Counting

CSE2315, Chapter 4-2

Counting

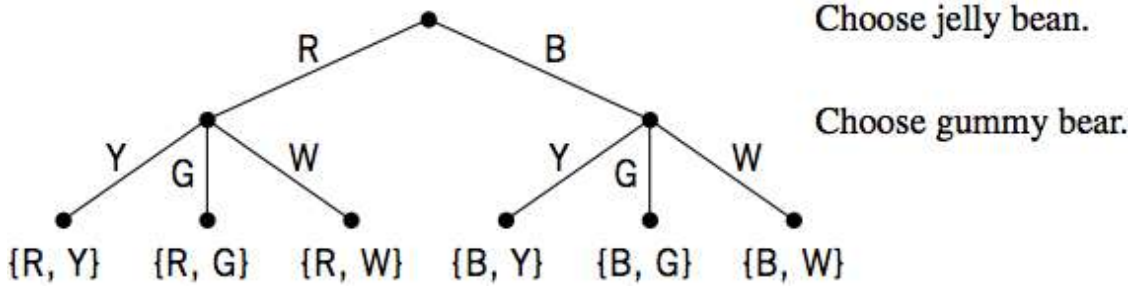
- **Combinatorics** is the branch of mathematics that deals with counting
- The idea is to find out how many members are present in a finite set. Principles of counting answer the following kind of questions:
  - How many four digit numbers can there be if repetition of numbers are allowed and if repetition of numbers are not allowed?
  - If a man has 4 suits, 8 shirts and 5 ties, how many outfits can he put together?

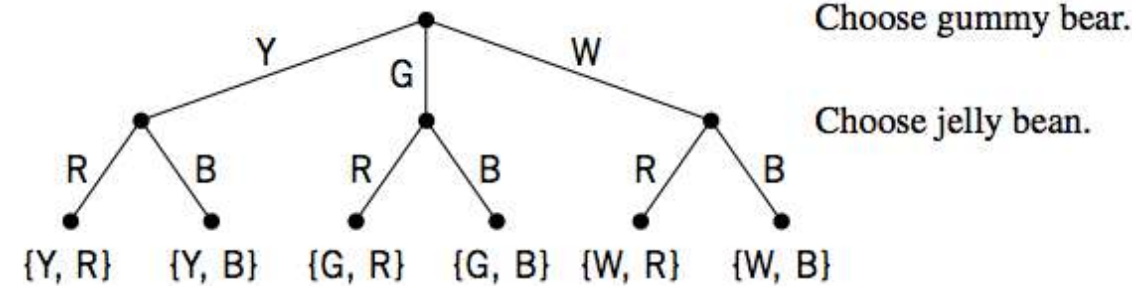
Example: Multiplication Principle

- A child is allowed to choose one jellybean out of two jellybeans, one red and one black, and one gummy bear out of three gummy bears, yellow, green, and white. How many different sets of candy can the child have?

Example: Multiplication Principle

- There are $2\times3=6$ or $3\times2=6$ possible outcomes as seen from the following figures





Multiplication Principle

- Multiplication Principle: If there are *m* possible outcomes for a first event and *n* possible outcomes for a second event, then there are $m \cdot n$ possible outcomes for the sequence of two events.
- Hence, from the multiplication principle, it follows that for two sets A and B

$$|A \times B| = |A| \cdot |B|$$

Addition Principle

- Addition Principle: If A and B are disjoint events with *m* and *n* outcomes, respectively, then the total number of possible outcomes for event "A or B" is *m+n*.
- If A and B are disjoint sets, then $|A \cup B| = |A| + |B|$ using the addition principle.
- (ex32) Prove that if A and B are finite sets then

  $|A-B| = |A| - |A \cap B|$ and $|A-B| = |A| - |B|$ if $B \subseteq A$

  $(A-B) \cup (A \cap B) = (A \cap B') \cup (A \cap B)$

  $= A \cap (B' \cup B)$

  $= A \cap U$

  $= A$
- Also, A-B and $A \cap B$ are disjoint sets, therefore using the addition principle

  $|A| = |(A-B) \cup (A \cap B)| = |A-B| + |A \cap B|$
- Hence, $|A-B| = |A| - |A \cap B|$
- If $B \subseteq A$, then $A \cap B = B$
- Hence, $|A-B| = |A| - |B|$

Class Exercise

- How many four-digit numbers begin with a 4 or a 5 (ex34)?

- How many three-digit integers (numbers between 100 and 999 inclusive) are even (ex36)?