Economic Analysis of Blackjack: An Application of Prospect Theory[*]

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May 17, 2009

Abstract

With prior gains, a stock trader becomes more risk-seeking until the gains are cancelled out by further losses. A losing hockey team tends to pull out the goalie in the final minute of the game even this increases the chance of losing more scores. The first example is called "house money effect" and the second "break even effect". These behaviours are conformable with the implications of prospect theory developed by Daniel Kahneman and Amos Tversky. These hypotheses are usually tested by behavioural economists under laboratory environments. On the contrary, we analyse the actual behaviours of blackjack players using data collected in a casino. Our results indicate that less than half of the gamblers follow the optimal strategies in the game. But they increase their effort when the wagers are higher. More interestingly, only a moderate fraction of the gamblers exhibits the house money effect and/or the break even effect.

JEL Classification Code: D81

1 Introduction

Since the expected utility theory (EUT) was proposed by John von Neumann and Oskar Morgenstein some six decades ago, it has been the work horse of many economic applications for choices under risky situations. The theory implies that the overall cardinal utility of a decision maker facing a gamble is a convex combination of the direct utilities of all the possible outcomes, using the probability of each

outcome as the weight. Over the years, there have been numerous empirical observations which work against the original theory. The linear structure of EUT is too restrictive when comparing the choices of different gambles, creating a number of now famous "paradoxes". For example, a decision maker cannot be both risk-averse and risk-seeking at the same time.

A number of new theories have been proposed to relax the restrictive structure of EUT. Most of these models replace the independent axiom of EUT with a more flexible one, such as the "betweenness" axiom proposed by Chew (1983).[1] The new theories emphasize the mathematical properties of the models to resolve the paradoxes using the axiomatic approach. A common characteristic of these models is the so-called "first order risk aversion", which means that the risk premium of a gamble is proportional to the standard deviation of the gamble. The EUT, on the other hand, implies that the risk premium is proportional to the variance. The new theories have been applied to a number of practical applications with satisfactory results. Examples include Chew and Epstein's (1990) analysis of intertemporal consumption, Epstein and Zin's (1989) dynamic capital asset pricing model, and Yu's (2008) measurement of lottery output.

Based on extensive laboratory experiments, Daniel Kahneman and Amos Tversky have developed the prospect theory.[2] Unlike the other new theories described above, prospect theory emphasizes the psychology and behavioural foundation of decision making under risk and uncertainty. The theory has been tested extensively in the laboratory environment with impressive results. A number of applications and testings are described in Thaler (1991). Applications of prospect theory involve two steps. The first one is called the editing phase, where the decision maker put a risky or uncertain prospect in his or her mental framework. The second phase,

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[1]Epstein (1992), Machina (1987), and Starmer (2000) provide comprehensive surveys.

[2]See Kahneman and Tversky (1979) and Tversky and Kahneman (1981, 1992).

called valuation, uses a weighting function and a value function to evaluate the overall utility of the prospect. Kahneman and Tversky's theoretical development mainly concentrate on the valuation phase. Various hypotheses of behaviours in the editing phase have been proposed by Thaler and Johnson (1990). They conclude that in a multiple decision making process, prior gains and losses are incorporated into the editing phase of a subsequent prospect. In particular, prior losses may sensitize the chance of further losses and increase risk averseness, while prior gains are treated as "house money" and promotes risk-seeking behaviours. On the other hand, if a decision maker with prior losses is given the chance to "break even" in a subsequent prospect, he or she may turn risk-seeking.

There are difficulties in testing behaviours involving losses in a laboratory setting. First, since the subjects of the experiments are typically college students, the possibility of real monetary losses poses an ethical problem. Second, many students may be reluctant to participate in the study, thus creating a selection bias. Experiments carried out by Thaler and Johnson (1990) are designed to minimize the probability and the amount of losses. Their results are based on small-stake gambles and may not be realistic in practice.

In this paper we test Thaler and Johnson's editing rules of prospect theory using observed behaviours of blackjack players in a casino. Therefore the usual qualifications of the results obtained under the laboratory experimental setting do not apply. What we observed are the behaviours of real gamblers in a casino. We find a number of surprising results. First, the majority of gamblers are not strictly rational in the sense that they follow the rules of optimal strategies in playing the game. Second, only a moderate percentage of the players exhibit the house-money and break-even editing rules.

The structure of the paper is as follows. Section 2 briefly reviews prospect theory

and the editing rules proposed by Thaler and Johnson (1990). Section 3 describes the game of blackjack played in casinos. The rational strategy of playing for optimal gain is discussed. Data collection and analysis is presented in Section 4. This is followed by the conclusions in Section 5.

2   A Brief Review of Prospect Theory

A prospect is defined as a set of n outcomes $X = \{x_1, \ldots, x_n\}$, with corresponding probabilities $p_1, \ldots, p_n$ and $p_1 + \cdots + p_n = 1$. The prospect is usually represented in the form $(x_1, p_1; x_2, p_2; \ldots; x_n, p_n)$, with $x_1 < x_2 < \cdots < x_n$.[3] For example, $(-x, 1/2; x, 1/2)$ represents a simple lottery of winning or losing x with equal chances. In the editing phase, the decision maker performs a number of mental operations on the prospects. These operations include

1. Coding: Outcomes are seen as gains and losses from a reference point instead of from an overall wealth level view. The reference point can be affected by framing effects or expectations.

2. Combination: Probabilities with identical outcomes are combined.

3. Segregation: Common riskless components are segregated from the risky components. For example, if $0 < x < y$, $(x, p; y, 1 - p)$ can be decomposed into a sure gain of x and a risky prospect $(y - x, 1 - p)$.

4. Cancellation: Ignore stages of sequential games that are common to subsequent prospects (isolation effects).

5. Simplification: $(101, 0.49)$ is seen as $(100, 1/2)$, extremely unlikely outcomes are either discarded or over-represented.

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[3]In Tversky and Kahneman (1992) the negative outcomes are treated separately from the positive outcomes.

6. Detection of Dominance: Dominated alternatives are rejected without further evaluation.

   In the evaluation phase, the overall value V of a simple prospect $(x, p; y, q)$ is expressed in two scales: a decision-weight function $\pi : [0, 1] \rightarrow [0, 1]$ on probabilities p and q, and a value function $v : X \rightarrow \mathbb{R}$ which maps the outcomes into real numbers.

   A regular prospects is defined as either $p + q < 1$, or $x \geq 0 \geq y$, or $x \leq 0 \leq y$. In this case

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y),$$

with $v(0) = 0$, $\pi(0) = 0$, and $\pi(1) = 1$. On the other hand, a monotone prospect is defined as $p + q = 1$, and either $x > y > 0$, or $x < y < 0$. The overall value is

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)].$$

   For comparison, EUT specifies a von Neumann-Morgenstein utility function $u(w + z)$ where w is the current wealth level of the decision maker and z is any outcome in a prospect. The above prospect is evaluated by its expected utility

$$U(x, p; y, q) = pu(x + w) + qu(y + w).$$

Several characteristics of the value function v in prospect theory are distinguishable from the von Neumann-Morgenstein utility function u in EUT:

1. Prospects are evaluated using current wealth level w as a reference point. The value function v, however, can change with w.

2. The value function v is increasing and concave in the positive domain, and increasing and convex in the negative domain.

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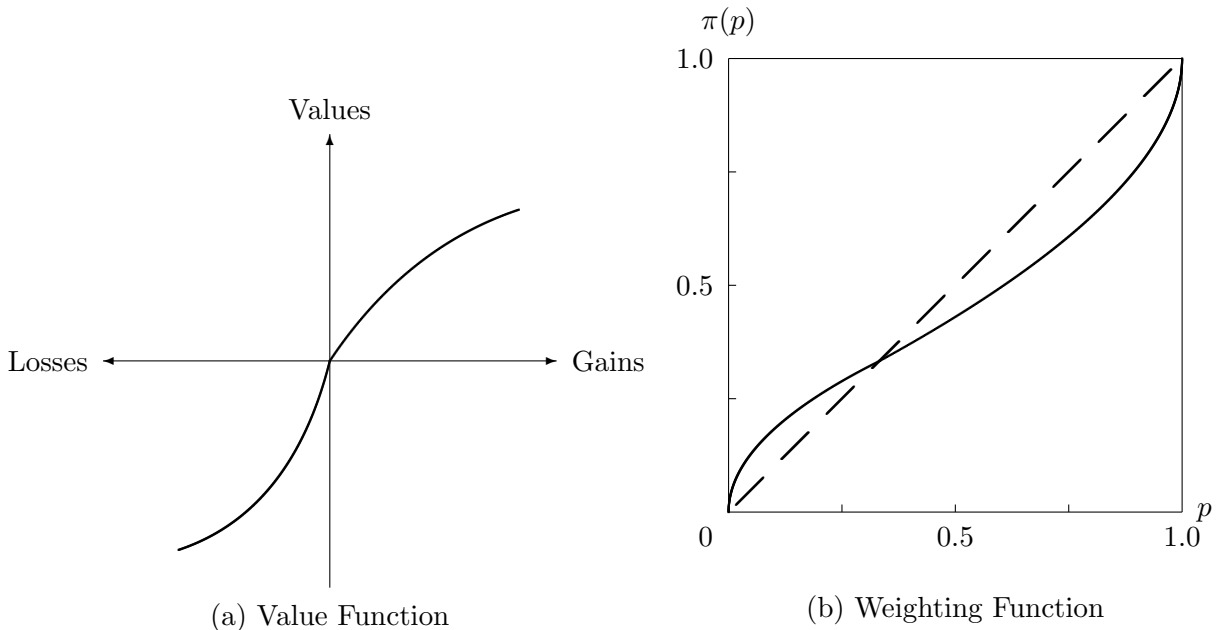|	
(a) Value Function	(b) Weighting Function

Figure 1: Value Function and Weighting Function

3. The slope of v is steeper for losses than for gains, i.e., for $x > 0$, $v(x) < -v(-x)$ and $v'(x) < v'(-x)$. This property reflects the commonly observed behaviour of loss aversion.

Figure 1(a) depicts the shape of a typical value function. Notice the kink at the origin due to loss aversion.

Decision makers often impose their own subjectivity on the probabilities of events, even when the objective probabilities are well-known. The differences are particularly salient for events with very small probabilities or for near certain events. For example, the probability of winning the jackpot in most government lotteries is extremely low (one in fourteen million for Lotto 6/49). Pictures of smiling winners promoted by the lottery corporations, however, give the impression that winning is not a remote thought. On the other hand, rare events that we do not see very often such as earthquakes, tsunamis, and terrorist attacks are ignored. These are examples of what Tversky and Kahneman (1974, p. 1127) call "biases due to the retrievability of instances". The weighting function π tries to incorporate this kind of mental

accounting in decision making. Figure 1(b) shows the graph of a typical weighting function proposed by Tversky and Kahneman (1992) satisfying the requirements $\pi(0) = 0$, and $\pi(1) = 1$. The two end points serve as the natural boundaries for π, which are impossibility and certainty. Low probabilities are over-weighted resulting in a concave curve, where moderate and high probabilities are under-weighted, represented by a convex curve.

Experimental results involving one-stage gambles reported by Tversky and Kahneman (1992) reveal the following behavioural patterns, which are compatible with prospect theory:

1. risk seeking in low probabilities for gains,

2. risk averse in moderate to high probabilities for gains,

3. risk averse in low probabilities for losses,

4. risk seeking in moderate to high probabilities for losses.

While the single-stage prospect theory put the reference point for gains and losses at the current wealth level, the reference may shift due to prior experience. Kahneman and Tversky (1979) suggest that, for example, incomplete adaptation to recent losses may cause the decision maker to be more risk-seeking. Thaler and Johnson (1990) maintain that prior experience should be incorporated into the editing phase of a subsequent prospect. They propose a number of editing rules which include

- Decisions with memory

- Decisions without memory

- Slovic's (1972, p. 9) concreteness principle, which states that "a judge or decision maker tends to use only the information that is explicitly displayed in the stimulus object and will use it only in the form in which it is displayed."

- Hedonic editing, in which decision makers seek the optimal rules to maximize the overall value of a prospect.

In a series of experiments involving two-stage gambles with prior gains or losses, Thaler and Johnson (1990) reject all the above hypotheses. They conclude that the risk attitude of the majority of the subjects conforms to what they call quasi-hedonic editing rules. Their findings can be summarized as follows.

First, the convexity of value function in the negative domain implies that unpleasant feeling can be mitigated by combining losses. That is, $v(-x)+v(-y) < v(-x-y)$. Quasi-hedonic editing, however, suggests that prior losses are not integrated with potential losses, causing an increase in risk aversion.

Second, prior gains are integrated with subsequent losses, mitigating loss aversion and facilitating risk-seeking. In other words, prior gains are treated as "house money" and cause a decision maker to be more adventurous.

Third, if a decision maker with prior losses faces a gamble with a moderate probability of winning an amount equal to or great than the prior losses, he or she may become more risk-seeking. This "break-even" effect therefore can switch a gambler's attitude toward risk.

Given the limitation of laboratory experiments involving losses discussed above, we are interested in testing these three editing rules in the field. Our targets are blackjack players in a Canadian casino. The game is introduced in the next section.

3   The Game of Blackjack

3.1   Casino Games and House Edges

According to Hayano (1982), gambling is controlled by the dimension of skill versus luck. Typical casino games like roulette, scraps, slot machines, and lotteries are

governed completely by the laws of chance. For these games, outcomes are neither predictable nor controllable by bettors. On the other hand, chess is a game of pure strategy and skill, independent of chance and luck. Blackjack is a game which lies between the two extremes of luck and skill, although it is hard to separate and measure the precise proportion of luck and skill. Occasionally the worst player at the table may end up with more chips than the best player.

Players are statistically in a disadvantage at each games against the casino because the house takes a percentage called the vigorish from each bet. In other words, all casino games are not actuarially fair. This advantage can be measured by the house edge, which is defined as the ratio of the average loss to the original wager.[4] Table 1 lists the house edges of some selected casino games. The house edge for blackjack is 0.28%, that is, the player will lose 28 cents on the average for every $100 original wager. Blackjack in fact has one of the lowest house edge among casino games. The combination of chance, skill, and low house edge makes it a popular game.

3.2   Rules of Blackjack

Blackjack is not a genuine game in the usual game-theoretic sense because one of the players' strategies is fixed — the dealer has a a set of strict rules to follow. From this point of view, if there is one player on the table, it becomes a one-person game. It is the only casino game that a player is able to attain a positive mathematical expectation from time to time. Therefore, unlike other casino games such as roulette and craps, blackjack is deemed to be a beatable game. Epstein (1995) also identifies other attractive characteristics of the game:

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[4]For some games such as blackjack, Let It Ride, and Caribbean stud poker, it is possible for a bettor to increase the wager in the midst of a game. The additional money wagered, however, is not used to calculate the house edge.

Table 1: House Edges of Selected Casino Games

Game	Bet/Rules	House Edge (percent)	Standard Deviation
Baccarat	Banker	1.06	0.93
Big Six	$1	11.11	0.99
Bonus Six	No insurance	10.42	5.79
	With insurance	23.83	6.51
Blackjack	Liberal Vegas rules	0.28	1.15
Casino War	Go to war on ties	2.88	1.05
	Surrender on ties	3.70	0.94
	Bet on tie	18.65	8.32
Craps	Pass/Come	1.41	1
	Don't pass/Don't come	1.36	0.99
Double Down Stud		2.67	2.97
Keno		25–29	1.30–46.04
Let it Ride		3.51	5.17
Roulette	Single zero	2.70	varied
	Double zero	5.26	varied
Slot Machines		2–15	8.74
Three Card Poker	Pair plus	2.32	2.91

Source: The Wizard of Odds <wizardofodds.com/houseedge>

1. Within each round the cards are interdependent so that each card reveals new information.

2. There are optimal strategies for any revealed information.

3. The mental retentiveness of a player also has influence on the results because change of the player's mood might change the player's risk-taking behaviour.

A blackjack table usually has 6 or 7 betting areas (see Figure 2). A round of blackjack begins with each player placing a bet in the circle in front of them. The game can use up to eight ordinary decks of cards shuffled together. Each card is given a numerical value corresponding to its rank except for the face cards (Kings, Queens, and Jacks), which all have a value of 10. The Aces can be counted as either 1 or 11. When the value of an ace can either be counted as 1 or 11, the sum of
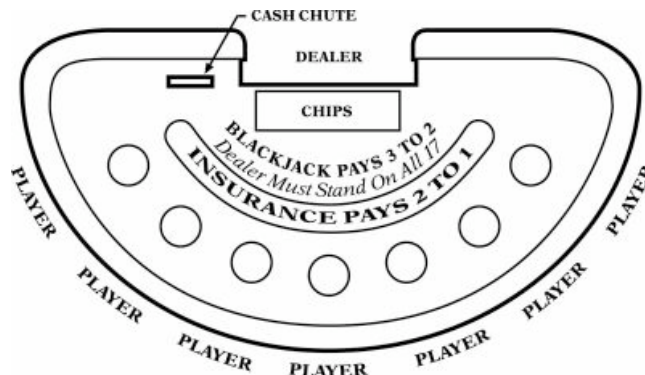
Figure 2: The Blackjack Table Layout

all card values in a hand is called the "soft total". Otherwise the sum is called the "hard total".

Each player is given two cards face up while the dealer has an up card and a hold card (face down). A player loses the round if he or she keeps drawing extra cards and the total point exceeds 21, which is called a break or bust. The dealer must draw on 16, and stand on 17 or above (either hard total or soft total). If the dealer exceeds 21, everyone remaining in the hand is paid. If neither a player nor the dealer exceeds 21, whoever gets the higher point wins. In the case when the dealer and a player get the same point, it is considered to be a push and the player does not lose his bet. If the player and the dealer both bust, then the player loses, which is part of the house advantage. All winning bets are paid at 1 to 1 odds except for a Blackjack, (original two cards equaling 21) which is paid 3 to 2. If the dealer has any ten-valued card as the up card, then he or she has to check for a Blackjack before the game proceeds. If the dealer does have a Blackjack, all players lose except for those who also have a Blackjack. If the dealer does not have a Blackjack, the game continues as usual.

When the dealer has an ace as the up card, an insurance wager is offered to the players. When a player accept the insurance, they are betting that the dealer has a Blackjack. If the dealer indeed has a Blackjack, all players lose their original wagers except for the plays that also have Blackjack, but those who bet on the insurance

will be paid 2 to 1. If the dealer does not have a Blackjack, the players lose all insurance bets and the hand continues as usual.

Players may ask the dealer for even money (1 to 1) before the dealer checks for Blackjack. If the player does not want even money and the dealer has a Blackjack, he will only get a push.

After it has been confirmed that the dealer does not have a Blackjack, the players can take turns to play their hands. The following options are available for the players.

1. Stand: If a player is satisfied with his hand, he or she chooses a stand pat, a hand signal waving over the table.

2. Hit: The players can draw as many cards as they want before exceeding a total of 21 points. The hand signal for "hit" is a knock on the table.

3. Split: Whenever the initial two cards dealt to a player are of equal value, the player has the option to split the cards into two separate hands. To signal a split, the player places another wager next to the original wager of equal value with the hand signal of holding two fingers up.

4. Double down: With the initial two cards, a player can double the original wager and draw one and only one additional card.

3.3   Strategies for Blackjack

3.3.1   Basic Strategies

Due to differences in the gamblers' mathematical backgrounds, attitudes toward risk, cognition distinction, and other related factors such as age and gender, each individual may have a different understanding of the game and therefore perform differently.

The mathematical and statistical properties of casino blackjack games are first studied by Baldwin *et al.* (1956) and Thorp (1961). The development of basic strategies for players has forced casinos to modified their rules in the 1970s (Manson *et al.*, 1975). The basic strategies here refer to how individuals play each hand differently against the dealer's variable up card. The strategies that casino players can refer to are shown in Figure 3. Calculation of these optimal strategies are based on the hypothesis that the dealer's and player's probabilities to obtain their respective totals by subsequent drawing of cards are independent. Dependency exists because the cards are drawn sequentially without replacing the used cards back. There is evidence, however, that the errors raised from the assumption of independence is negligible so that the optimal strategies can be derived from the assumption that each draw is independent. Epstein (1995, p. 223) reports that the error in expectation created by the assumption of independence is only 0.003 after a test of over four million hands of blackjack by computer simulation.

For a statistically independent game like blackjack, a player's optimal strategies depend on the mathematical expectation of gain per unit amount wagered.[5] For example, a simple prospect $(kx, p; -x, 1-p)$ has an expectation per dollar wagered equal to

$$\frac{pkx - (1-p)x}{x} = (1+k)p - 1.$$

Therefore a fair game with equal chances of winning or losing $(k = 1, p = 1/2)$ has expectation equal to zero. Table 2 list the differences in expectations between drawing and standing given the dealer's up card value and the player's hand total. A positive number means that drawing a new card has a better chance of beating the dealer than standing. Figure 3 is a facsimile of a card-size table published by the Ontario Lottery and Gaming Corporation and is available for any player in Ontario

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[5]See Theorem 1 in Epstein (1995, p. 53).

Table 2: Differences in Expectations Between Drawing and Standing

					dealer's Up-Card					
Player's Hard Total	A	2	3	4	5	6	7	8	9	10
12	0.2476	0.0393	0.0149	-0.0139	-0.0404	-0.023	0.2089	0.1885	0.1415	0.1537
13	0.2212	-0.0161	-0.044	-0.0796	-0.1124	-0.0932	0.1651	0.1483	0.1429	0.1173
14	0.1948	-0.0714	-0.1055	-0.1476	-0.1844	-0.1633	0.1214	0.1487	0.1063	0.0809
15	0.1684	-0.1295	-0.1695	-0.2154	-0.256	-0.2332	0.1184	0.1113	0.0696	0.0445
16	0.1384	-0.1902	-0.2334	-0.2833	-0.3274	-0.2619	0.0776	0.0739	0.033	0.0083
17	-0.0933	-0.4006	-0.4377	-0.4922	-0.4867	-0.5079	-0.3647	-0.1047	-0.1351	-0.1616
18	-0.5452	-0.7639	-0.7933	-0.7931	-0.8256	-0.8819	-0.9856	-0.685	-0.4248	-0.4732
19	-1.0314	-1.1404	-1.1207	-1.139	-1.1805	-1.2128	-1.3285	-1.3024	-0.9861	-0.8041
20	-1.5458	-1.4875	-1.4968	-1.5057	-1.5341	-1.5544	-1.624	-1.6407	-1.6035	-1.4072
Player's Soft Total										
17	0.2869	0.1417	0.1318	0.115	0.1375	0.119	0.1578	0.315	0.2644	0.2254
18	-0.0091	-0.0742	-0.0764	-0.0544	-0.0521	-0.0931	-0.2345	-0.0683	0.083	0.0295
19	-0.3118	-0.2845	-0.2526	-0.2377	-0.2393	-0.2596	-0.4029	-0.4496	-0.2838	-0.1742
20	-0.6075	-0.4611	-0.4448	-0.4224	-0.4119	-0.4216	-0.5292	-0.6112	-0.6599	-0.5559

Source: Epstein (1995, Chapter 7)

STRATEGIES FOR BLACKJACK
multiple deck game

Player's Hand	Dealer's up card									
	2	3	4	5	6	7	8	9	10	A
8 or Less	H	H	H	H	H	H	H	H	H	H
9	H	D	D	D	D	H	H	H	H	H
10	D	D	D	D	D	D	D	D	H	H
11	D	D	D	D	D	D	D	D	D	H
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
16	S	S	S	S	S	H	H	H	H	H
Ace-2	H	H	H	D	D	H	H	H	H	H
Ace-3	H	H	H	D	D	H	H	H	H	H
Ace-4	H	H	D	D	D	H	H	H	H	H
Ace-5	H	H	D	D	D	H	H	H	H	H
Ace-6	H	D	D	D	D	H	H	H	H	H
Ace-7	S	D	D	D	D	S	S	H	H	H
Ace-8	S	S	S	S	S	S	S	S	S	S
Ace-9	S	S	S	S	S	S	S	S	S	S
2 - 2	SP	SP	SP	SP	SP	SP	H	H	H	H
3 - 3	SP	SP	SP	SP	SP	SP	H	H	H	H
4 - 4	H	H	H	SP	SP	H	H	H	H	H
5 - 5	D	D	D	D	D	D	D	D	H	H
6 - 6	SP	SP	SP	SP	SP	H	H	H	H	H
7 - 7	SP	SP	SP	SP	SP	SP	H	H	H	H
8 - 8	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
9 - 9	SP	SP	SP	SP	SP	S	SP	SP	S	S
10 - 10	S	S	S	S	S	S	S	S	S	S
A - A	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
H = Hit	S = Stand		D = Double		SP = Split					

Figure 3: Basic Strategies for Blackjack

casinos. It provides information on the optimal strategies for a player with any combination at hand. For example, given the dealer's up card is 8 and a player has an Ace and a 7, the best strategy is S (stand). These strategies are based on comparing the mathematical expectations among the probable actions assuming card independence. Therefore they are statistically the best actions a player can take to beat the dealer in the long-run under the assumption of card independence. Epstein (1995, p. 225) classifies this technique as zero-memory strategies. Straight adherence to these strategies yields a slight advantage over the dealer by a slight positive expectation of 0.001. It is the only casino game with an overall positive expectation.[6] Alternatively, there are strategies with memory in which incorporate card dependence. They require the player to use the all the information of the up cards on the table and to follow an optimal strategy based on the particular odds.

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[6] Optimal strategies and expectations for 4-deck blackjack are computed by Manson *et al.* (1975).

The techniques, however, require extensive training and practice, with marginal improvement in the overall odds. It is not surprising that even the a skillful player use the basic strategies described in Figure 3 most of the time. In our analysis we assume that players at best follow the no-memory strategies. Our emphasis is not to analyze the mathematics properties of the basic strategy, but to study how players follow the basic strategies in different ways and their consequences in winning and losing.

3.3.2 Betting Strategies

At the beginning of each game each player decides on the wager before the dealer starts drawing any card. Players may choose to bet the same wager for every game or adjust their wager according to prior wins or losses.

Rogers (1998) describes an ubiquitous phenomenon in the real gambling world, called the gambler's fallacy or Monte Carlo fallacy. It states that gamblers adjust their wager according to the result of each hand. This is based on a belief that a particular outcome is more likely to occur simply because it has not occurred for a period of time. Gamblers who believe in the fallacy may adjust their wager according to the previous outcomes. This is of course an erroneous reasoning because the probability of an event in a random sequence is independent of preceding events.

The insurance bet is a special bet in blackjack and calls for special attention. Table 3 shows that the house edge on an insurance bet increases with the number of decks used. Obviously, all house edges on insurance are higher than the normal game (0.28%). Therefore the best choice for the players is not to take insurance, even if they have a Blackjack.

Table 3: House Edges on Insurance

Number of Decks	House Edge (Percent)
1	5.882
2	6.796
4	7.246
6	7.395
8	7.470

Source: The Wizard of Odds <wizardofodds.com/blackjack>

4 Data Collection and Analysis

4.1 Data

Data were collected by one of the authors, who worked in a casino in Ontario as a blackjack card dealer for six months. Training on game rules, card dealing techniques, and customer relationship was provided by the casino prior to job commencement. After three months on the job, behaviours of recognizable players are observed and recorded during break times. The whole data collection period spanned about three months. Players are identified on a daily basis with the following information recorded:

- Gender

- Minimum wager of the blackjack table

- Playing strategies

- Betting strategies

- Time spent at the blackjack table

- Gambling result at the end of the session

- Table number

- Date and time

First, players are categorized into four groups according to their playing strategies:

1. Strict Strategists: These are players who follow the basic strategies most of the time. Individuals who temporally abandon the basic strategies for only a few hands are still included.

2. Partial Strategists: Players who follow parts of the basic strategies. They usually stand on 16 when the dealer's up card shows 7, 8, 9, 10, and A, and do not hit 12 when dealer's up card shows 2 and 3.

3. Occasional Strategists: Players who follow the basic strategies to some extent. They not only stand on 16, but also stand on some of 12, 13, 14, or 15 when the dealer's up card shows 7, 8, 9, 10, and A, are classified into this group. Similar to group 2, individuals in this group usually do not hit 12 when the dealer's up card shows 2 and 3 as well.

4. Non-strategists: Players who do not follow the basic strategies and play the game in a random manner. They alternate between risk-seeking and risk-averse behaviours.

Therefore strategic behaviours in the above list are in a decreasing order. The similarity among the first three groups is that players tend to know more about the game relative to those in group 4. A possible explanation for the different playing styles and strategies may due to the differences of their attitude toward risk. Taking group 1 as the benchmark, players in group 2 are more risk-averse, and individuals in group 3 are the most risk-averse among these three groups. In particular, the distinction among groups 1, 2, and 3 lies on the reactions of players to the following two situations:

1. the decision to stand or hit on 16 when the dealer's up card shows 7, 8, 9, or 10,

2. the decision to stand or hit on 12 when the dealer's up card shows 2 and 3.

The reason for these criteria is that 12 and 16 are commonly considered as the two most troublesome hands for players. In Table 2, the closer the difference of expectations between drawing and standing is to zero, the harder it is for players make the decision.

Under the above mentioned situation 1, the differences of expectations between drawing and standing is considerably small compared with other cases. This means that the players have a good chance to exceed 21 when hitting 16. The risk-averse player would rather stand the hand in order to remain in the game, hoping for the dealer to go bust. Similarly, under situation 2, the differences are also noticeably small. The risk-averse players would rather stand and leave the chance of busting to the dealer. If a player is more risk-averse, he or she will try to avoid breaking his hand on more occasions. As a result, this kind of players will end up in group 3.

Since the samples are collected on a daily basis, a player may be counted more than once in the data set. Also, the player may change his or her strategies on different days and end up in different groups. A partial strategist on one day may become a strict strategist on another day, and vice versa. Our goal is to estimate the probabilities of winning for the four groups. As a result, even though one player presents consistent gambling strategy all the time, if he or she appears on the table for more than once, then this player will still be recorded more than once in this data set. For example, if one player consistently follow the basic strategy of the game, and he or she comes to play the game three times during the period of time that all the data are collected, then this player will be recorded three times in this data set.

It should be pointed out that the status of winning or losing is not the result of the whole betting day, but only for the certain hour on the certain table. In this case, the reference point for each individual is defined as the point whenever they start on a table. Whether one player wins or loses depends on the difference between the amount of chips the player brings to the table and the amount when he or she leave. For these reasons, some qualifications apply to our data set. First, the status of winning or losing for all the players is based on some personal judgements of the data collector and thus occasional inaccuracy may result. Second, for privacy reasons, the data do not contain the information about the exact amount of winning or losing for each player. Therefore our analysis is based on the percentage of winners and losers instead of the amount of wining or losing of individual gamblers.

In passing we should mention that at the project planning stage we are concerned about the accuracy of the data collection process. We soon discover that dealing blackjack in a casino is not unlike learning a foreign language. At first the dealer concentrates on carding counting and avoiding mistakes, like learning the grammar and vocabulary of the new language. After being on the job for a month, fluency in the techniques allows the dealer to understand a player's style, strategies, and body language after only a few hands. In fact, the results of our analysis are intuitively familiar to any experienced card dealer.

4.2 Winning and Losing

With the amount of chips a player bring to the table as the reference point, Table 4 reports the number of winners and losers when they leave the table. There are a total of 748 observations, with 644 male players (86%) and 104 female players (14%). In general, female gamblers prefer games that require less skill, such as slot machines. Overall, 40.8% of players win at the end of the sessions. The percentage of winners in

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Table 4: Descriptive Statistics of Male and Female Players

| | Group | | | | |
	1	2	3	4	total
Male					
win	143	62	37	19	261
lose	133	103	62	85	383
total	276	165	99	104	644
% winning	51.8	37.6	37.4	18.3	40.5
Female					
win	28	8	5	3	44
lose	17	25	8	10	60
total	45	33	13	13	104
% of winning	62.2	24.2	38.5	23.1	42.3
Combined					
win	171	70	42	22	305
lose	150	128	70	95	443
total	321	198	112	117	748
% winning	53.3	35.4	37.5	18.8	40.8

each group increases with the degree of strategies used. For example, 51.8% of the male strict strategists (Group 1) win, while only 18.3% of non-strategists (Group 4) do. One exception is the female occasional strategists (Group 3), who have a higher winning percentage than the female partial strategists (Group 2). Overall, the percentage of winners in Groups 2 and 3 are close. The conclusion here is that it pays to follow the basic strategies but only 321 out of 748 players (43%) strictly adopt them. A surprising result from our sample is that a player on the average can beat the house if he or she follows the basic strategies all the time, which makes blackjack a unique casino game.

We surmise the reasons why players do not strictly follow the basic strategies. The rules presented in Figure 3 require efforts in understanding and practice to follow. Some occasional gamblers may not even know that such strategies exist. Moreover, visitors of casinos may have objectives other than winning. Some of them are there for social gathering or for spending leisure time (Eadington, 1999).

Table 5: Descriptive Statistics of Different Minimum Wagers

	Group				
	1	2	3	4	total
$5 minimum					
win	98	42	36	19	195
lose	96	90	62	71	319
total	194	132	98	90	514
% of winning	50.5	31.8	36.7	21.1	37.9
$10 minimum					
win	63	24	6	1	94
lose	44	33	5	21	103
total	107	57	11	22	197
% of winning	58.9	42.1	54.6	4.6	47.76
$25 minimum					
win	10	4	0	2	16
lose	10	5	3	3	21
total	20	9	3	5	37
% of winning	50.0	44.4	0.0	40.0	43.2

The casino had set various minimum wagers for different blackjack tables. The minimum wagers in our data set are $5, $10, and $25 per hand. Table 5 shows that the $5 tables are the most popular, and the number of players decreases with the minimum wager. Winning percentages mainly follow the patterns in Table 4. Strict strategists have a advantage over all the other groups. Again the performance of Groups 2 and 3 are similar.

For tables with $5 minimum, 194 out of 514 players are strict strategist, which translates into 38%. The proportion for the $10 and $25 tables are both 54%. Therefore the higher stake stimulates more mental effort and promotes rational behaviours. As a result, the overall chance of winning at the $10 tables is higher than that at the $5 tables. The sample size at the $25 tables is not large enough to study the significance of the higher wagers.

Table 6: Betting Behaviours

Type	Characteristic	Players	Percentage
I	House money	54	7.2
II	Break even	78	10.4
III	Both	62	8.3
IV	Stay the course	554	74.1
Total		748	100

4.3   Betting Behaviours

Players can also be classified into four types according to their betting behaviours. Type I refers to players who increase bet when winning and become more risk-seeking. Type II refers to players who are losing but increase their bets at the last hand. Type III players possess the characteristics of both Type I and Type II. For example, a player can win money first and increases the wager but loses money later on and increase the wager at the last hand. Type IV players never change their bets.

This classification is used to test the house money effect and the break-even effect. According to the quasi-hedonic editing rules, players of Type I bet with "house money", while Type II players increase the stake at the last hand trying to "break even". Type III players are faithful followers of both of the quasi-hedonic rules. For players of Type IV, it seems that the prior gains or losses do not affect them much. We should point out that not all Type II players try to strictly break even since some have incurred large losses at the end. But they at least increase the bets substantially by a substantial amount.

According to Table 6, 15.5% (7.2% + 8.3%) of players exhibit the house money effect, and 18.7% (10.4% + 8.3%) of players exhibit the break even effect. Combining Type I, II, and III, a total of 25.9% of the players use the quasi-hedonic editing rules of "house money" effect and/or "break even" effect. Some of the "house-money" players, may end up losing money. Nevertheless, if we assume that all of them are winners at the end, and recall from Table 4 that the overall winning rate is 40.8%,

the "conditional" percentage of winning players who display house money effect is $15.5\%/0.408 = 38.0\%$. In other words, up to 38% of the winning gamblers play with house money. Similarly, the maximum "conditional" percentage for the break even effect is $18.7\%/(1 - 0.408) = 31.6\%$. That is, up to about one-third of the gamblers with a losing streak increase their bet in the last hand trying to break even.

The majority of the players, 74.1%, do not change their wagers throughout the sessions. It should be noted that some of the Type IV players may become more risk-averse after a losing streak and leave the game. But from the data we have no way to tell whether a player leave the table because of increasing risk-averseness or other reasons.

There are some important differences between gambling in a casino and participating in laboratory experiments. First, under the laboratory setting the wagers in the gambles are predetermined by the designers of the experiments. In the casino environment, however, players have to make a decision to increase their wagers to break even. This extra layer of decision making may have an unknown impact on the mental accounting process. Second, the amount of money involved in the casino is on average hundreds of dollar, whereas the maximum amount of money in the Thaler and Johnson (1990) experiments is $30 for gains and $9.75 for losses. For example, if a gambler has lost $200 before the last hand, increasing the wager to $200 for just one game is extremely risky. Third, casino gamblers are by definition risk-seeking, therefore our results is running the risk of selection bias.

5   Conclusions

The expected utility theory, being prescriptive and normative, does not describe economic behaviours very well. Prospect theory employs a psychological approach and therefore possesses more explanatory power. Nevertheless, we agree with Tversky

and Kahneman (1992, p. 317) that "[t]heories of choice are at best approximate and incomplete." In this paper we observe and analyse behaviours of blackjack players under the framework of prospect theory and the editing rules put forward by Thaler and Johnson (1990).

From an individual player's perspective, blackjack is a one-person repeated game involving risk, with each game having the same rules and risk structure. Given the well-publisized optimal strategies, a rational player can behave mechanistically like a computer algorithm. But the decision made at each call give a false sense of control. As a result, any playing or betting strategy is pure mental accounting.

Rationality assumptions dictate that players always follow the optimal strategies. In our observation, less than half (43%) of the players adhere to the strategies most of the time. High stake players, that is, those at a high minimum wager table, seems to exercise more mental effort by employing the optimal strategies to a greater extent. The strict strategists also bear out the mathematical fact that they have a much higher chance of winning than those with no strategy (53.3% versus 18.8%).

Notwithstanding the fact that casino gamblers are already risk-seeking, our results indicate that a fair proportion of the players behave according to the quasi-hedonic rules. Up to 38% of the winners become more risk-seeking and increase their wagers as if they are betting with house money. Less than one-third of the losing winners, on the other hand, try to recover part of their losses by increasing the wagers at the end. The behaviours of these players do conform with the quasi-hedonic editing rules.

Designing laboratory experiments involving monetary losses is a tricky business. It poses an ethical dilemma and many subjects refuse to participate. It is our hope that this project will encourage more future studies with field observations.

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