

P8160 - Breast Cancer Data: To lasso or to not lasso

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2022-03-28

Motivation

Diagnosing breast cancer is extremely important.

According to NIH there has been an estimated:

- ▶ 281,550 new cases of breast cancer in women in 2021,
- ▶ 43,600 breast cancer in women related deaths in 2021.

American Cancer Society Guideline for Breast Cancer Screening:

- ▶ Women between ages 25-40 should have an annual clinical breast examination.
- ▶ Women between ages 40-44 should begin annual screening via mammogram
- ▶ Women between ages 45-54 should screened annually via mammogram

Goal

With using all the collected image data we want to develop an algorithm to predict diagnosis. Since diagnosis is a binary outcome a logistic regression will be utilized.

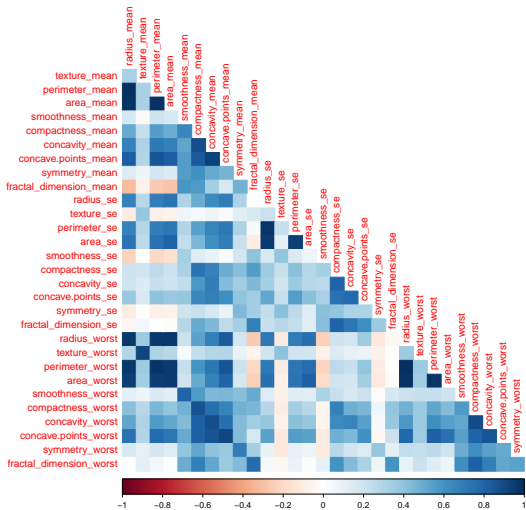
Methods:

- ▶ Newton-Raphson Algorithm (Full Model)
- ▶ Logistic LASSO Algorithm (Optimal Model)

Data

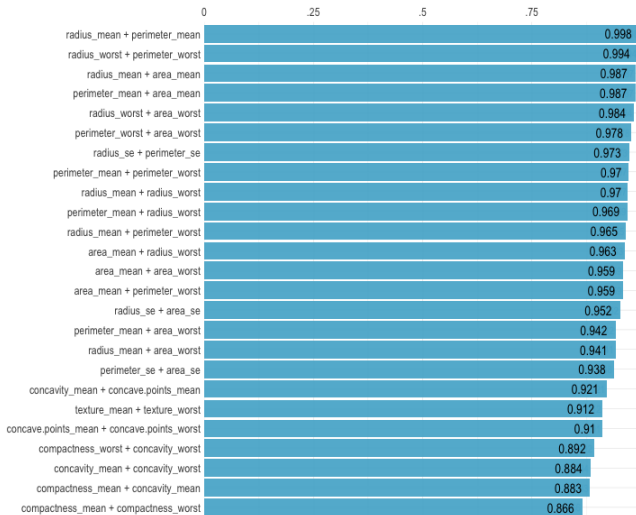
- ▶ 569 rows and 31 columns all related to breast tissue images
- ▶ Outcome of interest: Diagnosis (B or M)
 - ▶ 357 benign (B) cases and 212 malignant (M) cases
- ▶ The covariates include measurements such as radius, texture, perimeter, area, smoothness, compactness, concavity, concave points, symmetry, and fractal dimension.

Correlation Heat Plot of all Covariates



Ranking Cross-Correlations

25 most relevant



Identifying Equivalence Groups

25 most relevant



Best Proxy: radius_worst

Identifying Equivalence Groups

25 most relevant



Best Proxy: radius_se

Finally Selected Variables

Variable	Diagnosis Received		
	B, N = 357 [†]	M, N = 212 [†]	p-value [‡]
texture_mean	17.91 (4.00)	21.60 (3.78)	<0.001
smoothness_mean	0.09 (0.01)	0.10 (0.01)	<0.001
compactness_mean	0.08 (0.03)	0.15 (0.05)	<0.001
concave points_mean	0.03 (0.02)	0.09 (0.03)	<0.001
symmetry_mean	0.17 (0.02)	0.19 (0.03)	<0.001
fractal_dimension_mean	0.06 (0.01)	0.06 (0.01)	0.5
radius_se	0.28 (0.11)	0.61 (0.35)	<0.001
texture_se	1.22 (0.59)	1.21 (0.48)	0.6
smoothness_se	0.01 (0.00)	0.01 (0.00)	0.2
compactness_se	0.02 (0.02)	0.03 (0.02)	<0.001
concavity_se	0.03 (0.03)	0.04 (0.02)	<0.001
concave points_se	0.01 (0.01)	0.02 (0.01)	<0.001
symmetry_se	0.02 (0.01)	0.02 (0.01)	0.028
fractal_dimension_se	0.00 (0.00)	0.00 (0.00)	<0.001
radius_worst	13.38 (1.98)	21.13 (4.28)	<0.001
smoothness_worst	0.12 (0.02)	0.14 (0.02)	<0.001
compactness_worst	0.18 (0.09)	0.37 (0.17)	<0.001
concavity_worst	0.17 (0.14)	0.45 (0.18)	<0.001
symmetry_worst	0.27 (0.04)	0.32 (0.07)	<0.001
fractal_dimension_worst	0.08 (0.01)	0.09 (0.02)	<0.001

[†] Statistics presented: Mean (SD)

[‡] Statistical tests performed: Wilcoxon rank-sum test

Full Model (Newton-Raphson)

Consider the following log-likelihood, gradient, and hessian matrix.
First, let

$$\pi_i = P(Y_i = 1 | x_{i,1}, \dots, x_{i,p}) = \frac{e^{\beta_0 + \sum_{j=1}^p \beta_j x_{i,j}}}{1 + e^{\beta_0 + \sum_{j=1}^p \beta_j x_{i,j}}}.$$

The log-likelihood:

$$l(\mathbf{X} | \vec{\beta}) = \sum_{i=1}^n \left[y_i \left(\beta_0 + \sum_{j=1}^p \beta_j x_{i,j} \right) - \log \left(1 + \exp \left(\beta_0 + \sum_{j=1}^p \beta_j x_{i,j} \right) \right) \right]$$

The gradient:

$$\nabla l(\mathbf{X} | \vec{\beta}) = \left[\sum^n y_i - \pi_i \quad \sum^n x_{i,1}(y_i - \pi_i) \quad \dots \quad \sum^n x_{i,p}(y_i - \pi_i) \right]_{1 \times (p+1)}^T$$

The hessian: produces a matrix $(p + 1 \times p + 1)$

$$\nabla^2 l(\mathbf{X} | \vec{\beta}) = - \sum_{i=1}^n \begin{pmatrix} 1 \\ \mathbf{x} \end{pmatrix} \begin{pmatrix} 1 & \mathbf{x} \end{pmatrix} \pi_i (1 - \pi_i)$$

Optimal Model (Logistic LASSO)

Lemma 1. Consider the optimization problem

$$\min_{x \in \mathbb{R}} \left\{ \frac{1}{2}(x - b)^2 + c|x| \right\}$$

for $b \in \mathbb{R}$ and $c \in \mathbb{R}_{++}$. It follows that the minimizer is given by

$$\hat{x} = S(b, c),$$

where S is the soft-thresholding operator.

Lemma 2. Consider the optimization problem

$$\min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^n w_i \left(z_i - \sum_{j=1}^p \beta_j x_{ij} \right)^2 \right\}$$

for some $k \in \{1, \dots, p\}$. It follows that the minimizer is given by

$$\hat{\beta}_k = \left(\sum_{i=1}^n w_i x_{ik}^2 \right)^{-1} \sum_{i=1}^n w_i x_{ik} \left(z_i - \sum_{j \neq k} \beta_j x_{ij} \right).$$

Optimal Model (Logistic LASSO)

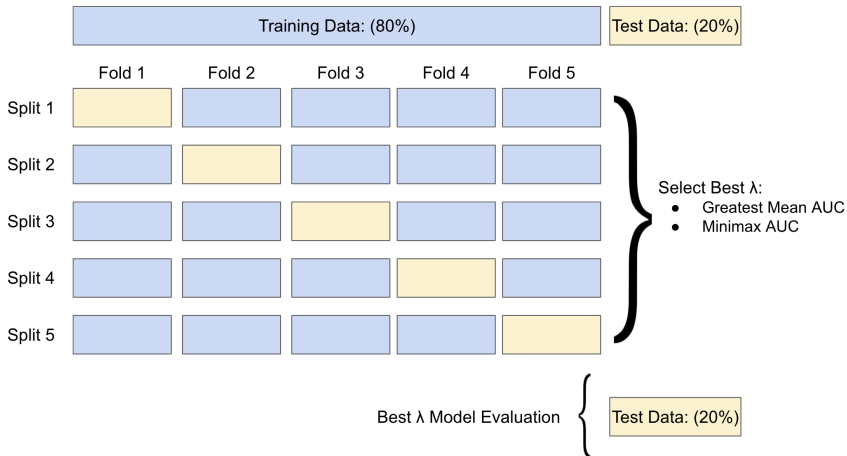
Lemma 3. With $\hat{\beta}_k$ defined as above,

$$\begin{aligned} \min_{\beta_k \in \mathbb{R}} & \left\{ \frac{1}{2n} \sum_{i=1}^n w_i \left(z_i - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p |\beta_j| \right\} \\ &= \min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2} (\beta_k - \hat{\beta}_k)^2 + \left(\frac{1}{n} \sum_{i=1}^n w_i x_{ik}^2 \right)^{-1} \lambda |\beta_k| \right\}. \end{aligned}$$

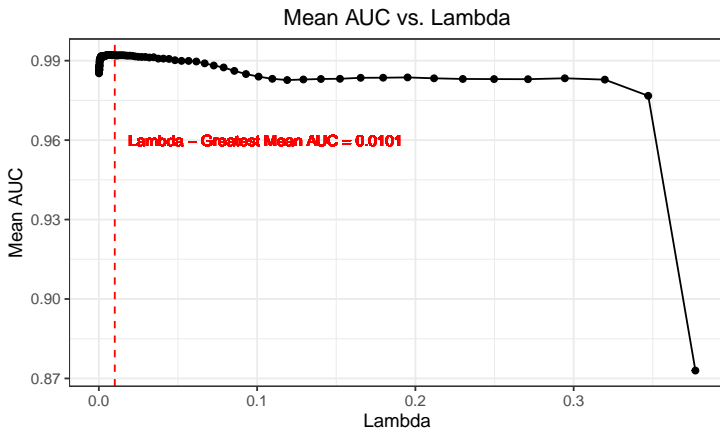
By Lemma 1 and Lemma 3,

$$\begin{aligned} \arg \min_{\beta_k \in \mathbb{R}} & \left\{ \frac{1}{2n} \sum_{i=1}^n w_i \left(z_i - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p |\beta_j| \right\} \\ &= S \left(\hat{\beta}_k, \left(\frac{1}{n} \sum_{i=1}^n w_i x_{ik}^2 \right)^{-1} \lambda \right) \end{aligned}$$

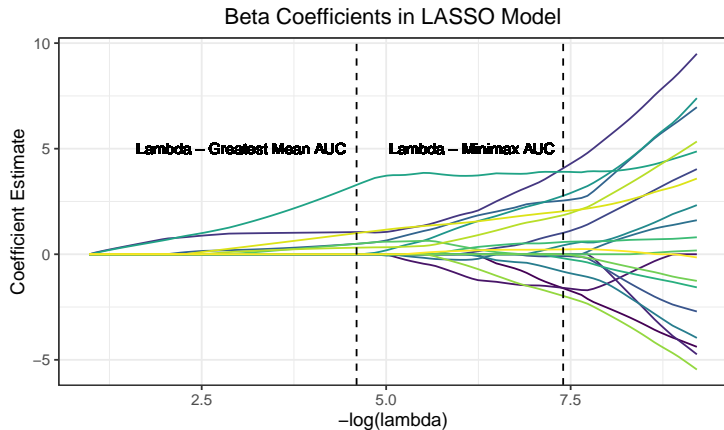
5-Fold Cross Validation



Cross Validation Results: Selecting Best Lambda



Cross Validation Results: LASSO Coefficients



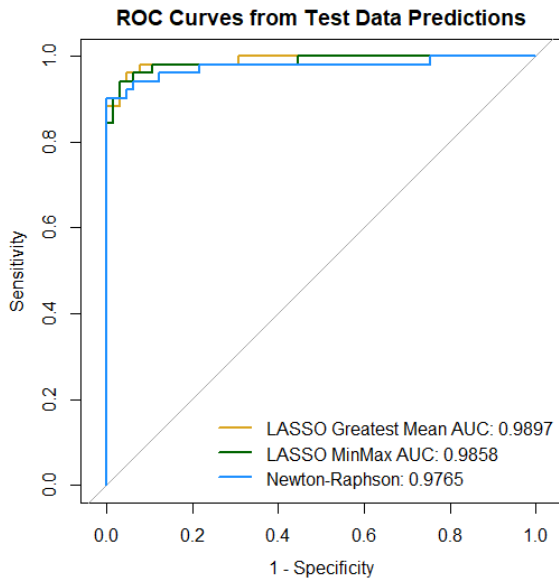
Coefficients Comparison: All Estimates

	NewtonRaphson	LASSO_GreatestMeanAUC	LASSO_MinimaxAUC
intercept	-0.0881	-0.9302	-1.2291
texture_mean	8.2691	0.9994	2.0355
smoothness_mean	-2.5447	0.0000	-0.1919
compactness_mean	-10.9297	0.0000	-1.6093
concave points_mean	20.9342	1.0512	4.0737
symmetry_mean	-1.9716	0.0000	0.0000
fractal_dimension_mean	3.9221	0.0000	0.0000
radius_se	18.5308	0.0000	2.7696
texture_se	-2.0504	0.0000	0.2269
smoothness_se	1.2753	0.0000	0.5802
compactness_se	4.7358	0.0000	-1.5925
concavity_se	-6.4914	0.0000	-0.0893
concave points_se	8.9092	0.0000	1.0126
symmetry_se	-13.8273	0.0000	-1.9717
fractal_dimension_se	-12.2152	0.0000	-0.8241
radius_worst	9.2654	3.2848	3.9047
smoothness_worst	0.6705	0.4955	0.0000
compactness_worst	-14.3912	0.0000	0.0000
concavity_worst	14.9611	0.4943	2.5436
symmetry_worst	12.3102	0.3112	1.8506
fractal_dimension_worst	7.7655	0.0000	0.4242

Coefficients Comparison: Model Performance

Measures	NewtonRaphson	LASSO_GreatestMeanAUC	LASSO_MinimaxAUC
Specificities	0.2462	0.6923	0.5538
AUC	0.9765	0.9897	0.9858
Selected Lambda	0	0.0101	0.0006
Number of Variables (w/o Intercept)	20	6	16

ROC Plot



Discussion

- ▶ Goal is accurately classify every patient
- ▶ Balancing Sensitivity vs. Specificity.
 - ▶ In first screening cases want to catch every case. Maximize Sensitivity.

Resources

Cancer Stat Facts: Female Breast Cancer. *National Cancer Institute*
- *NIH* <https://seer.cancer.gov/statfacts/html/breast.html>

American Cancer Society. (2019). Breast cancer facts & figures 2019–2020. Am Cancer Soc, 1-44.