The Discrimination of Visual Number
Author(s): E. L. Kaufman, M. W. Lord, T. W. Reese and J. Volkmann
Source: *The American Journal of Psychology*, Vol. 62, No. 4 (Oct., 1949), pp. 498-525
Published by: University of Illinois Press
Stable URL: http://www.jstor.org/stable/1418556
Accessed: 20-07-2018 02:42 UTC

THE DISCRIMINATION OF VISUAL NUMBER

By E. L. KAUFMAN, M. W. LORD, T. W. REESE, and J. VOLKMANN,
Mount Holyoke College

Suppose that there are two collections or groups of objects—coins, trees, beans, or aircraft—and we do not know how many objects there are. Suppose further that for some reason we cannot count the number of objects in either group. Still, some property of each group makes it possible for a person to say that one of these groups is greater-than, less-than, or equal-to the other group. It is this property of a collection of objects that we define as *numerousness*.[1] We might say that numerousness is that property of a group of objects which we can discriminate, without counting, under instruction to judge how many objects the group contains. We shall wish to modify this definition later as a result of the experiments reported in this paper, but it is adequate for the present discussion of the problem.

The judgment of 'numerousness' may be made in several different ways: (a) it may be comparative—more numerous or less numerous, larger or smaller, etc.; (b) or it may be 'absolute.' There is one special form that the absolute judgment of numerousness can take. It is called the direct reporting of number. In this method of reporting, a numeral is assigned to represent how many things there are in any given collection of objects. After a brief look—so brief that counting is impossible—we say 10, 23, or 250 to indicate that we estimate that the group contained 10, 23, or 250 members.

This paper is concerned with the study of 'numerousness' by the direct method of reporting. Before turning to this problem, however, we review the more recent experiments in the field.

Taves' study, reported in 1941, is pertinent to our problem.[2] He presented fields of dots, one at a time in random order. The number of dots on his stimulus-fields ranged from 2 to 180. The time of presentation was 1/5 sec. He used 133 *Ss* who were instructed to report the number of dots and also the degree of confidence in

* Accepted for publication March 8, 1949. These experiments were carried out under subcontract with Systems Research, The Johns Hopkins University, operating under contract N5-ori-166, Task Order I, with Special Devices Center, Office of Naval Research. This is report No. 166-I-MHC 5 under this subcontract.

[1] This term was first discussed and defined by S. S. Stevens, On the problem of scales for the measurement of psychological magnitudes, *J. Unified Sci.*, 9, 1938, 94-99.

[2] E. H. Taves, Two mechanisms for the perception of visual numerousness, *Arch. Psychol.*, 37, 1941 (no. 265), 1-47.

the correctness of their reports. Confidence was estimated on a 6-point scale ranging from 0 to 5. Zero meant no confidence, a sheer guess; and 5 meant complete certainty.

Taves found that the reports were accurate up to and including the 6-dot field. Somewhere between 6 and 8 dots the reports became inaccurate,[3] *i.e.* after 6 dots the median of his *Ss'* reports no longer followed the line representing the 'true' median.

Fig. 1 shows the median confidence of the reports of Taves' *Ss* plotted against the number of dots presented. The first part of the curve, representing the *Ss'* confidence in their report of 2, 4, and 6 dots, is almost a straight line with a confidence of 5. In short, up to 6 dots the *Ss* were certain of their reports. At 8 dots, however,

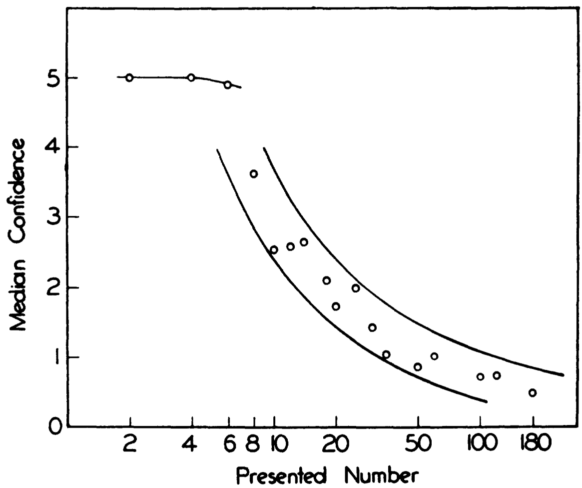

FIG. 1. MEDIAN CONFIDENCE AS A FUNCTION OF THE NUMBER OF DOTS IN THE STIMULUS-FIELD

The abscissa is logarithmic. (After Taves)

something happened. *Ss'* confidence fell rapidly and was very variable from then on. The points on the curve in Fig. 1 form a wide band decreasing quickly at first, then slowly, until at 180 dots the *Ss* had practically no confidence in the correctness of their reports. Again, something seemed to happen to confidence between 6 and 8 dots just as it happened at that point to accuracy.

Taves claims, in explanation of his results, that the discontinuity between 6 and 8 dots indicates two 'mechanisms' for the discrimination of visual numerousness.[4]

[3] We do not know whether they first became inaccurate at 7 or 8 because Taves did not present a field of 7 dots.

[4] Such sharp and sudden changes in the direction of a graph are referred to as discontinuities. Such a break or discontinuity in the graph usually indicates discontinuity of the first or higher derivatives of the function but not necessarily a discontinuity of the function. To describe these discontinuities we introduce the following notation. Let $h(x)$ be a function of x. We define $lim_{x \to a^+} h(x)$ to be the limit of $h(x)$ as x approaches a through values greater than a and $lim_{x \to a^-} h(x)$ to be the limit of $h(x)$ as x approaches a through values less than a.

Let $y = F(x)$ be the equation of Curve C fitted to data relating a variable y

A small number of things, from 1 to 7 or 8, are discriminated in one way and larger numbers are discriminated in another way. An 'adequate' perception of number is defined as one that agrees almost all the time with the number of objects as measured by another operation—in this case the method of counting. Even in an adequate perception there will be occasional errors, but they will never be large nor many. We can see, then, that the first 'mechanism' leads to an adequate perception of number. The second 'mechanism' leads to an inadequate perception of number; the errors are many and large and confidence is lacking.

Taves reports more experimental evidence in support of his thesis, *e.g.* he found that the graphs for variability of direct reports were also discontinuous. This brief account of his experimental findings is typical, however, of his other results.

The most recent study bearing on our problem was published by Saltzman and Garner in 1948,[5] who studied the effect of a large number of variables on this discrimination. They wished to find out whether it was affected by such things as: (a) *S*'s knowledge of stimulus-range; (b) practice; (c) regularity of the spacing of the stimulus-objects; (d) *S*'s distance from the stimulus-objects; (e) brightness of the background on which the stimulus-objects appeared; and (f) size of the stimulus-objects.

---

with a variable x. If $\lim_{x\to a^+}F(x) \neq \lim_{x\to a^-}F(x)$, then the function $F(x)$ is discontinuous at the point a, or a is a point of discontinuity of the function. A discontinuity of the function is clearly associated with a break in the Curve C as is illustrated in Fig. A.

To take a different example, shown in Fig. B, let us assume that the function $F(x)$—and therefore also the Curve C, which is the graph of the equation $y = F(x)$

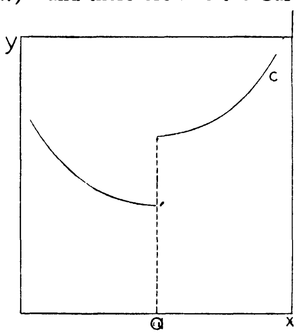

FIG. A. FUNCTION C, DISCONTINUOUS AT POINT *a*

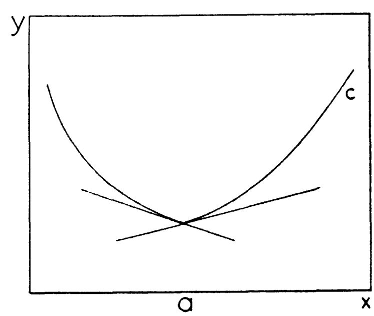

FIG. B. DERIVATIVE OF FUNCTION C, DISCONTINUOUS AT POINT *a*

—is continuous. Let $F'(x)$ be the derivative of $F(x)$ with respect to x. If $\lim_{x\to a^+}F'(x) \neq \lim_{x\to a^-}F'(x)$, then a is a point of discontinuity of the derivative of the function. Since $F'(x)$ is the slope of the tangent line to C at the point x, a point of discontinuity of the derivative of $F(x)$ is a point at which the right hand tangent line T_R and the left hand tangent line T_L differ in position.

In practice the data may be plotted on rectangular, semi-log, log-log, or hyperbolic coördinate paper. It can be shown that the properties of continuity and discontinuity of $F(x)$ and of $F'(x)$ are independent of these coordinate systems. For assistance in this discussion we are indebted to Dr. F. L. Kiokemeister, Department of Mathematics, Mount Holyoke College.

[5] I. J. Saltzman and W. R. Garner, Reaction-time as a measure of span of attention, *J. Psychol.*, 25, 1948, 227-241.

They used two different methods: (1) The tachistoscopic method, in which the stimulus-fields were exposed for 1/2-sec., and in which *S*'s accuracy of discrimination was recorded. This was essentially the same as Taves' procedure, except that Taves used a 1/5-sec. exposure. (2). The reaction-time method, in which the stimulus-fields were exposed until *S* responded. As *S* was instructed for accuracy, this procedure encouraged counting. *S*'s reaction-time—the interval between the exposure of the stimulus-field and the beginning of *S*'s verbal response—was measured.

In most of their experiments, the stimulus-objects (concentric circles) varied in number from 2 to 10. The circles were so drawn that the distance between them was equal to the radius of the smallest circles. The diameter of the outermost circle was held constant for a given experiment regardless of the total number of circles

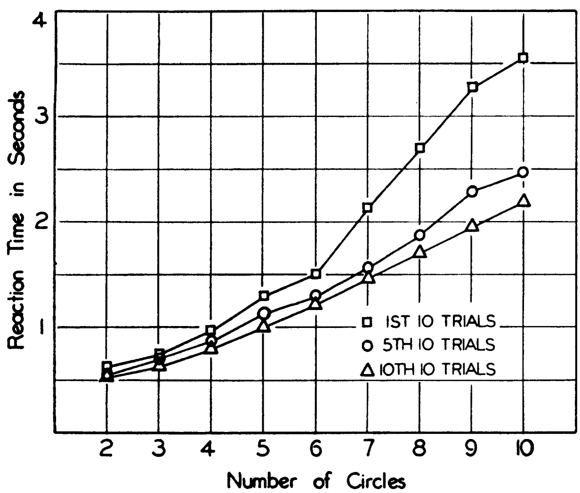

FIG. 2. REACTION-TIME (IN SEC.) AS A FUNCTION OF THE NUMBER OF CONCENTRIC CIRCLES PRESENTED

The coördinates are arithmetic (After Saltzman and Garner)

presented. Using the tachistoscopic method, they found that for the first 10 trials, no more than 3 circles were correctly identified 100% of the time. With repetition the reports became more accurate. They believed that this was due to increased familiarity with the stimulus-materials. They also discovered that repetition has relatively little effect on the accuracy of judgments for 6 circles and above. Incidentally, this is interesting in view of Taves' finding that something special seems to happen around 6.

Their results with the reaction-time method are plotted in Fig. 2. The curve shows that the reaction-time increases with the number of circles presented. The increase occurs even at the lower end of the stimulus-range when two and three circles are exposed. They also found a decrease in reaction-time with continued practice; but, as Fig. 2 shows, most of this occurred during the early part of the experiment.

They performed additional experiments with dots, rather than circles, varying in number from 1 to 10. In one of these, the dots were regularly spaced, being placed equidistant from each other on a horizontal line. In another experiment the dots were randomly spaced within a circular frame. Their results with dots were similar to those with circles. Reaction-time increased and accuracy decreased with the number of stimulus-objects. There was an increase in reaction-time even from 1 to 2 dots. When the dots were irregularly spaced, the reaction-times were greater; but the general shape of the curve was the same—a continuous increase in reaction-time with an increase in the number of dots.

They found in their study, that all of the variables considered—knowledge of stimulus-range, practice, regularity in spacing, brightness, and size—affected both accuracy and reaction-time. Their most important finding, from the point of view of the present study, was that reaction-time increased with the number of stimulus-objects presented. Apparently it takes even more time to see two things accurately than it does to see one thing!

They concluded from their results that there is no span of apprehension if span of apprehension be defined as the constant number of things which one can take in 'immediately.' They reached this conclusion (a) because all the variables studied affected the accuracy of reporting; and (b) because it takes more time to see more things.

Are these two sets of experimental findings incompatible? Taves found that numbers of objects up to 6 were discriminated adequately, with minimum variability and nearly maximum confidence. Beyond 6 the adequacy of the discrimination breaks down, variability suddenly increases, and confidence rapidly decreases. In short, he found that all three of these graphs are discontinuous between 6 and 8. He explains this by saying that numbers of objects up to 6 are perceived by a different mechanism from numbers of objects beyond this point.

Saltzman and Garner found that numerous variables affect both the accuracy of reporting and the report-time (reaction-time) for all numbers up to 10. Further, they found that reaction-time increases steadily as the number of objects increases. From this they argue against the concept of a span of apprehension defined as an *immediate and adequate perception of number.*

The argument for the existence of two mechanisms in the report of numerousness is based upon the discontinuities found by Taves in the graphs of median report, confidence, and variability. This argument does not state that there are no errors in the reporting of numbers below 6—only that the graph is discontinuous between 6 and 8. Further, this argument does not deny that reaction-time increases with an increase in the number to be reported, but it would predict that reaction-time is discontinuous if the other graphs are discontinuous, and that it will be discontinuous at the *same point* as the other graphs.

Perhaps the findings of Saltzman and Garner can be considered in relation to Taves' first mechanism, the one that permits the discrimination of numbers below 6 or 8. These findings should dispose of any ready mentalistic interpretation; for example, that there is some power of the mind that immediately grasps the truth about a given number of objects in the external world. What is just as important, Saltzman and Garner have given us a method for finding out about the discrimination of low numbers, and a striking set of data.

To avoid confusion, note that we are dealing only with the discrimination of the sheer *number* of objects, and not with their identity or any other characteristic. As the analysis proceeds, note also that we are not concerned with the frequency of correct reports of numbers, nor with the range of attention or apprehension derived from the frequency of correct reports.[6] During much of the study, the primary datum will be the median number of objects reported, whether that number is correct or not. This datum has at least one real advantage: we can follow its course in detail when using medium and high stimulus-numbers, where the frequency of correct reports is usually an unvarying 0%. Furthermore, we have found this measure more revealing when examining the changes in the entire function, from low stimulus-values to high ones. Similar advantages appear in the other principal data of our study, the speed of reporting and confidence in the report.

Experimental Conditions

Stimulus-material. The stimulus-objects were groups of dots projected on a large screen. The film was prepared in the following way. We drew small, solid-black

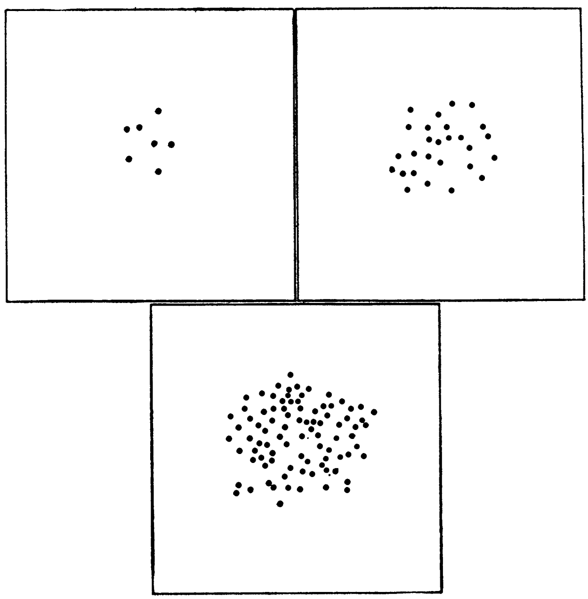

Fig. 3. Representative Stimulus-Fields of 7, 28, and 89 Dots

circles on white paper. Regular or meaningful patterns were avoided. We tried to keep the apparent density of the dots about the same from one drawing to the next, but to vary the density somewhat in the different parts of any one field. Fig. 3 shows dot-fields of 7, 28, and 89 dots.

Two series of stimulus-fields were prepared. In each there were 35 fields. The

[6] The range of attention or apprehension has been defined as that number of stimulus-objects correctly reported 50% of the time. For the proposal that the range be thus defined, see S. W. Fernberger, this Journal, 32, 1921, 121-133. For a bibliography of studies on range, see N. F. Gill and K. M. Dallenbach, this Journal, 37, 1926, 247-256.

number of dots in the first 15 fields progressed by ones from 1-15. The remaining fields contained the following numbers of dots: 17, 19, 22, 25, 28, 32, 37, 42, 49, 57, 66, 77, 89, 103, 118, 134, 152, 170, 191, and 210. From 1 to 15, the series was as dense as it could be, since a fractional number of dots can not be presented. Above 15 dots, the increment was less than 1 *jnd*.[7] These drawings were photographed in random order on 35-mm. negative strip film and then turned upside down and photographed again. This was done to decrease the possibility that an *S* would recognize a field as one that she had previously seen. Since the dots were photographed on negative film, they appeared white when projected.

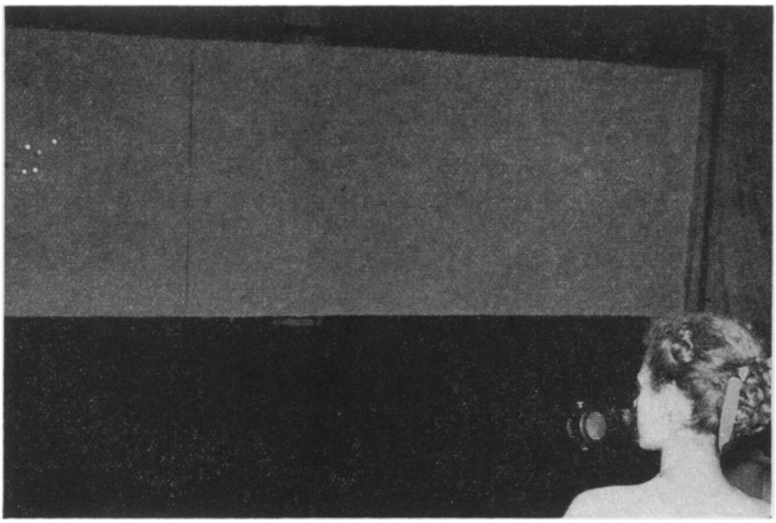

FIG. 4. AN *S* SERVING IN THE EXPERIMENT

Instructions. *S*'s task was to report the number of dots flashed on the screen. The instructions to one group of *S*s emphasized speed, and to another group *accuracy*. These important features of the instructions were often repeated during the experiment. *S* was not informed about the correctness of her reports.

After every report of number of dots seen, *S* indicated her degree of confidence upon a 5-point scale: 5 meaning that she was absolutely certain of the accuracy of her report, and 1 meaning that she was absolutely uncertain—that her report was a sheer guess.[8]

Subjects. Eight women and one man served in this study. None of them had served in experiments of this kind before. Four were instructed for speed, and the other

[7] To achieve this, we plotted Taves' averaged measurements of the *jnd* for numerousness (*op. cit.,* 40) against the standard number of dots, and drew a smooth curve. Another curve was drawn parallel to the first curve, but below it, and the increments were read from this second curve.

[8] This crude but effective verbal scale was replaced in subsequent experiments by a visual indicating device. *S* now sets a pointer on a convenient graphical scale, and *E* reads its position.

five for accuracy. Every *S* served for approximately 3 hr. in sessions lasting from 1 to 1¼ hr. In all, every *S* made about 21 reports of each of the 35 stimulus-values—over 700 reports.

Experimental conditions. The experiment was conducted in an auditorium. Fig. 4 shows an *S* serving in the study. She has a microphone in front of her and faces a large white screen 32 ft. away. The screen is 45 ft. long, 7½ ft. high, and a center space 8½ ft. wide is marked off by vertical, black lines. The stimulus-dots appeared in the approximate center of this space. As projected on the screen, each

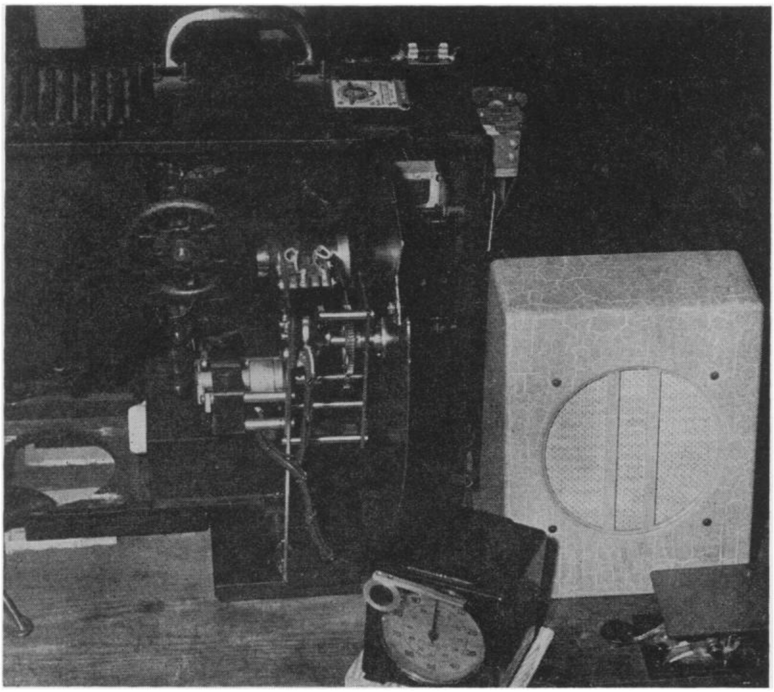

FIG. 5. E'S DESK SHOWING THE APPARATUS
Flash projector, circular disk, loudspeaker, telegraph key, and chronoscope

dot was 1¼ in. in diameter. Its brightness was 0.108 apparent footcandles as measured with a Macbeth illuminometer. Two 15-w. lamps, carefully shaded and located in the corners of the auditorium behind *S*, dimly illuminated the background. The level of this illumination at the screen was less than 0.02 footcandles. The background was illuminated for two reasons: (1) to show *S* where the dots were to be projected, and (2) to reduce after-images.

Fig. 5 shows *E*'s desk which was behind *S*. The apparatus shown here provided a 'ready-signal,' exposed the film, and measured the time between the exposure and *S*'s report. The various parts worked in the following way. When *E* pressed the telegraph-key shown at the right in the figure, the circular disk in the center began to turn. It was driven by a 1-rps. synchronous motor through a gear train. A cam, turning on the same shaft as the disk, closed a switch and sounded the ready-signal. After 1½ sec. an open sector in the disk then exposed the dot-pattern for 1/5 sec. At the same time that the dots flashed on the screen, a cam closed a holding relay (not shown in the figure). This relay started the chronoscope. After the exposure, another cam interrupted the current to the synchronous motor and stopped the disk.

When *S* spoke her report into the microphone, the speech-signal was amplified and used to operate a sensitive relay. This relay opened the holding relay, and so stopped the chronoscope. In this way the chronoscope measured the time between the exposure of the dots and *S*'s vocal response. The amplifier also fed *S*'s voice-signal to a loudspeaker, so that *E* could write down *S*'s report as well as the report-time and the confidence with which the report was given.

RESULTS

Group reports. The first measure we shall look at is the median number of dots reported by the *S*s for each stimulus-number presented to them. The reports for the four *S*s, instructed for accuracy, have been combined to give group-medians. The reports for the other five *S*s, who were instructed for speed, have also been combined. The two sets of group-medians appear in Table I, and in Figs. 6 and 7.

In Fig. 6, the median reported number is plotted against presented number for the accuracy-instruction. The solid line in the graph represents the adequate report—for example, a median report of 49 dots when 49 dots are presented. The data look very much like Taves'. When the *S*s are shown 1 to 5 dots, they usually report the correct number. From 6 to 9 dots, the median reports are a little too high. Above 10 dots they are a little too low and at larger numbers of dots become still lower.

The data for the speed-group, in Fig. 7, are very similar. This method of plotting is not sensitive enough, however, to show any small differences between the group instructed for speed and the group instructed for accuracy. The data have, consequently, been plotted differently. Figs. 8 and 9 use as a different measure the percentage error of reporting: the median number of dots reported minus the number presented, taken as a percentage of the number presented.

These graphs make it still easier to see that small numbers of dots are reported adequately; that some larger numbers up to 10 or 12 are reported as slightly too high; that still larger numbers of dots are reported as too low. The speed-data vary between +14% and −42% error. The accuracy-data vary between slightly narrower limits: +9% and −35%. The average deviation of the points in Fig. 9 (speed-data), taken from the line of zero percentage error, is 16.1%. The corresponding average deviation for the accuracy-data is 12.6%. These findings are what might

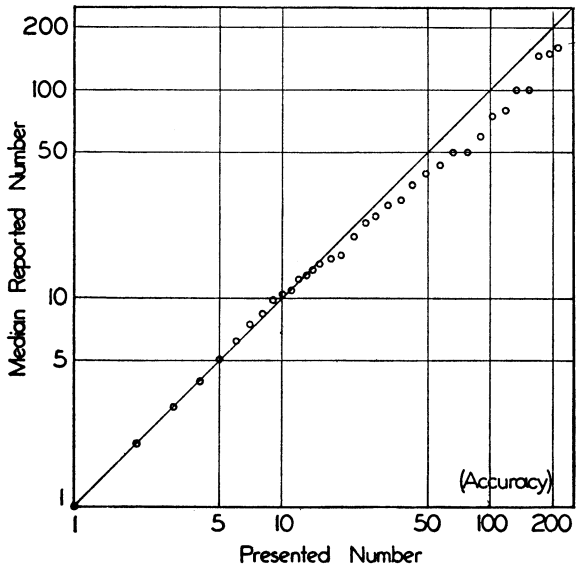

FIG. 6. MEDIAN NUMBER REPORTED AS A FUNCTION OF NUMBER OF DOTS PRESENTED

S instructed for accuracy; cördinates logarithmic; group results.

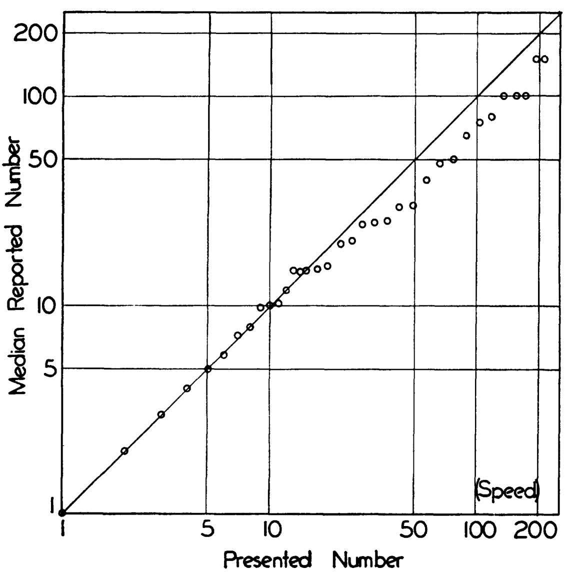

FIG. 7. MEDIAN NUMBER REPORTED AS A FUNCTION OF NUMBER OF DOTS PRESENTED

S instructed for speed; coördinates logarithmic; group results.

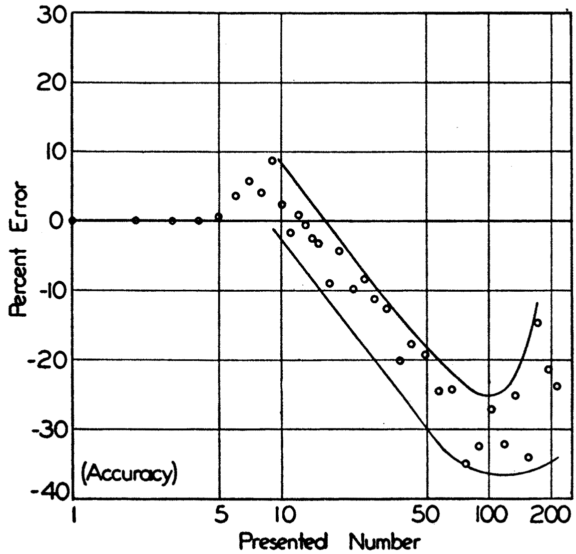

FIG. 8. PERCENTAGE ERROR AS A FUNCTION OF NUMBER OF DOTS PRESENTED
S instructed for accuracy; coördinates semi-logarithmic; group results.

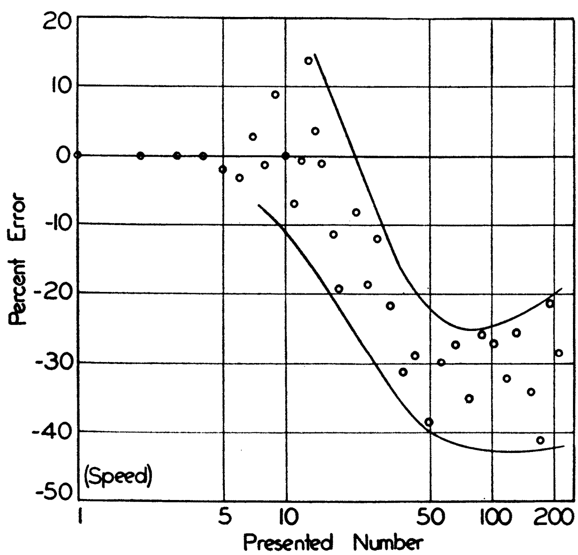

FIG. 9. PERCENTAGE ERROR AS A FUNCTION OF NUMBER OF DOTS PRESENTED
S instructed for speed; coördinates semi-logarithmic; group results.

be expected—instruction for accuracy should result in greater accuracy. The speed- and accuracy-data, it must be recalled, come from different groups of *Ss*, and our evidence is, therefore, not as clear as it might be. In addition, the population at large may be more variable than our small sample of *Ss*.

The points plotted in Figs. 8 and 9 show considerable scatter. One reason for this scatter is the well-known tendency to estimate in round numbers. For example, our *Ss* were much more likely to say 100 dots than 98 or 104. If the stimulus-field

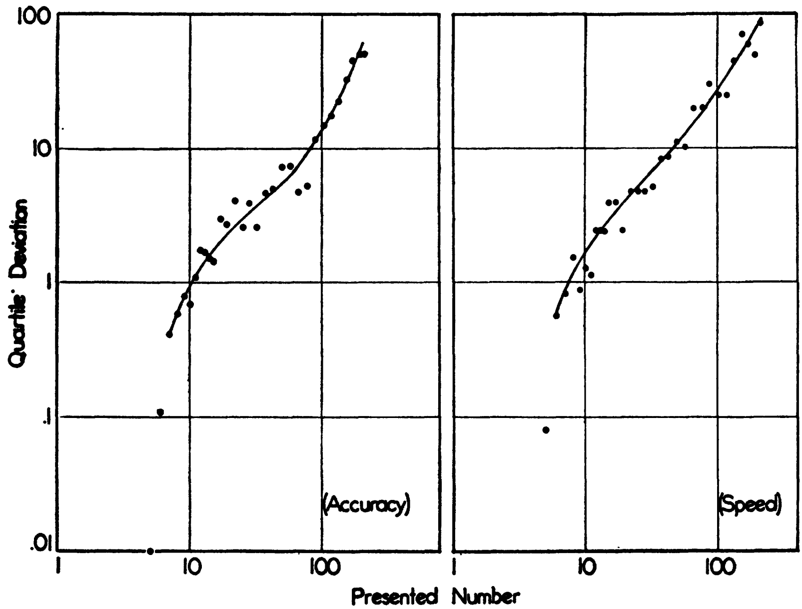

FIG. 10. QUARTILE DEVIATIONS AS A FUNCTION OF NUMBER OF DOTS PRESENTED

S instructed for accuracy; coördinates logarithmic; group results.

FIG. 11. QUARTILE DEVIATIONS AS A FUNCTION OF NUMBER OF DOTS PRESENTED

S instructed for speed; coördinates logarithmic; group results.

actually had contained 100 dots, frequent reports of 100 would produce a small error. If the stimulus-field had contained 89 or 103 dots, reports of 100 would produce a larger error. The error, therefore, is sometimes made small and sometimes large by this 'round-number' tendency. Because the tendency occurs in reports of all sorts, it deserves an intensive study in its own right.

The scatter of points is greater under the speed instruction (Fig. 9) than under the accuracy instruction (Fig. 8). We can suppose that the attempt to hurry facilitates round-numbered estimates. Note that in both Figs. 8 and 9 the negative percentage of errors increase up to 50-70 dots. Above this region they remain roughly constant. We do not know why.

Variability. To measure the variability of *Ss*' reports about the median, we calculated the quartile deviation (Table I). Figs. 10 and 11 show how this measure varies with the presented number of dots under the instruc-

TABLE I

MEDIAN, PERCENTAGE ERROR, AND QUARTILE DEVIATION OF REPORTED NUMBER, AND MEDIAN CONFIDENCE, FOR THE GROUPS INSTRUCTED FOR ACCURACY AND SPEED RESPECTIVELY

Four *Ss* were instructed for accuracy and five for speed. Each group gave 21 reports for every stimulus-field. Negative signs indicate percentage underestimated; the other figures, percentage overestimated.

No. dots	Accuracy instruction				Speed instruction			
	Median reports	% error	Quartile deviation	Median confidence	Median reports	% error	Quartile deviation	Median confidence
1	1.00	0.00	0.00	5.00	1.00	0.00	0.00	5.00
2	2.00	0.00	0.00	5.00	2.00	0.00	0.00	5.00
3	3.00	0.00	0.00	5.00	3.00	0.00	0.00	4.98
4	4.00	0.00	0.00	4.97	4.00	0.00	0.00	4.97
5	5.03	0.60	0.01	4.94	4.90	− 2.00	0.08	4.83
6	6.21	3.50	0.19	4.62	5.80	− 3.33	0.56	4.17
7	7.41	5.85	0.41	4.17	7.20	2.85	0.82	3.72
8	8.32	4.00	0.58	3.95	7.90	− 1.25	1.53	3.63
9	9.78	8.66	0.79	3.79	9.80	8.88	0.87	3.39
10	10.23	2.30	0.69	3.59	10.00	0.00	1.27	2.85
11	10.80	− 1.81	1.09	3.47	10.30	− 7.00	1.14	2.92
12	12.11	0.91	1.73	3.32	11.90	− 0.83	2.44	2.71
13	12.92	− 0.61	1.68	3.16	14.80	13.84	2.43	2.88
14	13.62	− 2.57	1.50	3.05	14.50	3.57	2.40	2.77
15	14.50	− 3.33	1.42	3.08	14.84	− 1.06	3.93	2.71
17	15.44	− 9.17	2.93	2.98	15.05	−11.47	3.98	2.46
19	18.16	− 4.42	2.69	2.89	15.31	−19.42	2.46	2.50
22	19.81	− 9.95	4.06	2.82	19.82	− 8.27	4.73	2.32
25	22.90	− 8.40	2.57	2.79	20.34	−18.64	4.72	2.42
28	24.86	−11.21	3.85	2.79	24.50	−12.50	4.73	2.28
32	27.94	−12.68	2.57	2.74	25.05	−21.71	5.09	2.14
37	29.55	−20.13	4.61	2.79	25.42	−31.29	8.44	1.91
42	34.57	−17.69	4.92	2.50	29.84	−28.95	8.75	1.78
49	39.50	−19.38	7.27	2.68	30.11	−38.55	11.11	1.89
57	43.00	−24.56	7.33	2.50	40.00	−29.82	10.14	1.69
66	50.00	−24.24	4.65	2.42	48.00	−27.27	19.79	1.74
77	50.00	−35.06	5.11	2.22	50.00	−35.06	20.00	1.53
89	60.00	−32.58	11.85	2.05	65.00	−25.84	30.00	1.41
103	75.00	−27.18	14.77	2.00	75.00	−27.18	25.00	1.40
118	80.00	−32.20	17.29	1.98	80.00	−32.20	25.00	1.37
134	100.00	−25.37	22.25	1.66	100.00	−25.37	45.00	1.21
152	100.00	−34.21	32.50	1.62	100.00	−34.21	70.00	1.17
170	145.00	−14.70	45.00	1.47	100.00	−41.17	60.00	1.12
191	150.00	−21.46	50.00	1.54	150.00	−21.46	50.00	1.05
210	160.00	−23.80	50.00	1.37	150.00	−28.57	85.50	1.08

tions for accuracy and speed respectively. Both axes are logarithmic. The variability increases quite regularly from 5 or 6 dots to 210, the largest number presented.

Report-times. The median time in seconds between the exposure of the dots and *S*'s report appears in Tables II and III and Figs. 12 and 13. The times first increase slowly, then rapidly, to about 6 dots. It seems that Saltzman and Garner were right.

Above 6 dots, in Fig. 12, the times increase irregularly and fall into a

TABLE II

MEDIAN REPORT TIME (IN SEC.) FOR EACH STIMULUS-FIELD, FOR FOUR *Ss* INDIVIDUALLY AND AS A GROUP—ACCURACY INSTRUCTION

Every *S* gave 21 reports for every stimulus-field

No. dots	*Ss*				Group median times
	Re	*Mc*	*Ro*	*Wy*	
1	0.96	1.00	1.20	1.15	1.09
2	1.18	1.21	1.19	1.12	1.18
3	1.05	1.20	1.30	1.20	1.20
4	1.15	1.21	1.29	1.29	1.23
5	2.04	1.31	1.70	1.55	1.61
6	3.20	1.89	2.27	3.36	2.70
7	3.40	1.92	2.30	3.85	2.78
8	3.40	1.76	2.85	4.35	3.15
9	4.44	1.68	3.05	3.67	3.14
10	4.50	1.70	3.58	4.12	3.86
11	4.72	1.63	3.23	3.68	3.82
12	4.50	1.53	3.82	3.68	3.82
13	4.89	1.74	3.99	3.43	3.79
14	4.94	1.78	4.05	3.24	3.97
15	5.02	1.70	4.00	2.84	3.91
17	4.66	1.62	4.00	2.83	3.64
19	5.70	1.66	4.59	4.01	4.01
22	5.30	1.65	4.85	3.12	4.03
25	4.92	1.47	5.06	2.77	3.71
28	5.20	1.53	5.05	2.99	3.32
32	5.26	1.57	3.81	2.66	3.32
37	6.39	1.62	4.33	2.89	3.58
42	5.40	1.60	4.33	2.67	3.42
49	5.92	1.73	4.40	3.38	3.71
57	6.20	1.81	4.44	3.15	3.88
66	5.91	1.59	4.73	3.11	3.79
77	7.56	1.69	4.40	2.97	3.26
89	7.40	1.50	4.88	2.83	3.38
103	7.82	1.80	4.40	2.86	3.30
118	8.02	1.60	4.47	3.09	3.80
134	7.39	1.52	5.01	2.70	3.72
152	9.09	1.81	3.12	3.05	3.20
170	9.52	1.57	6.30	3.12	3.66
191	9.77	1.44	3.60	3.10	3.47
210	10.72	1.80	3.00	2.94	3.12

broad, curved band. In Fig. 13, the report times are nearly constant for 6 dots and above. Both of these graphs suggest, however, that we may wish to describe the data with two curves instead of one. They recall Taves' evidence of discontinuity, and his conclusion that there are two mechanisms for visual numerousness.

The graph of the speed-data in Fig. 13 falls quite obviously into two branches. The accuracy-data in Fig. 12 do not make so clear a picture.

We wish to see what goes on in the crucial region of the graph, between 5 and 10 stimulus-dots. One device for examining the shape of a function is to rectify some

TABLE III

MEDIAN REPORT TIME (IN SEC.) FOR EACH STIMULUS-FIELD, FOR FIVE Ss INDIVIDUALLY AND AS A GROUP—SPEED INSTRUCTION

Every *S* gave 21 reports for every stimulus-field

No. dots	Ss: *Sc*	*Ju*	*Se*	*Ce*	*Tu*	Group median times
1	0.66	0.64	0.74	0.66	0.65	0.67
2	0.72	0.76	0.84	0.70	0.81	0.77
3	0.78	0.80	0.87	0.76	0.77	0.79
4	0.87	0.92	1.01	0.82	0.84	0.91
5	1.08	1.11	1.37	0.97	1.14	1.12
6	1.39	1.66	1.77	1.15	1.45	1.45
7	1.49	1.56	1.84	1.16	1.42	1.48
8	1.52	1.74	1.72	1.15	1.56	1.51
9	1.57	1.56	1.76	1.12	1.55	1.44
10	1.40	1.53	1.80	1.03	1.51	1.49
11	1.44	1.74	2.06	1.08	1.58	1.51
12	1.51	1.55	1.90	1.09	1.58	1.52
13	1.47	1.55	1.98	1.09	1.70	1.54
14	1.45	1.48	1.74	1.03	1.50	1.47
15	1.53	1.40	1.88	1.10	1.77	1.52
17	1.49	1.35	1.82	1.02	1.62	1.47
19	1.46	1.47	1.96	1.00	1.66	1.53
22	1.55	1.40	1.93	1.06	1.79	1.52
25	1.50	1.35	1.82	0.98	1.53	1.46
28	1.41	1.53	1.89	1.08	1.60	1.50
32	1.41	1.48	1.78	1.04	1.54	1.44
37	1.64	1.45	1.82	1.00	1.55	1.60
42	1.49	1.49	1.80	1.04	1.54	1.50
49	1.52	1.43	1.60	0.96	1.58	1.52
57	1.52	1.50	1.84	0.95	1.74	1.48
66	1.66	1.58	1.82	0.86	1.48	1.60
77	1.63	1.55	1.93	0.89	1.43	1.53
89	1.44	1.50	1.85	0.90	1.43	1.44
103	1.58	1.72	1.93	0.87	1.56	1.56
118	1.69	1.86	1.96	0.85	1.57	1.69
134	1.52	1.71	1.90	0.80	1.65	1.55
152	1.40	1.67	1.78	0.83	1.59	1.50
170	1.54	1.76	1.60	0.86	1.42	1.52
191	1.48	1.68	1.51	0.92	1.68	1.50
210	1.60	1.81	1.52	0.94	1.54	1.54

selected portion of it and to look at it again. The first apparent branch (from 1 to 6 dots) has, consequently, been rectified by plotting a suitable function of the median time, rather than the median time itself. It is important to note that the same function has been applied to *all* of the dots, not just to those in the first apparent branch. Here is the process: from a plot of median time against stimulus-number, on arithmetic coördinates, we extrapolate to the y-intercept. This is the time that it would take *S* to report zero dots. This *time* is subtracted from each median time. The reciprocal of the difference is the function plotted in Fig. 14. The data for the speed instructions have also been rectified and appear in Fig. 15. The lines in these figures were fitted by eye.

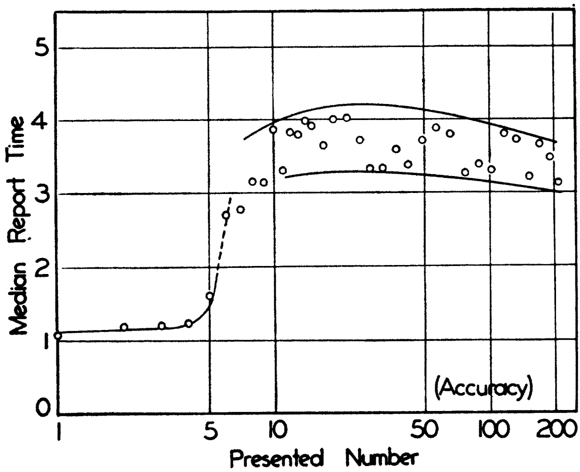

FIG. 12. MEDIAN REPORT-TIME AS A FUNCTION OF NUMBER OF DOTS PRESENTED
S instructed for accuracy; coördinates semi-logarithmic; group results.

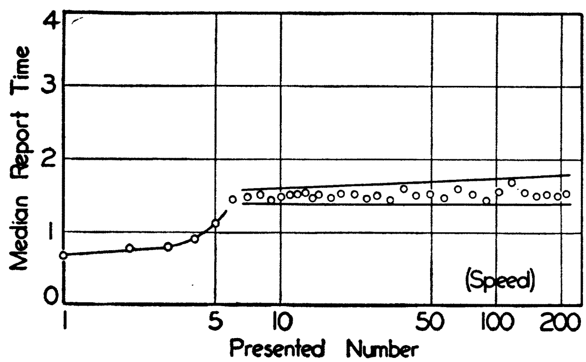

FIG. 13. MEDIAN REPORT-TIME AS A FUNCTION OF NUMBER OF DOTS PRESENTED
S instructed for speed; coördinates semi-logarithmic; group results.

From these graphs we see that there are two branches. The data are discontinuous with respect to slope, the second kind of discontinuity discussed in a preceding section. Fig. 15 is especially convincing, because both segments of the graph look rectilinear. So the evidence from report-time leads us to infer two mechanisms for the discrimination of visual numerousness. Taves, too, was right.

He did not know exactly where the point of discontinuity was, but placed it somewhere between 6 and 8 dots. Figs. 14 and 15 show that it lies at or near 6

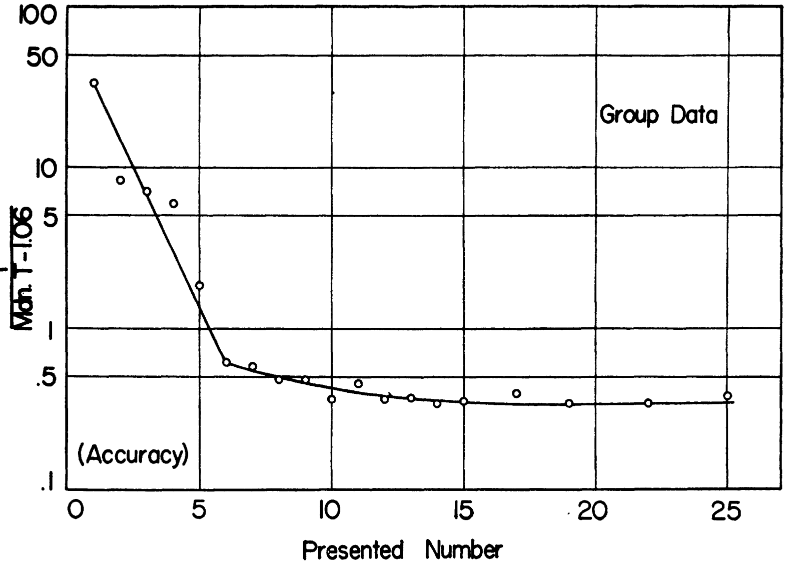

FIG. 14. A RECIPROCAL FUNCTION OF MEDIAN REPORT-TIME PLOTTED AGAINST NUMBER OF DOTS PRESENTED

S instructed for accuracy; coördinates semi-logarithmic; group results.

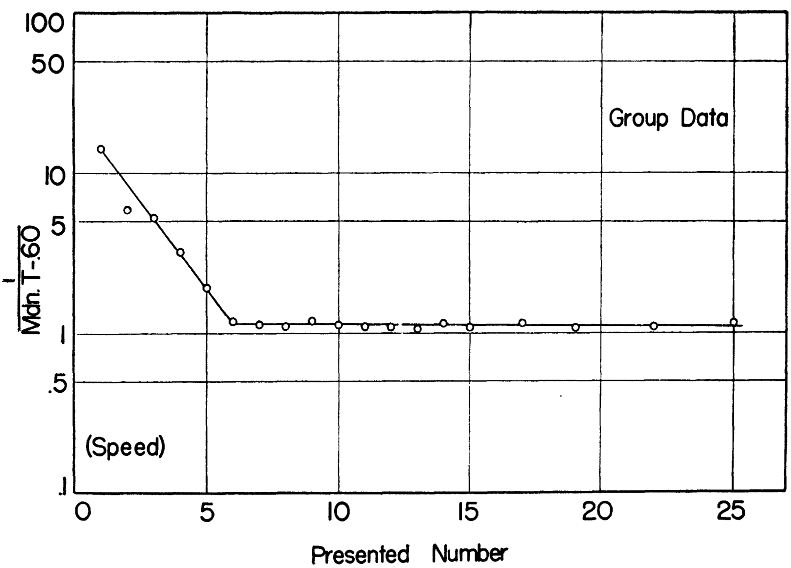

FIG. 15. A RECIPROCAL FUNCTION OF MEDIAN REPORT-TIME PLOTTED AGAINST NUMBER OF DOTS PRESENTED

S instructed for speed; coördinates semi-logarithmic; group results.

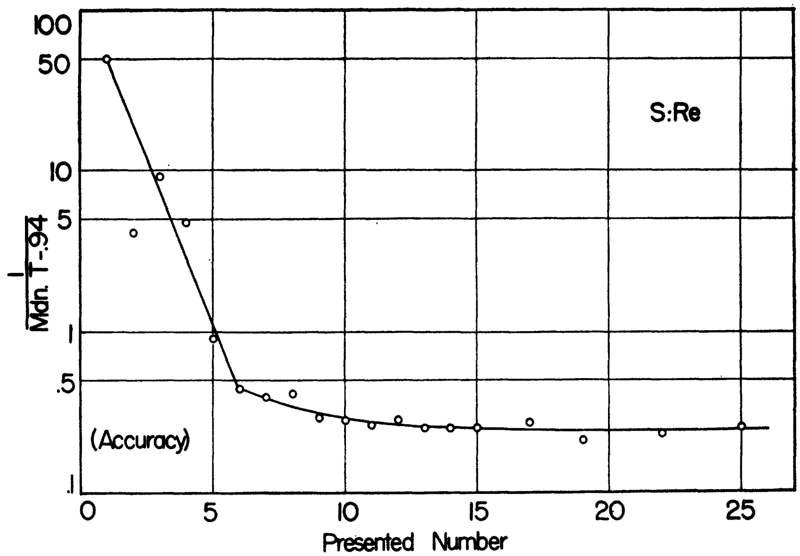

FIG. 16. A RECIPROCAL FUNCTION OF MEDIAN REPORT-TIME PLOTTED AGAINST NUMBER OF DOTS PRESENTED

Data for one *S* instructed for accuracy; coördinates semi-logarithmic.

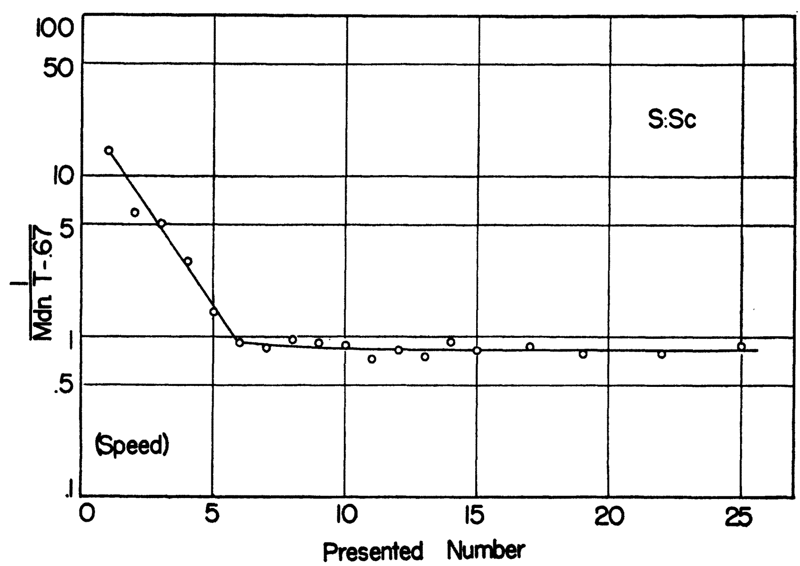

FIG. 17. A RECIPROCAL FUNCTION OF MEDIAN REPORT-TIME PLOTTED AGAINST NUMBER OF DOTS PRESENTED

Data for one *S* instructed for speed; coördinates semi-logarithmic.

dots. The question remains how 6 dots are discriminated, and this question will be discussed below.

Individual median times. The results of individual *Ss*, like the group-

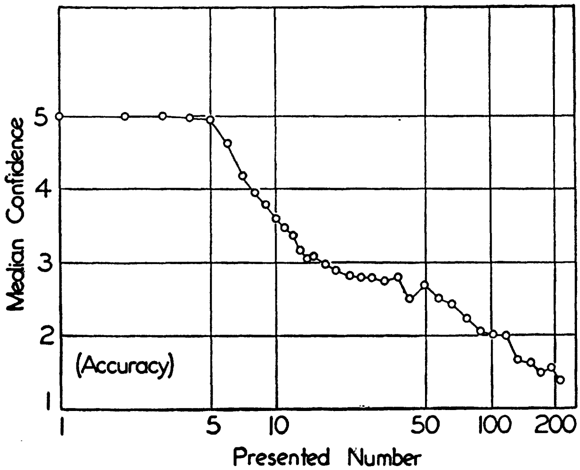

FIG. 18. MEDIAN CONFIDENCE AS A FUNCTION OF NUMBER OF DOTS PRESENTED
S instructed for accuracy; coördinates semi-logarithmic; group results.

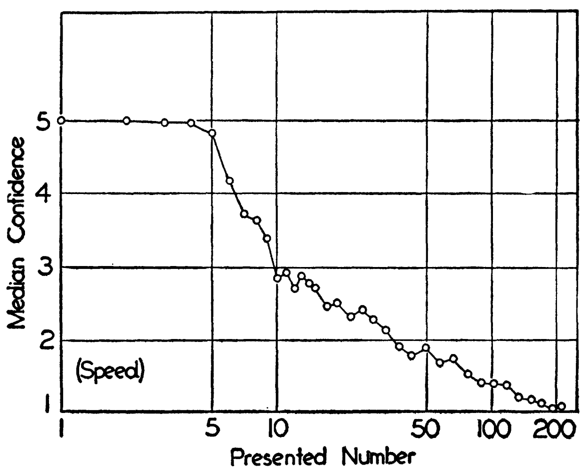

FIG. 19. MEDIAN CONFIDENCE AS A FUNCTION OF NUMBER OF DOTS PRESENTED
S instructed for speed; coördinates semi-logarithmic; group results.

results, lead us to infer two discriminatory mechanisms. Figs. 16 and 17 show the median times of *Re* for accuracy, and *Sc* for speed. Both graphs have been rectified in the same way as Figs. 14 and 15.

Confidence. The median confidence that accompanied the reports of number appears in Tables I, IV, and V and Figs. 18 and 19. The graphs resemble Taves', except that his *Ss* seem to have reported 6 dots with more confidence than ours did. In order to examine the data more closely, the

TABLE IV

MEDIAN CONFIDENCE FOR FOUR Ss INDIVIDUALLY AND AS A GROUP—ACCURACY INSTRUCTION

No. dots	Ss *Re*	*Mc*	*Ro*	*Wy*	Group median confidence
1	5.00	5.00	5.00	5.00	5.00
2	4.98	5.00	5.00	5.00	5.00
3	4.98	4.98	5.00	5.00	5.00
4	4.92	4.98	5.00	5.00	4.97
5	4.80	4.88	5.00	5.00	4.94
6	4.62	4.06	4.94	4.62	4.62
7	4.00	3.80	4.78	4.20	4.17
8	4.02	3.36	4.69	4.09	3.95
9	4.54	3.45	4.18	3.78	3.79
10	3.04	3.29	4.02	3.54	3.59
11	3.27	3.13	4.05	3.23	3.47
12	2.72	3.18	4.00	3.12	3.32
13	2.92	3.05	3.82	3.00	3.16
14	2.69	3.00	3.73	2.90	3.05
15	2.82	2.94	3.92	2.82	3.08
17	2.70	2.94	3.62	2.73	2.98
19	2.35	3.00	3.62	2.38	2.89
22	2.23	2.98	3.18	2.54	2.82
25	2.23	3.00	2.97	2.46	2.79
28	2.25	2.98	3.14	2.46	2.79
32	2.00	2.92	3.06	2.63	2.74
37	2.03	3.00	2.95	2.79	2.79
42	1.82	2.97	2.62	2.42	2.50
49	1.96	3.03	2.88	2.62	2.68
57	1.30	2.94	2.46	2.68	2.50
66	1.12	3.02	2.54	2.30	2.42
77	1.08	2.94	2.02	2.62	2.22
89	1.02	2.92	2.04	2.15	2.05
103	1.05	2.92	1.97	2.04	2.00
118	1.08	2.94	1.86	2.06	1.98
134	1.12	2.62	1.38	1.75	1.66
152	1.05	2.38	1.29	1.86	1.62
170	1.05	2.45	1.22	1.65	1.47
191	1.00	2.84	1.32	1.76	1.54
210	1.00	2.62	1.11	1.54	1.37

first apparent branch has been rectified. The process in detail was to subtract each median confidence from 5.00 (maximum confidence possible), and to plot the result on a logarithmic scale as a function of presented number. Figs. 20 and 21 are the outcome. Both graphs reveal a discontinuity in slope at or near 6 stimulus-dots. In neither graph do the points below 6 lie on the curve of the points above 6, as reasonably extrapolated. Nor do the points above 6 lie on the line of the points below 6.

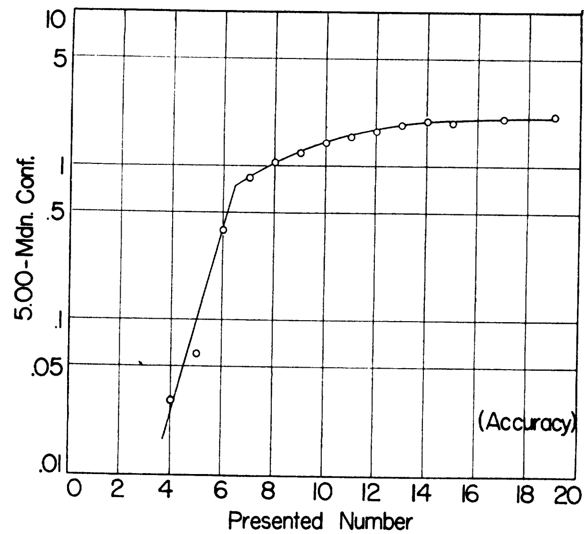

FIG. 20. A FUNCTION OF MEDIAN CONFIDENCE PLOTTED AGAINST NUMBER OF DOTS PRESENTED

S instructed for accuracy; coördinates semi-logarithmic; group results.

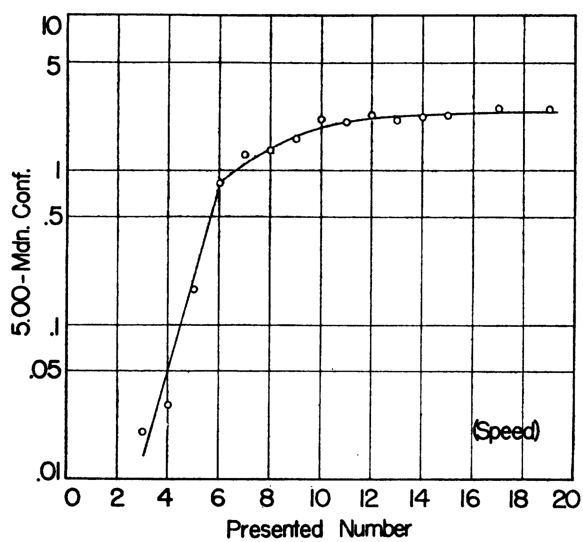

FIG. 21. A FUNCTION OF MEDIAN CONFIDENCE PLOTTED AGAINST NUMBER OF DOTS PRESENTED

S instructed for speed; coördinates semi-logarithmic; group results.

The evidence from confidence, then, confirms the evidence from report time. There are two discriminatory mechanisms, and one gives way to the other at or near 6 stimulus-dots.

TABLE V

MEDIAN CONFIDENCE FOR FIVE Ss INDIVIDUALLY AND AS A GROUP—SPEED INSTRUCTION

No. dots	Ss					Group median confidence
	Sc	*Ju*	*Se*	*Ce*	*Tu*	
1	5.00	5.00	5.00	5.00	5.00	5.00
2	5.00	5.00	5.00	5.00	5.00	5.00
3	5.00	5.00	5.00	5.00	5.00	4.98
4	4.90	5.00	5.00	5.00	5.00	4.97
5	4.80	5.00	4.90	4.20	4.80	4.83
6	4.30	5.00	3.80	2.30	4.10	4.17
7	3.40	4.80	3.70	1.40	3.90	3.72
8	3.70	4.30	3.40	1.30	3.70	3.63
9	3.50	4.10	3.20	1.20	3.40	3.39
10	3.10	4.00	2.50	1.20	2.90	2.85
11	3.20	4.00	2.50	1.30	2.90	2.92
12	2.80	3.90	3.40	1.20	2.30	2.71
13	3.10	3.90	2.80	1.50	2.70	2.88
14	2.90	3.90	3.00	1.20	2.40	2.77
15	2.90	3.90	2.80	1.10	2.30	2.71
17	2.60	3.80	2.40	1.10	2.30	2.46
19	2.90	3.60	2.20	1.10	2.40	2.50
22	2.40	3.80	2.40	1.10	2.10	2.32
25	2.50	3.60	2.90	1.00	2.10	2.42
28	2.70	3.30	2.20	1.10	2.00	2.28
32	2.30	3.20	2.50	1.00	1.80	2.14
37	1.90	3.40	2.00	1.00	1.70	1.91
42	1.80	3.10	2.10	1.00	1.30	1.78
49	1.80	3.10	2.40	1.00	1.40	1.89
57	1.70	2.80	2.20	1.00	1.30	1.69
66	1.40	2.90	2.30	1.00	1.40	1.74
77	1.40	2.80	1.90	1.00	1.50	1.53
89	1.30	2.30	1.80	1.00	1.10	1.41
103	1.10	2.10	1.80	1.00	1.50	1.40
118	1.20	2.10	2.00	1.00	1.20	1.37
134	1.00	1.80	1.70	1.00	1.10	1.21
152	1.10	1.50	1.30	1.00	1.10	1.17
170	1.00	1.40	1.20	1.00	1.10	1.12
191	1.10	1.10	1.10	1.00	1.10	1.05
210	1.00	1.20	1.30	1.00	1.20	1.08

DISCUSSION

Estimating, subitizing, and counting. We can now describe several quite different ways in which people discriminate the number of objects. To do this most clearly, let us consider only the visual discrimination of dots of uniform size and shape. What *S* in an experiment does with these dots depends on certain stimulus-variables and instructional variables. The stimulus-variables are as follows.

(1) Spacing. The dots may be shown in the same place (at different times) or in discriminably different places (at different times or at the same time). The spatial arrangement of the dots is a class of variables that will not be considered here.

(2) Rate of stimulation. If the dots are presented in the same place at a high rate, they fuse. If they are presented at different places at a very high rate, they are seen as simultaneous.

(3) Duration. The exposure-time of each dot-stimulus.

(4) Number. The number of dots presented.

(5) Brightness. The brightness of each dot-stimulus. It will be assumed here that the brightness of each dot is far above threshold.

The experiments described in this paper used one common combination of these stimulus-variables, *i.e.* (a) dots in different places, in random arrangements; (b) simultaneous presentation; (c) a short duration of presentation; (d) a varying number of dots. The *S*s were instructed to report the number of dots. The basic experimental operations, then, included instructing the *S*s and presenting the stimulus-material in the ways just specified.

When we do this, however, there appear in the results certain characteristic discontinuities. To give a clear description, we shall need different names for what happens above 6 dots and what happens at 6 and below. The term *estimating* is commonly applied to a report of numerousness, no matter what the stimulus-number may be. We propose that it be reserved for the discrimination of stimulus-numbers greater than 6. If it be, a new term is needed for the discrimination of stimulus-numbers of 6 and below. We wish to avoid terms now in use, having other meanings, and terms with the misleading connotations of estimating, counting, or grasping by intuition. The term proposed is *subitize.*[9]

The results already presented in this paper show some characteristic differences between subitizing and estimating. Subitizing is, on the average, more accurate and more rapid than estimating, and it is done with more confidence. These differences, although real, are still statistical. The discontinuities that appear in our results are discontinuities of slope. This means that there is at least one point at which subitizing and estimating are equally rapid, accurate, and confident. This point is where the two branches of the function meet.

[9] The derivation of this term is suggestive and not misleading: the classical Latin adjective *subitus,* meaning *sudden,* and the medieval Latin verb *subitare,* meaning *to arrive suddenly.* The ending *ize* is Greek, but its use is not without precedent in medieval Latin. *Subitate* would be all Latin, but it can be taken too easily for a malapropism. We are indebted to Dr. Cornelia C. Coulter, the Department of Classical Languages and Literatures, Mount Holyoke College, for suggesting this term.

There is another difference between subitizing and estimating that is not shown in the results. It concerns the effect of anchoring. If the *S*s are shown an auxiliary field of dots and are told that this field contains 49 dots, for example, their *estimations* will be affected in accuracy, speed, and confidence. We may call this auxiliary field an anchoring stimulus.[10] There is now evidence that an anchoring stimulus of 6 dots does not act similarly upon the *subitizing* of 6 dots or less.[11] We believe that most anchoring stimuli will affect estimating and that no anchoring stimuli will affect subitizing.

We intend that *subitizing* and *estimating* be operationally defined, so that any person (otherwise able to do so) can carry out the operations and produce either subitizing or estimating as he chooses. As applied to the visual discrimination of numerousness in a briefly and simultaneously presented, randomly arranged field of dots: *estimating* is what occurs when the stimulus-number is greater than 6; *subitizing* is what occurs when the stimulus-number is less than 6.[12]

Estimating and *subitizing* are similar in meaning because the following concrete operations are common to both: a brief and simultaneous presentation of a randomly arranged field of dots, and an instruction to report the number of dots. The two terms differ in meaning, because to produce the process of *estimating* we present more than 6 dots; to produce *subitizing* we present 6 or less. This difference is surely an identifiable difference in operations. It might be a trivial difference, but the results tell us that it is not. If no discontinuities had appeared in the results, no distinction between subitizing and estimating could have been drawn. In general, we must look to the data to tell us whether a class of operations should be sub-divided or taken as a whole.

Because the data have led us to divide the defining operations into two sub-classes, we must be prepared to revise the operational definition of numerousness given at the beginning of this paper. That definition was: "Numerousness is that property of a group of objects which we can discriminate, without counting, under instruction to judge how many objects the group contains." Strictly speaking, however, there will be two kinds of numerousness. One kind will be associated with the process of subitizing

[10] A bibliography on anchoring may be found in H. R. McGarvey, Anchoring effects in the absolute judgment of verbal materials, *Arch. Psychol.*, 39, 1943 (no. 281), 1-80.

[11] E. C. Reed and K. Safford, The effect of anchoring on the visual discrimination of number. (In press.)

[12] See below, however, the separate discussion of the discrimination of 6 dots. More recent experiments suggest that there are operations other than those stated here which favor the *estimation* rather than the *subitizing* of 6 or fewer dots.

and the presentation of stimulus-numbers of 6 or less. The other kind will be associated with estimating and the presentation of stimulus-numbers greater than 6. Whether the two kinds of numerousness are different phenomenologically is, of course, an entirely different question.

Counting is a process that is quite different from either subitizing or estimating. It occurs (under instruction) in any one of the following three situations: when the stimulus-dots are presented in one place at a sufficiently low rate; when they are simultaneously presented in different places and are kept there; when they appear successively, in different places, and at a low rate.

Counting, when actually carried out by a person, enables him to determine what Stevens calls the property of *numerosity,* equivalent in our discussion to the number of stimulus-dots. "In order to specify the numerosity of any group we have merely to pair successively each object in the group with a numeral from the numeral-series, beginning of course with the first numeral in the series. This operation we call *counting.*"[13] In this case the person who is counting gives one verbal response for each single object in a group of N objects. In these words Stevens describes counting by *ones.* In subitizing or estimating, on the other hand, the person gives only one response for the entire group of N objects. Counting by twos, threes, or larger sub-groups furnishes an interesting combination of subitizing and counting. For each sub-group of *n* objects, the person counting gives one verbal response until the whole group of N has been exhausted.

Counting-behavior raises some interesting questions. Is it more efficient, for example, to count a random field by ones, twos, threes, or still larger sub-groups? No matter how it is performed, counting will not always be accurate. What degree of accuracy is to be expected, and on what conditions does it depend?

Simultaneous presentation is only one general condition in which counting, subitizing, or estimating can occur. The successive presentation of stimuli in one place has been explored by Taubman[14] and Jerome,[15] with interesting results. Successive presentation in different places and at different rates involves some technical difficulties, but it still should be attempted nevertheless.

Parsimony. After reading our description of subitizing and estimating, someone might very naturally object to it, invoking the principle of parsimony. Admittedly, it would be neater to account for the discrimination of numerousness with one hypothetized process rather than two. There are several answers to the objection, beginning with the most obvious one that the data, as analyzed, indicate the presence of two processes.

---

[13] *Op. cit.,* 9, 1938, 95.

[14] R. E. Taubman, *Studies in Judged Number,* Doctoral dissertation, Columbia University. (To be published.)

[15] E. A. Jerome, An unpublished study from the Columbia University Psychology Laboratory.

The principle of parsimony is an uncertain guide in any actual scientific case. When a theory has to be built in the light of the facts, those facts may call for a less or a more complicated theory. The facts of numerousness present certain complications. We shall eventually need to explain in detail the accuracy, time, and confidence functions obtained in research on numerousness. It now appears much more feasible to divide the problem of explanation into two parts and to produce a different set of assumptions for each of the two parts. In fact, anyone who wants to find one set of assumptions, rather than two, is welcome to the job.

Next we note that in many types of discrimination the stimuli vary through a wide range, covering several log cycles. To deal with this wide range, more than one discriminatory mechanism may be required. At the very least, we should not be surprised when evidence of more than one mechanism turns up. The classical example of more than one mechanism is, of course, the rod-cone duplexity encountered in vision.

Anatomical correlation. The duplexity theory of vision presents an example of a neat relation between functional and anatomical findings. There is no such relation in numerousness; we do not know of separate organs or pathways for *subitizing* on the one hand and *estimating* on the other. Should we therefore wait until an anatomical correlation appears, before postulating separate discriminatory mechanisms?

In saying *no,* we are thinking about psychophysics and psychophysiology in general, and about two contrasted examples in particular. In the first example, we return to the duplicity theory. Dozens of functional facts in vision lead directly to a distinction between scotopic and photopic vision. The evidence is so overwhelming that the distinction would stand even if nobody had ever identified the retinal rods and cones as separate receptors.

One well-known piece of evidence for duplexity is a discontinuity of slope[16]—the same kind of evidence that has led us to speak of two discriminatory mechanisms for visual numerousness.

Note, however, that there is no corresponding duplexity theory of audition. This is because the functional findings concerning pitch and loudness discrimination do not suggest duplexity. It is not for lack of possible anatomical correlates. There are, in fact, two sets of receptor cells, the external and internal hair-cells in the organ of Corti, that might by some stretch of the imagination serve as anatomical correlates for a duplexity theory.

In these examples, and in general, the functional evidence is decisive. Psychophysics and psychophysiology have their own analytic methods and modes of inference, and cannot wait for some other branch of science to catch up with them. From another point of view, the man who seeks an anatomical correlate for a functional fact might like to know what sort of thing he is looking for. Sometimes only the most sensitive and thoroughgoing use of functional science can tell him.

Individual differences. Taves' *S*s, and our own, produced functions that were discontinuous at very nearly the same point, 6 dots. This finding will seem strange to most psychologists, and strange even to those who know at first hand the precision that modern psychophysics can achieve. They all would expect to find sizeable

[16] S. Hecht, The nature of the photoreceptor process, *Handbook of General Experimental Psychology* (C. Murchison, Ed.), 1934, 727, Fig. 11.

individual differences in any psychological performance. In the discrimination of number they would expect to find a distribution of ability positively correlated with other abilities. Being good empiricists, they would regard the discrimination of number as having been learned. Presumably it has been learned by some people better than by others. In addition, some cultural environments have favored the learning, and others have not.

These views suggest some fairly specific questions, and we too would like to know the answers. How does the discrimination of number develop in children? What are the effects of practice with and without explicit differential reinforcement (telling *S* the correct answer)? Will *S*s of relatively low intelligence give about the same results as our highly selected college students?

Surely the variables of age, practice, and level of ability will produce effects of some kind, but they need not necessarily alter the point at which our functions are discontinuous in slope. Explicit differential reinforcement, for example, will probably increase the accuracy, speed, and confidence of both subitizing and estimating. It is still possible, however, that the point of discontinuity will not move up to 8 or 9 dots, but will stay near 6. The experiments necessary to test this hypothesis have not been done.

From one systematic point of view, subitizing, estimating, and counting are all examples of discriminated verbal operant behavior. This behavior is 'learned' in the sense that it has changed in strength as a result of reinforcement or non-reinforcement. It is still subject to control by specific discriminatory mechanisms, however, and it is this fact that has interested us most.

Discrimination of six dots. The statement that our functions show discontinuity of slope "at or near 6 dots" has left open the question how 6 dots are dealt with. Are they subitized or estimated?

To answer this question, we plotted the data of individual *S*s against several different sets of coördinates, each of which approximately rectified the first branch of the function. We then looked to see whether the plotted point corresponding to 6 dots fell on the first branch of the function (subitizing), or the second branch (estimating).

This inspection led to the following conclusions. (1) For these *S*s, the problem can be limited to 6 dots, because (under the conditions of our experiment) 5 dots were subitized and 7 dots were estimated, by all of the *S*s. (2) Six dots can be subitized and often are. Taves' *S*s subitized 6 dots regularly. Two of our *S*s, working under instructions for accuracy, subitized 6 dots. This is why, in the definition of subitizing given above, 6 has been included in the range of stimulus-numbers discriminated by subitizing. (3) Working under an instruction for speed, however, most of our *S*s *estimated* 6 dots. Their time-functions, as rectified and extrapolated, show that subitizing 6 dots would have taken more time, on the average, than estimating them. Perhaps that is why they were estimated.

To follow this inquiry would require more *S*s, and somewhat more reliable measures. The same *S*s should receive, in turn, the instructions for accuracy and for speed. We should test the effects of certain stimulus-variables, like brightness, duration of exposure, and the average spatial separation between dots. But the discrimination of 6 dots soon becomes a minor topic. The important thing is that our functions are discontinuous in slope, and that the discontinuity occurs at some point near 6 dots.

Conclusions

Our *Ss* received brief, simultaneous, visual presentations of randomly arranged fields of dots. They were instructed to report the number of these dots. Some *Ss* were further instructed for maximum *accuracy* of report, and others for maximum *speed*. The actual number of stimulus-dots varied between 1 and 210.

The results of the experiment are in terms of the percentage error and the variability of the reported number, the time between the stimulus and the report, and the confidence with which the report was given. Analysis of the results, including analysis by graphical rectification, leads us to the following conclusions:

(1) The functional relations between time and stimulus-number, and confidence and stimulus-number are discontinuous in slope. Taves has already drawn the inference that there are two mechanisms for the discrimination of visual numerousness. That inference is fully supported by our results.

(2) The functions for both time and confidence are discontinuous in slope at nearly the same point. This point is close to 6 stimulus-dots.

(3) The instructions (a) for speed and for accuracy yield functions that are similar in shape; (b) for speed produce more speed; (c) for accuracy produce slightly more accuracy and less variability.

(4) The time between stimulation and report increases regularly from 1 to 5 or 6 stimulus-dots. From parallel evidence, Saltzman and Garner have argued that if the term *span of apprehension* is defined as the immediate cognition of number, there is no such thing. We agree.

(5) On operational grounds, useful distinctions can be drawn between *subitizing* (a new term), *estimating*, and *counting*.

(6) On the average, *subitizing* is a considerably more accurate, more rapid, and more confident process than *estimating*.