P8160 - Less is More: Comparing Logistic and Lasso-Logistic Regression in Breast Cancer Diagnosis

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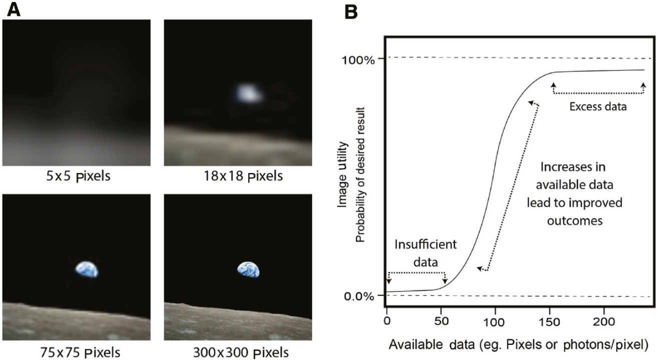
Motivation

As breast cancer is one of the most common kinds of cancer in the United States, great efforts have been made to aid in early and accurate detection.

Improvements in tumor imaging technology used in screening procedures have allow us access to more data than ever before, ideally to construct better ways to evaluate disease severity.

However... data $\neq$ information.

Imaging Data and Information Overload



From: Duncan, J., *Diagnosis*, 2017

Goal

We want to investigate two questions:

Does having more data always correspond to an advantage in diagnosis prediction?

Can we reduce the amount of information we need to collect while maintaining (or increasing) predictive power?

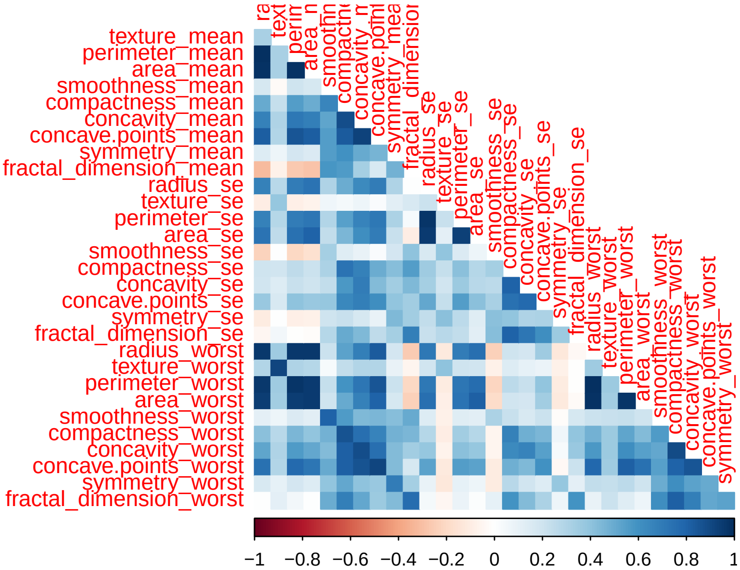
Toward this end, we will develop and evaluate the performance of two predictive models.

- Newton-Raphson Algorithm (Full Model)
- Logistic LASSO Algorithm (Optimal Model)

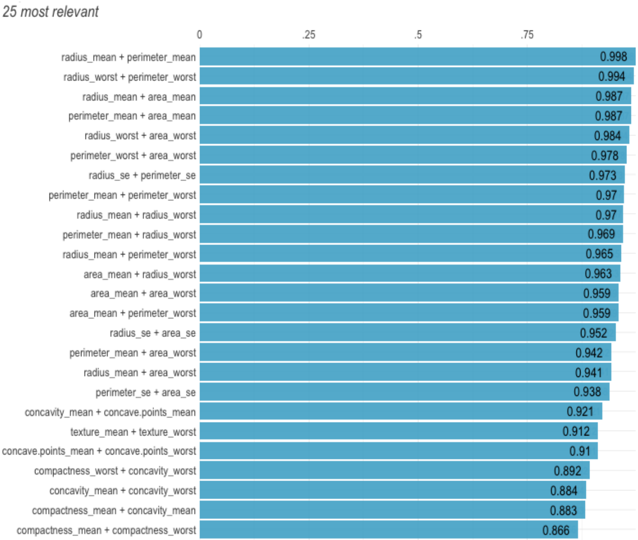
Data

- 569 rows and 31 columns related to breast tissue images
- Outcome of interest: Diagnosis (B or M)
  - 357 benign (B) cases and 212 malignant (M) cases
- The available predictors include summary statistics for tumor characteristics such as radius, texture, perimeter, area, smoothness, compactness, concavity, concave points, symmetry, and fractal dimension.

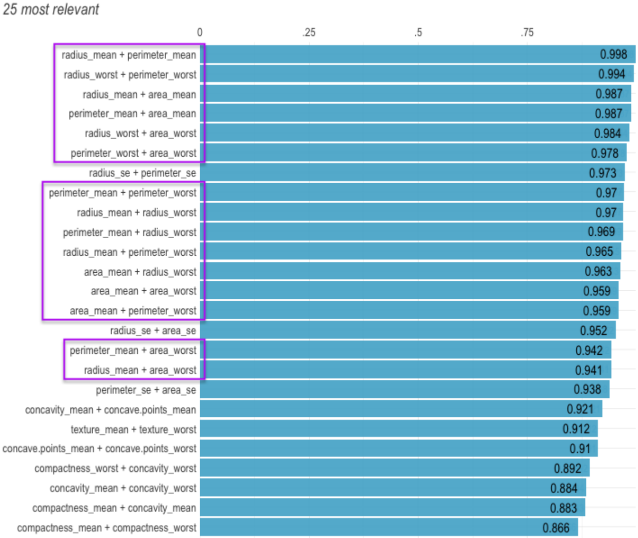
Correlation Heat Plot of all Covariates



Ranking Cross-Correlations

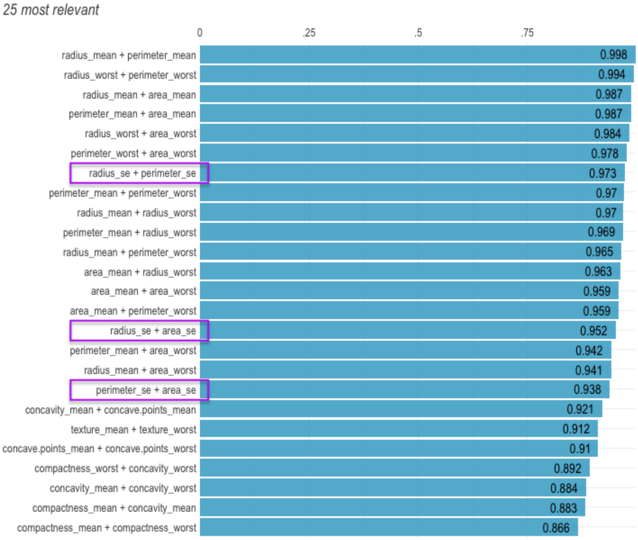


Identifying Equivalence Groups



Best Proxy: `radius_worst`

Identifying Equivalence Groups



Best Proxy: `radius_se`

Selected Variables

	Diagnosis Received		
Variable	**B**, N = 357[1]	**M**, N = 212[1]	**p-value**[2]
texture_mean	17.91 (4.00)	21.60 (3.78)	<0.001
smoothness_mean	0.09 (0.01)	0.10 (0.01)	<0.001
compactness_mean	0.08 (0.03)	0.15 (0.05)	<0.001
concave points_mean	0.03 (0.02)	0.09 (0.03)	<0.001
symmetry_mean	0.17 (0.02)	0.19 (0.03)	<0.001
fractal_dimension_mean	0.06 (0.01)	0.06 (0.01)	0.5
radius_se	0.28 (0.11)	0.61 (0.35)	<0.001
texture_se	1.22 (0.59)	1.21 (0.48)	0.6
smoothness_se	0.01 (0.00)	0.01 (0.00)	0.2
compactness_se	0.02 (0.02)	0.03 (0.02)	<0.001
concavity_se	0.03 (0.03)	0.04 (0.02)	<0.001
concave points_se	0.01 (0.01)	0.02 (0.01)	<0.001
symmetry_se	0.02 (0.01)	0.02 (0.01)	0.028
fractal_dimension_se	0.00 (0.00)	0.00 (0.00)	<0.001
radius_worst	13.38 (1.98)	21.13 (4.28)	<0.001
smoothness_worst	0.12 (0.02)	0.14 (0.02)	<0.001
compactness_worst	0.18 (0.09)	0.37 (0.17)	<0.001
concavity_worst	0.17 (0.14)	0.45 (0.18)	<0.001
symmetry_worst	0.27 (0.04)	0.32 (0.07)	<0.001
fractal_dimension_worst	0.08 (0.01)	0.09 (0.02)	<0.001

[1] Statistics presented: Mean (SD)

[2] Statistical tests performed: Wilcoxon rank-sum test

Table 1

Full Model: Newton-Raphson

Consider the following log-likelihood, gradient, and hessian matrix. First, let

$$\pi_i = P(Y_i = 1 | x_{i,1}, \dots x_{i,p}) = \frac{e^{\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j}}}{1 + e^{\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j}}}.$$

The log-likelihood:

$$l(\mathbf{X}|\vec{\beta}) = \sum_{i=1}^{n} \left[y_i \left(\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j} \right) - \log \left(1 + \exp \left(\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j} \right) \right) \right]$$

The gradient:

$$\nabla l(\mathbf{X}|\vec{\beta}) = \left[\sum^n y_i - \pi_i \quad \sum^n x_{i,1}(y_i - \pi_i) \quad \dots \quad \sum^n x_{i,p}(y_i - \pi_i) \right]^T_{1 \times (p+1)}$$

The hessian: produces a matrix $(p + 1 \times p + 1)$

$$\nabla^2 l(\mathbf{X}|\vec{\beta}) = -\sum_{i=1}^{n} \begin{pmatrix} 1 \\ X \end{pmatrix} \begin{pmatrix} 1 & X \end{pmatrix} \pi_i (1 - \pi_i)$$

Optimal Model: Logistic LASSO

For vector $\boldsymbol{\alpha} \in \mathbb{R}^{p+1}$, define $g : \mathbb{R}^{p+1} \to \mathbb{R}$ to be

$$g(\beta) \equiv -\frac{1}{2n} \sum_{i=1}^{n} w_i (z_i - \mathbf{X}_i^t \beta)^2 + O(\boldsymbol{\alpha}),$$

the Taylor expansion of our log-likelihood centered around $\boldsymbol{\alpha}$, where

$$\begin{aligned} z_i &\equiv \mathbf{X}_i^t \boldsymbol{\alpha} + \frac{y_i - \pi_i}{w_i}, && \text{(effective response)} \\ w_i &\equiv \pi_i (1 - \pi_i), \text{ and} && \text{(effective weights)} \\ \pi_i &\equiv \frac{e^{\mathbf{X}_i^t \boldsymbol{\alpha}}}{1 + e^{\mathbf{X}_i^t \boldsymbol{\alpha}}} \end{aligned}$$

for $i \in \{1, \ldots, n\}$.

Optimal Model: Logistic LASSO

It follows that for any $\lambda \in \mathbb{R}_+$,

$$\underset{\beta_k \in \mathbb{R}}{\arg\min} \left\{ g(\boldsymbol{\beta}) + \lambda \sum_{j=1}^{p} |\beta_j| \right\} = S\left(\hat{\beta}_k, \lambda_k\right), \text{ where}$$

$$\hat{\beta}_k \equiv \left(\sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \sum_{i=1}^{n} w_i x_{ik} \left(z_i - \sum_{j \neq k} \beta_j x_{ij} \right),$$

$$\lambda_k \equiv \left(\frac{1}{n} \sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \lambda,$$

and S is the soft-thresholding (or *shrinkage*) function. This is analogous to a penalized, weighted Gaussian regression.

Optimal Model: Logistic LASSO

Our coordinate descent algorithm proceeds as follows.

- Outer Loop: Decrement over $\lambda \in (\lambda_{\text{max}}, \ldots, \lambda_{\text{min}})$
- Middle Loop: Update $\boldsymbol{\alpha} = \boldsymbol{\beta}$ and Taylor expand g around $\boldsymbol{\alpha}$.
- Inner Loop: Update $\beta_k = S\left(\hat{\beta}_k, \lambda_k\right)$ sequentially for $k \in \{0, 1, \ldots, p, 0, 1, \ldots, p, 0, 1, \ldots\}$ until convergence.

Note: the middle loop terminates when a given Taylor expansion no longer yields updates (within the specified tolerance) to $\boldsymbol{\beta}$ in the inner loop.

Cross Validation: Setting Initial λ Range

λ_{max}: smallest penalty for which $\beta_k = 0$ for all $k \in \{1, \ldots, p\}$.

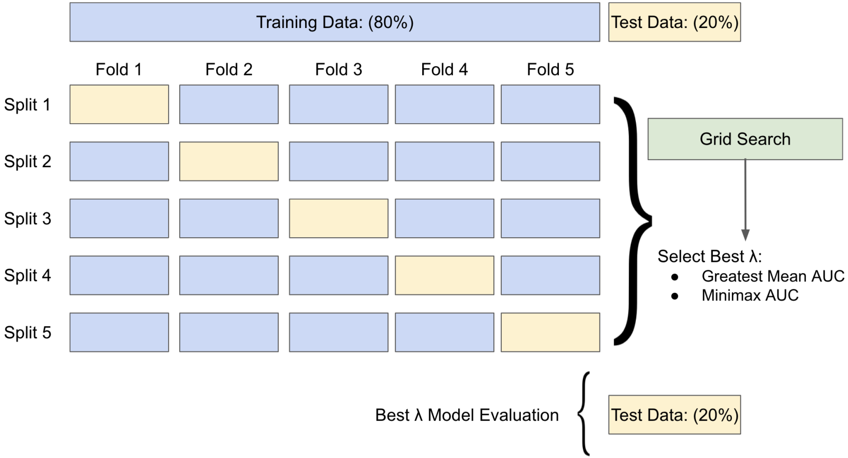
- Produces Null Model

$\lambda_{min} = \lambda_{max}/1000$.

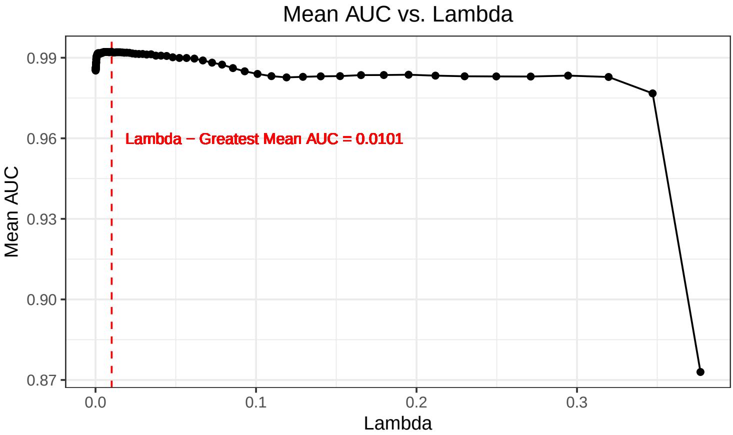
- Produces Full Model

Step size selected so we have 100 values, on a log scale.

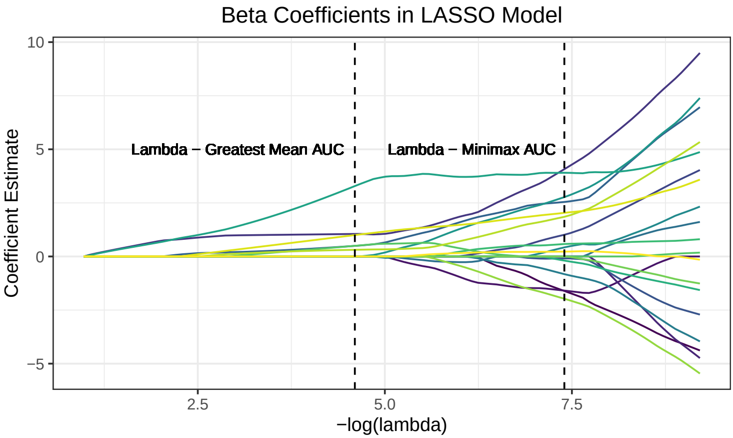
Cross Validation: Full Process



Cross Validation Results: Selecting Best Lambda



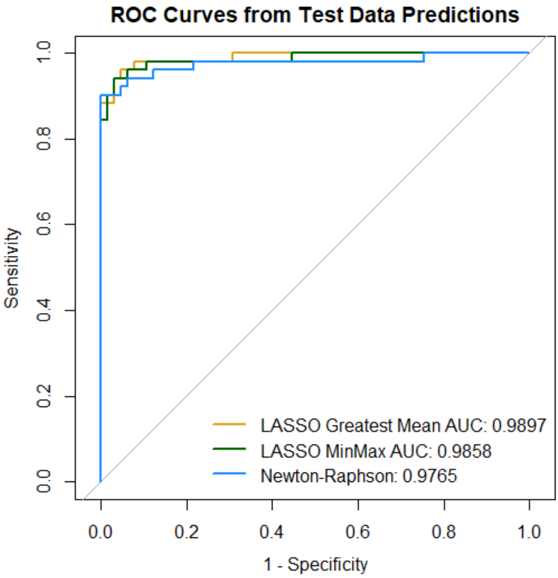
Cross Validation Results: LASSO Coefficients



Coefficients Comparison: All Estimates

	NewtonRaphson	LASSO_GreatestMeanAUC	LASSO_MinimaxAUC
intercept	-0.0881	-0.9302	-1.2291
texture_mean	8.2691	0.9994	2.0355
smoothness_mean	-2.5447	0.0000	-0.1919
compactness_mean	-10.9297	0.0000	-1.6093
concave points_mean	20.9342	1.0512	4.0737
symmetry_mean	-1.9716	0.0000	0.0000
fractal_dimension_mean	3.9221	0.0000	0.0000
radius_se	18.5308	0.0000	2.7696
texture_se	-2.0504	0.0000	0.2269
smoothness_se	1.2753	0.0000	0.5802
compactness_se	4.7358	0.0000	-1.5925
concavity_se	-6.4914	0.0000	-0.0893
concave points_se	8.9092	0.0000	1.0126
symmetry_se	-13.8273	0.0000	-1.9717
fractal_dimension_se	-12.2152	0.0000	-0.8241
radius_worst	9.2654	3.2848	3.9047
smoothness_worst	0.6705	0.4955	0.0000
compactness_worst	-14.3912	0.0000	0.0000
concavity_worst	14.9611	0.4943	2.5436
symmetry_worst	12.3102	0.3112	1.8506
fractal_dimension_worst	7.7655	0.0000	0.4242

ROC Plot


ROC Curves from Test Data Predictions
Sensitivity
1 - Specificity
0.0
0.2
0.4
0.6
0.8
1.0
LASSO Greatest Mean AUC: 0.9897
LASSO MinMax AUC: 0.9858
Newton-Raphson: 0.9765

Model Performance

Measures	NewtonRaphson	LASSO_GreatestMeanAUC	LASSO_MinimaxAUC
Specificities	0.2462	0.6923	0.5538
AUC	0.9765	0.9897	0.9858
Selected Lambda	0	0.0101	0.0006
Number of Variables (w/o Intercept)	20	6	16

Discussion

- Lasso-logistic model, with fewer predictors, out-performed the logistic model with all selected predictors.
- Ideal performance is to accurately classify every patient
- Balancing sensitivity and specificity
  - False positives vs false negatives
  - Decision boundaries
- Future work

Resources

Duncan, J. R. (2017, September 1). Information overload: When less is more in medical imaging. De Gruyter. https://www.degruyter.com/document/doi/10.1515/dx-2017-0008/html?lang=en

Cancer Stat Facts: Female Breast Cancer. *National Cancer Institute - NIH* https://seer.cancer.gov/statfacts/html/breast.html

American Cancer Society. (2019). Breast cancer facts & figures 2019–2020. Am Cancer Soc, 1-44.