P8160 - Bayesian Modeling of Hurricane Trajectories

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MCMC for Hierarchical Bayesian Model:Data Partition

Data Partition

- For each hurricane, 80% of records were randomly assigned to the training set and the remaining 20% were assigned to testing set.
- Hurricanes with less than 7 records were removed: at least 5 observations are included in the training set and 1 observation is included in the testing set.

MCMC for Hierarchical Bayesian Model: Method

Distribution Derivation:

- Gibbs sampler is used to generate random variables from given distribution:
- Let $\Theta = (\boldsymbol{B}^T, \beta^T, \boldsymbol{\sigma}^2, \boldsymbol{\Sigma})$, the posterior distribution can be written as:

$$\begin{aligned}P(\Theta \mid Y) &\propto f(Y|\Theta)P(\Theta) \\ &= f(Y \mid \boldsymbol{B}, \beta, \sigma^2, \boldsymbol{\Sigma}) f(\boldsymbol{B} \mid \beta, \boldsymbol{\Sigma}) P(\beta) P(\sigma^2) P(\boldsymbol{\Sigma}^{-1})\end{aligned}$$

MCMC for Hierarchical Bayesian Model: Method

Conditional Distribution of Θ:

- $Y_i \sim MVN(X_i\beta_i^T, \sigma^2 I_{n_i})$, where n_i is the number of observation of the i^{th} hurricane.

$$f(Y \mid \boldsymbol{B}, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\Sigma}) = \prod_{i=1}^{N} f(Y_i|B, \beta, \Sigma, \sigma^2)$$
$$= \prod_{i=1}^{N} (2\pi)^{-\frac{n_i}{2}} \left|\sigma^2 I_{n_i}\right|^{-\frac{1}{2}} \exp(-\frac{1}{2}(Y_i - X_i\beta_i^T)^T(\sigma^2 I_{n_i})^{-1}$$

- $\beta_i \sim \mathrm{MVN}(\mu, \Sigma)$:

$$f(B|\beta, \Sigma) = \prod_{i=1}^{N} (2\pi)^{-\frac{5}{2}} |\sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\beta_i - \mu)^T \Sigma^{-1}(\beta_i - \mu)\right)$$

MCMC for Hierarchical Bayesian Model: Method

Conditional Distribution of Θ:

- the posterior distribution of Θ, where $A = \Sigma^{-1}$:

$$P(\Theta \mid Y) \propto$$

$$\sigma^{-\sum_{i=1}^{N} n_i - 2} |A|^{d + \frac{N}{2} + 1}$$

$$\exp\left[-\frac{1}{2} \sum_{i=1}^{N} \left[(Y_i - X_i \beta_i^T)^T (\sigma^2 I_{n_i})^{-1} (Y_i - X_i \beta_i^T) + (\beta_i - \mu) \right. \right.$$

$$\left. -\frac{1}{2} tr(A) \right]$$

MCMC for Hierarchical Bayesian Model: Method

Conditional Distribution of each parameter:

- $\beta_i \sim MVN(V^{-1}M, V^{-1})$ where, $V = \Sigma^{-1} + X_i^T \sigma^{-2} I_{ni} X_i$, $R = Y_i^T \sigma^{-2} I_{ni} Y_i + \mu^T \Sigma^{-1} \mu$, $M = Y_i^T \sigma^{-2} I_{ni} X_i + \mu^T \Sigma^{-1}$
- $\mu \sim MVN(V^{-1}M, V^{-1})$, $V = N\Sigma^{-1}$, $R = \sum_{i=1}^{N} \beta_i^T \Sigma^{-1} \beta_i$, $M = \sum_{i=1}^{N} \Sigma^{-1} \beta_i$
- $W \sim$ inverse Gamma$(\frac{\sum_{i=1}^{N} n_i}{2}, \frac{1}{2} \sum_{i=1}^{N} \sum_{t=1}^{n_i} (y_{it} - x_{it} \beta_i^T)^2)$, $W = \sigma^2$
- $A \sim$ Wishart $\left(3d + N + 3,\ (I + \sum_{i=1}^{N} (\beta_i - \mu)(\beta_i - \mu)^T)^{-1}\right)$, $A = \Sigma^{-1}$