$$L_X(\theta) = \sum \left[y_i \log(p_i) + (1-y_i)\log(1-p_i) \right]$$

$$f(B_0, B_1, \ldots B_{30}) = \sum_{i=1}^{n} \left(Y_i \left(B_0 + B_1 x_i + \ldots + B_{30} x_i\right) - \log\left(1 + e^{(B_0 + \ldots + B_{30} x_i)}\right) \right)$$

$$\nabla f(B_0, \ldots, B_{30}) = \begin{pmatrix} \sum Y_i - p_i \\ \sum_{i=1}^{n} x_i (Y_i - p_i) \\ \vdots \\ \sum_{i=1}^{n} x_i (Y_i - p_i) \end{pmatrix} = \sum x_i (y_i - p_i) = X^T (y - p)$$

$$\nabla^2 f(B_0, \ldots, B_{30}) = -\sum_i \pi_i (1 - \pi_i) x_i x_i^T$$

$$= -X^T \mathrm{diag}(\pi_i (1 - \pi_i)) X = -X^T W X$$

where $\pi_i = \mathrm{sigm}(x_i, \theta)$

$$B_{new} = B_{old} - \frac{\nabla f}{\nabla^2 f}$$

X: 455×31 (fixed)

p: 455×1 (updated)

y: 455×1 (fixed)

Logistic Newton Raphson Full Model

Hun

3/17/2022

```
cancer_df <- read.csv("~/Downloads/breast-cancer.csv") %>% janitor::clean_names()
```

```
data <-
  cancer_df %>% dplyr::select(-id, -x) %>%
  mutate(diagnosis = ifelse(diagnosis == "M", 1, 0)) %>% distinct()

set.seed(7777)
split <- initial_split(data, prop = 0.8)

training_df <- split %>% training()

testing_df <- split %>% testing()
```

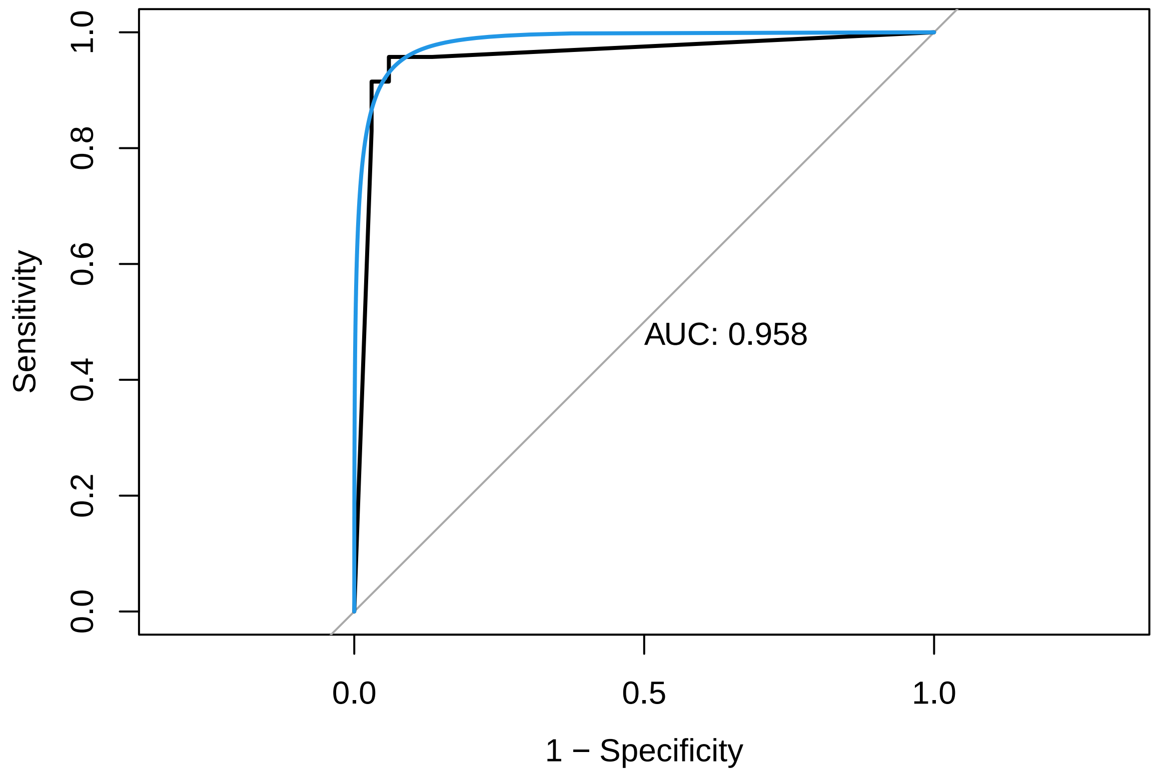
```
training_df_5p <- training_df %>% dplyr::select(1:5)
training_df_31p <- training_df

model_5p <- glm(diagnosis ~ ., data = training_df_5p, family = "binomial")
model_31p <- glm(diagnosis ~ ., data = training_df_31p, family = "binomial")
```

```
beta1 <- model_5p$coefficients %>% round(digits = 3) %>% broom::tidy()
```

```
beta2 <- model_31p$coefficients %>% round(digits = 3) %>% broom::tidy()
```

```
test_pred_prob <- predict(model_31p, testing_df, type = "response")
roc.glm <- roc(testing_df$diagnosis, test_pred_prob)
plot(roc.glm, legacy.axes = TRUE, print.auc = TRUE)
plot(smooth(roc.glm), col = 4, add = TRUE)
```



Function for log likelihood, gradient, and Hessian

```
logisticstuff <- function(X, y, beta) {
  p <- exp(X %*% beta) / (1+ exp(X %*% beta)) %>% as.vector()
  for (i in 1:length(p)) {
    if (p[i] == 1) {
      p[i] <- 1-1e-8
    }
  }
  loglik <- t(y) %*% log(p) + t(1-y)  %*% log(1-p)
  grad <- t(X) %*% (y-p)
  W <- diag(c(p*(1-p)))
  Hess <- -t(X) %*% W %*% X
  return(list(loglik = loglik, grad = grad, Hess = Hess))
}
```

Newton Raphson with 5 parameters

```
X <- model.matrix(diagnosis~., training_df_5p)
y <- as.matrix(training_df$diagnosis)

NewtonRaphson <- function(X, y, logit_func, start, tol=1e-10, maxiter = 200) {
   i <- 0
   cur_beta <- start
   stuff <- logit_func(X, y, cur_beta)
   asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
   res <- c(i, stuff$loglik, asc_dir_check, cur_beta)
   prevloglik <- -Inf # To make sure it iterates

   while (i < maxiter && abs(stuff$loglik - prevloglik) > tol) {
     i <- i + 1
     prevloglik <- stuff$loglik
     prev_beta <- cur_beta
     cur_beta <- prev_beta - (solve(stuff$Hess) %*% stuff$grad) #update beta
     stuff <- logit_func(X, y, cur_beta) #update log likelihood, gradient, Hessia
     asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
     res <- rbind(res, c(i, stuff$loglik, asc_dir_check, cur_beta))
     colnames(res) <- c("Number of trial", "Log_likelihood", "asc_dir_check", paste0("Beta", 0:4))

   }
   return(res)
}

coef <- rep(0,ncol(X)) # Randomly assigned coefficients (starting point)

ans <- NewtonRaphson(X, y, logisticstuff, coef) %>% data.frame() %>% `rownames<-`( NULL )
ans %>% kbl(caption = "Newton Raphson result with 5 parameters") %>%
   kable_styling(font_size = 8, latex_options = "HOLD_position")
```

Table 1: Newton Raphson result with 5 parameters

Number.of.trial	Log_likelihood	asc_dir_check	Beta0	Beta1	Beta2	Beta3	Beta4
0	-315.38197	297.4027390	0.0000000	0.000000	0.0000000	0.0000000	0.0000000
1	-141.92524	64.4620788	-8.2741164	-1.259642	0.0795466	0.2861187	-0.0034815
2	-100.99962	25.2067494	-12.8134881	-2.880231	0.1358457	0.5775260	-0.0044236
3	-85.08700	9.5366502	-13.7360236	-4.995863	0.1916826	0.8709006	-0.0001595
4	-79.07361	3.3687360	-8.6473294	-7.581528	0.2344300	1.1188065	0.0123355
5	-77.06852	0.4321716	-0.2645781	-10.005234	0.2577985	1.2831011	0.0289108
6	-76.83723	0.0057185	2.8702542	-10.980122	0.2685915	1.3503646	0.0357635
7	-76.83435	0.0000010	3.1959332	-11.094010	0.2700253	1.3588134	0.0365377
8	-76.83435	0.0000000	3.2001911	-11.095519	0.2700442	1.3589267	0.0365478
9	-76.83435	0.0000000	3.2001918	-11.095520	0.2700442	1.3589267	0.0365478

```
## True Beta0: -3.2 , Beta1: -11.096 , Beta2: 0.27 , Beta3: 1.359 , Beta4: 0.037
```

Newton Raphson with all 31 parameters

```
logisticstuff <- function(X, y, beta) {
  p <- exp(X%*%beta) / (1 + exp(X%*%beta)) %>% as.vector()
  for (i in 1:length(p)) {
    if (p[i] == 1) {
      p[i] <- 1-2e-8
    }
  }
  loglik <- t(y) %*% log(p) + t(1-y)  %*% log(1-p)
  grad <- t(X) %*% (y-p)
  W <- diag(c(p*(1-p)))
  Hess <- -t(X) %*% W %*% X
  return(list(loglik = loglik, grad = grad, Hess = Hess))
}

X <- model.matrix(diagnosis~., training_df_31p)
y <- as.matrix(training_df$diagnosis)

NewtonRaphson <- function(X, y, logit_func, start, tol=1e-10, maxiter = 43) {
   i <- 0
   cur_beta <- start
   stuff <- logit_func(X, y, cur_beta)
   asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
   res <- c(i, stuff$loglik, asc_dir_check, cur_beta)
   prevloglik <- -Inf # To make sure it iterates

   while (i < maxiter && abs(stuff$loglik - prevloglik) > tol) {
     i <- i + 1
     prevloglik <- stuff$loglik
     prev_beta <- cur_beta
     cur_beta <- prev_beta - (solve(stuff$Hess) %*% stuff$grad) #update beta
     stuff <- logit_func(X, y, cur_beta) #update log likelihood, gradient, Hessia
     asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
     res <- rbind(res, c(i, stuff$loglik, asc_dir_check, cur_beta))
     colnames(res) <- c("Number_of_trial", "Log_likelihood", "asc_dir_check", paste0("Beta", 0:30))
   }
   return(res)
}

coef <- rep(0,ncol(X)) # Randomly assigned coefficients (starting point)
ans <- NewtonRaphson(X, y, logisticstuff, coef) %>% data.frame() %>% `rownames<-`( NULL )
ans %>% kbl(caption = "Newton Raphson result with 31 parameters") %>%
   kable_styling(font_size = 8, latex_options = "HOLD_position")
```

Table 2: Newton Raphson result with 31 parameters

Number_of_trial	Log_likelihood	asc_dir_check	Beta0	Beta1	Beta2	Beta3	Beta4	Beta5
0	-315.3819672	363.3863080	0.00000	0.0000000	0.0000000	0.0000000	0.0000000	0.00000
1	-105.0649679	72.6858805	-10.22033	-0.9360000	0.0252396	0.0861849	0.0023765	3.64103
2	-58.3133087	34.7575625	-17.49844	-0.9011090	0.0147350	0.0982375	0.0023645	11.36727
3	-35.6372927	18.8281790	-25.45537	-0.9217526	-0.0361649	0.1557306	0.0014547	19.33863
4	-23.2348067	12.0995319	-36.38389	-1.3710840	-0.0871143	0.2339244	0.0016870	24.70603
5	-15.0488775	10.0318418	-56.77922	-1.9465817	-0.0753320	0.2490561	0.0028944	44.85133
6	-8.2985550	7.2261060	-91.13019	-3.8130106	0.0045095	0.2249788	0.0152424	101.57375
7	-3.6072702	3.4217703	-143.73478	-6.9205335	0.1232558	0.3089762	0.0326358	173.14779
8	-1.4255469	1.3794906	-204.19664	-10.2275614	0.1985325	0.4651119	0.0466945	268.69662
9	-0.5489105	0.5370347	-265.83920	-14.7424919	0.2491523	0.7565496	0.0641260	380.75184
10	-0.2081442	0.2050172	-327.39849	-20.8328954	0.2917701	1.1944761	0.0883038	507.67285
11	-0.0781367	0.0769959	-387.87498	-28.5404951	0.3403048	1.7432844	0.1223028	643.62385
12	-0.0292694	0.0285672	-448.15400	-37.3858578	0.4003243	2.3821901	0.1625516	781.60031
13	-0.0110700	0.0105491	-511.49230	-47.1248196	0.4628012	3.2150915	0.1999765	924.99196
14	-0.0042974	0.0039108	-581.39421	-58.5908834	0.5219444	4.4926386	0.2274242	1080.59892
15	-0.0017537	0.0014675	-659.95049	-71.8607992	0.5704151	6.3975175	0.2331574	1235.62022
16	-0.0007773	0.0005637	-743.86695	-84.8716500	0.5851627	8.7517464	0.2063349	1360.26572
17	-0.0003896	0.0002290	-825.33800	-96.1825906	0.5380378	11.2189731	0.1563408	1449.67579
18	-0.0002265	0.0001058	-894.49775	-104.8654488	0.4549612	13.2799599	0.1084977	1515.13021
19	-0.0001500	0.0000577	-948.24398	-111.1803630	0.3844162	14.8097315	0.0717586	1564.34692
20	-0.0001088	0.0000362	-989.42316	-115.9561635	0.3380406	15.9423143	0.0450139	1601.37158
21	-0.0000838	0.0000251	-1021.77072	-119.7020744	0.3101736	16.8003212	0.0254486	1629.61455
22	-0.0000673	0.0000187	-1048.12706	-122.7243020	0.2944686	17.4688006	0.0107932	1651.59365
23	-0.0000557	0.0000147	-1070.34397	-125.2189261	0.2862967	18.0036925	-0.0004920	1669.30957
24	-0.0000471	0.0000120	-1089.57958	-127.3208215	0.2827493	18.4417318	-0.0093852	1684.14261
25	-0.0000405	0.0000101	-1106.57767	-129.1241741	0.2821007	18.8073906	-0.0165144	1696.96844
26	-0.0000353	0.0000087	-1121.84171	-130.6944755	0.2833342	19.1173456	-0.0223029	1708.34483
27	-0.0000311	0.0000077	-1135.72707	-132.0787108	0.2858292	19.3833668	-0.0270463	1718.63472
28	-0.0000277	0.0000069	-1148.49342	-133.3115746	0.2891929	19.6140282	-0.0309581	1728.08208
29	-0.0000249	0.0000062	-1160.33592	-134.4193253	0.2931695	19.8157464	-0.0341966	1736.85646
30	-0.0000225	0.0000057	-1171.40474	-135.4222510	0.2975886	19.9934334	-0.0368822	1745.07935
31	-0.0000204	0.0000053	-1181.88530	-136.3696944	0.3029300	20.1582094	-0.0392283	1752.73140
32	-0.0000187	0.0000050	-1191.80581	-137.2400652	0.3081387	20.3055441	-0.0411818	1760.10577
33	-0.0000171	0.0000047	-1201.23849	-138.0441882	0.3134401	20.4380993	-0.0428086	1767.14707
34	-0.0000158	0.0000044	-1210.24761	-138.7906760	0.3188614	20.5579467	-0.0441587	1773.88132
35	-0.0000146	0.0000042	-1218.88663	-139.4865934	0.3243979	20.6667661	-0.0452721	1780.34116
36	-0.0000136	0.0000040	-1227.20008	-140.1378168	0.3300422	20.7659482	-0.0461820	1786.55470
37	-0.0000127	0.0000039	-1235.22547	-140.7492787	0.3357891	20.8566603	-0.0469160	1792.54497
38	-0.0000119	0.0000037	-1243.04230	-141.3264902	0.3406743	20.9406962	-0.0475476	1798.31437
39	-0.0000111	0.0000036	-1250.61131	-141.8726644	0.3461432	21.0181468	-0.0480439	1803.88539
40	-0.0000105	0.0000035	-1257.96833	-142.3895255	0.3518382	21.0896134	-0.0484238	1809.28189
41	-0.0000099	0.0000034	-1265.13967	-142.8796968	0.3576664	21.1557643	-0.0487040	1814.51722
42	-0.0000093	0.0000033	-1272.14572	-143.3456263	0.3636008	21.2171900	-0.0488987	1819.60084
43	-0.0000089	0.0000032	-1279.00343	-143.7894612	0.3696339	21.2744032	-0.0490201	1824.54059

Table 3: Failed Newton Rapshon for 31 parameters

Result	values
Number_of_trial	42
Log_likelihood	-0.000009341111
asc_dir_check	0.000003304651
Beta0	-1272.146
Beta1	-143.3456
Beta2	0.3636008
Beta3	21.21719
Beta4	-0.04889868
Beta5	1819.601
Beta6	-1630.311
Beta7	929.9978
Beta8	342.2007
Beta9	-442.6396
Beta10	2262.871
Beta11	-28.39532
Beta12	-22.86274
Beta13	-3.83439
Beta14	2.290926
Beta15	12389.41
Beta16	5063.34
Beta17	-1606.132
Beta18	471.5062
Beta19	-5288.991
Beta20	-45056.05
Beta21	121.76
Beta22	5.032851
Beta23	-5.146386
Beta24	-0.7058646
Beta25	-923.8527
Beta26	-379.2067
Beta27	124.5956
Beta28	502.6894
Beta29	703.1726
Beta30	2613.257

Table 4: True coefficients of 31 parameters

names	x
(Intercept)	-1293.417
radius_mean	-172.482
texture_mean	1.066
perimeter_mean	13.475
area_mean	0.718
smoothness_mean	2785.956
compactness_mean	-1775.940
concavity_mean	1424.757
concave_points_mean	473.400
symmetry_mean	-345.545
fractal_dimension_mean	3672.362
radius_se	-254.806
texture_se	-42.316
perimeter_se	-7.104
area_se	4.773
smoothness_se	15391.485
compactness_se	5711.477
concavity_se	-2162.101
concave_points_se	4968.934
symmetry_se	-7212.775
fractal_dimension_se	-55050.387
radius_worst	178.104
texture_worst	6.587
perimeter_worst	-6.294
area_worst	-1.114
smoothness_worst	-1623.965
compactness_worst	-402.553
concavity_worst	160.429
concave_points_worst	134.278
symmetry_worst	875.003
fractal_dimension_worst	3118.667

Modified Newton Raphson with 5 number of parameters

```r
logisticstuff <- function(X, y, beta) {
  p <- exp(X%*%beta) / (1+ exp(X%*%beta)) %>% as.vector()
  for (i in 1:length(p)) {
    if (p[i] == 1) {
      p[i] <- 1-1e-8
    }
  }
  loglik <- t(y) %*% log(p) + t(1-y)  %*% log(1-p)
  grad <- t(X) %*% (y-p)
  W <- diag(c(p*(1-p)))
  Hess <- -t(X) %*% W %*% X
  return(list(loglik = loglik, grad = grad, Hess = Hess))
}

X <- model.matrix(diagnosis~., training_df_5p)
y <- as.matrix(training_df$diagnosis)

NewtonRaphson_mod <- function(X, y, logit_func, start, tol=1e-10, maxiter = 200) {
   i <- 0
   cur_beta <- start
   stuff <- logit_func(X, y, cur_beta)
   asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
   res <- c(i, stuff$loglik, asc_dir_check, cur_beta)
   prevloglik <- -Inf # To make sure it iterates
   lambda <- 1 #initial random lambda

   while (i < maxiter && abs(stuff$loglik - prevloglik) > tol) {
   i <- i + 1
   prev_beta <- cur_beta

   #checking if direction is ascent. If not, transform Hessian into negative definite.
   if (asc_dir_check < 0) {
      stuff$Hess = stuff$Hess - (max(stuff$Hess) + 5)
      prev_beta <- prev_beta - lambda * (solve(stuff$Hess) %*% stuff$grad)
      stuff <- logit_func(X, y, prev_beta)
      prevloglik <- stuff$loglik
   }

   else {
      prev_beta <- prev_beta - lambda * (solve(stuff$Hess) %*% stuff$grad)
      stuff <- logit_func(X, y, prev_beta)
      prevloglik <- stuff$loglik
   }

   cur2_beta <- prev_beta - lambda * (solve(stuff$Hess) %*% stuff$grad)
   stuff2 <- logit_func(X, y, cur2_beta)

   #condition check before step halving process
   if (stuff2$loglik > prevloglik) {
      cur_beta = cur2_beta
      stuff = stuff2
```

```
    asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
  }

  #step halving process
  else {
    repeat {
    lambda2 = lambda/2
    cur_beta = prev_beta - lambda2 * (solve(stuff$Hess) %*% stuff$grad)
    stuff <- logit_func(X, y, cur_beta)
    if (stuff$loglik > prevloglik) {
    cur_beta = cur_beta
    stuff = stuff
    asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
    }
    break}
    }

  res <- rbind(res, c(i, stuff$loglik, asc_dir_check, cur_beta))
  colnames(res) <- c("Number of trial", "Log_likelihood", "asc_dir_check", paste0("Beta", 0:4))
  }
  return(res)
}

coef <- rep(0,ncol(X)) # Randomly assigned coefficients (starting point)

ans <- NewtonRaphson_mod(X, y, logisticstuff, coef) %>% data.frame() %>% `rownames<-`( NULL )
ans %>% kbl(caption = "Newton Raphson result with 5 parameters") %>%
  kable_styling(font_size = 8, latex_options = "HOLD_position")
```

Table 5: Newton Raphson result with 5 parameters

Number.of.trial	Log_likelihood	asc_dir_check	Beta0	Beta1	Beta2	Beta3	Beta4
0	-315.38197	297.4027390	0.000000	0.000000	0.0000000	0.000000	0.0000000
1	-100.99962	25.2067494	-12.813488	-2.880231	0.1358457	0.577526	-0.0044236
2	-79.07361	3.3687360	-8.647329	-7.581528	0.2344300	1.118807	0.0123355
3	-76.83723	0.0057185	2.870254	-10.980122	0.2685915	1.350365	0.0357635
4	-76.83435	0.0000000	3.200191	-11.095519	0.2700442	1.358927	0.0365478
5	-76.83435	0.0000000	3.200192	-11.095520	0.2700442	1.358927	0.0365478

Modified Newton Raphson with all 31 parameters

```
logisticstuff <- function(X, y, beta) {
  p <- exp(X%*%beta) / (1 + exp(X%*%beta)) %>% as.vector()
  for (i in 1:length(p)) {
    if (p[i] == 1) {
      p[i] <- 1-2e-8
    }
  }
  loglik <- t(y) %*% log(p) + t(1-y)  %*% log(1-p)
  grad <- t(X) %*% (y-p)
  W <- diag(c(p*(1-p)))
```

```
  Hess <- -t(X) %*% W %*% X
  return(list(loglik = loglik, grad = grad, Hess = Hess))
}

X <- model.matrix(diagnosis~., training_df_31p)
y <- as.matrix(training_df$diagnosis)

NewtonRaphson_mod <- function(X, y, logit_func, start, tol=1e-10, maxiter = 21) {
   i <- 0
   cur_beta <- start
   stuff <- logit_func(X, y, cur_beta)
   asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
   res <- c(i, stuff$loglik, asc_dir_check, cur_beta)
   prevloglik <- -Inf # To make sure it iterates
   lambda <- 1 #initial random lambda

   while (i < maxiter && abs(stuff$loglik - prevloglik) > tol) {
   i <- i + 1
   prev_beta <- cur_beta

   #checking if direction is ascent. If not, transform Hessian into negative definite.
   if (asc_dir_check < 0) {
      stuff$Hess = stuff$Hess - (max(stuff$Hess) + 5)
      prev_beta <- prev_beta - lambda * (solve(stuff$Hess) %*% stuff$grad)
      stuff <- logit_func(X, y, prev_beta)
      prevloglik <- stuff$loglik
   }

   else {
      prev_beta <- prev_beta - lambda * (solve(stuff$Hess) %*% stuff$grad)
      stuff <- logit_func(X, y, prev_beta)
      prevloglik <- stuff$loglik
   }

   cur2_beta <- prev_beta - lambda * (solve(stuff$Hess) %*% stuff$grad)
   stuff2 <- logit_func(X, y, cur2_beta)

   #condition check before step halving process
   if (stuff2$loglik > prevloglik) {
      cur_beta = cur2_beta
      stuff = stuff2
      asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
   }

   #step halving process
   else {
      repeat {
      lambda2 = lambda/2
      cur_beta = prev_beta - lambda2 * (solve(stuff$Hess) %*% stuff$grad)
      stuff <- logit_func(X, y, cur_beta)
      if (stuff$loglik > prevloglik) {
      cur_beta = cur_beta
      stuff = stuff
```

```r
      asc_dir_check <- -t(stuff$grad) %*% solve(stuff$Hess) %*% stuff$grad
      }
      break}
      }

   res <- rbind(res, c(i, stuff$loglik, asc_dir_check, cur_beta))
   colnames(res) <- c("Number of trial", "Log_likelihood", "asc_dir_check", paste0("Beta", 0:30))
   }
   return(res)
}

coef <- rep(0,ncol(X)) # Randomly assigned coefficients (starting point)

ans <- NewtonRaphson_mod(X, y, logisticstuff, coef) %>% data.frame() %>% `rownames<-`( NULL )
ans %>% kbl(caption = "Newton Raphson result with 5 parameters") %>%
   kable_styling(font_size = 8, latex_options = "HOLD_position")
```

Table 6: Newton Raphson result with 5 parameters

Number.of.trial	Log_likelihood	asc_dir_check	Beta0	Beta1	Beta2	Beta3	Beta4	Beta5	
0	-315.3819672	363.3863080	0.00000	0.000000	0.0000000	0.0000000	0.0000000	0.00000	
1	-58.3133087	34.7575625	-17.49844	-0.901109	0.0147350	0.0982375	0.0023645	11.36727	-
2	-23.2348067	12.0995319	-36.38389	-1.371084	-0.0871143	0.2339244	0.0016870	24.70603	-
3	-8.2985550	7.2261060	-91.13019	-3.813011	0.0045095	0.2249788	0.0152424	101.57375	-
4	-1.4255469	1.3794906	-204.19664	-10.227561	0.1985325	0.4651119	0.0466945	268.69662	-2
5	-0.2081442	0.2050172	-327.39849	-20.832895	0.2917701	1.1944761	0.0883038	507.67285	-3
6	-0.0292694	0.0285672	-448.15400	-37.385858	0.4003243	2.3821901	0.1625516	781.60031	-5
7	-0.0042974	0.0039108	-581.39421	-58.590883	0.5219444	4.4926386	0.2274242	1080.59892	-7
8	-0.0007773	0.0005637	-743.86695	-84.871650	0.5851627	8.7517464	0.2063349	1360.26572	-9
9	-0.0002265	0.0001058	-894.49775	-104.865449	0.4549612	13.2799599	0.1084977	1515.13021	-11
10	-0.0001088	0.0000362	-989.42316	-115.956164	0.3380406	15.9423143	0.0450139	1601.37158	-12
11	-0.0000673	0.0000187	-1048.12706	-122.724302	0.2944686	17.4688006	0.0107932	1651.59365	-13
12	-0.0000471	0.0000120	-1089.57958	-127.320821	0.2827493	18.4417318	-0.0093852	1684.14261	-14
13	-0.0000353	0.0000087	-1121.84171	-130.694476	0.2833342	19.1173456	-0.0223029	1708.34483	-14
14	-0.0000277	0.0000069	-1148.49342	-133.311575	0.2891929	19.6140282	-0.0309581	1728.08208	-14
15	-0.0000225	0.0000057	-1171.40474	-135.422251	0.2975886	19.9934334	-0.0368822	1745.07935	-15
16	-0.0000187	0.0000050	-1191.80581	-137.240065	0.3081387	20.3055441	-0.0411818	1760.10577	-15
17	-0.0000158	0.0000044	-1210.24761	-138.790676	0.3188614	20.5579467	-0.0441587	1773.88132	-15
18	-0.0000136	0.0000040	-1227.20008	-140.137817	0.3300422	20.7659482	-0.0461820	1786.55470	-15
19	-0.0000119	0.0000037	-1243.04230	-141.326490	0.3406743	20.9406962	-0.0475476	1798.31437	-15
20	-0.0000105	0.0000035	-1257.96833	-142.389525	0.3518382	21.0896134	-0.0484238	1809.28189	-16
21	-0.0000093	0.0000033	-1272.14572	-143.345626	0.3636008	21.2171900	-0.0488987	1819.60084	-16

Conundrum:

It is to be observed most of the β_i increase as Newton Raphson algorithm proceeds. This causes some of the elements in p vector to be very close to 1, leading some of the elements in log(1-p) vector to be negative infinity and hence the next log likelihood to diverge to negative infinity. As a result, Newton Raphson algorithm cannot go further till its convergence of maximum likelihood estimation.

Proof for why Newton Raphson algorithm cannot reach convergence

```r
true_beta_vector <- beta2[2] %>% pull()

p <- exp(X %*% true_beta_vector) / (1 + exp(X %*% true_beta_vector))

which(p == 1)
```

```
##   [1]   3  15  19  21  25  26  29  30  33  37  38  39  41  42  44  45  48  52
##  [19]  58  60  62  68  70  71  72  76  84  88  92  95  97  99 100 103 104 107
##  [37] 108 109 110 111 113 117 119 121 122 128 129 133 138 139 141 146 150 153
##  [55] 161 169 171 185 186 188 190 196 197 198 199 201 202 204 206 210 222 224
##  [73] 227 239 244 260 261 263 265 266 268 270 271 274 277 281 285 287 288 289
##  [91] 290 294 296 300 305 307 309 325 327 328 331 334 335 339 343 345 348 352
## [109] 356 361 368 371 373 374 375 381 383 387 389 391 392 394 398 399 402 403
## [127] 406 411 412 418 421 423 426 429 430 432 437 438 439 441 444 453 455
```

```r
log_likelihood <- t(y) %*% log(p) + t(1-y)  %*% log(1-p)

log_likelihood
```

```
##      [,1]
## [1,]  NaN
```