Logistic Lasso optimization

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```r
cancer_df <- read.csv("~/Downloads/breast-cancer.csv") %>% janitor::clean_names()
```

```r
data <-
  cancer_df %>% dplyr::select(-id, -x) %>%
  mutate(diagnosis = ifelse(diagnosis == "M", 1, 0)) %>% distinct()

set.seed(1)
split <- initial_split(data, prop = 0.8)

training_df <- split %>% training()

testing_df <- split %>% testing()
```

Standardization

```r
for (i in 2:length(training_df)) {
  training_df[,i] = (training_df[,i] - mean(training_df[,i]))/sd(training_df[,i])
}
```

Checking standardization

```r
summary <- skimr::skim_without_charts(training_df) %>% data.frame()
summary[1:6] %>% kable()
```

skim_type	skim_variable	n_missing	complete_rate	numeric.mean	numeric.sd
numeric	diagnosis	0	1	0.3714286	0.4837186
numeric	radius_mean	0	1	0.0000000	1.0000000
numeric	texture_mean	0	1	0.0000000	1.0000000
numeric	perimeter_mean	0	1	0.0000000	1.0000000
numeric	area_mean	0	1	0.0000000	1.0000000
numeric	smoothness_mean	0	1	0.0000000	1.0000000
numeric	compactness_mean	0	1	0.0000000	1.0000000
numeric	concavity_mean	0	1	0.0000000	1.0000000
numeric	concave_points_mean	0	1	0.0000000	1.0000000
numeric	symmetry_mean	0	1	0.0000000	1.0000000
numeric	fractal_dimension_mean	0	1	0.0000000	1.0000000
numeric	radius_se	0	1	0.0000000	1.0000000
numeric	texture_se	0	1	0.0000000	1.0000000
numeric	perimeter_se	0	1	0.0000000	1.0000000
numeric	area_se	0	1	0.0000000	1.0000000
numeric	smoothness_se	0	1	0.0000000	1.0000000
numeric	compactness_se	0	1	0.0000000	1.0000000
numeric	concavity_se	0	1	0.0000000	1.0000000
numeric	concave_points_se	0	1	0.0000000	1.0000000
numeric	symmetry_se	0	1	0.0000000	1.0000000
numeric	fractal_dimension_se	0	1	0.0000000	1.0000000
numeric	radius_worst	0	1	0.0000000	1.0000000
numeric	texture_worst	0	1	0.0000000	1.0000000
numeric	perimeter_worst	0	1	0.0000000	1.0000000
numeric	area_worst	0	1	0.0000000	1.0000000
numeric	smoothness_worst	0	1	0.0000000	1.0000000
numeric	compactness_worst	0	1	0.0000000	1.0000000
numeric	concavity_worst	0	1	0.0000000	1.0000000
numeric	concave_points_worst	0	1	0.0000000	1.0000000
numeric	symmetry_worst	0	1	0.0000000	1.0000000
numeric	fractal_dimension_worst	0	1	0.0000000	1.0000000

```
X <- training_df %>% dplyr::select(-diagnosis) %>% as.matrix()
y <- training_df$diagnosis %>% as.matrix()

dim(X)
```

```
## [1] 455  30
```

```
dim(y)
```

```
## [1] 455   1
```

Helper Functions

```
# logistic function
logistic <- function(x) 1 / (1 + exp(-x))

# soft threshold
```

```r
S <- function(beta, gamma) {
  if (beta > 0 && gamma < abs(beta)) {
    beta - gamma
  } else if (beta < 0 && gamma < abs(beta)) {
    beta + gamma
  } else {
    0
  }
}


# probability adjustment function
p_adj <- function(p, epsilon) {
  if (p < epsilon) {
    0
  } else if (p > 1 - epsilon) {
    1
  } else {
    p
  }
}

# weight adjustment function
w_adj <- function(p, epsilon) {
  if ((p < epsilon) | (p > 1 - epsilon)) {
    epsilon
  } else {
    p * (1 - p)
  }
}
```

Computing lambda max in two ways and defining lambda sequence

```r
alpha = 1
n = length(y)

lambda_max <- max(abs(t(X) %*% y))/n

same_lambda_max <- max(abs(t(y - mean(y)*(1 - mean(y))) %*% X ))/(alpha*n)

lambda_max
```

```
## [1] 0.386795
```

```r
same_lambda_max
```

```
## [1] 0.386795
```

```r
lambda_max <- lambda_max %>% round(digits = 2)
```

```
epsilon = 1e-5
lambda_seq <- seq(lambda_max, 1e-5*lambda_max, length = 100)
```

Toy example

```
set.seed(1)
n     <- 1000
X     <- scale(matrix(rnorm(3 * n), c(n, 3)))
X     <- as.matrix(cbind(rep(1, n), X))
y     <- 1 * (runif(n) > 0.5)

lambda_max <- round(max(abs(t(X) %*% y))/n,2)
epsilon <- 1e-5

lambda_seq <- seq(lambda_max, epsilon*lambda_max, length = 100)

pb1 <- progress_bar$new(format = "Lasso optimization [:bar] :percent eta: :eta",
                        total = length(lambda_seq))

# initialize parameters
beta <- list()
beta_old <- list()
b_k_temp <- list()

# creating initial beta list
for (i in 1:length(lambda_seq)) {
  beta[[i]] <- rep(0, ncol(X))
  beta_old[[i]] <- rep(NA, ncol(X))
}

# outer loop for lambda decrement
for (i in 1:length(lambda_seq)) {
  pb1$tick()
  lambda <- lambda_seq[i]
# inner loop for updating quadratic approximation ingredient
  for (inner in 1:2) {
    p <- map_dbl(logistic(X %*% beta[[i]]), p_adj, epsilon)
    w <- map_dbl(p, w_adj, epsilon)
    z <- X %*% beta[[i]] + (y - p) / w

    terminate <- 0
    iter <- 1
    # while loop for beta convergence
    while (terminate < 1) {
      beta_old[[i]] <- beta[[i]]
      # running coordinate descending algorithm
      for (k in 1:ncol(X)) {
        x_k    <- X[ , k]
        x_notk <- X[ , -k]
        b_notk <- beta[[i]][-k]
```

```r
        # un-penalized coefficient update
        b_k_temp <- sum(w * (z - x_notk %*% b_notk) * x_k) / sum(w * x_k^2)
        # shrinkage update
        b_k           <- S(b_k_temp, lambda * (k > 1) / mean(w * x_k^2))
        #b_k       <- S(sum(w*x_k*(z - x_notk %*% b_notk)), lambda * (k > 1)) / sum(w * x_k^2)

        # update beta list along with other parameters
        beta[[i]][k] <- b_k
        iter <- iter + 1
        }
      if (iter == 200 | max(abs(beta[[i]] - beta_old[[i]])) < 1e-10) {
        terminate <- 1
      }
    }
  }
}
# Estimates from Coordinate Descent
beta <- data.frame(beta)

for (i in 1:length(lambda_seq)) {
  colnames(beta)[i] <- paste0(lambda_seq[i] %>% round(digits = 6))
}

beta <- t(beta) %>% data.frame()

beta %>% mutate(lambda = rownames(beta)) %>% relocate(lambda) %>% `rownames<-`(NULL)
```

```
##        lambda        X1         X2          X3          X4
## 1        0.53 0.1040939 0.00000000 0.000000000  0.000000000
## 2    0.524647 0.1040939 0.00000000 0.000000000  0.000000000
## 3    0.519293 0.1040939 0.00000000 0.000000000  0.000000000
## 4     0.51394 0.1040939 0.00000000 0.000000000  0.000000000
## 5    0.508586 0.1040939 0.00000000 0.000000000  0.000000000
## 6    0.503233 0.1040939 0.00000000 0.000000000  0.000000000
## 7    0.497879 0.1040939 0.00000000 0.000000000  0.000000000
## 8    0.492526 0.1040939 0.00000000 0.000000000  0.000000000
## 9    0.487172 0.1040939 0.00000000 0.000000000  0.000000000
## 10   0.481819 0.1040939 0.00000000 0.000000000  0.000000000
## 11   0.476465 0.1040939 0.00000000 0.000000000  0.000000000
## 12   0.471112 0.1040939 0.00000000 0.000000000  0.000000000
## 13   0.465758 0.1040939 0.00000000 0.000000000  0.000000000
## 14   0.460405 0.1040939 0.00000000 0.000000000  0.000000000
## 15   0.455051 0.1040939 0.00000000 0.000000000  0.000000000
## 16   0.449698 0.1040939 0.00000000 0.000000000  0.000000000
## 17   0.444344 0.1040939 0.00000000 0.000000000  0.000000000
## 18   0.438991 0.1040939 0.00000000 0.000000000  0.000000000
## 19   0.433637 0.1040939 0.00000000 0.000000000  0.000000000
## 20   0.428284 0.1040939 0.00000000 0.000000000  0.000000000
## 21    0.42293 0.1040939 0.00000000 0.000000000  0.000000000
## 22   0.417577 0.1040939 0.00000000 0.000000000  0.000000000
## 23   0.412223 0.1040939 0.00000000 0.000000000  0.000000000
## 24    0.40687 0.1040939 0.00000000 0.000000000  0.000000000
## 25   0.401516 0.1040939 0.00000000 0.000000000  0.000000000
```

```
## 26   0.396163 0.1040939 0.00000000 0.000000000  0.000000000
## 27   0.390809 0.1040939 0.00000000 0.000000000  0.000000000
## 28   0.385456 0.1040939 0.00000000 0.000000000  0.000000000
## 29   0.380103 0.1040939 0.00000000 0.000000000  0.000000000
## 30   0.374749 0.1040939 0.00000000 0.000000000  0.000000000
## 31   0.369396 0.1040939 0.00000000 0.000000000  0.000000000
## 32   0.364042 0.1040939 0.00000000 0.000000000  0.000000000
## 33   0.358689 0.1040939 0.00000000 0.000000000  0.000000000
## 34   0.353335 0.1040939 0.00000000 0.000000000  0.000000000
## 35   0.347982 0.1040939 0.00000000 0.000000000  0.000000000
## 36   0.342628 0.1040939 0.00000000 0.000000000  0.000000000
## 37   0.337275 0.1040939 0.00000000 0.000000000  0.000000000
## 38   0.331921 0.1040939 0.00000000 0.000000000  0.000000000
## 39   0.326568 0.1040939 0.00000000 0.000000000  0.000000000
## 40   0.321214 0.1040939 0.00000000 0.000000000  0.000000000
## 41   0.315861 0.1040939 0.00000000 0.000000000  0.000000000
## 42   0.310507 0.1040939 0.00000000 0.000000000  0.000000000
## 43   0.305154 0.1040939 0.00000000 0.000000000  0.000000000
## 44     0.2998 0.1040939 0.00000000 0.000000000  0.000000000
## 45   0.294447 0.1040939 0.00000000 0.000000000  0.000000000
## 46   0.289093 0.1040939 0.00000000 0.000000000  0.000000000
## 47    0.28374 0.1040939 0.00000000 0.000000000  0.000000000
## 48   0.278386 0.1040939 0.00000000 0.000000000  0.000000000
## 49   0.273033 0.1040939 0.00000000 0.000000000  0.000000000
## 50   0.267679 0.1040939 0.00000000 0.000000000  0.000000000
## 51   0.262326 0.1040939 0.00000000 0.000000000  0.000000000
## 52   0.256972 0.1040939 0.00000000 0.000000000  0.000000000
## 53   0.251619 0.1040939 0.00000000 0.000000000  0.000000000
## 54   0.246265 0.1040939 0.00000000 0.000000000  0.000000000
## 55   0.240912 0.1040939 0.00000000 0.000000000  0.000000000
## 56   0.235559 0.1040939 0.00000000 0.000000000  0.000000000
## 57   0.230205 0.1040939 0.00000000 0.000000000  0.000000000
## 58   0.224852 0.1040939 0.00000000 0.000000000  0.000000000
## 59   0.219498 0.1040939 0.00000000 0.000000000  0.000000000
## 60   0.214145 0.1040939 0.00000000 0.000000000  0.000000000
## 61   0.208791 0.1040939 0.00000000 0.000000000  0.000000000
## 62   0.203438 0.1040939 0.00000000 0.000000000  0.000000000
## 63   0.198084 0.1040939 0.00000000 0.000000000  0.000000000
## 64   0.192731 0.1040939 0.00000000 0.000000000  0.000000000
## 65   0.187377 0.1040939 0.00000000 0.000000000  0.000000000
## 66   0.182024 0.1040939 0.00000000 0.000000000  0.000000000
## 67    0.17667 0.1040939 0.00000000 0.000000000  0.000000000
## 68   0.171317 0.1040939 0.00000000 0.000000000  0.000000000
## 69   0.165963 0.1040939 0.00000000 0.000000000  0.000000000
## 70    0.16061 0.1040939 0.00000000 0.000000000  0.000000000
## 71   0.155256 0.1040939 0.00000000 0.000000000  0.000000000
## 72   0.149903 0.1040939 0.00000000 0.000000000  0.000000000
## 73   0.144549 0.1040939 0.00000000 0.000000000  0.000000000
## 74   0.139196 0.1040939 0.00000000 0.000000000  0.000000000
## 75   0.133842 0.1040939 0.00000000 0.000000000  0.000000000
## 76   0.128489 0.1040939 0.00000000 0.000000000  0.000000000
## 77   0.123135 0.1040939 0.00000000 0.000000000  0.000000000
## 78   0.117782 0.1040939 0.00000000 0.000000000  0.000000000
## 79   0.112428 0.1040939 0.00000000 0.000000000  0.000000000
```

```
## 80   0.107075 0.1040939 0.00000000 0.000000000   0.000000000
## 81   0.101721 0.1040939 0.00000000 0.000000000   0.000000000
## 82   0.096368 0.1040939 0.00000000 0.000000000   0.000000000
## 83   0.091014 0.1040939 0.00000000 0.000000000   0.000000000
## 84   0.085661 0.1040939 0.00000000 0.000000000   0.000000000
## 85   0.080308 0.1040939 0.00000000 0.000000000   0.000000000
## 86   0.074954 0.1040939 0.00000000 0.000000000   0.000000000
## 87   0.069601 0.1040939 0.00000000 0.000000000   0.000000000
## 88   0.064247 0.1040939 0.00000000 0.000000000   0.000000000
## 89   0.058894 0.1040939 0.00000000 0.000000000   0.000000000
## 90    0.05354 0.1040939 0.00000000 0.000000000   0.000000000
## 91   0.048187 0.1040939 0.00000000 0.000000000   0.000000000
## 92   0.042833 0.1040939 0.00000000 0.000000000   0.000000000
## 93    0.03748 0.1040939 0.00000000 0.000000000   0.000000000
## 94   0.032126 0.1040939 0.00000000 0.000000000   0.000000000
## 95   0.026773 0.1040939 0.00000000 0.000000000   0.000000000
## 96   0.021419 0.1040939 0.00000000 0.000000000   0.000000000
## 97   0.016066 0.1040939 0.00000000 0.000000000   0.000000000
## 98   0.010712 0.1040956 0.00000000 0.000000000  -0.008051621
## 99   0.005359 0.1041167 0.00000000 0.000000000  -0.029552983
## 100     5e-06 0.1041748 0.02045426 0.006292914  -0.052235163
```

```r
# True estimates from GLM
#as.vector(glm(y ~ X[ , -1], family = binomial)$coefficients)

# True estimates from GLMNET
fit <- glmnet(X, y, family = "binomial", standardize = FALSE, lambda = lambda_seq, thresh = 1e-10)

fit_result <- fit$beta %>% as.matrix() %>% t() %>% data.frame() %>% `rownames<-`(NULL)

fit_result %>% mutate(lambda = fit$lambda %>% round(digits = 6)) %>% relocate(lambda)
```

```
##        lambda V1         V2          V3         V4
## 1    0.530000  0 0.00000000 0.000000000  0.00000000
## 2    0.524647  0 0.00000000 0.000000000  0.00000000
## 3    0.519293  0 0.00000000 0.000000000  0.00000000
## 4    0.513940  0 0.00000000 0.000000000  0.00000000
## 5    0.508586  0 0.00000000 0.000000000  0.00000000
## 6    0.503233  0 0.00000000 0.000000000  0.00000000
## 7    0.497879  0 0.00000000 0.000000000  0.00000000
## 8    0.492526  0 0.00000000 0.000000000  0.00000000
## 9    0.487172  0 0.00000000 0.000000000  0.00000000
## 10   0.481819  0 0.00000000 0.000000000  0.00000000
## 11   0.476465  0 0.00000000 0.000000000  0.00000000
## 12   0.471112  0 0.00000000 0.000000000  0.00000000
## 13   0.465758  0 0.00000000 0.000000000  0.00000000
## 14   0.460405  0 0.00000000 0.000000000  0.00000000
## 15   0.455051  0 0.00000000 0.000000000  0.00000000
## 16   0.449698  0 0.00000000 0.000000000  0.00000000
## 17   0.444344  0 0.00000000 0.000000000  0.00000000
## 18   0.438991  0 0.00000000 0.000000000  0.00000000
## 19   0.433637  0 0.00000000 0.000000000  0.00000000
## 20   0.428284  0 0.00000000 0.000000000  0.00000000
## 21   0.422930  0 0.00000000 0.000000000  0.00000000
```

```
## 22   0.417577   0 0.00000000 0.000000000  0.00000000
## 23   0.412223   0 0.00000000 0.000000000  0.00000000
## 24   0.406870   0 0.00000000 0.000000000  0.00000000
## 25   0.401516   0 0.00000000 0.000000000  0.00000000
## 26   0.396163   0 0.00000000 0.000000000  0.00000000
## 27   0.390809   0 0.00000000 0.000000000  0.00000000
## 28   0.385456   0 0.00000000 0.000000000  0.00000000
## 29   0.380103   0 0.00000000 0.000000000  0.00000000
## 30   0.374749   0 0.00000000 0.000000000  0.00000000
## 31   0.369396   0 0.00000000 0.000000000  0.00000000
## 32   0.364042   0 0.00000000 0.000000000  0.00000000
## 33   0.358689   0 0.00000000 0.000000000  0.00000000
## 34   0.353335   0 0.00000000 0.000000000  0.00000000
## 35   0.347982   0 0.00000000 0.000000000  0.00000000
## 36   0.342628   0 0.00000000 0.000000000  0.00000000
## 37   0.337275   0 0.00000000 0.000000000  0.00000000
## 38   0.331921   0 0.00000000 0.000000000  0.00000000
## 39   0.326568   0 0.00000000 0.000000000  0.00000000
## 40   0.321214   0 0.00000000 0.000000000  0.00000000
## 41   0.315861   0 0.00000000 0.000000000  0.00000000
## 42   0.310507   0 0.00000000 0.000000000  0.00000000
## 43   0.305154   0 0.00000000 0.000000000  0.00000000
## 44   0.299800   0 0.00000000 0.000000000  0.00000000
## 45   0.294447   0 0.00000000 0.000000000  0.00000000
## 46   0.289093   0 0.00000000 0.000000000  0.00000000
## 47   0.283740   0 0.00000000 0.000000000  0.00000000
## 48   0.278386   0 0.00000000 0.000000000  0.00000000
## 49   0.273033   0 0.00000000 0.000000000  0.00000000
## 50   0.267679   0 0.00000000 0.000000000  0.00000000
## 51   0.262326   0 0.00000000 0.000000000  0.00000000
## 52   0.256972   0 0.00000000 0.000000000  0.00000000
## 53   0.251619   0 0.00000000 0.000000000  0.00000000
## 54   0.246265   0 0.00000000 0.000000000  0.00000000
## 55   0.240912   0 0.00000000 0.000000000  0.00000000
## 56   0.235559   0 0.00000000 0.000000000  0.00000000
## 57   0.230205   0 0.00000000 0.000000000  0.00000000
## 58   0.224852   0 0.00000000 0.000000000  0.00000000
## 59   0.219498   0 0.00000000 0.000000000  0.00000000
## 60   0.214145   0 0.00000000 0.000000000  0.00000000
## 61   0.208791   0 0.00000000 0.000000000  0.00000000
## 62   0.203438   0 0.00000000 0.000000000  0.00000000
## 63   0.198084   0 0.00000000 0.000000000  0.00000000
## 64   0.192731   0 0.00000000 0.000000000  0.00000000
## 65   0.187377   0 0.00000000 0.000000000  0.00000000
## 66   0.182024   0 0.00000000 0.000000000  0.00000000
## 67   0.176670   0 0.00000000 0.000000000  0.00000000
## 68   0.171317   0 0.00000000 0.000000000  0.00000000
## 69   0.165963   0 0.00000000 0.000000000  0.00000000
## 70   0.160610   0 0.00000000 0.000000000  0.00000000
## 71   0.155256   0 0.00000000 0.000000000  0.00000000
## 72   0.149903   0 0.00000000 0.000000000  0.00000000
## 73   0.144549   0 0.00000000 0.000000000  0.00000000
## 74   0.139196   0 0.00000000 0.000000000  0.00000000
## 75   0.133842   0 0.00000000 0.000000000  0.00000000
```

```
## 76   0.128489  0 0.00000000 0.000000000  0.00000000
## 77   0.123135  0 0.00000000 0.000000000  0.00000000
## 78   0.117782  0 0.00000000 0.000000000  0.00000000
## 79   0.112428  0 0.00000000 0.000000000  0.00000000
## 80   0.107075  0 0.00000000 0.000000000  0.00000000
## 81   0.101721  0 0.00000000 0.000000000  0.00000000
## 82   0.096368  0 0.00000000 0.000000000  0.00000000
## 83   0.091014  0 0.00000000 0.000000000  0.00000000
## 84   0.085661  0 0.00000000 0.000000000  0.00000000
## 85   0.080308  0 0.00000000 0.000000000  0.00000000
## 86   0.074954  0 0.00000000 0.000000000  0.00000000
## 87   0.069601  0 0.00000000 0.000000000  0.00000000
## 88   0.064247  0 0.00000000 0.000000000  0.00000000
## 89   0.058894  0 0.00000000 0.000000000  0.00000000
## 90   0.053540  0 0.00000000 0.000000000  0.00000000
## 91   0.048187  0 0.00000000 0.000000000  0.00000000
## 92   0.042833  0 0.00000000 0.000000000  0.00000000
## 93   0.037480  0 0.00000000 0.000000000  0.00000000
## 94   0.032126  0 0.00000000 0.000000000  0.00000000
## 95   0.026773  0 0.00000000 0.000000000  0.00000000
## 96   0.021419  0 0.00000000 0.000000000  0.00000000
## 97   0.016066  0 0.00000000 0.000000000  0.00000000
## 98   0.010712  0 0.00000000 0.000000000 -0.00805162
## 99   0.005359  0 0.00000000 0.000000000 -0.02955296
## 100  0.000005  0 0.02045426 0.006292915 -0.05223517
```

Cancer data

```r
data <-
  read_csv("data/breast-cancer.csv", show_col_types = FALSE) %>%
  mutate(diagnosis = 1 * (diagnosis == "M"))


n     <- nrow(data)
X     <- data[ , -c(1, 2)]
X     <- as.matrix(cbind(rep(1, n), X))
y     <- data$diagnosis



lambda_max <- round(max(abs(t(X) %*% y))/n,2)
epsilon <- 1e-4

lambda_seq <- seq(lambda_max, epsilon*lambda_max, length = 100)


pb1 <- progress_bar$new(format = "Lasso optimization1 [:bar] :percent eta: :eta",
                        total = length(lambda_seq))

pb2 <- progress_bar$new(format = "Lasso optimization2 [:bar] :percent eta: :eta",
                        total = ncol(X))

# initialize parameters
beta <- list()
```

```r
beta_old <- list()
b_k_temp <- list()

# creating initial beta list
for (i in 1:length(lambda_seq)) {
  beta[[i]] <- rep(0, ncol(X))
  beta_old[[i]] <- rep(NA, ncol(X))
}

# outer loop for lambda decrement
for (i in 1:length(lambda_seq)) {
  pb1$tick()
  lambda <- lambda_seq[i]
# inner loop for updating quadratic approximation ingredient
  for (inner in 1:8) {
    p <- map_dbl(logistic(X %*% beta[[i]]), p_adj, epsilon)
    w <- map_dbl(p, w_adj, epsilon)
    z <- X %*% beta[[i]] + (y - p) / w
    terminate <- 0
    iter <- 1
    # while loop for beta convergence
    while (terminate < 1) {
      beta_old[[i]] <- beta[[i]]
      # running coordinate descending algorithm
      for (k in 1:ncol(X)) {
        x_k    <- X[ , k]
        x_notk <- X[ , -k]
        b_notk <- beta[[i]][-k]

        # un-penalized coefficient update
        b_k_temp <- sum(w * (z - x_notk %*% b_notk) * x_k) / sum(w * x_k^2)
        # shrinkage update
        b_k           <- S(b_k_temp, lambda * (k > 1) / mean(w * x_k^2))
        #b_k      <- S(sum(w*x_k*(z - x_notk %*% b_notk)), lambda * (k > 1)) / sum(w * x_k^2)

        # update beta list along with other parameters
        beta[[i]][k] <- b_k
        iter <- iter + 1
        }
      if (iter == 200 | max(abs(beta[[i]] - beta_old[[i]])) < 1e-10) {
        terminate <- 1
      }
    }
  }
}
# Estimates from Coordinate Descent
beta <- data.frame(beta)

for (i in 1:length(lambda_seq)) {
  colnames(beta)[i] <- paste0(lambda_seq[i] %>% round(digits = 6))
}

beta <- t(beta) %>% data.frame()
```

```r
beta %>% mutate(lambda = rownames(beta)) %>% relocate(lambda) %>% `rownames<-`(NULL)
```

```
##          lambda         X1 X2 X3 X4          X5 X6 X7 X8 X9 X10 X11 X12 X13 X14
## 1        529.92 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 2    524.567808 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 3    519.215616 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 4    513.863424 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 5    508.511232 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 6     503.15904 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 7    497.806848 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 8    492.454656 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 9    487.102464 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 10   481.750272 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 11    476.39808 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 12   471.045888 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 13   465.693696 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 14   460.341504 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 15   454.989312 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 16    449.63712 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 17   444.284928 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 18   438.932736 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 19   433.580544 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 20   428.228352 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 21    422.87616 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 22   417.523968 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 23   412.171776 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 24   406.819584 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 25   401.467392 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 26     396.1152 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 27   390.763008 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 28   385.410816 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 29   380.058624 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 30   374.706432 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 31    369.35424 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 32   364.002048 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 33   358.649856 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 34   353.297664 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 35   347.945472 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 36    342.59328 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 37   337.241088 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 38   331.888896 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 39   326.536704 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 40   321.184512 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 41    315.83232 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 42   310.480128 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 43   305.127936 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 44   299.775744 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 45   294.423552 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 46    289.07136 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 47   283.719168 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 48   278.366976 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 49   273.014784 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
## 50   267.662592 -0.5211495  0  0  0  0.00000000  0  0  0  0   0   0   0   0   0
```

```
## 51     262.3104  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 52   256.958208  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 53   251.606016  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 54   246.253824  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 55   240.901632  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 56    235.54944  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 57   230.197248  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 58   224.845056  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 59   219.492864  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 60   214.140672  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 61    208.78848  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 62   203.436288  -0.5211495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 63   198.084096  -0.5645798  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 64   192.731904  -0.6261928  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 65   187.379712  -0.6875513  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 66    182.02752  -0.7489478  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 67   176.675328  -0.8106658  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 68   171.323136  -0.8729809  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 69   165.970944  -0.9361619  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 70   160.618752  -1.0004723  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 71    155.26656  -1.0661727  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 72   149.914368  -1.1335233  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 73   144.562176  -1.2027878  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 74   139.209984  -1.2742370  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 75   133.857792  -1.3481532  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 76     128.5056  -1.4248358  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 77   123.153408  -1.5046064  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 78   117.801216  -1.5878161  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 79   112.449024  -1.6748532  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 80   107.096832  -1.7661532  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 81    101.74464  -1.8622108  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 82    96.392448  -1.9635957  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 83    91.040256  -2.0709726  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 84    85.688064  -2.1851276  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 85    80.335872  -2.3070031  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 86     74.98368  -2.4377454  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 87    69.631488  -2.5787693  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 88    64.279296  -2.7318495  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 89    58.927104  -2.8992513  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 90    53.574912  -3.0839052  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 91     48.22272  -3.2897732  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 92    42.870528  -3.5221718  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 93    37.518336  -3.7885921  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 94    32.166144  -4.0999912  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 95    26.813952  -4.4730612  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 96     21.46176  -4.9346679  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 97    16.109568  -5.5315132  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 98    10.757376  -6.3543366  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 99     5.405184  -7.6091041  0  0  0   0.00000000  0  0  0  0   0   0   0   0   0
## 100    0.052992 -23.0878024  0  0  0  -0.01838479  0  0  0  0   0   0   0   0   0
##           X15 X16 X17 X18 X19 X20 X21 X22       X23       X24        X25 X26
## 1  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 2  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 3  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
```

```
## 4   0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 5   0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 6   0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 7   0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 8   0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 9   0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 10  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 11  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 12  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 13  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 14  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 15  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 16  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 17  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 18  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 19  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 20  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 21  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 22  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 23  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 24  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 25  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 26  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 27  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 28  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 29  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 30  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 31  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 32  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 33  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 34  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 35  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 36  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 37  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 38  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 39  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 40  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 41  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 42  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 43  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 44  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 45  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 46  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 47  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 48  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 49  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 50  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 51  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 52  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 53  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 54  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 55  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 56  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 57  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
```

```
## 58  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 59  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 60  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 61  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 62  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 63  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.920976e-05   0
## 64  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.186748e-04   0
## 65  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.875073e-04   0
## 66  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.561024e-04   0
## 67  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.248391e-04   0
## 68  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.940816e-04   0
## 69  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.641828e-04   0
## 70  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 5.354862e-04   0
## 71  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 6.083300e-04   0
## 72  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 6.830512e-04   0
## 73  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 7.599903e-04   0
## 74  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 8.394967e-04   0
## 75  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 9.219348e-04   0
## 76  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.007691e-03   0
## 77  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.097181e-03   0
## 78  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.190859e-03   0
## 79  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.289229e-03   0
## 80  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.392856e-03   0
## 81  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.502383e-03   0
## 82  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.618554e-03   0
## 83  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.742236e-03   0
## 84  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.874460e-03   0
## 85  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.016461e-03   0
## 86  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.169747e-03   0
## 87  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.336178e-03   0
## 88  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.518096e-03   0
## 89  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.718490e-03   0
## 90  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.941233e-03   0
## 91  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.191560e-03   0
## 92  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.476503e-03   0
## 93  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.805964e-03   0
## 94  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.194411e-03   0
## 95  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.663846e-03   0
## 96  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 5.249580e-03   0
## 97  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 6.012713e-03   0
## 98  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 7.071163e-03   0
## 99  0.00000000   0   0   0   0   0   0   0 0.0000000 0.0000000 8.689907e-03   0
## 100 0.05124424   0   0   0   0   0   0   0 0.1801632 0.1647047 1.279928e-02   0
##      X27 X28 X29 X30 X31
## 1      0   0   0   0   0
## 2      0   0   0   0   0
## 3      0   0   0   0   0
## 4      0   0   0   0   0
## 5      0   0   0   0   0
## 6      0   0   0   0   0
## 7      0   0   0   0   0
## 8      0   0   0   0   0
## 9      0   0   0   0   0
## 10     0   0   0   0   0
```

```
## 11        0        0        0        0        0
## 12        0        0        0        0        0
## 13        0        0        0        0        0
## 14        0        0        0        0        0
## 15        0        0        0        0        0
## 16        0        0        0        0        0
## 17        0        0        0        0        0
## 18        0        0        0        0        0
## 19        0        0        0        0        0
## 20        0        0        0        0        0
## 21        0        0        0        0        0
## 22        0        0        0        0        0
## 23        0        0        0        0        0
## 24        0        0        0        0        0
## 25        0        0        0        0        0
## 26        0        0        0        0        0
## 27        0        0        0        0        0
## 28        0        0        0        0        0
## 29        0        0        0        0        0
## 30        0        0        0        0        0
## 31        0        0        0        0        0
## 32        0        0        0        0        0
## 33        0        0        0        0        0
## 34        0        0        0        0        0
## 35        0        0        0        0        0
## 36        0        0        0        0        0
## 37        0        0        0        0        0
## 38        0        0        0        0        0
## 39        0        0        0        0        0
## 40        0        0        0        0        0
## 41        0        0        0        0        0
## 42        0        0        0        0        0
## 43        0        0        0        0        0
## 44        0        0        0        0        0
## 45        0        0        0        0        0
## 46        0        0        0        0        0
## 47        0        0        0        0        0
## 48        0        0        0        0        0
## 49        0        0        0        0        0
## 50        0        0        0        0        0
## 51        0        0        0        0        0
## 52        0        0        0        0        0
## 53        0        0        0        0        0
## 54        0        0        0        0        0
## 55        0        0        0        0        0
## 56        0        0        0        0        0
## 57        0        0        0        0        0
## 58        0        0        0        0        0
## 59        0        0        0        0        0
## 60        0        0        0        0        0
## 61        0        0        0        0        0
## 62        0        0        0        0        0
## 63        0        0        0        0        0
## 64        0        0        0        0        0
```

```
## 65     0   0   0   0   0
## 66     0   0   0   0   0
## 67     0   0   0   0   0
## 68     0   0   0   0   0
## 69     0   0   0   0   0
## 70     0   0   0   0   0
## 71     0   0   0   0   0
## 72     0   0   0   0   0
## 73     0   0   0   0   0
## 74     0   0   0   0   0
## 75     0   0   0   0   0
## 76     0   0   0   0   0
## 77     0   0   0   0   0
## 78     0   0   0   0   0
## 79     0   0   0   0   0
## 80     0   0   0   0   0
## 81     0   0   0   0   0
## 82     0   0   0   0   0
## 83     0   0   0   0   0
## 84     0   0   0   0   0
## 85     0   0   0   0   0
## 86     0   0   0   0   0
## 87     0   0   0   0   0
## 88     0   0   0   0   0
## 89     0   0   0   0   0
## 90     0   0   0   0   0
## 91     0   0   0   0   0
## 92     0   0   0   0   0
## 93     0   0   0   0   0
## 94     0   0   0   0   0
## 95     0   0   0   0   0
## 96     0   0   0   0   0
## 97     0   0   0   0   0
## 98     0   0   0   0   0
## 99     0   0   0   0   0
## 100    0   0   0   0   0
```

```r
# True estimates from GLM
#as.vector(glm(y ~ X[ , -1], family = binomial)$coefficients)

# True estimates from GLMNET
fit <- glmnet(X, y, family = "binomial", standardize = FALSE, lambda = lambda_seq, thresh = 1e-10)

fit_result <- fit$beta %>% as.matrix() %>% t() %>% data.frame() %>% `rownames<-`(NULL)

for (i in 1:ncol(fit_result)) {
  colnames(fit_result)[i] <- paste0("X",i)
}

fit_result %>% mutate(lambda = fit$lambda %>% round(digits = 6)) %>% relocate(lambda)
```

```
##          lambda X1 X2 X3 X4         X5 X6 X7 X8 X9 X10 X11 X12 X13 X14
## 1    529.920000  0  0  0  0 0.00000000  0  0  0  0   0   0   0   0   0
## 2    524.567808  0  0  0  0 0.00000000  0  0  0  0   0   0   0   0   0
```

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```
## 3    519.215616  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 4    513.863424  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 5    508.511232  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 6    503.159040  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 7    497.806848  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 8    492.454656  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 9    487.102464  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 10   481.750272  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 11   476.398080  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 12   471.045888  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 13   465.693696  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 14   460.341504  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 15   454.989312  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 16   449.637120  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 17   444.284928  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 18   438.932736  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 19   433.580544  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 20   428.228352  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 21   422.876160  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 22   417.523968  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 23   412.171776  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 24   406.819584  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 25   401.467392  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 26   396.115200  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 27   390.763008  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 28   385.410816  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 29   380.058624  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 30   374.706432  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 31   369.354240  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 32   364.002048  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 33   358.649856  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 34   353.297664  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 35   347.945472  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 36   342.593280  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 37   337.241088  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 38   331.888896  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 39   326.536704  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 40   321.184512  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 41   315.832320  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 42   310.480128  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 43   305.127936  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 44   299.775744  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 45   294.423552  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 46   289.071360  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 47   283.719168  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 48   278.366976  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 49   273.014784  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 50   267.662592  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 51   262.310400  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 52   256.958208  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 53   251.606016  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 54   246.253824  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 55   240.901632  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
## 56   235.549440  0  0  0  0  0.00000000  0  0  0  0  0  0  0  0  0
```

```
## 57   230.197248   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 58   224.845056   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 59   219.492864   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 60   214.140672   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 61   208.788480   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 62   203.436288   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 63   198.084096   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 64   192.731904   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 65   187.379712   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 66   182.027520   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 67   176.675328   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 68   171.323136   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 69   165.970944   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 70   160.618752   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 71   155.266560   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 72   149.914368   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 73   144.562176   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 74   139.209984   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 75   133.857792   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 76   128.505600   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 77   123.153408   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 78   117.801216   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 79   112.449024   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 80   107.096832   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 81   101.744640   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 82    96.392448   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 83    91.040256   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 84    85.688064   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 85    80.335872   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 86    74.983680   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 87    69.631488   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 88    64.279296   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 89    58.927104   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 90    53.574912   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 91    48.222720   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 92    42.870528   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 93    37.518336   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 94    32.166144   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 95    26.813952   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 96    21.461760   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 97    16.109568   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 98    10.757376   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 99     5.405184   0   0   0   0   0.00000000   0   0   0   0     0     0     0     0     0
## 100    0.052992   0   0   0   0  -0.01839172   0   0   0   0     0     0     0     0     0
##               X15 X16 X17 X18 X19 X20 X21 X22       X23       X24         X25 X26
## 1     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 2     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 3     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 4     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 5     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 6     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 7     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 8     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
## 9     0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 0.000000e+00   0
```

```
## 10 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 11 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 12 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 13 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 14 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 15 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 16 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 17 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 18 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 19 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 20 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 21 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 22 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 23 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 24 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 25 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 26 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 27 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 28 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 29 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 30 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 31 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 32 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 33 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 34 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 35 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 36 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 37 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 38 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 39 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 40 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 41 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 42 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 43 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 44 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 45 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 46 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 47 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 48 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 49 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 50 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 51 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 52 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 53 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 54 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 55 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 56 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 57 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 58 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 59 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 60 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 61 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 62 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 0.000000e+00 0
## 63 0.0000000 0 0 0 0 0 0 0 0.0000000 0.0000000 4.920976e-05 0
```

```
## 64   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.186748e-04   0
## 65   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.875073e-04   0
## 66   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.561024e-04   0
## 67   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.248391e-04   0
## 68   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.940816e-04   0
## 69   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.641828e-04   0
## 70   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 5.354862e-04   0
## 71   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 6.083300e-04   0
## 72   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 6.830512e-04   0
## 73   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 7.599903e-04   0
## 74   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 8.394967e-04   0
## 75   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 9.219348e-04   0
## 76   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.007691e-03   0
## 77   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.097181e-03   0
## 78   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.190859e-03   0
## 79   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.289229e-03   0
## 80   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.392856e-03   0
## 81   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.502383e-03   0
## 82   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.618554e-03   0
## 83   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.742236e-03   0
## 84   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 1.874460e-03   0
## 85   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.016461e-03   0
## 86   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.169746e-03   0
## 87   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.336178e-03   0
## 88   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.518095e-03   0
## 89   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.718489e-03   0
## 90   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 2.941252e-03   0
## 91   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.191567e-03   0
## 92   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.476507e-03   0
## 93   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 3.805994e-03   0
## 94   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.194478e-03   0
## 95   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 4.663923e-03   0
## 96   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 5.249648e-03   0
## 97   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 6.012874e-03   0
## 98   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 7.071424e-03   0
## 99   0.0000000   0   0   0   0   0   0   0 0.0000000 0.0000000 8.690592e-03   0
## 100 0.0512712   0   0   0   0   0   0   0 0.1802206 0.1647303 1.280587e-02   0
##      X27 X28 X29 X30 X31
## 1      0   0   0   0   0
## 2      0   0   0   0   0
## 3      0   0   0   0   0
## 4      0   0   0   0   0
## 5      0   0   0   0   0
## 6      0   0   0   0   0
## 7      0   0   0   0   0
## 8      0   0   0   0   0
## 9      0   0   0   0   0
## 10     0   0   0   0   0
## 11     0   0   0   0   0
## 12     0   0   0   0   0
## 13     0   0   0   0   0
## 14     0   0   0   0   0
## 15     0   0   0   0   0
## 16     0   0   0   0   0
```

```
## 17    0    0    0    0    0
## 18    0    0    0    0    0
## 19    0    0    0    0    0
## 20    0    0    0    0    0
## 21    0    0    0    0    0
## 22    0    0    0    0    0
## 23    0    0    0    0    0
## 24    0    0    0    0    0
## 25    0    0    0    0    0
## 26    0    0    0    0    0
## 27    0    0    0    0    0
## 28    0    0    0    0    0
## 29    0    0    0    0    0
## 30    0    0    0    0    0
## 31    0    0    0    0    0
## 32    0    0    0    0    0
## 33    0    0    0    0    0
## 34    0    0    0    0    0
## 35    0    0    0    0    0
## 36    0    0    0    0    0
## 37    0    0    0    0    0
## 38    0    0    0    0    0
## 39    0    0    0    0    0
## 40    0    0    0    0    0
## 41    0    0    0    0    0
## 42    0    0    0    0    0
## 43    0    0    0    0    0
## 44    0    0    0    0    0
## 45    0    0    0    0    0
## 46    0    0    0    0    0
## 47    0    0    0    0    0
## 48    0    0    0    0    0
## 49    0    0    0    0    0
## 50    0    0    0    0    0
## 51    0    0    0    0    0
## 52    0    0    0    0    0
## 53    0    0    0    0    0
## 54    0    0    0    0    0
## 55    0    0    0    0    0
## 56    0    0    0    0    0
## 57    0    0    0    0    0
## 58    0    0    0    0    0
## 59    0    0    0    0    0
## 60    0    0    0    0    0
## 61    0    0    0    0    0
## 62    0    0    0    0    0
## 63    0    0    0    0    0
## 64    0    0    0    0    0
## 65    0    0    0    0    0
## 66    0    0    0    0    0
## 67    0    0    0    0    0
## 68    0    0    0    0    0
## 69    0    0    0    0    0
## 70    0    0    0    0    0
```

```
## 71    0    0    0    0    0
## 72    0    0    0    0    0
## 73    0    0    0    0    0
## 74    0    0    0    0    0
## 75    0    0    0    0    0
## 76    0    0    0    0    0
## 77    0    0    0    0    0
## 78    0    0    0    0    0
## 79    0    0    0    0    0
## 80    0    0    0    0    0
## 81    0    0    0    0    0
## 82    0    0    0    0    0
## 83    0    0    0    0    0
## 84    0    0    0    0    0
## 85    0    0    0    0    0
## 86    0    0    0    0    0
## 87    0    0    0    0    0
## 88    0    0    0    0    0
## 89    0    0    0    0    0
## 90    0    0    0    0    0
## 91    0    0    0    0    0
## 92    0    0    0    0    0
## 93    0    0    0    0    0
## 94    0    0    0    0    0
## 95    0    0    0    0    0
## 96    0    0    0    0    0
## 97    0    0    0    0    0
## 98    0    0    0    0    0
## 99    0    0    0    0    0
## 100   0    0    0    0    0
```