P8160 - Bayesian Modeling of Hurricane Trajectories

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Hurricane Ida

CAPITAL WEATHER GANG

Ida's impact from the Gulf Coast to Northeast — by the numbers

The storm caused more than 40 deaths in the Northeast, brought tornadoes in six states and unleashed 172 mph winds in Louisiana

By Ian Livingston

September 3, 2021 at 11:17 a.m. EDT

From: Livingston, I., *The Washington Post*, 2021

Saffir-Simpson Wind Scale

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Saffir-Simpson Hurricane Wind Scale

Climatology | Names | Wind Scale | Extremes | Models | Breakpoints

The Saffir-Simpson Hurricane Wind Scale is a 1 to 5 rating based only on a hurricane's maximum sustained wind speed. **This scale does not take into account other potentially deadly hazards such as storm surge, rainfall flooding, and tornadoes.**

The Saffir-Simpson Hurricane Wind Scale estimates potential property damage. While all hurricanes produce life-threatening winds, hurricanes rated Category 3 and higher are known as major hurricanes*. Major hurricanes can cause devastating to catastrophic wind damage and significant loss of life simply due to the strength of their winds. Hurricanes of all categories can produce deadly storm surge, rain-induced floods, and tornadoes. These hazards require people to take protective action, including evacuating from areas vulnerable to storm surge.

From: *NHC NOAA*

Proposed Hierarchical Bayesian Model

The following hierarchical Bayesian model was proposed to predict the wind speed of the i^{th} hurricane at time $t+6$:

$$Y_i(t+6) = \beta_{0,i} + \beta_{1,i} Y_i(t) + \beta_{2,i}\Delta_{i,1}(t) + \beta_{3,i}\Delta_{i,2}(t) + \beta_{4,i}\Delta_{i,3}(t) + \varepsilon_i(t),$$

where $Y_i(t)$ is the wind speed at time t, $\Delta_{i,1}(t)$, $\Delta_{i,2}(t)$, $\Delta_{i,3}(t)$ are the changes in latitude, longitude, and wind speed between times t and $t-6$, $\varepsilon_i(t)$ is the random error associated with each $Y_i(t+6)$

We want to estimate the random coefficients, $\beta_i = (\beta_{1,i}, \beta_{2,i}, \beta_{3,i}.\beta_{4,i})$, for each hurricane.

Assumed Prior Distributions

The prior distributions for each of these parameters are assumed to be as follows:

$$\epsilon_i(t) \sim N(0, \sigma^2), \text{ which are independent across } t$$

$$P\left(\sigma^2\right) \propto \frac{1}{\sigma^2}$$

$$P(\boldsymbol{\mu}) \propto 1$$

$$P\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \exp\left(-\frac{1}{2}\Sigma^{-1}\right), \text{ where } d \text{ is the dimension of } \beta_i$$

$$\beta_i \sim N(\mu, \Sigma)$$

Goals

1. Construct an MCMC algorithm from which we can sample from a posterior distribution to estimate $\Theta = (\mathbf{B}, \mu, \sigma^{\mathbf{2}}, \circ)$.
2. Conduct analysis using estimated parameters to understand their properties.
    a. Seasonal changes in any of the coefficients
    b. Predictive influence of these coefficients on forecasting hurricane impact.

Data

ID: ID of hurricanes

Year: In which the hurricane occurred

Month: In which the hurricane occurred

Nature: Nature of the hurricane

- ET: Extra Tropical
- DS: Disturbance
- NR: Not Rated
- SS: Sub Tropical
- TS: Tropical Storm

Time: dates and time of the record

Latitude and **Longitude**: The location of a hurricane check point

Wind.kt: Maximum wind speed (in Knot) at each check point

MCMC for Hierarchical Bayesian Model:Data Partition

Data Partition

- For each hurricane, 80% of records were randomly assigned to the training set and the remaining 20% were assigned to testing set.
- Hurricanes with less than 7 records were removed: at least 5 observations are included in the training set and 1 observation is included in the testing set.

MCMC for Hierarchical Bayesian Model: Method

Distribution Derivation:

- Gibbs sampler is used to generate random variables from given distribution:
- Let $\Theta = (\boldsymbol{B}^T, \beta^T, \boldsymbol{\sigma}^2, \boldsymbol{\Sigma})$, the posterior distribution can be written as:

$$
\begin{aligned}
P(\Theta \mid Y) &\propto f(Y|\Theta)P(\Theta) \\
&= f(Y \mid \boldsymbol{B}, \beta, \sigma^2, \boldsymbol{\Sigma}) f(\boldsymbol{B} \mid \beta, \boldsymbol{\Sigma}) P(\beta) P(\sigma^2) P(\boldsymbol{\Sigma}^{-1})
\end{aligned}
$$

MCMC for Hierarchical Bayesian Model: Method

Posterior Distribution of Θ:

- $Y_i \sim MVN(X_i\beta_i^T, \sigma^2 I_{n_i})$, where n_i is the number of observation of the i^{th} hurricane.

$$f(Y \mid \boldsymbol{B}, \boldsymbol{\beta}, \sigma^2, \boldsymbol{\Sigma}) = \prod_{i=1}^{N} f(Y_i|B, \beta, \Sigma, \sigma^2)$$
$$= \prod_{i=1}^{N} (2\pi)^{-\frac{n_i}{2}} \left|\sigma^2 I_{n_i}\right|^{-\frac{1}{2}} \exp(-\frac{1}{2}(Y_i - X_i\beta_i^T)^T(\sigma^2 I_{n_i})^{-1}$$

- $\beta_i \sim \mathrm{MVN}(\mu, \Sigma)$:

$$f(B|\beta, \Sigma) = \prod_{i=1}^{N} (2\pi)^{-\frac{5}{2}} \left|\sigma\right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\beta_i - \mu)^T \Sigma^{-1}(\beta_i - \mu)\right)$$

MCMC for Hierarchical Bayesian Model: Method

Posterior Distribution of Θ:

- the posterior distribution of Θ, where $A = \Sigma^{-1}$:

$$P(\Theta \mid Y) \propto$$

$$\sigma^{-\sum_{i=1}^{N} n_i - 2} |A|^{d + \frac{N}{2} + 1}$$

$$\exp\left[-\frac{1}{2}\sum_{i=1}^{N}\left[(Y_i - X_i\beta_i^T)^T(\sigma^2 I_{n_i})^{-1}(Y_i - X_i\beta_i^T) + (\beta_i - \mu)\right.\right.$$

$$\left. -\frac{1}{2}tr(A)\right]$$

MCMC for Hierarchical Bayesian Model: Method

Conditional Distribution of each parameter:

- $\beta_i \sim MVN(V^{-1}M, V^{-1})$, where$V = \Sigma^{-1} + X_i^T \sigma^{-2} I_{ni} X_i$, $M = Y_i^T \sigma^{-2} I_{ni} X_i + \mu^T \Sigma^{-1}$
- $\mu \sim MVN(V^{-1}M, V^{-1})$, where $V = N\Sigma^{-1}$, $M = \sum_{i=1}^{N} \Sigma^{-1} \beta_i$
- $W \sim$ inverse Gamma$(\frac{\sum_{i=1}^{N} n_i}{2}, \frac{1}{2} \sum_{i=1}^{N} \sum_{t=1}^{n_i} (y_{it} - x_{it} \beta_i^T)^2)$, $W = \sigma^2$
- $A \sim$ Wishart $\left(3d + N + 3,\ (I + \sum_{i=1}^{N} (\beta_i - \mu)(\beta_i - \mu)^T)^{-1}\right)$, $A = \Sigma^{-1}$

Gibbs Sampling

Initialize $\Theta_0 = (\mathbf{B}_0, \boldsymbol{\mu}_0, \sigma_0^2, \Sigma_0)$

for iteration i = 1,2,... **do**

Sample $\mathbf{B}_i \sim \pi(\mathbf{B}|\boldsymbol{\mu}_{i-1}, \sigma_{i-1}^2, \Sigma_{i-1}, \mathbf{Y})$

Sample $\boldsymbol{\mu}_i \sim \pi(\boldsymbol{\mu}|\mathbf{B}_i, \sigma_{i-1}^2, \Sigma_{i-1}, \mathbf{Y})$

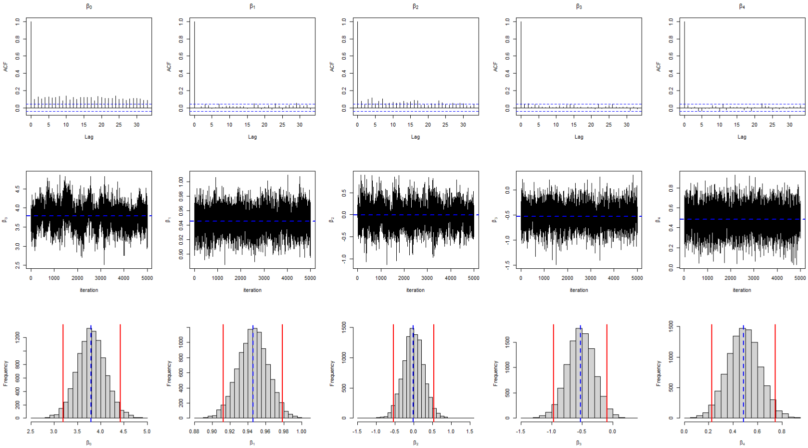
Sample $\sigma_i^2 \sim \pi(\sigma^2|\mathbf{B}_i, \boldsymbol{\mu}_i, \Sigma_{i-1}, \mathbf{Y})$

Sample $\Sigma_i^{-1} \sim \pi(\Sigma^{-1}|\mathbf{B}_i, \boldsymbol{\mu}_i, \sigma_i^2, \mathbf{Y})$

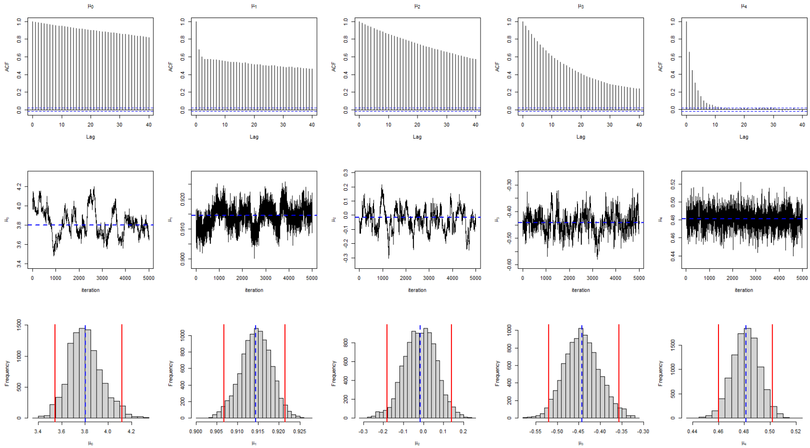
take inverse

end for

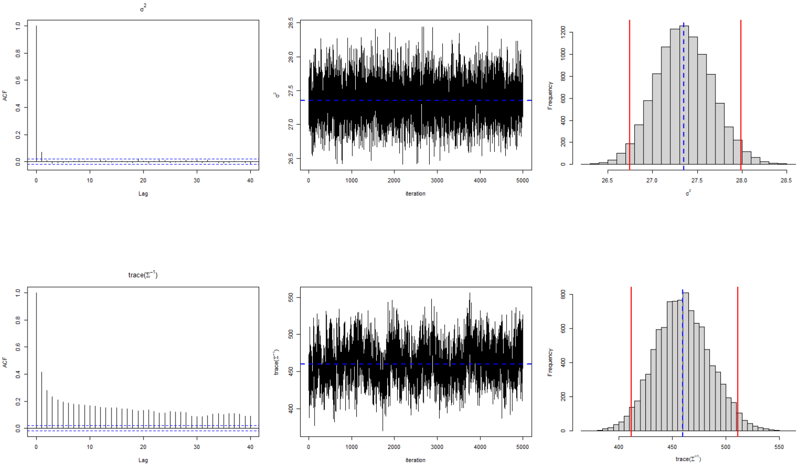
Convergence Plots and Distributions of **B**



Convergence Plots and Distributions of $\boldsymbol{\mu}$



Convergence Plots and Distributions of σ^2 and Σ^{-1}



Estimates for $\mathbf{B}$ and $\boldsymbol{\mu}$

Hurricane	β_0	β_1	β_2	β_3	β_4
ABLE.1950	3.79[3.19,4.42]	0.95[0.92,0.97]	-0.12[-0.65,0.4]	-0.53[-0.98,-0.1]	0.54[0.32,0.77]
BAKER.1950	3.79[3.2,4.4]	0.92[0.89,0.95]	-0.1[-0.65,0.44]	-0.39[-0.84,0.07]	0.68[0.49,0.87]
CHARLIE.1950	3.78[3.17,4.38]	0.94[0.92,0.97]	-0.01[-0.53,0.51]	-0.42[-0.85,0.04]	0.45[0.18,0.71]
DOG.1950	3.81[3.22,4.43]	0.96[0.94,0.97]	-0.06[-0.58,0.45]	-0.39[-0.79,-0.01]	0.53[0.31,0.76]
EASY.1950	3.8[3.19,4.43]	0.92[0.88,0.95]	-0.01[-0.53,0.53]	-0.43[-0.89,0.02]	0.54[0.33,0.74]
FOX.1950	3.79[3.16,4.4]	0.95[0.93,0.98]	-0.1[-0.66,0.42]	-0.56[-1.02,-0.11]	0.56[0.31,0.81]
GEORGE.1950	3.81[3.21,4.4]	0.95[0.93,0.98]	-0.03[-0.56,0.49]	-0.38[-0.78,0.02]	0.46[0.19,0.73]
HOW.1950	3.82[3.22,4.43]	0.89[0.82,0.96]	-0.02[-0.55,0.52]	-0.43[-0.89,0.03]	0.47[0.17,0.79]
ITEM.1950	3.83[3.23,4.45]	0.92[0.88,0.97]	-0.05[-0.57,0.49]	-0.45[-0.91,0.01]	0.5[0.3,0.71]
JIG.1950	3.83[3.23,4.45]	0.95[0.92,0.98]	-0.02[-0.55,0.5]	-0.47[-0.92,-0.03]	0.48[0.22,0.75]

μ_0	μ_1	μ_2	μ_3	μ_4
3.82[3.59,4.12]	0.91[0.91,0.92]	-0.03[-0.2,0.12]	-0.44[-0.52,-0.36]	0.48[0.46,0.5]

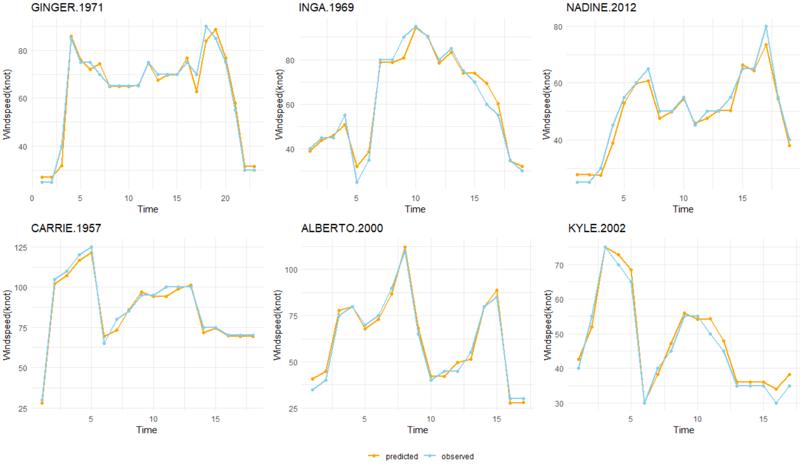
Estimates for σ^2 and Σ^{-1}

$\sigma^2 = 27.36[26.76, 27.98]$

Σ^{-1} :

17.0148877	7.740045	-0.1368787	0.8033023	1.8493396
7.7400455	360.894502	5.5883769	3.7666486	-9.9431197
-0.1368787	5.588377	19.0227527	1.0456746	0.5528947
0.8033023	3.766649	1.0456746	22.7116754	-3.4877498
1.8493396	-9.943120	0.5528947	-3.4877498	40.9917065

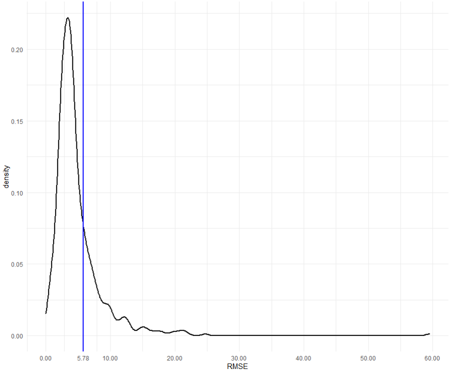
Prediction Performance



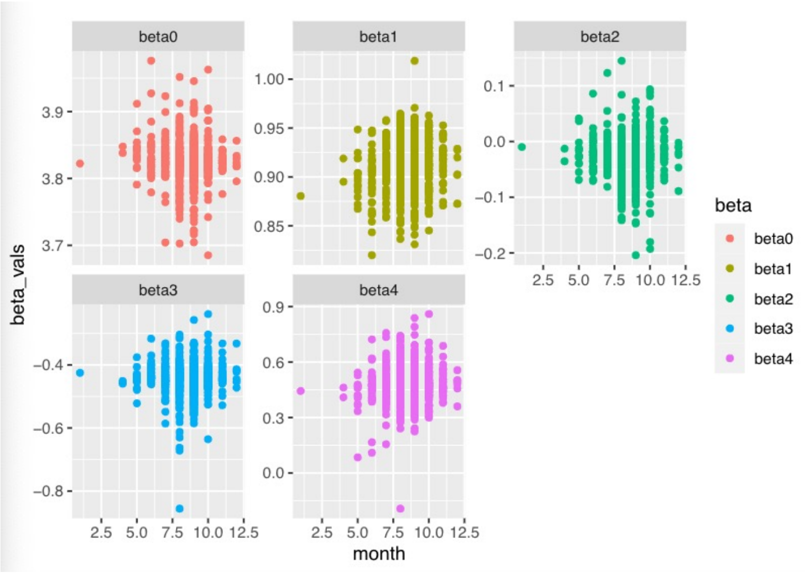
Prediction Performance

RMSE $= 5.78$

Hurricane	RMSE
ABLE.1950	3.142189
BAKER.1950	6.665961
CHARLIE.1950	2.420988
DOG.1950	3.352041
EASY.1950	7.954826
FOX.1950	3.360563
GEORGE.1950	3.966812
HOW.1950	3.212678
ITEM.1950	15.515327
JIG.1950	2.198730

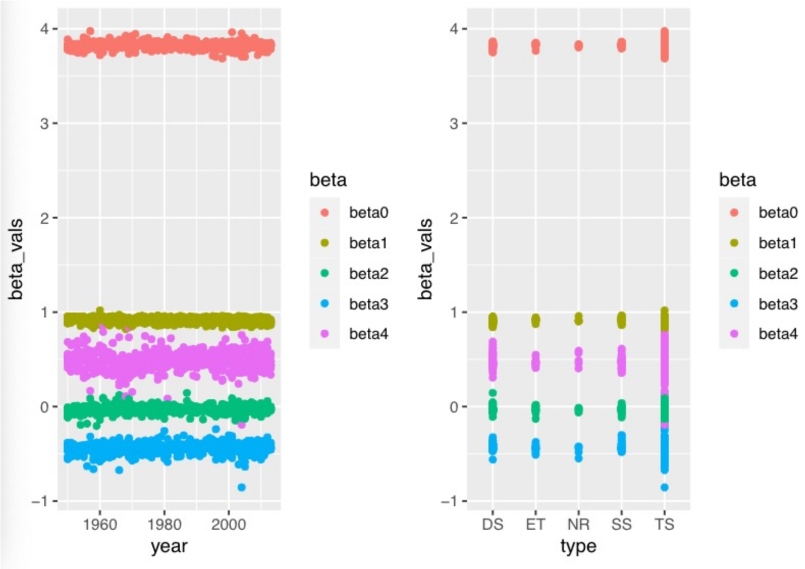


EDA of β_i



beta0
beta1
beta2
beta3
beta4
beta_vals
month
3.9
3.8
3.7
1.00
0.95
0.90
0.85
0.1
0.0
−0.1
−0.2
−0.4
−0.6
−0.8
0.9
0.6
0.3
0.0
2.5 5.0 7.5 10.0 12.5
beta
beta0
beta1
beta2
beta3
beta4

EDA of β_i


beta_vals
year
1960
1980
2000
-1
0
1
2
3
4
beta
beta0
beta1
beta2
beta3
beta4
beta_vals
type
DS
ET
NR
SS
TS
-1
0
1
2
3
4
beta
beta0
beta1
beta2
beta3
beta4

Seasonal Analysis

Model 1:

$\beta_i = \alpha_0 + \alpha_{1m} I(\text{Month} = M) + \alpha_2 \times \text{Year} + \alpha_{3n} I(\text{Type} = N)$

M: April-December

N: ET, NR, SS, TS

Coefficients of Model 1 for β_i

	Beta0		Beta1		Beta2		Beta3		Beta4	
	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
(Intercept)	3.8722305	0.0000000	1.4200232	0.0000000	-0.0807485	0.6123701	-0.7395516	0.0012896	0.8052773	0.0590557
factor(month)4	0.0211436	0.5897652	0.0316515	0.3472330	-0.0115682	0.7896176	-0.0234552	0.7064830	-0.0053003	0.9635270
factor(month)5	0.0202412	0.5405684	0.0281544	0.3215509	-0.0113447	0.7563959	-0.0120033	0.8192595	-0.0390352	0.6897011
factor(month)6	0.0159306	0.6239449	0.0246013	0.3779221	-0.0148705	0.6789563	0.0098662	0.8483886	0.0112067	0.9071129
factor(month)7	0.0078141	0.8090619	0.0404591	0.1453332	-0.0165819	0.6428643	-0.0027461	0.9573635	0.0199011	0.8350861
factor(month)8	0.0019068	0.9527290	0.0425000	0.1241205	-0.0260312	0.4643250	-0.0112584	0.8255944	0.0229217	0.8095080
factor(month)9	0.0009337	0.9768273	0.0472980	0.0868820	-0.0233893	0.5105760	-0.0075900	0.8818256	0.0389302	0.6820459
factor(month)10	0.0074737	0.8163761	0.0411045	0.1371659	-0.0168883	0.6351370	-0.0007253	0.9886799	0.0268667	0.7776519
factor(month)11	0.0057884	0.8588527	0.0448708	0.1086605	-0.0079430	0.8253327	0.0043686	0.9326594	0.0387334	0.6872898
factor(month)12	0.0048248	0.8874129	0.0308019	0.2926123	-0.0208686	0.5797518	0.0072339	0.8936869	0.0283150	0.7786590
year	-0.0000290	0.6794636	-0.0002769	0.0000050	0.0000378	0.6260392	0.0001587	0.1544308	-0.0001713	0.4088111
factor(type)ET	0.0075408	0.4379949	0.0086401	0.3006911	-0.0108462	0.3131136	-0.0192894	0.2118151	-0.0222770	0.4382889
factor(type)NR	0.0005575	0.9705947	0.0072156	0.5784605	-0.0132239	0.4292334	-0.0418405	0.0819304	0.0070952	0.8739144
factor(type)SS	0.0074733	0.2505823	0.0082071	0.1417607	-0.0038667	0.5906999	0.0003254	0.9748646	-0.0225589	0.2407159
factor(type)TS	0.0057948	0.2474418	0.0009877	0.8182988	-0.0024415	0.6592917	-0.0141969	0.0746373	-0.0108813	0.4623952

Seasonal Analysis

Model 2:

$Beta_i = \alpha_0 + \alpha_{1s} I(\text{Season} = S) + \alpha_2 \times \text{Year} + \alpha_{3n} I(\text{Type} = N)$

S: Spring, Summer, Winter

N: ET, NR, SS, TS

Coefficients of Model 2 for β_i

	Beta 0		Beta 1		Beta 2		Beta 3		Beta 4	
	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
(Intercept)	3.8777185	0.0000000	1.4515958	0.0000000	-0.1108749	0.4777627	-0.7432739	0.0009368	0.8246599	0.0469629
factor(season)spring	0.0165743	0.0386414	-0.0161701	0.0195181	0.0080903	0.3613959	-0.0095303	0.4529259	-0.0700505	0.0029939
factor(season)summer	0.0017442	0.4921921	-0.0054774	0.0126824	-0.0021923	0.4356701	-0.0020772	0.6061277	-0.0145208	0.0520294
factor(season)winter	0.0004419	0.9695014	-0.0175939	0.0782636	0.0012626	0.9214169	0.0101974	0.5781775	-0.0099913	0.7687012
year	-0.0000297	0.6721280	-0.0002706	0.0000093	0.0000429	0.5814228	0.0001589	0.1539417	-0.0001639	0.4270083
factor(type)ET	0.0090086	0.3383681	0.0086688	0.2860439	-0.0058182	0.5765098	-0.0167604	0.2616162	-0.0206719	0.4547698
factor(type)NR	0.0017339	0.9077767	0.0079185	0.5401551	-0.0073224	0.6586863	-0.0384250	0.1059825	0.0085343	0.8462012
factor(type)SS	0.0077248	0.2318002	0.0080589	0.1486481	-0.0023967	0.7374924	0.0006525	0.9492157	-0.0222179	0.2419930
factor(type)TS	0.0047623	0.3404235	0.0024950	0.5629051	-0.0029696	0.5913088	-0.0157040	0.0477883	-0.0093683	0.5233984

Forcasting Hurricane Impact

ID: ID of the hurricanes
Season: In which year the hurricane occurred
Month: In which month the hurricane occurred
Nature: Nature of the hurricane
Damage: Financial loss (in Billion U.S. dollars) caused by hurricanes
Deaths: Number of death caused by hurricanes
Maxspeed: Maximum recorded wind speed of the hurricane
Meanspeed: Average wind speed of the hurricane
Maxpressure: Maximum recorded central pressure of the hurricane
Meanpressure: Average central pressure of the hurricane
Hours: Duration of the hurricane in hours
Total.Pop: Total affected population
Percent.Poor: % affected population that reside in low GDP counties
Percent.USA: % affected population that reside in the United States

LASSO Model for Damage

	Coefficients
(Intercept)	-533.5099174
season	3.3837824
deaths	0.0000000
monthJuly	0.0000000
monthJune	0.0000000
monthNovember	0.0000000
monthOctober	0.0000000
monthSeptember	0.0000000
natureNR	0.0000000
natureTS	0.0000000
maxspeed	1.2117851
meanspeed	0.0000000
maxpressure	0.0000000
meanpressure	0.0000000
hours	0.0000000
total_pop	0.3187361
percent_poor	0.0000000
percent_usa	0.7073409
beta0	0.0000000
beta1	0.0000000
beta2	0.0000000
beta3	0.0000000
beta4	0.0000000

Refitted Linear Regression Model

Model: $Y = \gamma_0 + \gamma_1$ x season $+ \gamma_2$ x maxspeed $+ \gamma_3$ x total_pop $+ \gamma_4$ x percent_usa

	Coefficients
(Intercept)	-1316.7386136
season	0.6485139
maxspeed	0.1968674
total_pop	0.0000033
percent_usa	0.1356486

Poisson Model for Deaths

$y_i \sim \text{Poisson}(\mu_i)$, where $\mu_i = hours_i * \lambda_i$, λ_i is the number of deaths per hour

Model: $log(\lambda_i) = \boldsymbol{X_i^T}\gamma + log(hours_i)$

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-199.5331639	11.8792784	-16.7967411	0.0000000
season	-0.0404185	0.0028048	-14.4104991	0.0000000
damage	0.0220163	0.0005679	38.7649762	0.0000000
monthJuly	-10.2286750	0.1604645	-63.7441688	0.0000000
monthJune	0.3928062	0.0989170	3.9710698	0.0000716
monthNovember	1.8733767	0.1664682	11.2536625	0.0000000
monthOctober	-1.6041896	0.0787720	-20.3649754	0.0000000
monthSeptember	1.2490015	0.0575033	21.7205350	0.0000000
natureNR	2.0903864	0.1371766	15.2386495	0.0000000
natureTS	-1.1903051	0.1118619	-10.6408484	0.0000000
maxspeed	0.0035207	0.0013988	2.5168778	0.0118400
meanspeed	-0.1978651	0.0039977	-49.4953412	0.0000000
maxpressure	0.0048106	0.0075485	0.6372945	0.5239331
meanpressure	0.0021204	0.0001759	12.0515409	0.0000000
total_pop	0.0000009	0.0000000	31.4237737	0.0000000
percent_poor	0.0873434	0.0010058	86.8433730	0.0000000
percent_usa	-0.0080185	0.0004884	-16.4173000	0.0000000
beta0	41.3531048	0.5634443	73.3934206	0.0000000
beta1	132.8784572	1.9305164	68.8305252	0.0000000
beta2	-10.7339527	0.5001340	-21.4621524	0.0000000
beta3	-0.4736994	0.5091748	-0.9303277	0.3522014
beta4	4.4919244	0.1971025	22.7897893	0.0000000

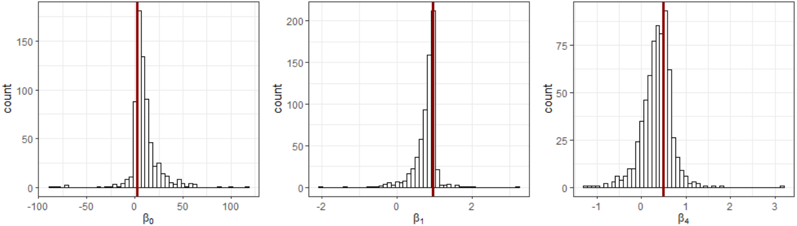
Discussion

Strength & Limitation of MCMC methods

- Bypass coefficient optimization process and directly sample coefficients from their assumed distributions
- Often computationally expensive and can be inefficient
- Convergence is not guaranteed

Why Non-convergence?

- $\beta_i \sim N(\beta, \Sigma)$ may be a too strong of an assumption



Distribution of β_is obtained by performing OLS for each hurricane (red line: β obtained by performing OLS on the whole training dataset)

- **Future work**: use a more adequate distribution assumption of β_i which can account for skewness

References

1. Livingston, I. (2021, September 3). Ida's impact from the Gulf Coast to northeast - by the numbers. The Washington Post. https://www.washingtonpost.com/weather/2021/09/03/hurricane-ida-numbers-surge-wind-pressure-damage/
2. Saffir-Simpson Hurricane Wind Scale. (n.d.). https://www.nhc.noaa.gov/aboutsshws.php