# formula for posterior distribution

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## Posterior dist of mu

$$
\begin{aligned}
f(\mu|B,\Sigma,\sigma^2,Y) &\propto exp(-\frac{1}{2}\sum_{i=1}^{N}(\beta_i-\mu)^T\Sigma^{-1}(\beta i-\mu)) \\
&\propto exp(\frac{1}{2}(\sum_{i=1}^{N}\beta_i^T\Sigma^{-1}\beta_i+\mu^T N\Sigma^{-1}\mu-2\sum_{i=1}^{N}\beta_i^T\Sigma^{-1}\mu)) \\
&\propto exp(R+\beta V\beta-2M\beta) \\
&\propto exp((\beta-V^{-1}M)^T V(\beta-V^{-1}M)) \\
&\implies \mu\sim MVN(V^{-1}M,V^{-1}) \\
\text{where: } V &= N\Sigma^{-1} \\
R &= \sum_{i=1}^{N}\beta_i^T\Sigma^{-1}\beta_i \\
M &= \sum_{i=1}^{N}\Sigma^{-1}\beta_i
\end{aligned}
$$

## Posterior dist of beta

$$
\begin{aligned}
f(\beta_i|\mu,\Sigma,\sigma^2,Y) &\propto exp(-\frac{1}{2}[(Y_i-X_i\beta_i^T)^T(\sigma^2 I_{n_i})^{-1}(Y_i-X_i\beta_i^T)+(\beta_i-\mu)^T\Sigma^{-1}(\beta i-\mu)]) \\
&= Y_i^T\sigma^{-2}I_{n_i}T_i+\mu^T\Sigma^{-1}\mu+\beta_i^T(\Sigma^{-1}+X_i^T\sigma^{-2}I_{n_i}X_i)\beta_i-2(Y_i^T\sigma^{-2}I_{n_i}X_i+\mu^T\Sigma^{-1})\beta_i \\
&= R+\beta_i^T V\beta_i-2M\beta_i \\
&\propto (\beta_i-V^{-1}M)^T V(\beta_i-V^{-1}M) \\
\implies \beta_i &\sim MVN(V^{-1}M,V^{-1}) \\
\text{where: } V &= \Sigma^{-1}+X_i^T\sigma^{-2}I_{n_i}X_i \\
R &= Y_i^T\sigma^{-2}I_{n_i}T_i+\mu^T\Sigma^{-1}\mu \\
M &= Y_i^T\sigma^{-2}I_{n_i}X_i+\mu^T\Sigma^{-1}
\end{aligned}
$$