Odle Math Club

2022-2023 Mathcounts Team Selection Test

Sprint (Problems 1-30)
40 minutes

Write your four-digit unique ID legibly and neatly on the line below. If you are unsure of your ID, please call a proctor for assistance. Do not write any other identifying information.

ID: ____________

Instructions:
DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

The sprint round consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books, or other aids during this test. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the answer sheet provided to you. If you complete the problems before time is called, use the remaining time to check your answers.

On this test, units are not required, but **must be correct if written**. Answer only in the unit that the question suggests. For instance, if the question asks for the answer in kilograms, the answer "3000 grams" will not be accepted even if it is correct.

Remember to write your ID on this sheet. **Do not write your name.** All answers must be written legibly or they may be graded incorrectly. Coaches retain the discretion to disqualify any individual who fails to comply with procedure.

DO NOT FILL OUT THE SCORING BOXES BELOW.

Total Points	Scorer's Initials

The answer sheet is on the next page. Do not fill out the scoring columns.

#	Answer	0/1	0/1	#	Answer	0/1	0/1
1				16			
2				17			
3				18			
4				19			
5				20			
6				21			
7				22			
8				23			
9				24			
10				25			
11				26			
12				27			
13				28			
14				29			
15				30			
#1-15 Subtotal				#15-30 Subtotal			
Total Points							

Problem 1. Calculate $12 + 122 + 1222 + 12222 + 122222$.

Problem 2. Daniel is going to be X days old tomorrow. X is a very special age, because it is the second smallest positive integer that is a perfect square, a perfect cube, and a perfect fourth power. How many days old is Daniel today?

Problem 3. If Ursula can write 1 problem an hour, Phoebe can write 10 problems an hour, Rachel can write 100 problems an hour, Monica can write 1000 problems an hour, and the Odle Mathcounts test has 9999 problems, how many hours will it take all 4 of them working together to write the test?

Problem 4. For an upcoming test, Bret needs a pair of lucky socks, a water bottle, two pencils, and one very large eraser. If Bret owns 6 pairs of socks, 2 water bottles, 9 pencils and 3 large erasers, in how many different ways can he pick his items to take the test?

Problem 5. Find the area of a triangle with side lengths $33, 44$, and 55.

Problem 6. Which positive integer under 125 has the most number of factors?

Problem 7. If 3 tennis balls can be traded for 2 frisbees, and 3 frisbees can be traded for 7 golf balls, with trades going both ways, how many tennis balls can one trade starting with 20 frisbees and 44 golf balls?

Problem 8. Maximilian can type 138 words per minute, but it uses 23 Joules of electricity a minute. Maximilian can handwrite 18 words per minute, using only 2 Joules of electricity a minute (to power his lamp). If Maximilian has 337 words to write and wants to use 49 Joules in total, for how many seconds should he handwrite?

Problem 9. If o, d, l, and e are consecutive positive integers such that

$$\frac{1}{o} + \frac{1}{d} + \frac{1}{l} + \frac{1}{e} < \frac{3}{4},$$

compute the smallest possible value of $odle$.

Problem 10. Jo, Josh, and Joshua are siblings. One day, Jo astutely remarks, "Our ages form an arithmetic sequence, and moreover, all of our ages are prime!" If Jo is 68 years older than Josh, compute $M - N$, where M is the maximum possible sum of their ages and N is the minimum possible sum of their ages.

Problem 11. Holden is trying to bake some cakes. For each cake, he needs 6 eggs, $1\frac{1}{2}$ cups of sugar, and $1\frac{3}{4}$ cups of flour. If he has 200 eggs, 48 cups of sugar, and 56 cups of flour, how many cakes can he make? He can not make fractional cakes and must have enough of all three ingredients to make a cake.

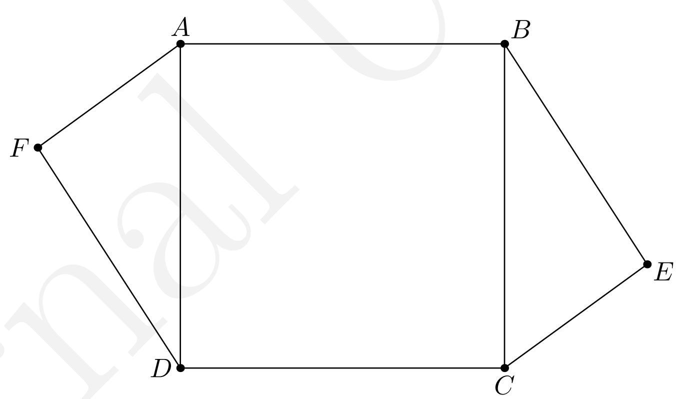
Problem 12. A heptagon, which is a seven-sided polygon, has five sides measuring $12, 23, 34, 45$, and 56. If the perimeter of the heptagon is 1000, and both other sides have positive integer lengths, find the maximum difference between the larger other side and the smaller other side.

Problem 13. For how many integers n where $3 \leq n \leq 623$ does there exist two perpendicular diagonals in the regular n-gon?

Problem 14. Given a rectangular mat of length 9 and width 14, let V_1 and V_2 be the volumes of the two cylinders produced by rolling the rectangular mat length-wise and width-wise. Assuming that the mat is rolled perfectly so there is no overlap between the layers, compute $\frac{\max(V_1,V_2)}{\min(V_1,V_2)}$.

Problem 15. Five friends, Anna, Bryan, Clarence, David, and Edna sit in a row of five chairs while watching a movie, in that order. They decide to pause the movie, and all get out of their seats to grab some popcorn and use the restroom. When they return, none of them want to sit back down in the seat they were originally in, and in addition, Anna refuses to ever sit in the middle seat. How many ways are there for them to re-seat themselves?

Problem 16. $ABCD$ is a square with side length 13, and E and F are constructed so that BEC and AFD are both right angles, and $CE = AF = 5$. Find the value of FC^2.



Problem 17. A snail is randomly placed in a 100×100 square. What is the probability that the snail is at most 30 units away from the perimeter of the square?

Problem 18. The "Jerry Mander Inn" is having an election for its new housekeeper, where the only two candidates are Jerry and Mander. The 121 staff will be split into 11 committees of 11, and in each committee, the 11 staff members will cast their vote (they must vote for either Jerry or Mander). For each committee that a candidate that gets the majority of votes in, they receive a point (for a maximum of 11 points), and the candidate with more points wins. What is the minimum possible number of staff members that voted for Jerry if he won?

Problem 19. Compute $\frac{1}{3} + \frac{1}{3^2} + \frac{2}{3^3} + \frac{3}{3^4} + \frac{5}{3^5} + \cdots$, where the numerators follow the Fibonacci sequence and the sequence continues indefinitely.

Problem 20. In triangle ABC, suppose there are points D on segment AB and E on segment AC such that $DECB$ is a cyclic quadrilateral, and $AE = 1, AD = DE = 2, EC = 4$. Find the perimeter of triangle ABC.

Problem 21. A triangle is called *kind of cute* if it is acute, but has at least one angle measuring more than 80 degrees. Consider 36 equally spaced points on a circle. How many *kind of cute* triangles exist with vertices as three of these points?

Problem 22. Ethan has a collection of magnifying glasses, which take on four types, scaling images up by $2, 3, 4$, and 6 times respectively. If he wants to increase the size of an image by a factor of 24, how many ways can he arrange his magnifying glasses? For example, he could first have a 4 times magnifier then a 6 times magnifier, or first a 6 times magnifier than a 4 times magnifier, which would be two different ways. Assume he has an infinite amount of each type.

Problem 23. Peter starts at the point $(0, 0)$ on the coordinate plane, and every minute he can choose to either move one unit up or one unit to the right. He wants to get to the point $(20, 22)$, but he really likes multiples of 5, so at all times either his x-coordinate or y-coordinate must be a multiple of 5 (or both). How many ways can Peter get to $(20, 22)$ from his starting point at the origin?

Problem 24. There exists equilateral triangle ABC with side length 2 and points D, E, F on minor arc AB of the circumcircle of ABC. What is the maximum possible value of the area of pentagon $ABDEF$?

Problem 25. If I roll a fair 20-sided dice three times to get the numbers a, b, and c, what is the probability that $|a - b| \geq 4, |b - c| \geq 4$, and $|a - c| \geq 4$?

Problem 26. Let $k = 1! + 2! + \cdots + 2023!$ and S be the set of one-digit positive integers that k is divisible by. Compute the sum of the numbers in S.

Problem 27. Consider the equation

$$\frac{1}{x-3} + \frac{1}{x-11} + \frac{1}{x-19} + \frac{1}{x-27} = 0,$$

which has r distinct roots that sum to s and multiply to p. What is $r + s + p$?

Problem 28. Let $f(x) = x^4 - 12x^3 + 36x^2 + 18x + 12$, and let a, b, c be real numbers such that $a + b + c = 12$. What is the minimum value of $f(a) + f(b) + f(c)$?

Problem 29. Let ABC be a triangle with $AB = 13, BC = 15$, and $CA = 14$. Let D be on BC such that $AD = CD$. Compute $\frac{CD}{BD}$.

Problem 30. Two subsets are called *disjoint* if they do not share any common elements. Compute the number of ordered tuples (A, B, C), where A, B, and C are subsets (not necessarily distinct or non-empty) of $\{1, 2, 3, 4, 5\}$ such that A and B are disjoint and B and C are disjoint.

You have reached the end of the sprint round. Please use the remaining time to check your answers.

Forms of Answers

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure, equivalent answers expressed in other units are *not* acceptable.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and $\gcd(a, b) = 1$. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and $\gcd(a, b) = 1$.

Ratios should be expressed as simplified common fractions unless otherwise specified.

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form.

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit should be expressed in the form (\$)$a.bc$, where a is an integer and b and c are digits. The only exception to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they both may be omitted. Answer in the form (\$)$a.bc$ should be rounded to the nearest cent, unless otherwise specified.

Do not make approximations for numbers (e.g. $\frac{2}{3}, \pi, 5\sqrt{3}$) unless otherwise specified. Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer.

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form. Thus, 25.0 will not be accepted for 25 and 25 will not be accepted for 25.0.