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A Classification Scheme for Basic Physics Concepts: Learning and Thinking from Neurodevelopmental Science Perspective

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Abstract. Conscious learning begins from the moment the neurosensors act on events in the physical-sociocultural (PSC) environment to create their mental representations. The sensors convert various forms of energies in the events into patterned electric signals; neural pathways carry those signals to the different parts of the brain to form new neural circuits and/or modify the existing circuits by leaving biochemical traces, which serve as memory; and the learning continues until the self-regulating "thought processes" associate new thoughts with the active circuits that contain the event information, in an emotional context. These thoughts, the thought processes, the corresponding neural circuits, and the actions continue to develop non-linearly, by following a set of transformation rules, from reflexes through abstract principles, which are the result of nested reciprocal influences, owing to further life-time experiences in the PSC environment. Using this frame work, the paper analyzes basic physics concepts to classify them into a hierarchically-complex scheme and proposes a mechanism for how a concept is associated with a set of patterns in the event-information, and how simpler concepts develop into more complex concepts using "Concepts Developmental Equations" (CDE).

1. Introduction

From the probabilistic epigenetic [1, 2] point-of-view, development of thought processes and thoughts is due to the nested reciprocal influences within and between the genetic, neural, behavioral, physical, social and cultural environment, and the structured actions of individuals in this environment. A self-regulating, holistic, psychological "structure" that has reached a level of equilibration with its environment, does not further undergo genetic changes [3], and is capable of performing reversible transformations, transforms the structured actions into schemes of internal-imaginings and back into actions [2]. These imaginings are thoughts, also called structured relations. Neuroscience has shown that learning involves neural firing to form or activate existing neural circuits [4, 5]. Learning is a survival mechanism triggered by the environmental affordances through which we learn to associate thoughts and feelings with the neural circuits [6]. The non-linear developmental theory [2, 6, 7] offers a systematic mechanism for associating thoughts and feelings with those activated circuits and their mutual interactions [8]. This paper briefly discusses the framework for the classification scheme of physics concepts and then analyzes the basic physics concepts that result in the proposal of the scheme.

2. Framework

The above briefly-discussed theories can holistically be represented as a block diagram shown in Figure 1. Each part of the diagram is explained below.
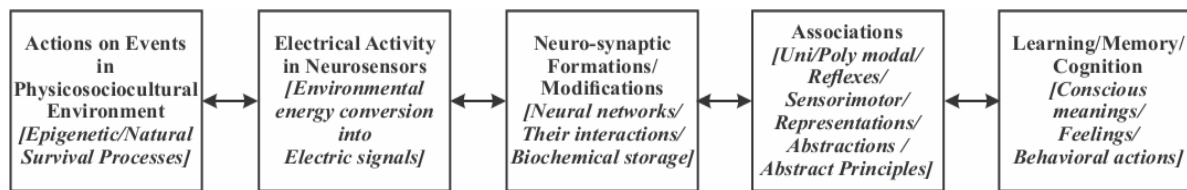
Figure 1. Learning, Thinking, and Memory. Each component influences and is influenced by the other components.

2.1. Actions (Acts of learning)

In the process of learning, individuals perform goal-oriented, intentional, structured actions on events in PSC environments [9]*,* which are simply represented as AE. The actions, events and the environments can be real or imagined. Human emotions play an important role in these actions [9]. Rewards received and the reward expectations increase the probability of actions, as rewards give us pleasure and satisfaction [8, 10, 11]. Thoughts have their origin in these structured actions, and they occur within the medium of the human body [12]. The actions result in the neurosensors capturing the information of an event in a given PSC environment.

2.2. Electrical activity

Sensors capture information from an event in various forms of energies, for example: electromagnetic by vision; chemical by taste and smell; and mechanical-vibrational by hearing. The parts of the sensors exposed to the environment convert those energy forms into electrical signals through an electro-chemical conversion process (stimulus energy changes the inner-and-outer membrane's Na^+, K^+, Ca^{++} and Cl^- ion imbalances, resulting in the generation of the electrical signals). These electrical signals travel through specified neural pathways to various brain areas to form or activate the existing neural circuits in those respective areas [13].

2.3. Neurosynaptic formations/modifications

These electrical signals fire and create synapses with appropriate target neurons in the brain to form neural circuits. The event information in the synapses is carried by several types of neurotransmitters, such as GABA, dopamine, epinephrine, and serotonin, which are responsible for our emotions. During the formation or activation of the existing neural circuits, biochemical processes (protein kinases and kinase-cascades responsible for gene transcriptions and protein synthesis) take place, which result in memory [13]. Rewards change the functioning of the neurons and increase the probability of synapse and circuital formations [8, 10].

While a single neuron is not intelligent, networks of neurons can think, feel, remember, perceive, accept rewards and generate many ideas in conjunction with emotions. These neural circuits, especially hippocampal neural circuits and their biochemical traces, are important for new learning and memories.

2.4. Associations

As "structured actions," AEs form or activate the existing neural circuits. Self-regulating, holistic psychological structures (called "thought processes"), which are in equilibration with the environment [3], associate internal, structured-imaginings (called "thoughts") with these networks. Thoughts, which are internal representations of the AEs, are expressed using semiotics. Words and sentences are one form of semiotics that can be used to express thoughts. Thoughts are composed of concepts. Words can be used to express concepts verbally, and the meanings are properties of those words [15, 16]. These associations are done in the context of prior experiences, and they occur in primary-association areas and in hippocampal areas [17, 18].

During *thought process*, at any given point-of-time and in an emotional context, motive-relevant appraisal processes, which are mostly unconscious, monitor the event-related information at different levels through neurosensors. At the same time, the appraisal processes continuously generate, modify and modulate the production of feelings and motive-actions. Depending on the relevance to the motives, the feelings amplify, organize and select certain monitoring-events for conscious attention [6,19]. Through the operation of the executive processes, individuals associate thoughts (structured relations),

by consciously coordinating the sources of variations in the information gathered from the conscious events [2, 6]. Thus the AEs that surface result in thoughts, which have neurobiological evidence [20, 25, 26]. Depending on our motive-appraisal evaluation of such events, we pay more or less attention to them by modifying the neurotransmitter flow. This adjustment changes the input sensory performance [8].

Thinking and associating thoughts are acquired skills, which are an individual's capacity to coordinate the variable elements in the thought process to produce those thoughts. The neural networks, the thought processes, thoughts and AEs continuously undergo 4-tier-4-cyclical-within-tier, non-linear, developmental changes through their nested reciprocal influences owing to further life-time experiences. This development takes place in spurts, optimally with scaffoldings, and has multiple paths, which explains multiple-knowledge domains [6]. The four macrodevelopmental tiers are: Reflex (Rf), which is coordinating inborn reflexes; Sensorimotor (Sm), which is coordinating smoothly sensorimotor actions; Representations (Rp), which is representing Sm actions through semiotics; and Abstractions (Ab) and Abstract principles (Ap), which are generalizing and producing intangible meanings, such as constancies and conservations. Within each tier, there are four cyclical microdevelopmental transformation sets: differentiated *single set*, in which each AE is a unit and is represented as set [*X*], or [*Y*]. Each of the basic elements in single sets *X*, *Y* represent AE or its structured internal imagining, which is a thought, and a set with more than one of such elements represent a skill; *mapping set* is the mapping of at least two *single sets* and represented as $\left[X - Y \right]$; *systems set* is mapping of two *mapping sets* with each one containing at least two *single sets* as subsets and represented as $\left[X_{1,2} \longleftrightarrow Y_{1,2} \right]$; *system of systems set*, which is the transformed single set of the next tier and is represented as mapping of at least two systems sets and represented as $\left[_{1,2}X_{3,4} \Leftrightarrow {}_{5,6}Y_{7,8} \right] \equiv \left[Z \right]$. Where $\left[_{1,2}X_{3,4} \right]$ and $\left[_{5,6}Y_{7,8} \right]$ are *systems sets* with each one containing at least two *mapping sets* as subsets and each of the *mapping sets* containing at least two *single sets* as subsets. $\left[Z \right]$ is the single set of the next tier, which has qualitatively different properties from its constituent subsets. Thus *system of systems* contains at least eight single sets; *systems* contain four; and *mapping* contains two single sets. Higher-level knowledge elements contain lower-level knowledge elements and are built from at least two of the simplest-knowledge element sets. The microdevelopmental sets result in macrodevelopmental sets. The complex-action sets develop from simple-action sets, and the mappings explain the relations among them. These mappings are thoughts and feelings; furthermore, actions and thoughts cannot be separated. The transformations between these levels and tiers are explained using four rules, three of which are explained below. Furthermore, the original rules are redefined in order to explain the concepts and skills of physics. I refer the readers to the original papers for more details [7, 21]. These rules represent mental operations an individual can perform on the information received during AEs to build thoughts and skills at various levels of complexity. Developmental researchers made an attempt to explain these cognitive operations using set theory and mathematical logic as tools [3, 7]. Cognitive operations include looking for correspondences (e.g., one to-one), comparing, shifting focus, and summing. Probably, the mathematical logic came from these cognitive operations.

2.4.1. Intercoordination: Develops next higher-level, complex sets of concepts and skills from two same lower-level sets, already learnt and perfected (macro level development)**.** The higher-level complex sets of thoughts possess the properties of each of the constituent lower-level sets, but the new ones show completely different properties, just as a molecule has the different properties from the constituent atoms.

The process is represented as

$$\left[a^l \right] \bullet \left[b^l \right] = \left[d^{(l+1)} \right] \tag{1}$$

The *a, b* at level *l* and *d* at level *(l+1)* are thoughts.

We redefine the original Intercoordination rule because development of certain physics concepts requires a rule that produces a more complex concept at higher-level from two different lower-level concepts. The redefined, transformation rule includes the Fischer's original rule as a special case. In fact, Fischer himself suggests that there may be more transformation rules: "These five transformation

rules are probably not exhaustive; future research will indicate whether additional transformation rules are required" (p 497, [7]).

Augmented Intercoordination: Develops a *(l+1)* level structure from two *l* and *k* level structures, where *k* is $Rp_1 \leq k \leq l$. The augmented rule equation is

$$\left[d^{\,l} \right] * \left[e^{\,k} \right] = \left[f^{\,(l+1)} \right] \qquad (2)$$

where " $*$ " represents Augmented intercoordination. It develops a more complex higher-level thought, *f* from the less complex lower-level thoughts, *d* and *e*. A new notation is introduced to represent mappings of any two different-level subsets, $[H_{igher} \Leftarrow L_{ower}]$ or $[L_{ower} \Rightarrow H_{igher}]$. The arrow points from lower-level to higher-level subsets. The new rule still produces a thought that is at immediate next, level *(l+1)*. The old rule is a special case of the augmented rule, *k=l,* when the old rule results.

2.4.2. Compounding: Develops, through actions, more complex concepts and skills sets at the same level as the previously-learnt sets of concepts and skills.

This microdevelopmental, transformation rule combines two thoughts *a* and *b*, of Level *l* to form a more complex thought "*c,*" at the same Level *l*. The compounding equation is

$$\left[a^{\,l} \right] + \left[b^{\,l} \right] = \left[c^{\,l} \right] \qquad (3)$$

This rule is also augmented to apply it to physics concepts and skills. Fischer also pointed out in his paper (p 499 note 9, [7]) that

$$\left[d^{\,l} \right] + \left[e^{\,(l+1)} \right] = \left[f^{\,(l+1)} \right] \qquad (4)$$

where structures *d* is at level *l* and *e* and *f* are at higher-level *(l+1)*. Similarly, the compounding is *augmented* to

$$\left[d^{\,k} \right] + \left[e^{\,l} \right] = \left[f^{\,l} \right] \qquad (5)$$

where *k is* $Rp_1 \leq k \leq l$. *k=l,* resulting in original compounding rule. Further research is needed to experimentally verify these augmented rules.

2.4.3. Shift of Focus: Represents a moment-to-moment behavioral shift to develop complex thoughts from simpler thoughts at the same level. When a shift in focus is consistently controlled, the transformation rule is represented as

$$Foc(e, f) = [e > f] \qquad (6)$$

2.4.4. Substitution: Replaces one component with another similar component in a set to produce similar thought. Substitution is represented as

$$Sub[d] = [d_1] \qquad (7)$$

where *d* and d_1 are at the same level.

2.5. Learning, memory and cognition

Simply put, learning results in changes in behaviors [7], thought processes and thoughts [6], biochemical traces [13], neuronal firing, neural networks [24, 25, 26], and in grey- and white-matter volumes [22, 23]. Memory results from biochemical traces in the circuits [13]. Learning takes place in cycles [16].

From the networks' point-of-view, *cognition* is a special kind of pattern formation, the interplay of functional segregation and integration of information, and it involves a continual emergence of dynamic structures that are molded by connectivity and subtly modified by external input and internal state [14]. Cognition is a neural network phenomenon, and it does not exist in a synapse or in a single neuron [24, 25, 26]. Thus the cognition has a physical substrate. which is a set of distinct large scale, neurocognitive networks spread over the entire brain that support different domains of cognitive and behavioral functions.

3. Application to physics

The above-discussed framework is used to predict and explain the development of more complex physics concepts and skills from simpler ones, and to define a hierarchical classification scheme for the same. The physics concepts and reasoning use both mathematics and language statements, and they are represented by both variable and constant symbols in the equations. The equations explain relations among the physics concepts that the variables represent in the equations, and they present the physics reasoning precisely. Higher-level mathematical tools and reasoning skills are needed to perform such tasks in physics. The required skills include: identifying and differentiating patterns in the event information that is captured during AEs; representing those patterns with appropriate differentiating variables; and controlling and coordinating those variables—by realizing their properties—to develop thoughts. Scaffolding environment helps to learn and improve these skills. Considering a specific context, such as a ball moving under a contact-force or in the gravitational field, helps the analysis. The analysis makes use of equations that the author calls "Concepts Developmental Equations" (CDE). The equations explain how more complex thoughts develop from simpler ones as result of mental-transformation operations described in the earlier sections. They also show the corresponding levels of complexity of skills needed to develop such thoughts, which are Ab1 *(Tier iv, level 1)*; Ab2 *(Tier iv, level 2)*; Ab3 *(Tier iv, level 3)*; Ab4 *(Tier iv, level 4)*/Ap1 *(Tier v, level 1)*; Ap2 *(Tier v, level 2)*; Ap3 *(Tier v, level 3)* and Ap4 *(Tier v, level 4)* as discussed in the non-linear developmental theory, where the thoughts develop through actions[26].

In general, a CDE can be represented as, $\left[{}^{tier}a^{level}_{subsets} \; {}^{c}\Re \; {}^{tier}b^{level}_{subsets} \right] = \left[{}^{tier}c^{level}_{subsets} \right]$, where *a, b* are the

learned concepts at certain *tiers* and *levels* that are built from subsets and $\overset{c}{\Re}$, which is a generalized representation for mapping, systems, or system of systems, and is a relation of *a* and *b* to the resulting new concept *c* at a certain *tier* and *level*. The RHS represents the new thought or concept set. These equations obey transformation rules.

The developmental research on mathematical concepts and skills shows that the conceptual representation of a number line with a variable *x*, starts at level Ab1 [2]. The skills of identifying the patterns in the moving-ball-event information, in a coordinate system and of representing those patterns using different variables for physics concepts—positions $(x, x_e, \text{ and } x_l)$, times $(t, t_e, \text{ and } t_l)$, and masses $(m, m_e, \text{ and } m_l)$—are at Ab1 level. The suffix "*e*" refers to earlier and *l* refers to later values. These variables assume a number at a given time on the number line with units. For example $x \in [\mathbb{R} : \text{meters}]$; $t \in [\text{Positive } \mathbb{R} : \text{seconds}]$; and $m \in [\text{Positive } \mathbb{R} : \text{kilograms}]$, where $\mathbb{R}$ represents the real number set.

As a result, variable representations for position, time, and mass fall in single sets, Ab1. Environmental scaffolding plays a very crucial role in all these tasks.

3.1. Ab1. Simple abstract concepts: position, time, and mass

$$\left[{}^{iv}x^1 \right], \left[{}^{iv}x_e^1 \right], \left[{}^{iv}x_l^1 \right]; \left[{}^{iv}t^1 \right], \left[{}^{iv}t_e^1 \right], \left[{}^{iv}t_l^1 \right]; \left[{}^{iv}m^1 \right], \left[{}^{iv}m_e^1 \right], \text{ and } \left[{}^{iv}m_l^1 \right] \tag{8}$$

The physics concepts $x, x_e, x_l, t, t_e, t_l, m, m_e$ and m_l are represented as single sets at Ab1 level. Intervals, such as, $x_{le}, t_{le},$ and $m_{le},$ result from comparing earlier values with later values as time flows and realizing that time as an implicit parameter. Furthermore, the x_{le} is an ordered change, which is the earlier value is subtracted from the later value. The later values of the physical quantities are related to the earlier ones through mappings. The mapping skills, in math research are at Ab2 level [2]. Therefore, the concepts and skills of the intervals fall in Ab2 level.

3.2. Ab2. Abstract mapped concepts: displacement, distance travelled, mass change and time interval

The displacement x_{le} can be considered as a mapping of set $x_e \in [\mathbb{R}_e : \text{meters}]$ and set $x_l \in [\mathbb{R}_l : \text{meters}]$, to $x_{le} = (x_l - x_e) \in [\text{finite } \mathbb{R}_{le} : \text{meters}]$, which is a finite set. Similarly, we can consider time interval $t_{le} = (t_l - t_e) \in [\text{finite } \mathbb{R}_{le} : \text{seconds}]$ and $m_{le} = (m_l - m_e) \in [\text{finite } \mathbb{R}_{le} : \text{kilograms}]$. Even though x_{le} has the same units as x_l or x_e, a new idea, "direction" is introduced. The + (positive) or – (negative) values of x_{le} represent direction of motion, and is the same for t_{le}, representing direction of time flow and an increase

or decrease for the mass, respectively. The sources of variations are x_{le}, t_{le} and m_{le} for skills development. The CDE for x_{le} is

$$\left[{}^{iv}x_l{}^1 \xrightarrow{a} {}^{iv}x_e{}^1 \right] = \left[{}^{iv}x_{le}{}^2 \right] \tag{9}$$

The physics concept (i.e., structured relation or thought as in the original theory) that develops by coordinating x_l and x_e, is the displacement "*a*," which is at Ab2 level. The intercoordination rule explains the mapping:

$$\left[{}^{iv}x_l{}^1 \right] \bullet \left[{}^{iv}x_e{}^1 \right] = \left[{}^{iv}x_{le}{}^2 \right] \tag{10}$$

The mapped set, $\left[{}^{iv}x_{le}{}^2 \right]$ contains two single subsets, $\left[{}^{iv}x_e{}^1 \right]$ and $\left[{}^{iv}x_l{}^1 \right]$. Similarly, in the time interval, the time the ball takes to travel from position x_e to x_l or to displace x_{le} is

$$\left[{}^{iv}t_l{}^1 \xrightarrow{b} {}^{iv}t_e{}^1 \right] = \left[{}^{iv}t_{le}{}^2 \right] \tag{11}$$

The physics concept developed is *b=time interval* or *time-of-travel*, which is at Ab2 level. Intercoordination rule is

$$\left[{}^{iv}t_l{}^1 \right] \bullet \left[{}^{iv}t_e{}^1 \right] = \left[{}^{iv}t_{le}{}^2 \right] \tag{12}$$

At this level, an individual may look at the intervals, x_{le} and t_{le}, separately and may not have skills to compare these two intervals. The comparison, which could be due to shift of focus behavior, may happen while developing the velocity concept to seek relation among them.
Additionally, distance travelled during a certain time is always a positive number, which is mathematically an absolute value $/x_{le}/=x_{l,e} \in$ [positive finite $\mathbb{R}$:meters].

Distance of x_e to x_l is

$$\left[{}^{iv}x_l{}^1 \xrightarrow{c} {}^{iv}x_e{}^1 \right] = \left[{}^{iv}x_{l,e}{}^2 \right] = \left[{}^{iv}x_{e,l}{}^2 \right] \tag{13}$$

where *c= distance* and the rule that produces is intercoordination

$$\left[{}^{iv}x_e{}^1 \right] \bullet \left[{}^{iv}x_l{}^1 \right] = \left[{}^{iv}x_{e,l}{}^2 \right] \tag{14}$$

Similarly, the mass change can be presented as

$$\left[{}^{iv}m_l{}^1 \xrightarrow{d} {}^{iv}m_e{}^1 \right] = \left[{}^{iv}m_{le}{}^2 \right] \tag{15}$$

where m_e is mass at time t_e, m_l is mass at time t_l, and the developed concept is *d=change of mass*.
The intercoordination develops this concept, which is represented as

$$\left[{}^{iv}m_l{}^1 \right] \bullet \left[{}^{iv}m_e{}^1 \right] = \left[{}^{iv}m_{le}{}^2 \right] \tag{16}$$

Associating physical meanings to sings (+ or −) is a skills and it may happen at Ab2 level.

3.3. Ab3. Abstract systems concepts: average velocity, average speed, instantaneous velocity and change in the velocity

Let us use a capital "*V*" for average velocities and small "*v*" for instantaneous velocities. Average velocity *(V_{le})*, which is a relation between displacement x_{le} and the time t_{le} for the displacement, has different properties and units from the individual properties and units of x_{le} and t_{le} but their properties are innate in the V_{le}. The average velocity is the mapping of ${}^{iv}x{}^2{}_{le} \in$ [finite $\mathbb{R}$: meters] and ${}^{iv}t{}^2{}_{le} \in$ [finite $\mathbb{R}$: seconds] resulting $V_{le} \in$ [finite $\mathbb{R}$: meters/seconds]. V_{le} should result from intercoordination, as it includes properties and units of displacement and time interval. Similarly, instantaneous velocity v_l or v_e–the velocity at a given instant of time.
The CDE for average velocity is

$$\left[{}^{iv}x^2{}_{le} \xleftrightarrow{a} {}^{iv}t^2{}_{le} \right] = \left[{}^{iv}V^3{}_{le} \right] \tag{17}$$

where the developed concept is *a=average velocity* or *displacement per unit time*. The RHS is V_{le} set.
The corresponding mental operation is intercoordination:

$$\left[{}^{iv}x^2{}_{le} \right] \bullet \left[{}^{iv}t^2{}_{le} \right] = \left[{}^{iv}V^3{}_{le} \right] \tag{18}$$

The V_{le} contains subsets x_{le} and t_{le}, which stem from the abstract simple-concept subsets x_l, x_e, and t_l, t_e respectively, and these four simple-concept sets are innate in V_{le}. The V_{le} depends only on x_{le} and t_{le} but does not directly depend on the simple sets. The instantaneous velocity is the velocity at a given time. The average velocity, $^{iv}V^3_{le}$, transforms into instantaneous velocity, $^{iv}v^3_e$ with $^{iv}t^2_{le}= \Delta t \to 0$ at t_e. The meaning of *tending to zero ($\to 0$)* is that the value is so small that it cannot be measured. Here an issue arises about the level of $^{iv}t^2_{le}$ as $\Delta t \to 0$. The t_{le} is at Ab2 level. When it is an infinitesimal and unmeasurably-small quantity representing value *at time t_e*, is it at the level Ab2 or Ab1? The experiments will give better answer. This ambiguity does not change the level of the developed concept due to the definition of Augmented compounding as the definition includes the range for the lower-level concept k, $Rp_1 \le k \le l$. The mathematical concept of *differentiation* is used for instantaneous velocity in physics. The concept of *tending to zero ($\to 0$)* and *differentiation* cannot be explained using regular algebraic operations. The mathematical concept of *differentiation*, which is $\lim\limits_{t_{le} \to 0} \dfrac{x_{le}}{t_{le}} = \dfrac{dx}{dt}$, is at Ab3 level [2].

The instantaneous velocity, *(v_e)*, could develop from intercoordination on $^{iv}x_{le}$ and t_{le}, with $^{iv}(t_{le} \to 0)^2$ or augmented compounding operation. The latter one is presented as

$$\left[{}^{iv}V^3_{le} \overset{v}{\Leftarrow} {}^{iv}(t_{le} \to 0)^2 \right] = \left[{}^{iv}v^3_e \right] \tag{19}$$

where the developed concept is *v=instantaneous velocity*. The RHS is set $[^{iv}v^3_e]$. The augmented compounding relation is

$$\left[{}^{iv}V^3_{le} \right] + \left[{}^{iv}(t_{le} \to 0)^2 \right] = \left[{}^{iv}v^3_e \right] \tag{20}$$

As previously discussed, the change in the instantaneous velocity, $v_{le}=(v_l-v_e) \in$ [finite $\mathbb{R}$: meters/seconds], is at the Ab3 level because v_e, v_l, and v_{le} have same properties and units. The CDE is

$$\left[{}^{iv}v^3_l \overset{c}{\longleftrightarrow} {}^{iv}v^3_e \right] = \left[{}^{iv}v^3_{le} \right] \tag{21}$$

where the developed concept is *c=ordered change* of velocity. The compounding rule is

$$\left[{}^{iv}v^3_l \right] + \left[{}^{iv}v^3_e \right] = \left[{}^{iv}v^3_{le} \right] \tag{22}$$

3.4. Ap1. Simple abstract principles concepts: momentum, impulse, average acceleration, and instantaneous acceleration

The concept of momentum is the product of mass and velocity. It plays a crucial role in force, acceleration and impulse concepts. Momentum *p* is mapping of mass set $m \in$ [Positive $\mathbb{R}$: kilograms] and velocity set $v \in$ [finite $\mathbb{R}$: meters/seconds] to result in *p* with $p \in$ [finite $\mathbb{R}$: kg m/s units], and has the properties and units different from both the constituent concepts, although they are innate in *p* set. The sources of variation and coordinating variables are mass and velocity and their combined variations. The CDE for *p* is

$$\left[{}^{iv}m^1 \overset{b}{\Rightarrow} {}^{iv}v^3_e \right] = \left[{}^{iv}p^4_e \right] \equiv \left[{}^{v}p^1_e \right] \tag{23}$$

where the developed concept is *b=momentum*, and the RHS represents momentum set *p*. The augmented intercoordination transformation rule develops a one level up concept *p*,

$$\left[{}^{iv}m^1 \right] * \left[{}^{iv}v^3_e \right] = \left[{}^{iv}p^4_e \right] \equiv \left[{}^{v}p^1_e \right] \tag{24}$$

Similarly, impulse or momentum change p_{le} is

$$\left[{}^{iv}m^2_{le} \overset{a}{\Rightarrow} {}^{iv}v^3_e \right] = \left[{}^{iv}p^4_{le} \right] \equiv \left[{}^{v}p^1_{le} \right] \tag{25}$$

where developed concept is *a=impulse or momentum change* due to a change in mass. The + or –values of m_{le} refer to addition (+) or subtraction (–) of mass to the system. The transformation rule is

$$\left[{}^{iv}m^2_{le} \right] * \left[{}^{iv}v^3_e \right] = \left[{}^{iv}p^4_{le} \right] \equiv \left[{}^{v}p^1_{le} \right] \tag{26}$$

There can also be a situation in which mass does not change and velocity does change, as shown below

$$\left[\, ^{iv}m_e^2 \overset{a}{\Rightarrow} \, ^{iv}v_{le}^3 \right] = \left[\, ^{iv}p_{le}^4 \right] \equiv \left[\, ^v p_{le}^1 \right] \tag{27}$$

where a=*impulse or momentum change* due to the change in the velocity. The transformation operation is

$$\left[\, ^{iv}m_e^1 \right] * \left[\, ^{iv}v_{le}^3 \right] = \left[\, ^{iv}p_{le}^4 \right] \equiv \left[\, ^v p_{le}^1 \right] \tag{28}$$

The average acceleration A_{le} is the relation between change in instantaneous velocities, v_{le} -Ab 3 and t_{le}-Ab2. The A_{le} has different properties from the individual properties of v_{le} and t_{le}, but their properties are innate, which gets developed by controlling and coordinating the v_{le} and t_{le} with the skills gained at Ab3 and Ab2. The units are also different but related to the units of v_{le} and t_{le}. The $A_{le} \in$ [finite $\mathbb{R}$: m/s^2 units]. The A_{le} is at one level higher than the change in the velocities v_{le}. The CDE is

$$\left[\, ^{iv}v_{le}^3 \overset{a}{\Longleftarrow} \, ^{iv}t_{le}^2 \right] = \left[\, ^{iv}A_{le}^4 \right] \equiv \left[\, ^v A_{le}^1 \right] \tag{29}$$

where the developed concept is *a=average acceleration* or *change in velocity per unit time*. The A_{le} builds from subsets v_{le} and t_{le}. The v_{le} contains subsets v_l and v_e, each of which contains subsets x_{le} with $t_{le} \to 0$ at l and x_{le} with $t_{le} \to 0$ at e. The time interval t_{le} contains subsets t_l and t_e. Thus one can see that the average acceleration, which is at a simple abstract principle concept level, develops from simple abstract concept level sets. Furthermore, all these simple abstract concept sets have their origin at the Sm level, which develops from Sm through Rp to Ap levels. This explains why learning is a cyclical process, and, when working on a task, an individual goes back-and-forth from Sm through Ap levels.

Augmented intercoordinate equation is

$$\left[\, ^{iv}v_{le}^3 \right] * \left[\, ^{iv}t_{le}^2 \right] = \left[\, ^{iv}A_{le}^4 \right] \equiv \left[\, ^v A_{le}^1 \right] \tag{30}$$

where A_{le} is the average acceleration during time interval t_{le}. The instantaneous acceleration is acceleration at a given time. It depends upon infinitesimal change in the velocity v_{le}. Using the same reasoning as the one used in the context of instantaneous velocity, we can consider the instantaneous acceleration developed from an augmented compound operation:

$$\left[\, ^v A_{le}^1 \right] + \left[\, ^{iv}(t_{le} \to 0)^2 \right] = \left[\, ^v a_e^1 \right] \tag{31}$$

The CDE is

$$\left[\, ^v A_{le}^1 \overset{a}{\Longleftarrow} \, ^{iv}(t_{le} \to 0)^2 \right] = \left[\, ^v a_e^1 \right] \tag{32}$$

where the developed concept is *a=instantaneous acceleration* or *rate of change of velocity*.

3.5. Ap2. Abstract principles mapped concepts: force

An object of mass *(m)* accelerates *(a)* or decelerates *(d)* when a force *(f)* is applied during a certain time because the force changes the momentum of the object. The controlling and coordinating variables are momentum change *(p$_{le}$)* and time interval *(t$_{le}$)* with the realization that the p_{le} can be due to mass change, m_{le}, or due to a velocity change, v_{le}. *Force f* $\in$ [finite $\mathbb{R}$: newtons = kg.m/s^2 as units] is a mapping of two variables, p_{le} and t_{le} sets. The force is at one-level higher than that of p_{le} and can be developed through augmented intercoordination. The CDEs are

$$\left[\, ^v p_{le}^1 \overset{f}{\Longleftarrow} \, ^{iv}(t_{le} \to 0)^2 \right] = \left[\, ^v f_e^2 \right] \tag{33}$$

where the developed concept is *f=force*. The RHS is force set. The augmented intercoordination relation is

$$\left[\, ^v p_{le}^1 \right] * \left[\, ^{iv}(t_{le} \to 0)^2 \right] = \left[\, ^v f_e^2 \right] \tag{34}$$

The concept of force can also be developed from

$$\left[\, ^{iv}m^1 \overset{f}{\Rightarrow} \, ^v a^1 \right] = \left[\, ^v f^2 \right] \tag{35}$$

where *f=force, acting on a mass*, which accelerates the mass. The augmented intercoordination is

$$\left[\,^{iv}m^1 \right] * \left[\,^{v}a^1 \right] = \left[\,^{v}f^2 \right] \tag{36}$$

Force acting in the direction *(f₊)* of motion accelerates the mass and force acting in the opposite direction *(f₋)* of motion decelerates the object. Both the accelerated *(a)* and decelerated *(d)* motions can be considered as stemming from shift of focus between the two types of motions:

$$Foc(a,d) = \left[a > d \right]; Foc \left(\left[\,^{iv}m_a^1 \Rightarrow \,^{v}f_+^2 \right], \left[\,^{iv}m_d^1 \Rightarrow \,^{v}f_-^2 \right] \right) = \left[\,^{iv}m_a^1 \Rightarrow \,^{v}f_+^2 \right] > \left[\,^{iv}m_d^1 \Rightarrow \,^{v}f_-^2 \right] \tag{37}$$

This can transit into the augmented compounding rule as

$$\left[\,^{iv}m_a^1 \Rightarrow \,^{v}f_\pm^2 \Leftarrow \,^{iv}m_d^1 \right] \tag{38}$$

where m_a refer to the accelerating mass and m_d refer to decelerating mass.

3.6. Ap3. Abstract principles systems concepts: work, energy that the force-agent spends, kinetic energy, potential energy, total energy, and conservation of mechanical energy

A force *(f)*, accelerates or decelerates a mass, while simultaneously displacing the mass, which an individual is unlikely to realize at the lower-level skills. The force, *(f)*, is actually exerted by another object or an entity, which is called the "agents of the force," and the force is then represented as *(f_{fa})*. Both the f_{fa} and *f* are the same, except that f_{fa} is usually represented simply as *f* by suppressing the suffix, *fa*, depending on the context of discussion, which an individual may be capable of realizing at this skills level through substitution operation. Mostly, the forces are either "contact forces," in which case the force-agent is in physical contact with the accelerating mass (e.g., pushing or pulling it), or "action-at-a-distance," in which the force-agent is not in physical contact with the mass (e.g., gravitational or electromagnetic forces). The concept of *work* is related to the force and the displacement. The force-agent performs *work* on the mass by spending some of its energy. The "spent" energy transfers as a change in the energy of the accelerating mass, and thus the energy is not lost, which could be realized with a simple shift of focus operation. The *work*, $W \in$ [finite $\mathbb{R}$: Joules], which is mapping of the *force*, $f \in$ [finite $\mathbb{R}$: Newtons], and the *displacement*, $x_{le} \in$ [finite $\mathbb{R}$: Meters], has properties different from its constituent concepts. The coordinating variables are f and x_{le}. As a result, *work* is a more complex concept than either force or displacement and can only be developed after forming both the concepts.

Thus it functions one level higher. The CDE is

$$\left[\,^{v}f^2 \overset{wk}{\Longleftarrow} \,^{iv}x_{le}^2 \right] = \left[\,^{v}W_{le}^3 \right] \tag{39}$$

where the f_{fa} or f does *work (wk)* on the mass to displace it by *(x_{le})*. The RHS is a work set. This is also same as the energy the force agent spends

$$\left[\,^{v}f^2 \overset{e}{\Longleftarrow} \,^{iv}x_{le}^2 \right] = \left[\,^{v}E_{fa}^3 \right] \tag{40}$$

where the developed concept is *e=energy* that the force agent spends to displace the mass by x_{le}. The RHS is the energy set. During the same time, the energy of the mass changes, which is represented as

$$\left[\,^{v}f^2 \overset{c}{\Longleftarrow} \,^{iv}x_{le}^2 \right] = \left[\,^{v}E_{le}^3 \right] \tag{41}$$

where the developed concept is *c=general change of energy* of the mass during the displacement. The RHS is an energy change set. If the force *f* is a contact force, it changes the kinetic energy as shown below:

$$\left[\,^{v}f^2 \overset{k}{\Longleftarrow} \,^{iv}x_{le}^2 \right] = \left[\,^{v}KE_{le}^3 \right] \tag{42}$$

The developed concept is *k=kinetic energy change* and the RHS is a *KE* change set, and *KE* is the energy associated with moving mass. The CDE is

$$\left[\,^{iv}m_M^1 \overset{p}{\Longrightarrow} \,^{v}E^3 \right] = \left[\,^{v}KE^3 \right] \tag{43}$$

The energy E that a moving mass ($^{iv}m^1_M$) possesses *(p)* due to its velocity is kinetic energy *(KE)*. The transformation rule is

$$\left[\,^{iv}m_M^1\,\right]+\left[\,^{v}E^3\,\right]=\left[\,^{v}KE^3\,\right] \tag{44}$$

If the force is an action-at-a-distance force, the weight is the gravitational force on mass, *m*. The *PE* of the mass changes, as shown by the CDE

$$\left[\,^{v}w^2 \stackrel{p}{\Longleftarrow}\,^{iv}y_{le}^2\,\right]=\left[\,^{v}PE_{le}^3\,\right] \tag{45}$$

where the developed concept is *p=potential energy change* from position y_e to y_l and the RHS is *PE* set. *PE* is the energy of a mass by virtue of its height from the ground and can be represented as

$$\left[\,^{iv}m_H^1 \stackrel{p}{\Longrightarrow}\,^{v}E^3\,\right]=\left[\,^{v}PE^3\,\right] \tag{46}$$

The energy, *E*, which a mass located at a height ($^{iv}m^1_H$) from the ground possesses *(p)*, is potential energy *(PE)*. The CDE is

$$\left[\,^{iv}m_H^1\,\right]+\left[\,^{v}E^3\,\right]=\left[\,^{v}PE^3\,\right] \tag{47}$$

The shift-of-focus rule helps individuals to realize that, in general, force does work on the mass*;* work is the energy the force-agent spends; if the force is a contact force, it changes the kinetic energy; if it is its own weight, *w,* it changes the potential energy; and all are connected to work as shown below:

$$Foc(wk,e)=[wk>e]\,;\,Foc(wk,c)=[wk>c]\,;\,Foc(wk,k)=[wk>k]\,;\,Foc(wk,p)=[wk>p] \tag{48}$$

The augmented intercoordination rule develops all these complex concepts:

$$\left[\,^{v}f^2\,\right]*\left[\,^{iv}x_{le}^2\,\right]=\left[\,^{v}W_{le}^3\,\right]=\left[\,^{v}E_{fa}^3\,\right]=\left[\,^{v}E_{le}^3\,\right]=\left[\,^{v}KE_{le}^3\,\right] \tag{49}$$

$$\left[\,^{v}w^2\,\right]*\left[\,^{iv}y_{le}^2\,\right]=\left[\,^{v}PE_{le}^3\,\right] \tag{50}$$

The sets E_{fa}, W_{le}, KE_{le}, and $PE_{le} \in$ [finite $\mathbb{R}$: Joules].

Lastly, the development of the concept of the energy conservation is considered. A mass moving under gravitational force possesses both *KE* and *PE*. Its total energy is *TE=KE+PE*, which is a fixed positive number for a given motion. The value of *TE* remains the same for that entire motion because an increase in one form of energy results in a decrease by the same amount in the other form, and vice versa. These ideas can be developed in a context; for example, when a ball is thrown up with an initial velocity. When it is moving up from one position (1) to another position (2), the value of the total energy at the two positions is the same number. The CDEs are

$$\left[\,^{v}KE_1^3 \stackrel{t}{\longleftrightarrow}\,^{v}PE_1^3\,\right]=\left[\,^{v}TE_1^3\,\right] \tag{51}$$

where *t= total energy TE₁* at Positon 1. The compounding rule explains this complex concept because the properties and the units are the same:

$$\left[\,^{v}KE_1^3\,\right]+\left[\,^{v}PE_1^3\,\right]=\left[\,^{v}TE_1^3\,\right] \tag{52}$$

Similar sets of equations can develop *TE₂* for Position 2. The shift of focus *(TE₁>TE₂ and TE₂>TE₁)* can help individuals comprehend that *TE₁=TE₂.* Furthermore, the same skill can also help them form the idea that an increase in one form of energy gets compensated for by a decrease in the other form of energy, in the same amount.

The CDE for change in kinetic energy is

$$\left[\,^{v}KE_l^3 \stackrel{b}{\longleftrightarrow}\,^{v}KE_e^3\,\right]=\left[\,^{v}KE_\pm^3\,\right] \tag{53}$$

where *b=ordered KE change*. The KE_{le} can be positive *(KE₊)* or negative *(KE₋)*. The compounding rule is

$$\left[\,^{v}KE_l^3\,\right]+\left[\,^{v}KE_e^3\,\right]=\left[\,^{v}KE_\pm^3\,\right] \tag{54}$$

Similar equations can be developed for a change in potential energy. The coordinating variables are $KE_\pm$ and $PE_\mp \in$ [finite $\mathbb{R}$: Joules]. Shift of focus can reveal that the *increase = -(decrease),* as such:

$$Foc\left(ke_+, pe_-\right) = \left[ke_+ > pe_-\right] \ \& \ Foc\left(ke_-, pe_+\right) = \left[ke_- > pe_+\right] \tag{55}$$

where ke_+, pe_+ indicate increases in kinetic and potential energies, and ke_-, pe_- are decreases in kinetic and potential energies. The CDEs are

$$\left[{}^{v}KE^3_\pm \xleftrightarrow{\ c\ } {}^{v}PE^3_\pm \right] = \left[\begin{matrix} {}^{v}\,ke_\pm\,{}^3 \\ E \\ pe_\mp \end{matrix} \right] \tag{56}$$

An increase in one form of energy gets compensated *(c)* for by a decrease in the other form of energy, and vice versa, so that the value of the *TE* is always the same number for a given motion, and E $\in$ [0: Joules].

3.7. Ap4. Systems of abstract principles system concepts: power

   The energy that the force-agent spends in doing work on the mass gets transferred to the mass in the form of *KE* or *PE*. Force-agents can transfer energy either slowly or quickly. The transfer rate of energy from the force-agent to the mass is described by another physics concept, *power*, which is rate of work done on the mass by the force-agent.

   Power, *P*, is mapping of work (${}^{v}W_{le}{}^3$) and time interval (${}^{iv}t_{le}{}^2$). It contains the properties of work (Joules) and time interval (seconds). The concept of power can develop by augmented intercoordination. The coordinating variables are W_{le} and t_{le}. The CDE is

$$\left[{}^{v}W^3_{le} \xleftarrow{\ p\ } {}^{iv}t^2_{le} \right] = \left[{}^{v}P^4_{le} \right] \tag{57}$$

where *p=rate of work done*. The RHS is the power set $P \in$ [finite $\mathbb{R}$: Joules/second=Watts]. The augmented intercoordination rule is

$$\left[{}^{v}W^3_{le} \right] * \left[{}^{iv}t^2_{le} \right] = \left[{}^{v}P^4_{le} \right] \tag{58}$$

4. Conclusions

   Using non-linear developmental theory, this paper proposes and explains how complex physics concepts can develop from simpler concepts and the skills required for this development. Development of complex concepts is explained using CDEs that follow the augmented transformation rules, which describe mental operations on concept sets. This work leads to a classification scheme that explains the hierarchical complexity of the skills and cognition involved in developing complex concepts from simple ones. The classification scheme is: *Simple abstract, Abstract mapped, Abstract systems, Simple abstract principles, Abstract principles mapped, Abstract principles systems, and Systems of abstract principles system concepts.* An analysis of other physics concepts may reveal more about the skills and concept tiers and levels. The validity of this work needs to be experimentally verified.

   The paper also explains the sources variations in AEs that needed to be coordinated for developing the complex concepts, which may help better learning and teaching. During the development of a "new" concept, an individual goes back-and-forth between the "new" concept, which is not yet constructed and whose validity will be assessed later, and the mastered concepts. The individual introduces a "skeleton structure" for the "new" concept out of the known concepts, which also serves as a place-holder to direct the "new" concept toward achieving the target; in other words, the skeleton structure does not contain the relevant concept yet, but contains the outline and properties of that concept. The skeleton structure also serves as scaffolding for construction of the "new" concept. The entire process of construction of the "new" concept happens in real time while working on the task. The individual functions at a lower-mastered level of the ongoing activity and at higher- level that will produce future learning. Thus, the process pulls development toward more advanced and relatively stable levels [16] and serves as a mechanism for developing a "new" concept.

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