	Multiplier
$tera$	T	10^{12}	$1,000,000,000,000$
$giga$	G	10^{9}	$1,000,000,000$
$mega$	M	10^{6}	$1,000,000$
$kilo$	k	10^{3}	$1,000$
$None$	–	10^{0}	1
$milli$	m	10^{-3}	0.001
$micro$	μ	10^{-6}	0.000001
$nano$	n	10^{-9}	0.000000001
$pico$	p	10^{-12}	0.000000000001

III. Ohm's Law

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points.

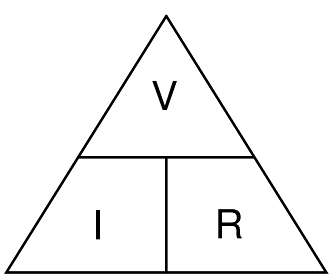


Fig. 1: Ohm's Law Triangle

The following equations are the relationships between V, R, and I. Looking at the triangle it is easily recognizable the for the value that you are trying to find you set that equal to whatever orientation the other two values are in.

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I} \tag{1}$$

IV. Resistors

A. Resistors in Series

To calculate the resistance of multiple resistors in series, simply take the sum of all the resistors. Please refer to Equation 11.

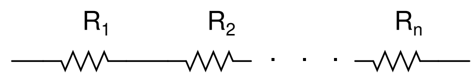


Fig. 2: Resistors in Series

$$R_T = R_1 + R_2 + ... \, R_n \tag{2}$$

B. Two (2) Resistors in Parallel

To calculate the resistance of two (2) resistors in series. Please refer to Equation 3.

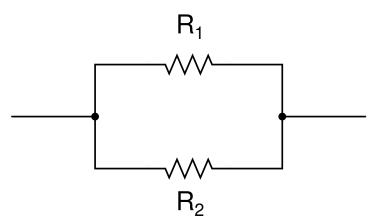


Fig. 3: Resistors in Parallel

$$R = \frac{R_1 R_2}{R_1 + R_2} \tag{3}$$

C. Multiple Resistors in Parallel

Please see Figure 4. You will notice that it has more than two (2) resistors. If there are more than two (2) resistors you will be unable to use Equation 3. So, the Equation 4 is used to calculate the total resistance. An important note to remember is that Equation 4 can still be used even when there are only two (2) resistors. I need to add more information here because I want the equation to be under the graphic like so, but that still is not enough information so I will continue to type until it is there. I don't understand why the equation won't post below this sentence.

$$R_P = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots \frac{1}{R_n}} \tag{4}$$

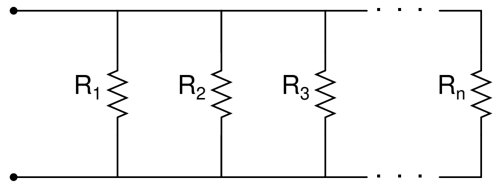
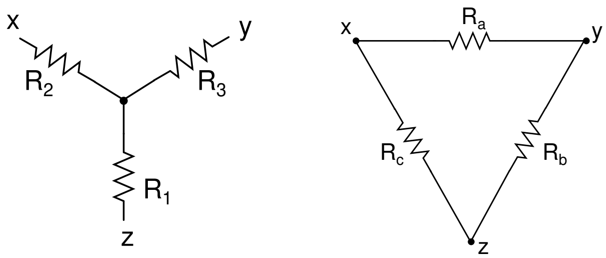


Fig. 4: Multiple resistors in parallel



(a) A subfigure (b) A subfigure

Fig. 5: A figure with two subfigures

D. Wye to Delta Transformation

The following figures and equations will illustrate how to convert from Wye to Delta and vice versa.

2

$$R_1 = \frac{R_c\,R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a\,R_c}{R_a + R_b + R_c} \tag{5}$$

$$R_3 = \frac{R_a\,R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1\,R_2 + R_2\,R_3 + R_3\,R_1}{R_1}$$

$$R_b = \frac{R_1\,R_2 + R_2\,R_3 + R_3\,R_1}{R_2} \tag{6}$$

$$R_c = \frac{R_1\,R_2 + R_2\,R_3 + R_3\,R_1}{R_3}$$

V. Current and Voltage Dividers

It is important to understand how to find the voltage and current in circuits. This because you need to figure out how to put the current into the circuit. I don't understand why the image isn't moving down. I still don't know why it's not moving down still. I wonder if I need to look it up online.

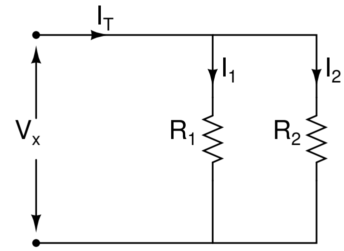


Fig. 6: Current Divider circuit

A. Current Divider

An example of how to use the current divider equation.

To find the current through on of the branches. First(1) find the R_p and plug it into equation (5) to find the current through the branch.

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \qquad I_1 = \frac{R_P}{R_1}(I_T) \tag{7}$$

The following equation is also a current divider equation but can only be used for circuits with two (2) resistors.

$$I_1 = \frac{I_T R_2}{R_1 + R_2} \tag{8}$$

B. Voltage Divider

An example of how to use the voltage divider equation

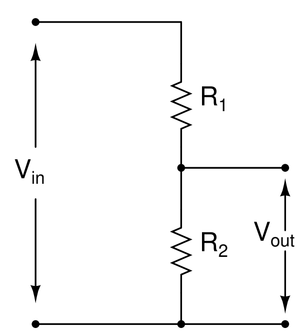


Fig. 7: Voltage Divider Circuit

$$V_{out} = \frac{R_2}{R_1 +\, R_2}\, V_{in} \tag{9}$$

VI. Capacitors

A. Capacitors in Series

Capacitors in Series. Basically just do the same as resistors in parallel but obviously use the values found for the capacitors.

$$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots \frac{1}{C_n}} \tag{10}$$

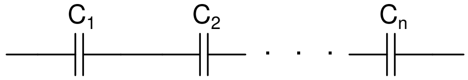


Fig. 8: Caption

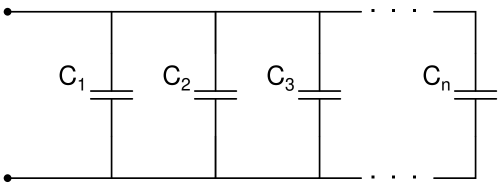


Fig. 9: Caption

B. *Capacitors in Parallel*

Capacitors in Parallel

$$C_P = C_1 + C_2 + ...\ C_n \tag{11}$$

VII. Operation Amplifiers (OP)

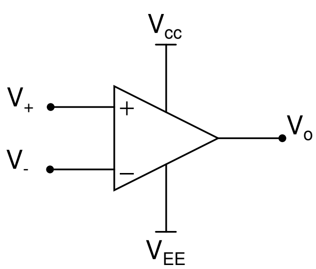


Fig. 10: Operation Amplifier

A. *Inverting Op-amp*

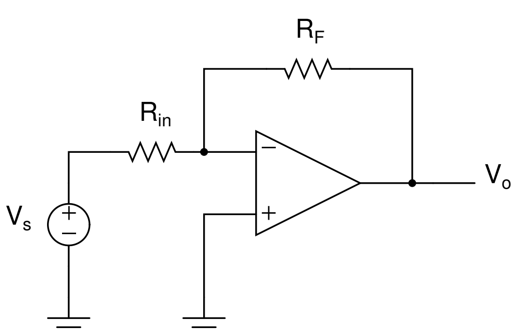


Fig. 11: Inverting Op amp

To find the voltage gain of an Inverting Op-amp use the following equation:

$$A_v = \frac{-R_F}{R_{in}} \tag{12}$$

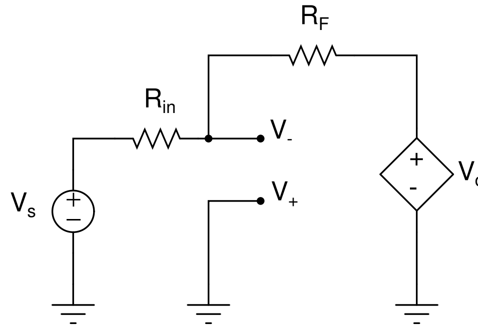


Fig. 12: Inverting Op amp Equivalent

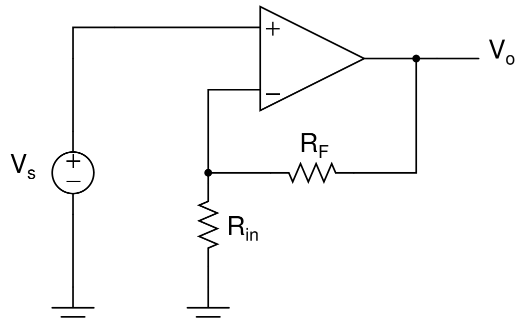


Fig. 13: Non-inverting Op amp

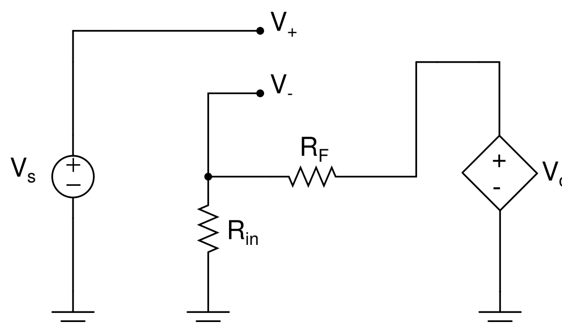


Fig. 14: Non-inverting Op amp equivalent

B. *Non Inverting Op-amp*

To find the voltage gain of a Non-inverting Op-amp use the following equation:

$$A_v = 1 + \frac{R_F}{R_i n} \tag{13}$$

C. *Ideal Op-Amp Voltage gain*

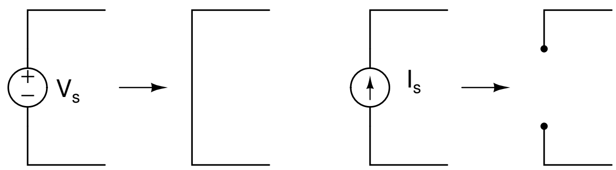
The ideal Op-Amp will have the following characteristics

$$\begin{aligned} A_o &\to \infty \\ R_{in} &\to \infty \\ R_o &\to 0 \\ i_+ = i_- &= 0 \\ V_- &= V_+ \end{aligned} \tag{14}$$

VIII. Additional Analysis Techniques

A. Superposition

To calculate superposition short circuit the voltage source, and calculate $V_0^{'}$. Then replace the voltage source and break the current source, then calculate $V_0^{''}$.



(a) Voltage Source (b) Current Source

Fig. 15: Superposition Calculations

B. Thévenin's Equivalent

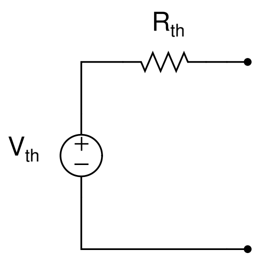


Fig. 16: Thevenin's Equivalent

C. Norton's Equivalent

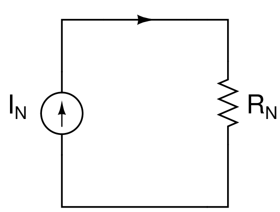


Fig. 17: Norton's Equivalent

D. Max Power xfer

Ohm's Law for Power

$$P = I^2 R \qquad P = VI \qquad R = \frac{V^2}{R} \tag{15}$$

Use the following relationship to gain max power

$$R_{th} = R_L \tag{16}$$

IX. Capacitance and Inductance

A. Capacitance

A capacitor is built out of parallel plates and a dielectric substance. Charge storage in a capacitor is found by using the following equation.

$$q = CV \tag{17}$$

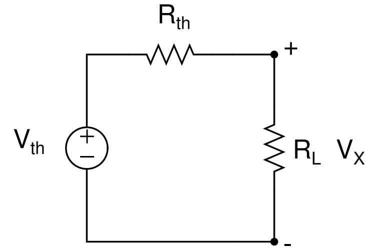


Fig. 18: Max Power xfer

B. Inductors

Inductors store energy.

C. Inductors in Series

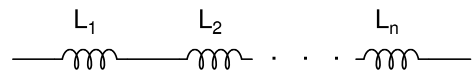


Fig. 19: Inductors in Series

$$L_{eq} = L_1 + L_2 + ...\ L_n \tag{18}$$

D. Inductors in Parallel

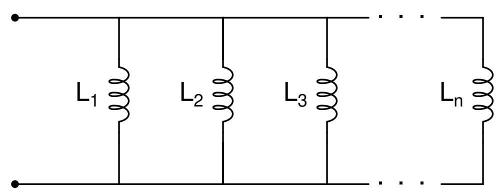


Fig. 20: Inductors in Parallel

$$L_P = \frac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3} ... \dfrac{1}{L_n}} \tag{19}$$

X. AC Circuit Analysis

A. Rectangular and Polar Coordinates

A complex number $z = x + jy$ is the ordered pair (x, y) of real numbers, which can be considered as the Cartesian coordinates of a point in the xy-plane.

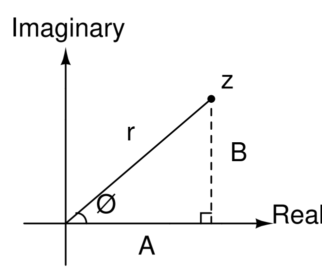


Fig. 21: Complex Plane

The following is the equation in rectangular coordinates:

$$z = A + jB \tag{20}$$

The following is the equation in polar coordinates:

$$z = r\angle\theta° \tag{21}$$

For the same phase of current I and voltage V_s, the anlge of Z_{total} must be zero (0).

$$I_m[Z_{total}] = 0 \tag{22}$$

Purely resistive circuit - the imaginary part of it must be zero (0).

To type polar coordinates into your calculator follow this guide:

Set your calculator to the following MODE:

- DEGREE
- POLAR
- a+bi

Type the following into your calculator to calculate in polar form:

$$5\angle\text{-}53°V = 5e^{-53\pi i/180} \tag{23}$$

B. Sinusoids

Let us begin our discussion of sinusoidal functions by considering the general expression for a sinusoidal function:

$$x(t) = X_M sin(wt + \theta) \tag{24}$$

By using the above function we can calculate the follow:

$$w = \frac{2\pi}{T} \qquad f = \frac{1}{T} \tag{25}$$

Sometimes you will be given an equation that you will need to convert to the other trig function, use the following formula to help:

$$\begin{aligned} v(t) &= X_M cos(wt + \theta)V \\ v(t) &= X_M sin(wt + \theta + 90°)V \end{aligned} \tag{26}$$

Same can be used for going from sin to cos, use the following equation:

$$\begin{aligned} v(t) &= X_M sin(wt + \theta)V \\ v(t) &= X_M cos(wt + \theta - 90°)V \end{aligned} \tag{27}$$

Use the following to help identify time domain and frequency domain:

TIME DOMAIN	FREQUENCY DOMAIN
$Acos(wt \pm \theta)$	$A\angle \pm \theta°$
$Asin(wt \pm \theta)$	$A\angle \pm \theta° - 90°$

The following are examples of converting from voltage functions to phasors.

$$\begin{aligned} v_1(t) = 12cos(377t - 425°)V &\longrightarrow V_1 = 12\angle - 425°V \\ v_2(t) = 18sin(2513t + 4.2°)V &\longrightarrow V_2 = 18\angle - 85.8°V \end{aligned} \tag{28}$$

The following are conversions from phasors to the time domain if the frequency was 400 Hz.

$$\begin{aligned} V_1 = 10\angle 20°V &\longrightarrow v_1(t) = 10cos(800\pi t + 20°)V \\ V_2 = 12\angle - 60°V &\longrightarrow v_2(t) = 12cos(800\pi t - 60°)V \end{aligned} \tag{29}$$

C. Impedance, Admittance, Reactants

Component	Impedance	Reactance
R	$\overrightarrow{Z_R} = R$	$X_R = 0$
L	$\overrightarrow{Z_L}$ =jwL	$X_L = wL$
C	$\overrightarrow{Z_C} = \frac{1}{jwC}$	$X_c = \frac{1}{wC}$

$$X_C = \frac{1}{jWC} \qquad X_C = \frac{1}{j2\pi fC} \tag{30}$$

$$X_L = jWL \qquad X_L = j2\pi fL \tag{31}$$

D. Steady-State Power

In general, the steady-state voltage and current for the network can be written as

$$\begin{aligned} v(t) &= V_M cos(wt + \theta_v) \\ i(t) &= I_M cos(wt + \theta_i) \end{aligned} \tag{32}$$

The instantaneous power is then found

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= V_M I_M cos(wt + \theta_v)cos(wt + \theta_i) \end{aligned} \tag{33}$$

Employing the following trigonometric identity, we find that the instantaneous power can be written as

$$p(t) = \frac{V_M I_M}{2}[cos(\theta_v - \theta_i) + cos(2wt + \theta_v + \theta_i)] \tag{34}$$

E. Max Power Transfer

The equation for average power at the load is

$$P_L = \frac{1}{2}V_L I_L cos(\theta_{vL} - \theta_{iL}) \tag{35}$$

Max power to the load is found with

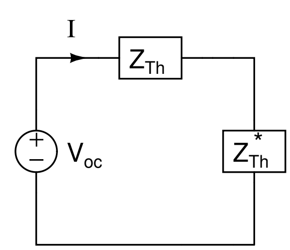


Fig. 22: Max Power

$$\boldsymbol{Z}_L = \boldsymbol{Z}_{TH}^* \tag{36}$$

The * means the complex conjugate. Simply just switch the ± sign for the complex number.

$$(C + jD)^* = (C - jD) \tag{37}$$

F. Effective or RMS Values

RMS - "root mean square"

On using the rms values for voltage and current, the average power can be written, in general, as

$$P = V_{rms} I_{rms} cos(\theta_v - \theta_i) \tag{38}$$

We now define the power factor (pf) as the ratio of the average power to the apparent power; that is,

$$pf = \frac{P}{V_{rms} I_{rms}} = cos(\theta_v - \theta_i) \tag{39}$$

where

$$cos(\theta_v - \theta_i) = cos\theta_{Z_L} \tag{40}$$

G. Complex Power

Complex power is defined to be:

$$\boldsymbol{S} = \boldsymbol{V}_{rms} \boldsymbol{I}_{rms}^* \tag{41}$$

where $\boldsymbol{I}_{rms}^*$ refers to the complex conjugate of $\boldsymbol{I}_{rms}$.
The complex power can be expressed in the form

$$\boldsymbol{S} = P = jQ \tag{42}$$

XI. Ideal Transformers

A. Example Problems

10.7 - Find $\boldsymbol{V}_o$ in the circuit. Go!

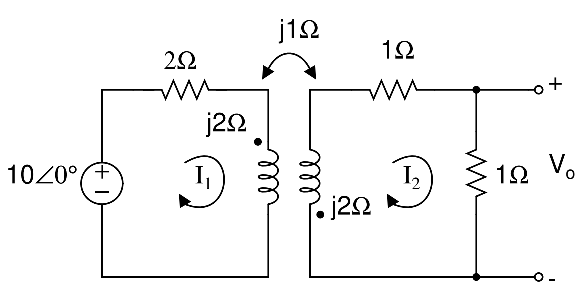


Fig. 23: 10.7

Apply the KVL for I_1 loop.

$$0 = -10\angle 0° + (2 + j2)I_1 + j1I_2$$

Apply the KVL for the I_2 loop.

$$0 = (2 + j2)I_2 + j1I_1$$

Rearrange the two (2) equations

$$(2 + j2)I_1 + j1I_2 = 10\angle 0°$$
$$(2 + j2)I_2 + j1I_1 = 0$$

Place the equations into your calculator using the following guide.

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

a - real number
b - imaginary number

The matrix for the two (2) equations should look like the following:

$$\begin{bmatrix} 2 & -2 & 0 & -1 & 10 & 0 \\ 2 & 2 & 1 & 0 & 0 & 10 \\ 0 & -1 & 2 & -2 & 0 & 0 \\ 1 & 0 & 2 & 2 & 0 & 0 \end{bmatrix}$$