distributions_math

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\begin{aligned}
Y_i(t+6) &= \beta_{0i} + \beta_{1i}Y_i(t) + \beta_{2i}\Delta_{i1}(t) + \beta_{3i}\Delta_{i2}(t) + \beta_{4i}\Delta_{i3}(t) + \varepsilon_i(t) \\
&= \eta_i(t)
\end{aligned}
$$

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\begin{aligned}
Y &= (Y_1, Y_2, ..., Y_n) \\
\text{Let } \eta &= (\eta_1, \eta_2, ..., \eta_n), \text{ where } \eta_i = X_i\beta_i^T \\
Y_i &\sim MVN(\eta_i, \sigma^2 I_{n_i}) \\
f(Y_i|B, \mu, \Sigma, \sigma^2) &= (2\pi)^{\frac{n_i}{2}} \left|\sigma^2 I_{n_i}\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left(Y_i - X_i\beta_i^T\right)(\sigma^2 I_{n_i})^{-1}(Y_i - X_i\beta_i^T)\right) \\
L(Y|B, \beta, \Sigma, \sigma^2) &= \prod_{i=1}^{N} f(Y_i|B, \beta, \Sigma, \sigma^2) \\
&= \prod_{i=1}^{N} (2\pi)^{-\frac{n_i}{2}} \left|\sigma^2 I_{n_i}\right|^{-\frac{1}{2}} \exp(-\frac{1}{2}(Y_i - X_i\beta_i^T)^T(\sigma^2 I_{n_i})^{-1}(Y_i - X_i\beta_i^T)^T) \\
B &= (\beta_1^T, \beta_2^T, ..., \beta_n^T)^T \\
\beta_i &\sim MVN(\mu, \Sigma) \\
f(B|\beta, \Sigma) &= \prod_{i=1}^{n} (2\pi)^{-\frac{5}{2}} \left|\sigma\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\beta_i - \mu)^T\Sigma^{-1}(\beta_i - \mu)\right) \\
f(\mu) &\propto 1 \\
f(\Sigma) &\propto |\Sigma|^{-(d+1)} \exp\left(-\frac{1}{2}\Sigma^{-1}\right), \text{ where } d = 5 \\
f(\sigma^2) &\propto \frac{1}{\sigma^2}
\end{aligned}
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\begin{aligned}
&\implies f(B, \mu, \Sigma, \sigma^2|Y) \propto L(Y|B, \beta, \Sigma, \sigma^2)f(B, \beta, \Sigma, \sigma^2) \\
&= L(Y|B, \beta, \Sigma, \sigma^2)f(B|\beta, \Sigma)f(\beta)f(\Sigma)f(\sigma^2) \\
&= \prod_{i=1}^{N} (2\pi)^{\frac{n_i}{2}} \left|\sigma^2 I_{n_i}\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(Y_i - X_i\beta_i)^T(\sigma^2 I_{n_i})^{-1}(Y_i - X_i\beta_i^T)\right) |\Sigma|^{-\frac{n}{2}} \exp\left(-\frac{1}{2}(\beta_i - \mu)^T\Sigma^{-1}(\beta_i - \mu)\right) |\Sigma|^{-(d+1)} \exp(- \\
&= \prod_{i=1}^{N} \left|\sigma^2 I_{n_i}\right|^{-\frac{1}{2}} |\Sigma|^{-(d+\frac{n}{2}+1)} \frac{1}{\sigma^2} \exp\left[-\frac{1}{2}\left[(Y_i - X_i\beta_i^T)^T(\sigma^2 I_{n_i})^{-1}(Y_i - X_i\beta_i^T) + (\beta_i - \mu)^T\Sigma^{-1}(\beta_i - \mu)\right]\right] \exp(-\frac{1}{2}\Sigma^{-1})
\end{aligned}
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