# P8160 - Breast Cancer Data: To lasso or to not lasso

Amy Pitts, Hun Lee, Jimmy Kelliher, Tucker Morgan, and Waveley Qiu

2022-03-28

# Motivation

Diagnosing breast cancer is extremely important.

According to NIH there has been an estimated:

- 281,550 new cases of breast cancer in women in 2021,
- 43,600 breast cancer in women related deaths in 2021.

American Cancer Society Guideline for Breast Cancer Screening:

- Women between ages 25-40 should have an annual clinical breast examination.
- Women between ages 40-44 should begin annual screening via mammogram
- Women between ages 45-54 should screened annually via mammogram

# Goal

With using all the collected imagine data we want to develop an algorithm to predict diagnosis. Since diagnosis is a binary outcome a logistic regression will be utilized.

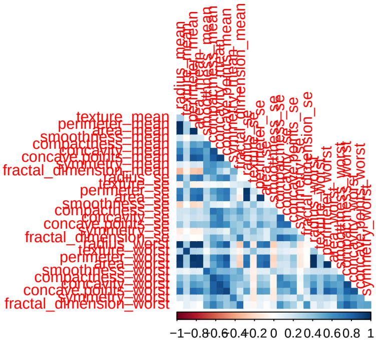
Methods:

- Newton-Raphson Algorithm (Full Model)
- Logistic LASSO Algorithm (Optimal Model)

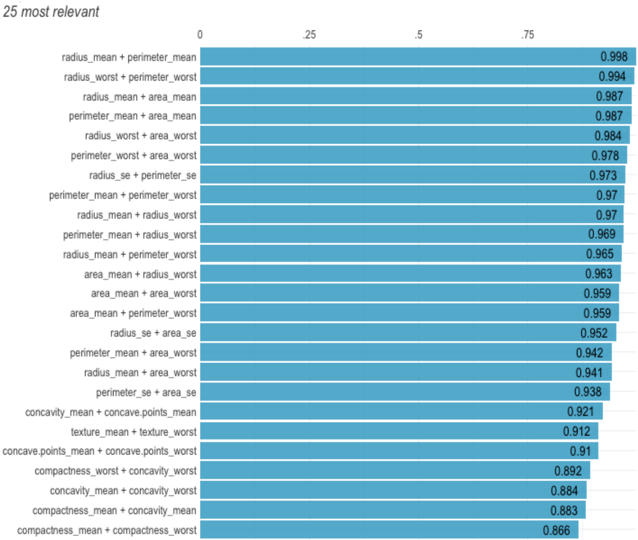
# Data

- 569 rows and 31 columns all related to breast tissue images
- Outcome of interest: Diagnosis (B or M)
  - 357 benign (B) cases and 212 malignant (M) cases
- The Covariates include information such as radius, texture, perimeter, area, smoothness, compactness, concavity, concave points, symmetry, and fractal dimension.

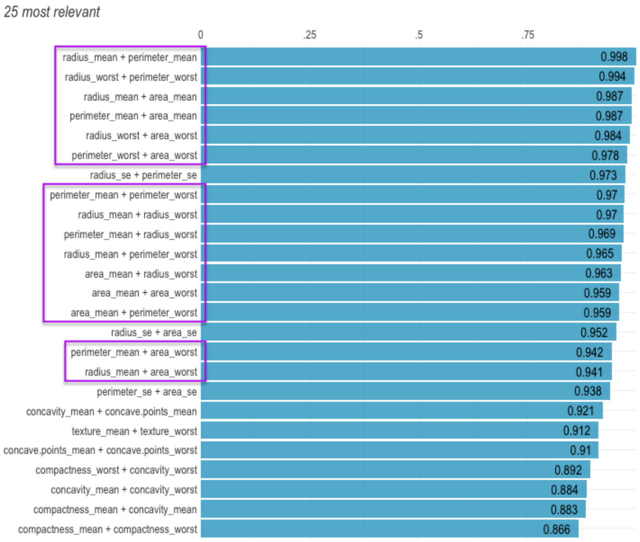
# Figure 1: Correlation Heat Plot of all Covariates



# Figure 1: Ranked Cross-Correlations

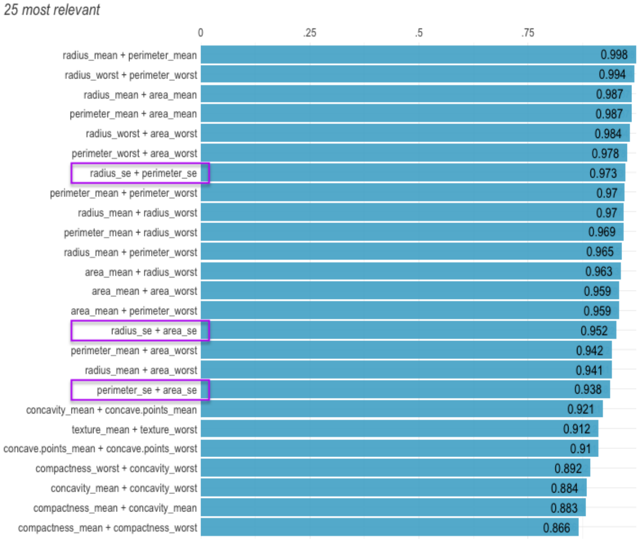


# Figure 1: Ranked Cross-Correlations




Best Representative `radius_worst`

# Figure 1: Ranked Cross-Correlations




Best Representative `radius_se`

# Table 1: Remaining Variables

| Variable | Diagnosis Received | | p-value[2] |
|---|---|---|---|
| | **B**, N = 357[1] | **M**, N = 212[1] | |
| texture_mean | 17.91 (4.00) | 21.60 (3.78) | <0.001 |
| smoothness_mean | 0.09 (0.01) | 0.10 (0.01) | <0.001 |
| compactness_mean | 0.08 (0.03) | 0.15 (0.05) | <0.001 |
| concave points_mean | 0.03 (0.02) | 0.09 (0.03) | <0.001 |
| symmetry_mean | 0.17 (0.02) | 0.19 (0.03) | <0.001 |
| fractal_dimension_mean | 0.06 (0.01) | 0.06 (0.01) | 0.5 |
| radius_se | 0.28 (0.11) | 0.61 (0.35) | <0.001 |
| texture_se | 1.22 (0.59) | 1.21 (0.48) | 0.6 |
| smoothness_se | 0.01 (0.00) | 0.01 (0.00) | 0.2 |
| compactness_se | 0.02 (0.02) | 0.03 (0.02) | <0.001 |
| concavity_se | 0.03 (0.03) | 0.04 (0.02) | <0.001 |
| concave points_se | 0.01 (0.01) | 0.02 (0.01) | <0.001 |
| symmetry_se | 0.02 (0.01) | 0.02 (0.01) | 0.028 |
| fractal_dimension_se | 0.00 (0.00) | 0.00 (0.00) | <0.001 |
| radius_worst | 13.38 (1.98) | 21.13 (4.28) | <0.001 |
| smoothness_worst | 0.12 (0.02) | 0.14 (0.02) | <0.001 |
| compactness_worst | 0.18 (0.09) | 0.37 (0.17) | <0.001 |
| concavity_worst | 0.17 (0.14) | 0.45 (0.18) | <0.001 |
| symmetry_worst | 0.27 (0.04) | 0.32 (0.07) | <0.001 |
| fractal_dimension_worst | 0.08 (0.01) | 0.09 (0.02) | <0.001 |

[1] Statistics presented: Mean (SD)

[2] Statistical tests performed: Wilcoxon rank-sum test

# Full Model (Newton-Raphson)

Consider the following log-likelihood, gradient, and hessian matrix. First, let

$$\pi_i = P(Y_i = 1|x_{i,1}, \ldots x_{i,p}) = \frac{e^{\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j}}}{1 + e^{\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j}}}.$$

**The log-likelihood:**

$$l(\mathbf{X}|\vec{\beta}) = \sum_{i=1}^{n} \left[ y_i \left( \beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j} \right) - \log \left( 1 + \exp \left( \beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j} \right) \right) \right]$$

**The gradient:**

$$\nabla l(\mathbf{X}|\vec{\beta}) = \left[ \sum^n y_i - \pi_i \quad \sum^n x_{i,1}(y_i - \pi_i) \quad \ldots \quad \sum^n x_{i,p}(y_i - \pi_i) \right]^T_{1\times(p+1)}$$

**The hessian:** produces a matrix $(p + 1 \times p + 1)$

$$\nabla^2 l(\mathbf{X}|\vec{\beta}) = -\sum_{i=1}^{n} \begin{pmatrix} 1 \\ X \end{pmatrix} \begin{pmatrix} 1 & X \end{pmatrix} \pi_i(1 - \pi_i)$$

# Optimal Model (Logistic LASSO)

**Lemma 1.** Consider the optimization problem

$$\min_{x \in \mathbb{R}} \left\{ \frac{1}{2}(x-b)^2 + c|x| \right\}$$

for $b \in \mathbb{R}$ and $c \in \mathbb{R}_{++}$. It follows that the minimizer is given by

$$\hat{x} = S(b, c),$$

where $S$ is the soft-thresholding operator.

**Lemma 2.** Consider the optimization problem

$$\min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^{n} w_i \left( z_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right\}$$

for some $k \in \{1, \ldots, p\}$. It follows that the minimizer is given by

$$\hat{\beta}_k = \left( \sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \sum_{i=1}^{n} w_i x_{ik} \left( z_i - \sum_{j \neq k} \beta_j x_{ij} \right).$$

# Optimal Model (Logistic LASSO)

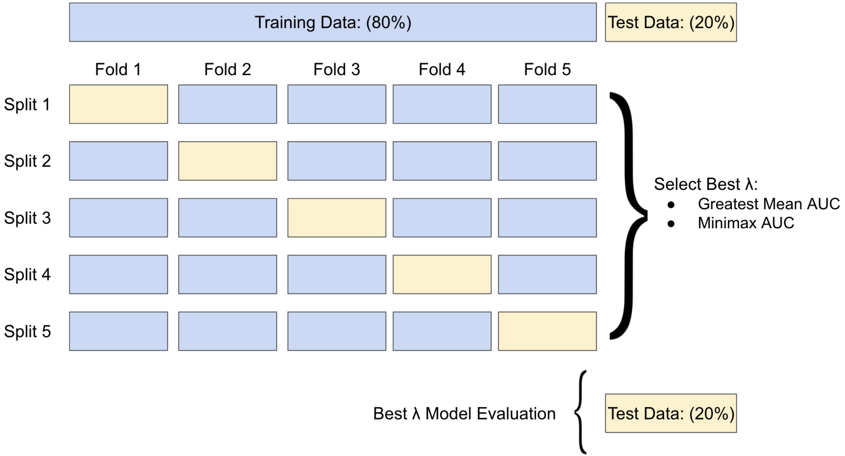
**Lemma 3.** With $\hat{\beta}_k$ defined as above,

$$\min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^{n} w_i \left( z_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}$$
$$= \min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2} (\beta_k - \hat{\beta}_k)^2 + \left( \frac{1}{n} \sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \lambda |\beta_k| \right\}.$$

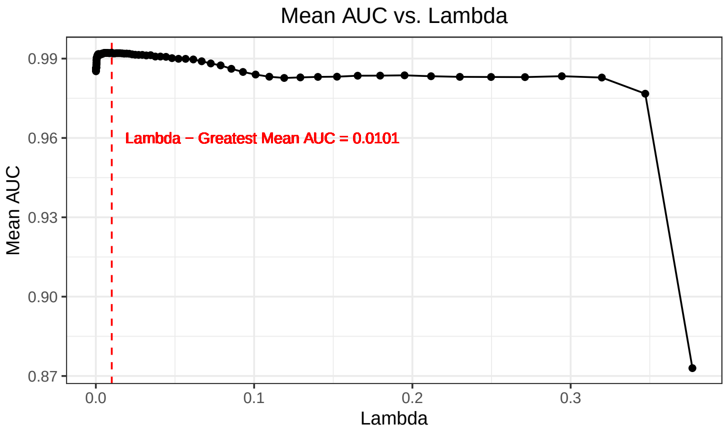
By Lemma 1 and Lemma 3,

$$\arg\min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^{n} w_i \left( z_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}$$
$$= S\left( \hat{\beta}_k, \left( \frac{1}{n} \sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \lambda \right)$$

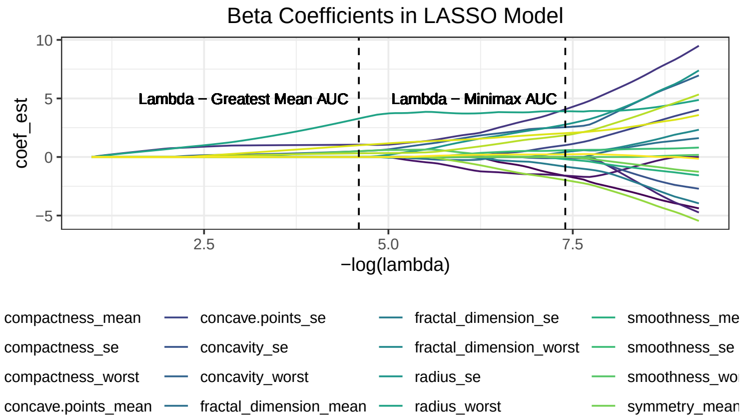
# Figure 2: 5-fold Cross Validation



# Cross Validation Results: Selecting Best Lambda



# Cross Validation Results: LASSO Coefficients



# Coefficients Comparison

Final Beta Coefficient Estimates

| Measures | NewtonRaphson | LASSO_GreatestMeanAUC | LASSO_MinimaxAUC |
|---|---|---|---|
| Specificities | 0.2462 | 0.6923 | 0.5538 |
| AUC | 0.9765 | 0.9897 | 0.9858 |
| Selected Lambda | 0 | 0.0101 | 0.0006 |
| Number of Variables (w/o Intercept) | 20 | 6 | 16 |

# Discussion

- Goal is accurately classify every patient
- Balancing Sensitivity vs. Specificity.
  - In first screening cases want to catch every case. Maximize Sensitivity.

# Resources

Cancer Stat Facts: Female Breast Cancer. *National Cancer Institute - NIH* https://seer.cancer.gov/statfacts/html/breast.html

American Cancer Society. (2019). Breast cancer facts & figures 2019–2020. Am Cancer Soc, 1-44.