

Logistic Lasso optimization

Hun, Jimmy

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```
cancer_df <- read.csv("~/Downloads/breast-cancer.csv") %>% janitor::clean_names()

data <-
  cancer_df %>% dplyr::select(-id, -x) %>%
  mutate(diagnosis = ifelse(diagnosis == "M", 1, 0)) %>% distinct()

set.seed(1)
split <- initial_split(data, prop = 0.8)

training_df <- split %>% training()

testing_df <- split %>% testing()
```

Standardization

```
for (i in 2:length(training_df)) {
  training_df[,i] = (training_df[,i] - mean(training_df[,i]))/sd(training_df[,i])
}
```

Checking standardization

```
summary <- skimr::skim_without_charts(training_df) %>% data.frame()
summary[1:6] %>% kable()
```

skim_type	skim_variable	n_missing	complete_rate	numeric.mean	numeric.sd
numeric	diagnosis	0	1	0.3714286	0.4837186
numeric	radius_mean	0	1	0.0000000	1.0000000
numeric	texture_mean	0	1	0.0000000	1.0000000
numeric	perimeter_mean	0	1	0.0000000	1.0000000
numeric	area_mean	0	1	0.0000000	1.0000000
numeric	smoothness_mean	0	1	0.0000000	1.0000000
numeric	compactness_mean	0	1	0.0000000	1.0000000
numeric	concavity_mean	0	1	0.0000000	1.0000000
numeric	concave_points_mean	0	1	0.0000000	1.0000000
numeric	symmetry_mean	0	1	0.0000000	1.0000000
numeric	fractal_dimension_mean	0	1	0.0000000	1.0000000
numeric	radius_se	0	1	0.0000000	1.0000000
numeric	texture_se	0	1	0.0000000	1.0000000
numeric	perimeter_se	0	1	0.0000000	1.0000000
numeric	area_se	0	1	0.0000000	1.0000000
numeric	smoothness_se	0	1	0.0000000	1.0000000
numeric	compactness_se	0	1	0.0000000	1.0000000
numeric	concavity_se	0	1	0.0000000	1.0000000
numeric	concave_points_se	0	1	0.0000000	1.0000000
numeric	symmetry_se	0	1	0.0000000	1.0000000
numeric	fractal_dimension_se	0	1	0.0000000	1.0000000
numeric	radius_worst	0	1	0.0000000	1.0000000
numeric	texture_worst	0	1	0.0000000	1.0000000
numeric	perimeter_worst	0	1	0.0000000	1.0000000
numeric	area_worst	0	1	0.0000000	1.0000000
numeric	smoothness_worst	0	1	0.0000000	1.0000000
numeric	compactness_worst	0	1	0.0000000	1.0000000
numeric	concavity_worst	0	1	0.0000000	1.0000000
numeric	concave_points_worst	0	1	0.0000000	1.0000000
numeric	symmetry_worst	0	1	0.0000000	1.0000000
numeric	fractal_dimension_worst	0	1	0.0000000	1.0000000

```
X <- training_df %>% dplyr::select(-diagnosis) %>% as.matrix()
y <- training_df$diagnosis %>% as.matrix()

dim(X)
```

```
## [1] 455 30
```

```
dim(y)
```

```
## [1] 455 1
```

Helper Functions

```
# logistic function
logistic <- function(x) 1 / (1 + exp(-x))

# soft threshold
```

```

S <- function(beta, gamma) {
  if (beta > 0 && gamma < abs(beta)) {
    beta - gamma
  } else if (beta < 0 && gamma < abs(beta)) {
    beta + gamma
  } else {
    0
  }
}

# probability adjustment function
p_adj <- function(p, epsilon) {
  if (p < epsilon) {
    0
  } else if (p > 1 - epsilon) {
    1
  } else {
    p
  }
}

# weight adjustment function
w_adj <- function(p, epsilon) {
  if ((p < epsilon) | (p > 1 - epsilon)) {
    epsilon
  } else {
    p * (1 - p)
  }
}

```

Computing lambda max in two ways and defining lambda sequence

```

alpha = 1
n = length(y)

lambda_max <- max(abs(t(X) %*% y))/n

same_lambda_max <- max(abs(t(y - mean(y)*(1 - mean(y))) %*% X ))/(alpha*n)

lambda_max

## [1] 0.386795

same_lambda_max

## [1] 0.386795

lambda_max <- lambda_max %>% round(digits = 2)

```

```
epsilon = 1e-5
lambda_seq <- seq(lambda_max, 1e-5*lambda_max, length = 100)
```

Toy example

```
set.seed(1)
n <- 1000
X <- scale(matrix(rnorm(3 * n), c(n, 3)))
X <- as.matrix(cbind(rep(1, n), X))
y <- 1 * (runif(n) > 0.5)

lambda_max <- round(max(abs(t(X) %*% y))/n, 2)
epsilon <- 1e-5

lambda_seq <- seq(lambda_max, epsilon*lambda_max, length = 100)

pb1 <- progress_bar$new(format = "Lasso optimization [:bar] :percent eta: :eta",
                        total = length(lambda_seq))

# initialize parameters
beta <- list()
beta_old <- list()
b_k_temp <- list()

# creating initial beta list
for (i in 1:length(lambda_seq)) {
  beta[[i]] <- rep(0, ncol(X))
  beta_old[[i]] <- rep(NA, ncol(X))
}

# outer loop for lambda decrement
for (i in 1:length(lambda_seq)) {
  pb1$tick()
  lambda <- lambda_seq[i]
  # inner loop for updating quadratic approximation ingredient
  for (inner in 1:2) {
    p <- map_dbl(logistic(X %*% beta[[i]]), p_adj, epsilon)
    w <- map_dbl(p, w_adj, epsilon)
    z <- X %*% beta[[i]] + (y - p) / w

    terminate <- 0
    iter <- 1
    # while loop for beta convergence
    while (terminate < 1) {
      beta_old[[i]] <- beta[[i]]
      # running coordinate descending algorithm
      for (k in 1:ncol(X)) {
        x_k <- X[, k]
        x_notk <- X[, -k]
        b_notk <- beta[[i]][-k]
```

```

# un-penalized coefficient update
b_k_temp <- sum(w * (z - x_notk %*% b_notk) * x_k) / sum(w * x_k^2)
# shrinkage update
b_k      <- S(b_k_temp, lambda * (k > 1) / mean(w * x_k^2))
#b_k     <- S(sum(w*x_k*(z - x_notk %*% b_notk)), lambda * (k > 1)) / sum(w * x_k^2)

# update beta list along with other parameters
beta[[i]][k] <- b_k
iter <- iter + 1
}
if (iter == 200 | max(abs(beta[[i]] - beta_old[[i]])) < 1e-10) {
  terminate <- 1
}
}
}
}

# Estimates from Coordinate Descent
beta <- data.frame(beta)

for (i in 1:length(lambda_seq)) {
  colnames(beta)[i] <- paste0(lambda_seq[i] %>% round(digits = 6))
}

beta <- t(beta) %>% data.frame()

beta %>% mutate(lambda = rownames(beta)) %>% relocate(lambda) %>% `rownames<-`(NULL)

```

```

##      lambda      X1      X2      X3      X4
## 1      0.53 0.1040939 0.00000000 0.00000000 0.00000000
## 2 0.524647 0.1040939 0.00000000 0.00000000 0.00000000
## 3 0.519293 0.1040939 0.00000000 0.00000000 0.00000000
## 4 0.51394 0.1040939 0.00000000 0.00000000 0.00000000
## 5 0.508586 0.1040939 0.00000000 0.00000000 0.00000000
## 6 0.503233 0.1040939 0.00000000 0.00000000 0.00000000
## 7 0.497879 0.1040939 0.00000000 0.00000000 0.00000000
## 8 0.492526 0.1040939 0.00000000 0.00000000 0.00000000
## 9 0.487172 0.1040939 0.00000000 0.00000000 0.00000000
## 10 0.481819 0.1040939 0.00000000 0.00000000 0.00000000
## 11 0.476465 0.1040939 0.00000000 0.00000000 0.00000000
## 12 0.471112 0.1040939 0.00000000 0.00000000 0.00000000
## 13 0.465758 0.1040939 0.00000000 0.00000000 0.00000000
## 14 0.460405 0.1040939 0.00000000 0.00000000 0.00000000
## 15 0.455051 0.1040939 0.00000000 0.00000000 0.00000000
## 16 0.449698 0.1040939 0.00000000 0.00000000 0.00000000
## 17 0.444344 0.1040939 0.00000000 0.00000000 0.00000000
## 18 0.438991 0.1040939 0.00000000 0.00000000 0.00000000
## 19 0.433637 0.1040939 0.00000000 0.00000000 0.00000000
## 20 0.428284 0.1040939 0.00000000 0.00000000 0.00000000
## 21 0.42293 0.1040939 0.00000000 0.00000000 0.00000000
## 22 0.417577 0.1040939 0.00000000 0.00000000 0.00000000
## 23 0.412223 0.1040939 0.00000000 0.00000000 0.00000000
## 24 0.40687 0.1040939 0.00000000 0.00000000 0.00000000
## 25 0.401516 0.1040939 0.00000000 0.00000000 0.00000000

```

## 26	0.396163	0.1040939	0.00000000	0.00000000	0.00000000
## 27	0.390809	0.1040939	0.00000000	0.00000000	0.00000000
## 28	0.385456	0.1040939	0.00000000	0.00000000	0.00000000
## 29	0.380103	0.1040939	0.00000000	0.00000000	0.00000000
## 30	0.374749	0.1040939	0.00000000	0.00000000	0.00000000
## 31	0.369396	0.1040939	0.00000000	0.00000000	0.00000000
## 32	0.364042	0.1040939	0.00000000	0.00000000	0.00000000
## 33	0.358689	0.1040939	0.00000000	0.00000000	0.00000000
## 34	0.353335	0.1040939	0.00000000	0.00000000	0.00000000
## 35	0.347982	0.1040939	0.00000000	0.00000000	0.00000000
## 36	0.342628	0.1040939	0.00000000	0.00000000	0.00000000
## 37	0.337275	0.1040939	0.00000000	0.00000000	0.00000000
## 38	0.331921	0.1040939	0.00000000	0.00000000	0.00000000
## 39	0.326568	0.1040939	0.00000000	0.00000000	0.00000000
## 40	0.321214	0.1040939	0.00000000	0.00000000	0.00000000
## 41	0.315861	0.1040939	0.00000000	0.00000000	0.00000000
## 42	0.310507	0.1040939	0.00000000	0.00000000	0.00000000
## 43	0.305154	0.1040939	0.00000000	0.00000000	0.00000000
## 44	0.2998	0.1040939	0.00000000	0.00000000	0.00000000
## 45	0.294447	0.1040939	0.00000000	0.00000000	0.00000000
## 46	0.289093	0.1040939	0.00000000	0.00000000	0.00000000
## 47	0.28374	0.1040939	0.00000000	0.00000000	0.00000000
## 48	0.278386	0.1040939	0.00000000	0.00000000	0.00000000
## 49	0.273033	0.1040939	0.00000000	0.00000000	0.00000000
## 50	0.267679	0.1040939	0.00000000	0.00000000	0.00000000
## 51	0.262326	0.1040939	0.00000000	0.00000000	0.00000000
## 52	0.256972	0.1040939	0.00000000	0.00000000	0.00000000
## 53	0.251619	0.1040939	0.00000000	0.00000000	0.00000000
## 54	0.246265	0.1040939	0.00000000	0.00000000	0.00000000
## 55	0.240912	0.1040939	0.00000000	0.00000000	0.00000000
## 56	0.235559	0.1040939	0.00000000	0.00000000	0.00000000
## 57	0.230205	0.1040939	0.00000000	0.00000000	0.00000000
## 58	0.224852	0.1040939	0.00000000	0.00000000	0.00000000
## 59	0.219498	0.1040939	0.00000000	0.00000000	0.00000000
## 60	0.214145	0.1040939	0.00000000	0.00000000	0.00000000
## 61	0.208791	0.1040939	0.00000000	0.00000000	0.00000000
## 62	0.203438	0.1040939	0.00000000	0.00000000	0.00000000
## 63	0.198084	0.1040939	0.00000000	0.00000000	0.00000000
## 64	0.192731	0.1040939	0.00000000	0.00000000	0.00000000
## 65	0.187377	0.1040939	0.00000000	0.00000000	0.00000000
## 66	0.182024	0.1040939	0.00000000	0.00000000	0.00000000
## 67	0.17667	0.1040939	0.00000000	0.00000000	0.00000000
## 68	0.171317	0.1040939	0.00000000	0.00000000	0.00000000
## 69	0.165963	0.1040939	0.00000000	0.00000000	0.00000000
## 70	0.16061	0.1040939	0.00000000	0.00000000	0.00000000
## 71	0.155256	0.1040939	0.00000000	0.00000000	0.00000000
## 72	0.149903	0.1040939	0.00000000	0.00000000	0.00000000
## 73	0.144549	0.1040939	0.00000000	0.00000000	0.00000000
## 74	0.139196	0.1040939	0.00000000	0.00000000	0.00000000
## 75	0.133842	0.1040939	0.00000000	0.00000000	0.00000000
## 76	0.128489	0.1040939	0.00000000	0.00000000	0.00000000
## 77	0.123135	0.1040939	0.00000000	0.00000000	0.00000000
## 78	0.117782	0.1040939	0.00000000	0.00000000	0.00000000
## 79	0.112428	0.1040939	0.00000000	0.00000000	0.00000000

```
## 80 0.107075 0.1040939 0.00000000 0.000000000 0.000000000
## 81 0.101721 0.1040939 0.00000000 0.000000000 0.000000000
## 82 0.096368 0.1040939 0.00000000 0.000000000 0.000000000
## 83 0.091014 0.1040939 0.00000000 0.000000000 0.000000000
## 84 0.085661 0.1040939 0.00000000 0.000000000 0.000000000
## 85 0.080308 0.1040939 0.00000000 0.000000000 0.000000000
## 86 0.074954 0.1040939 0.00000000 0.000000000 0.000000000
## 87 0.069601 0.1040939 0.00000000 0.000000000 0.000000000
## 88 0.064247 0.1040939 0.00000000 0.000000000 0.000000000
## 89 0.058894 0.1040939 0.00000000 0.000000000 0.000000000
## 90 0.05354 0.1040939 0.00000000 0.000000000 0.000000000
## 91 0.048187 0.1040939 0.00000000 0.000000000 0.000000000
## 92 0.042833 0.1040939 0.00000000 0.000000000 0.000000000
## 93 0.03748 0.1040939 0.00000000 0.000000000 0.000000000
## 94 0.032126 0.1040939 0.00000000 0.000000000 0.000000000
## 95 0.026773 0.1040939 0.00000000 0.000000000 0.000000000
## 96 0.021419 0.1040939 0.00000000 0.000000000 0.000000000
## 97 0.016066 0.1040939 0.00000000 0.000000000 0.000000000
## 98 0.010712 0.1040956 0.00000000 0.000000000 -0.008051621
## 99 0.005359 0.1041167 0.00000000 0.000000000 -0.029552983
## 100 5e-06 0.1041748 0.02045426 0.006292914 -0.052235163
```

```
# True estimates from GLM
#as.vector(glm(y ~ X[, -1], family = binomial)$coefficients)

# True estimates from GLMNET
fit <- glmnet(X, y, family = "binomial", standardize = FALSE, lambda = lambda_seq, thresh = 1e-10)

fit_result <- fit$beta %>% as.matrix() %>% t() %>% data.frame() %>% `rownames<-`(NULL)

fit_result %>% mutate(lambda = fit$lambda %>% round(digits = 6)) %>% relocate(lambda)
```

```
##      lambda V1      V2      V3      V4
## 1  0.530000 0 0.00000000 0.000000000 0.000000000
## 2  0.524647 0 0.00000000 0.000000000 0.000000000
## 3  0.519293 0 0.00000000 0.000000000 0.000000000
## 4  0.513940 0 0.00000000 0.000000000 0.000000000
## 5  0.508586 0 0.00000000 0.000000000 0.000000000
## 6  0.503233 0 0.00000000 0.000000000 0.000000000
## 7  0.497879 0 0.00000000 0.000000000 0.000000000
## 8  0.492526 0 0.00000000 0.000000000 0.000000000
## 9  0.487172 0 0.00000000 0.000000000 0.000000000
## 10 0.481819 0 0.00000000 0.000000000 0.000000000
## 11 0.476465 0 0.00000000 0.000000000 0.000000000
## 12 0.471112 0 0.00000000 0.000000000 0.000000000
## 13 0.465758 0 0.00000000 0.000000000 0.000000000
## 14 0.460405 0 0.00000000 0.000000000 0.000000000
## 15 0.455051 0 0.00000000 0.000000000 0.000000000
## 16 0.449698 0 0.00000000 0.000000000 0.000000000
## 17 0.444344 0 0.00000000 0.000000000 0.000000000
## 18 0.438991 0 0.00000000 0.000000000 0.000000000
## 19 0.433637 0 0.00000000 0.000000000 0.000000000
## 20 0.428284 0 0.00000000 0.000000000 0.000000000
## 21 0.422930 0 0.00000000 0.000000000 0.000000000
```

## 22	0.417577	0	0.00000000	0.00000000	0.00000000
## 23	0.412223	0	0.00000000	0.00000000	0.00000000
## 24	0.406870	0	0.00000000	0.00000000	0.00000000
## 25	0.401516	0	0.00000000	0.00000000	0.00000000
## 26	0.396163	0	0.00000000	0.00000000	0.00000000
## 27	0.390809	0	0.00000000	0.00000000	0.00000000
## 28	0.385456	0	0.00000000	0.00000000	0.00000000
## 29	0.380103	0	0.00000000	0.00000000	0.00000000
## 30	0.374749	0	0.00000000	0.00000000	0.00000000
## 31	0.369396	0	0.00000000	0.00000000	0.00000000
## 32	0.364042	0	0.00000000	0.00000000	0.00000000
## 33	0.358689	0	0.00000000	0.00000000	0.00000000
## 34	0.353335	0	0.00000000	0.00000000	0.00000000
## 35	0.347982	0	0.00000000	0.00000000	0.00000000
## 36	0.342628	0	0.00000000	0.00000000	0.00000000
## 37	0.337275	0	0.00000000	0.00000000	0.00000000
## 38	0.331921	0	0.00000000	0.00000000	0.00000000
## 39	0.326568	0	0.00000000	0.00000000	0.00000000
## 40	0.321214	0	0.00000000	0.00000000	0.00000000
## 41	0.315861	0	0.00000000	0.00000000	0.00000000
## 42	0.310507	0	0.00000000	0.00000000	0.00000000
## 43	0.305154	0	0.00000000	0.00000000	0.00000000
## 44	0.299800	0	0.00000000	0.00000000	0.00000000
## 45	0.294447	0	0.00000000	0.00000000	0.00000000
## 46	0.289093	0	0.00000000	0.00000000	0.00000000
## 47	0.283740	0	0.00000000	0.00000000	0.00000000
## 48	0.278386	0	0.00000000	0.00000000	0.00000000
## 49	0.273033	0	0.00000000	0.00000000	0.00000000
## 50	0.267679	0	0.00000000	0.00000000	0.00000000
## 51	0.262326	0	0.00000000	0.00000000	0.00000000
## 52	0.256972	0	0.00000000	0.00000000	0.00000000
## 53	0.251619	0	0.00000000	0.00000000	0.00000000
## 54	0.246265	0	0.00000000	0.00000000	0.00000000
## 55	0.240912	0	0.00000000	0.00000000	0.00000000
## 56	0.235559	0	0.00000000	0.00000000	0.00000000
## 57	0.230205	0	0.00000000	0.00000000	0.00000000
## 58	0.224852	0	0.00000000	0.00000000	0.00000000
## 59	0.219498	0	0.00000000	0.00000000	0.00000000
## 60	0.214145	0	0.00000000	0.00000000	0.00000000
## 61	0.208791	0	0.00000000	0.00000000	0.00000000
## 62	0.203438	0	0.00000000	0.00000000	0.00000000
## 63	0.198084	0	0.00000000	0.00000000	0.00000000
## 64	0.192731	0	0.00000000	0.00000000	0.00000000
## 65	0.187377	0	0.00000000	0.00000000	0.00000000
## 66	0.182024	0	0.00000000	0.00000000	0.00000000
## 67	0.176670	0	0.00000000	0.00000000	0.00000000
## 68	0.171317	0	0.00000000	0.00000000	0.00000000
## 69	0.165963	0	0.00000000	0.00000000	0.00000000
## 70	0.160610	0	0.00000000	0.00000000	0.00000000
## 71	0.155256	0	0.00000000	0.00000000	0.00000000
## 72	0.149903	0	0.00000000	0.00000000	0.00000000
## 73	0.144549	0	0.00000000	0.00000000	0.00000000
## 74	0.139196	0	0.00000000	0.00000000	0.00000000
## 75	0.133842	0	0.00000000	0.00000000	0.00000000


```
## 76 0.128489 0 0.00000000 0.00000000 0.00000000
## 77 0.123135 0 0.00000000 0.00000000 0.00000000
## 78 0.117782 0 0.00000000 0.00000000 0.00000000
## 79 0.112428 0 0.00000000 0.00000000 0.00000000
## 80 0.107075 0 0.00000000 0.00000000 0.00000000
## 81 0.101721 0 0.00000000 0.00000000 0.00000000
## 82 0.096368 0 0.00000000 0.00000000 0.00000000
## 83 0.091014 0 0.00000000 0.00000000 0.00000000
## 84 0.085661 0 0.00000000 0.00000000 0.00000000
## 85 0.080308 0 0.00000000 0.00000000 0.00000000
## 86 0.074954 0 0.00000000 0.00000000 0.00000000
## 87 0.069601 0 0.00000000 0.00000000 0.00000000
## 88 0.064247 0 0.00000000 0.00000000 0.00000000
## 89 0.058894 0 0.00000000 0.00000000 0.00000000
## 90 0.053540 0 0.00000000 0.00000000 0.00000000
## 91 0.048187 0 0.00000000 0.00000000 0.00000000
## 92 0.042833 0 0.00000000 0.00000000 0.00000000
## 93 0.037480 0 0.00000000 0.00000000 0.00000000
## 94 0.032126 0 0.00000000 0.00000000 0.00000000
## 95 0.026773 0 0.00000000 0.00000000 0.00000000
## 96 0.021419 0 0.00000000 0.00000000 0.00000000
## 97 0.016066 0 0.00000000 0.00000000 0.00000000
## 98 0.010712 0 0.00000000 0.00000000 -0.00805162
## 99 0.005359 0 0.00000000 0.00000000 -0.02955296
## 100 0.000005 0 0.02045426 0.006292915 -0.05223517
```

Cancer data

```
data <-
  read_csv("data/breast-cancer.csv", show_col_types = FALSE) %>%
  mutate(diagnosis = 1 * (diagnosis == "M"))

n <- nrow(data)
X <- data[, -c(1, 2)]
X <- as.matrix(cbind(rep(1, n), X))
y <- data$diagnosis

lambda_max <- round(max(abs(t(X) %*% y))/n, 2)
epsilon <- 1e-4

lambda_seq <- seq(lambda_max, epsilon*lambda_max, length = 100)

pb2 <- progress_bar$new(format = "Lasso optimization [:bar] :percent eta: :eta",
  total = length(lambda_seq))

# initialize parameters
beta <- list()
beta_old <- list()
b_k_temp <- list()

# creating initial beta list
```

```

for (i in 1:length(lambda_seq)) {
  beta[[i]] <- rep(0, ncol(X))
  beta_old[[i]] <- rep(NA, ncol(X))
}

# outer loop for lambda decrement
for (i in 1:length(lambda_seq)) {
  pb2$tick()
  lambda <- lambda_seq[i]
  # inner loop for updating quadratic approximation ingredient
  for (inner in 1:2) {
    p <- map_dbl(logistic(X %>% beta[[i]]), p_adj, epsilon)
    w <- map_dbl(p, w_adj, epsilon)
    z <- X %>% beta[[i]] + (y - p) / w

    terminate <- 0
    iter <- 1
    # while loop for beta convergence
    while (terminate < 1) {
      beta_old[[i]] <- beta[[i]]
      # running coordinate descending algorithm
      for (k in 1:ncol(X)) {
        x_k <- X[, k]
        x_notk <- X[, -k]
        b_notk <- beta[[i]][-k]

        # un-penalized coefficient update
        b_k_temp <- sum(w * (z - x_notk %>% b_notk) * x_k) / sum(w * x_k^2)
        # shrinkage update
        b_k <- S(b_k_temp, lambda * (k > 1) / mean(w * x_k^2))
        # b_k <- S(sum(w*x_k*(z - x_notk %>% b_notk)), lambda * (k > 1)) / sum(w * x_k^2)

        # update beta list along with other parameters
        beta[[i]][k] <- b_k
        iter <- iter + 1
      }
      if (iter == 200 | max(abs(beta[[i]] - beta_old[[i]])) < 1e-10) {
        terminate <- 1
      }
    }
  }
}

# Estimates from Coordinate Descent
beta <- data.frame(beta)

for (i in 1:length(lambda_seq)) {
  colnames(beta)[i] <- paste0(lambda_seq[i] %>% round(digits = 6))
}

beta <- t(beta) %>% data.frame()

beta %>% mutate(lambda = rownames(beta)) %>% relocate(lambda) %>% `rownames<-`(NULL)

```

##	lambda	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14
## 1	529.92	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 2	524.567808	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 3	519.215616	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 4	513.863424	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 5	508.511232	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 6	503.15904	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 7	497.806848	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 8	492.454656	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 9	487.102464	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 10	481.750272	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 11	476.39808	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 12	471.045888	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 13	465.693696	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 14	460.341504	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 15	454.989312	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 16	449.63712	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 17	444.284928	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 18	438.932736	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 19	433.580544	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 20	428.228352	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 21	422.87616	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 22	417.523968	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 23	412.171776	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 24	406.819584	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 25	401.467392	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 26	396.1152	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 27	390.763008	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 28	385.410816	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 29	380.058624	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 30	374.706432	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 31	369.35424	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 32	364.002048	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 33	358.649856	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 34	353.297664	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 35	347.945472	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 36	342.59328	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 37	337.241088	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 38	331.888896	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 39	326.536704	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 40	321.184512	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 41	315.83232	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 42	310.480128	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 43	305.127936	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 44	299.775744	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 45	294.423552	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 46	289.07136	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 47	283.719168	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 48	278.366976	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 49	273.014784	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 50	267.662592	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 51	262.3104	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 52	256.958208	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0
## 53	251.606016	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0	0

## 54	246.253824	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 55	240.901632	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 56	235.54944	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 57	230.197248	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 58	224.845056	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 59	219.492864	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 60	214.140672	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 61	208.78848	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 62	203.436288	-0.5211330	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 63	198.084096	-0.5645552	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 64	192.731904	-0.6261584	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 65	187.379712	-0.6875059	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 66	182.02752	-0.7488879	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 67	176.675328	-0.8105861	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 68	171.323136	-0.8728732	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 69	165.970944	-0.9360134	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 70	160.618752	-1.0002627	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 71	155.26656	-1.0658696	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 72	149.914368	-1.1330760	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 73	144.562176	-1.2021178	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 74	139.209984	-1.2732263	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 75	133.857792	-1.3466286	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 76	128.5056	-1.4225491	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 77	123.153408	-1.5012105	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 78	117.801216	-1.5828345	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 79	112.449024	-1.6676433	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 80	107.096832	-1.7558601	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 81	101.74464	-1.8477104	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 82	96.392448	-1.9434228	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 83	91.040256	-2.0432302	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 84	85.688064	-2.1473705	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 85	80.335872	-2.2560874	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 86	74.98368	-2.3696315	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 87	69.631488	-2.4882611	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 88	64.279296	-2.6122426	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 89	58.927104	-2.7418513	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 90	53.574912	-2.8773722	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 91	48.22272	-3.0190999	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 92	42.870528	-3.1673400	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 93	37.518336	-3.3224084	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 94	32.166144	-3.4846325	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 95	26.813952	-3.6543510	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 96	21.46176	-3.8319144	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 97	16.109568	-4.0176848	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 98	10.757376	-4.2120368	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 99	5.405184	-4.4153569	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 100	0.052992	-14.1634756	0	0	0	-0.00372259	0	0	0	0	0	0	0	0
##		X15	X16	X17	X18	X19	X20	X21	X22	X23	X24	X25		
## 1	0.00000000	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00		
## 2	0.00000000	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00		
## 3	0.00000000	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00		
## 4	0.00000000	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00		
## 5	0.00000000	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00		
## 6	0.00000000	0	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00		

[illegible]

## 61	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00
## 62	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	0.000000e+00
## 63	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	4.920152e-05
## 64	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.186592e-04
## 65	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.874848e-04
## 66	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.560710e-04
## 67	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.247948e-04
## 68	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.940182e-04
## 69	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	4.640894e-04
## 70	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	5.353445e-04
## 71	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	6.081091e-04
## 72	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	6.826995e-04
## 73	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	7.594248e-04
## 74	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	8.385881e-04
## 75	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	9.204877e-04
## 76	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.005419e-03
## 77	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.093676e-03
## 78	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.185551e-03
## 79	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.281339e-03
## 80	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.381334e-03
## 81	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.485836e-03
## 82	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.595147e-03
## 83	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.709575e-03
## 84	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.829434e-03
## 85	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	1.955047e-03
## 86	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.086743e-03
## 87	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.224863e-03
## 88	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.369754e-03
## 89	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.521778e-03
## 90	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.681306e-03
## 91	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	2.848721e-03
## 92	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.024420e-03
## 93	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.208810e-03
## 94	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.402314e-03
## 95	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.605366e-03
## 96	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	3.818417e-03
## 97	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	4.041927e-03
## 98	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	4.276374e-03
## 99	0.00000000	0	0	0	0	0	0	0	0.00000000	0.00000000	4.522248e-03
## 100	0.00622815	0	0	0	0	0	0	0	0.08462395	0.1449735	-2.327593e-03
##	X26 X27 X28 X29 X30 X31										
## 1	0	0	0	0	0	0					
## 2	0	0	0	0	0	0					
## 3	0	0	0	0	0	0					
## 4	0	0	0	0	0	0					
## 5	0	0	0	0	0	0					
## 6	0	0	0	0	0	0					
## 7	0	0	0	0	0	0					
## 8	0	0	0	0	0	0					
## 9	0	0	0	0	0	0					
## 10	0	0	0	0	0	0					
## 11	0	0	0	0	0	0					
## 12	0	0	0	0	0	0					
## 13	0	0	0	0	0	0					

## 14	0	0	0	0	0	0
## 15	0	0	0	0	0	0
## 16	0	0	0	0	0	0
## 17	0	0	0	0	0	0
## 18	0	0	0	0	0	0
## 19	0	0	0	0	0	0
## 20	0	0	0	0	0	0
## 21	0	0	0	0	0	0
## 22	0	0	0	0	0	0
## 23	0	0	0	0	0	0
## 24	0	0	0	0	0	0
## 25	0	0	0	0	0	0
## 26	0	0	0	0	0	0
## 27	0	0	0	0	0	0
## 28	0	0	0	0	0	0
## 29	0	0	0	0	0	0
## 30	0	0	0	0	0	0
## 31	0	0	0	0	0	0
## 32	0	0	0	0	0	0
## 33	0	0	0	0	0	0
## 34	0	0	0	0	0	0
## 35	0	0	0	0	0	0
## 36	0	0	0	0	0	0
## 37	0	0	0	0	0	0
## 38	0	0	0	0	0	0
## 39	0	0	0	0	0	0
## 40	0	0	0	0	0	0
## 41	0	0	0	0	0	0
## 42	0	0	0	0	0	0
## 43	0	0	0	0	0	0
## 44	0	0	0	0	0	0
## 45	0	0	0	0	0	0
## 46	0	0	0	0	0	0
## 47	0	0	0	0	0	0
## 48	0	0	0	0	0	0
## 49	0	0	0	0	0	0
## 50	0	0	0	0	0	0
## 51	0	0	0	0	0	0
## 52	0	0	0	0	0	0
## 53	0	0	0	0	0	0
## 54	0	0	0	0	0	0
## 55	0	0	0	0	0	0
## 56	0	0	0	0	0	0
## 57	0	0	0	0	0	0
## 58	0	0	0	0	0	0
## 59	0	0	0	0	0	0
## 60	0	0	0	0	0	0
## 61	0	0	0	0	0	0
## 62	0	0	0	0	0	0
## 63	0	0	0	0	0	0
## 64	0	0	0	0	0	0
## 65	0	0	0	0	0	0
## 66	0	0	0	0	0	0
## 67	0	0	0	0	0	0

```
## 68    0    0    0    0    0    0
## 69    0    0    0    0    0    0
## 70    0    0    0    0    0    0
## 71    0    0    0    0    0    0
## 72    0    0    0    0    0    0
## 73    0    0    0    0    0    0
## 74    0    0    0    0    0    0
## 75    0    0    0    0    0    0
## 76    0    0    0    0    0    0
## 77    0    0    0    0    0    0
## 78    0    0    0    0    0    0
## 79    0    0    0    0    0    0
## 80    0    0    0    0    0    0
## 81    0    0    0    0    0    0
## 82    0    0    0    0    0    0
## 83    0    0    0    0    0    0
## 84    0    0    0    0    0    0
## 85    0    0    0    0    0    0
## 86    0    0    0    0    0    0
## 87    0    0    0    0    0    0
## 88    0    0    0    0    0    0
## 89    0    0    0    0    0    0
## 90    0    0    0    0    0    0
## 91    0    0    0    0    0    0
## 92    0    0    0    0    0    0
## 93    0    0    0    0    0    0
## 94    0    0    0    0    0    0
## 95    0    0    0    0    0    0
## 96    0    0    0    0    0    0
## 97    0    0    0    0    0    0
## 98    0    0    0    0    0    0
## 99    0    0    0    0    0    0
## 100   0    0    0    0    0    0
```

```
# True estimates from GLM
#as.vector(glm(y ~ X[, -1], family = binomial)$coefficients)

# True estimates from GLMNET
fit <- glmnet(X, y, family = "binomial", standardize = FALSE, lambda = lambda_seq, thresh = 1e-10)

fit_result <- fit$beta %>% as.matrix() %>% t() %>% data.frame() %>% `rownames<-`(NULL)

for (i in 1:ncol(fit_result)) {
  colnames(fit_result)[i] <- paste0("X",i)
}

fit_result %>% mutate(lambda = fit$lambda %>% round(digits = 6)) %>% relocate(lambda)
```

```
##      lambda X1 X2 X3 X4      X5 X6 X7 X8 X9 X10 X11 X12 X13 X14
## 1  529.920000 0 0 0 0 0.00000000 0 0 0 0 0 0 0 0 0
## 2  524.567808 0 0 0 0 0.00000000 0 0 0 0 0 0 0 0 0
## 3  519.215616 0 0 0 0 0.00000000 0 0 0 0 0 0 0 0 0
## 4  513.863424 0 0 0 0 0.00000000 0 0 0 0 0 0 0 0 0
## 5  508.511232 0 0 0 0 0.00000000 0 0 0 0 0 0 0 0 0
```


## 6	503.159040	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 7	497.806848	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 8	492.454656	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 9	487.102464	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 10	481.750272	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 11	476.398080	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 12	471.045888	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 13	465.693696	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 14	460.341504	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 15	454.989312	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 16	449.637120	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 17	444.284928	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 18	438.932736	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 19	433.580544	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 20	428.228352	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 21	422.876160	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 22	417.523968	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 23	412.171776	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 24	406.819584	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 25	401.467392	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 26	396.115200	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 27	390.763008	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 28	385.410816	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 29	380.058624	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 30	374.706432	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 31	369.354240	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 32	364.002048	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 33	358.649856	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 34	353.297664	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 35	347.945472	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 36	342.593280	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 37	337.241088	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 38	331.888896	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 39	326.536704	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 40	321.184512	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 41	315.832320	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 42	310.480128	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 43	305.127936	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 44	299.775744	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 45	294.423552	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 46	289.071360	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 47	283.719168	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 48	278.366976	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 49	273.014784	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 50	267.662592	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 51	262.310400	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 52	256.958208	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 53	251.606016	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 54	246.253824	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 55	240.901632	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 56	235.549440	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 57	230.197248	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 58	224.845056	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0
## 59	219.492864	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0

## 60	214.140672	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 61	208.788480	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 62	203.436288	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 63	198.084096	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 64	192.731904	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 65	187.379712	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 66	182.027520	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 67	176.675328	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 68	171.323136	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 69	165.970944	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 70	160.618752	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 71	155.266560	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 72	149.914368	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 73	144.562176	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 74	139.209984	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 75	133.857792	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 76	128.505600	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 77	123.153408	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 78	117.801216	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 79	112.449024	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 80	107.096832	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 81	101.744640	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 82	96.392448	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 83	91.040256	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 84	85.688064	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 85	80.335872	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 86	74.983680	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 87	69.631488	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 88	64.279296	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 89	58.927104	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 90	53.574912	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 91	48.222720	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 92	42.870528	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 93	37.518336	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 94	32.166144	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 95	26.813952	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 96	21.461760	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 97	16.109568	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 98	10.757376	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 99	5.405184	0	0	0	0	0.00000000	0	0	0	0	0	0	0	0		
## 100	0.052992	0	0	0	0	-0.01839172	0	0	0	0	0	0	0	0		
##		X15	X16	X17	X18	X19	X20	X21	X22		X23		X24		X25	X26
## 1	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 2	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 3	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 4	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 5	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 6	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 7	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 8	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 9	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 10	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 11	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			
## 12	0.0000000	0	0	0	0	0	0	0	0	0.0000000	0.0000000	0.000000e+00	0			

[illegible]

## 67	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	3.248391e-04	0
## 68	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	3.940816e-04	0
## 69	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	4.641828e-04	0
## 70	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	5.354862e-04	0
## 71	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	6.083300e-04	0
## 72	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	6.830512e-04	0
## 73	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	7.599903e-04	0
## 74	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	8.394967e-04	0
## 75	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	9.219348e-04	0
## 76	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.007691e-03	0
## 77	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.097181e-03	0
## 78	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.190859e-03	0
## 79	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.289229e-03	0
## 80	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.392856e-03	0
## 81	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.502383e-03	0
## 82	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.618554e-03	0
## 83	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.742236e-03	0
## 84	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	1.874460e-03	0
## 85	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	2.016461e-03	0
## 86	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	2.169746e-03	0
## 87	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	2.336178e-03	0
## 88	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	2.518095e-03	0
## 89	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	2.718489e-03	0
## 90	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	2.941252e-03	0
## 91	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	3.191567e-03	0
## 92	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	3.476507e-03	0
## 93	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	3.805994e-03	0
## 94	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	4.194478e-03	0
## 95	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	4.663923e-03	0
## 96	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	5.249648e-03	0
## 97	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	6.012874e-03	0
## 98	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	7.071424e-03	0
## 99	0.0000000	0	0	0	0	0	0	0	0.0000000	0.0000000	8.690592e-03	0
## 100	0.0512712	0	0	0	0	0	0	0	0.1802206	0.1647303	1.280587e-02	0
##	X27 X28 X29 X30 X31											
## 1	0	0	0	0	0							
## 2	0	0	0	0	0							
## 3	0	0	0	0	0							
## 4	0	0	0	0	0							
## 5	0	0	0	0	0							
## 6	0	0	0	0	0							
## 7	0	0	0	0	0							
## 8	0	0	0	0	0							
## 9	0	0	0	0	0							
## 10	0	0	0	0	0							
## 11	0	0	0	0	0							
## 12	0	0	0	0	0							
## 13	0	0	0	0	0							
## 14	0	0	0	0	0							
## 15	0	0	0	0	0							
## 16	0	0	0	0	0							
## 17	0	0	0	0	0							
## 18	0	0	0	0	0							
## 19	0	0	0	0	0							

## 20	0	0	0	0	0
## 21	0	0	0	0	0
## 22	0	0	0	0	0
## 23	0	0	0	0	0
## 24	0	0	0	0	0
## 25	0	0	0	0	0
## 26	0	0	0	0	0
## 27	0	0	0	0	0
## 28	0	0	0	0	0
## 29	0	0	0	0	0
## 30	0	0	0	0	0
## 31	0	0	0	0	0
## 32	0	0	0	0	0
## 33	0	0	0	0	0
## 34	0	0	0	0	0
## 35	0	0	0	0	0
## 36	0	0	0	0	0
## 37	0	0	0	0	0
## 38	0	0	0	0	0
## 39	0	0	0	0	0
## 40	0	0	0	0	0
## 41	0	0	0	0	0
## 42	0	0	0	0	0
## 43	0	0	0	0	0
## 44	0	0	0	0	0
## 45	0	0	0	0	0
## 46	0	0	0	0	0
## 47	0	0	0	0	0
## 48	0	0	0	0	0
## 49	0	0	0	0	0
## 50	0	0	0	0	0
## 51	0	0	0	0	0
## 52	0	0	0	0	0
## 53	0	0	0	0	0
## 54	0	0	0	0	0
## 55	0	0	0	0	0
## 56	0	0	0	0	0
## 57	0	0	0	0	0
## 58	0	0	0	0	0
## 59	0	0	0	0	0
## 60	0	0	0	0	0
## 61	0	0	0	0	0
## 62	0	0	0	0	0
## 63	0	0	0	0	0
## 64	0	0	0	0	0
## 65	0	0	0	0	0
## 66	0	0	0	0	0
## 67	0	0	0	0	0
## 68	0	0	0	0	0
## 69	0	0	0	0	0
## 70	0	0	0	0	0
## 71	0	0	0	0	0
## 72	0	0	0	0	0
## 73	0	0	0	0	0

## 74	0	0	0	0	0
## 75	0	0	0	0	0
## 76	0	0	0	0	0
## 77	0	0	0	0	0
## 78	0	0	0	0	0
## 79	0	0	0	0	0
## 80	0	0	0	0	0
## 81	0	0	0	0	0
## 82	0	0	0	0	0
## 83	0	0	0	0	0
## 84	0	0	0	0	0
## 85	0	0	0	0	0
## 86	0	0	0	0	0
## 87	0	0	0	0	0
## 88	0	0	0	0	0
## 89	0	0	0	0	0
## 90	0	0	0	0	0
## 91	0	0	0	0	0
## 92	0	0	0	0	0
## 93	0	0	0	0	0
## 94	0	0	0	0	0
## 95	0	0	0	0	0
## 96	0	0	0	0	0
## 97	0	0	0	0	0
## 98	0	0	0	0	0
## 99	0	0	0	0	0
## 100	0	0	0	0	0