P8160 - Less is More: Comparing Logistic and Lasso-Logistic Regression in Breast Cancer Diagnosis

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Motivation

According to NIH, there have been an estimated:

- 281,550 new cases of breast cancer in women in 2021,
- 43,600 breast cancer in women related deaths in 2021.

American Cancer Society Guideline for Breast Cancer Screening:

- Women between ages 25-40 should have an annual clinical breast examination.
- Women between ages 40-44 should begin annual screening via mammogram
- Women between ages 45-54 should screened annually via mammogram

Imaging Data and Information Overload

Can we reduce the amount of information we need to collect while maintaining (or increasing) predictive power?

Goal

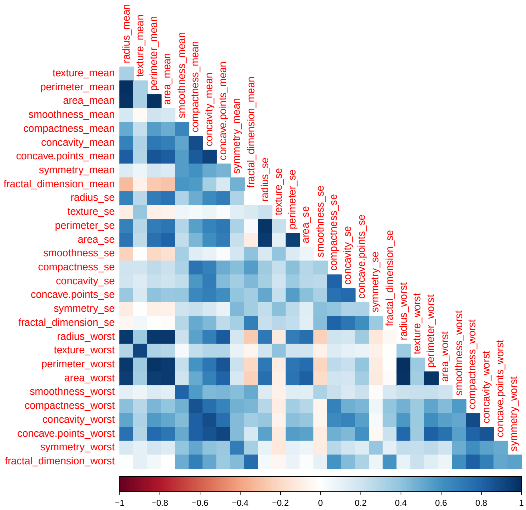
Toward this end, we want to develop and evaluate the performance of two predictive models.

- Newton-Raphson Algorithm (Full Model)
- Logistic LASSO Algorithm (Optimal Model)

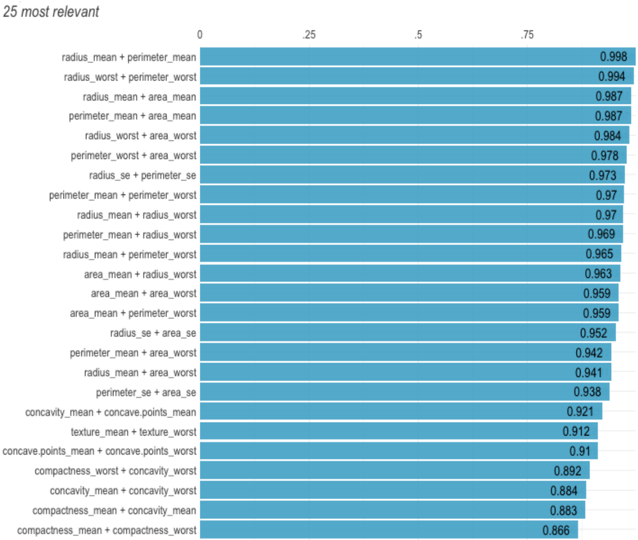
Data

- 569 rows and 31 columns related to breast tissue images
- Outcome of interest: Diagnosis (B or M)
  - 357 benign (B) cases and 212 malignant (M) cases
- The available predictors include measurements such as radius, texture, perimeter, area, smoothness, compactness, concavity, concave points, symmetry, and fractal dimension.

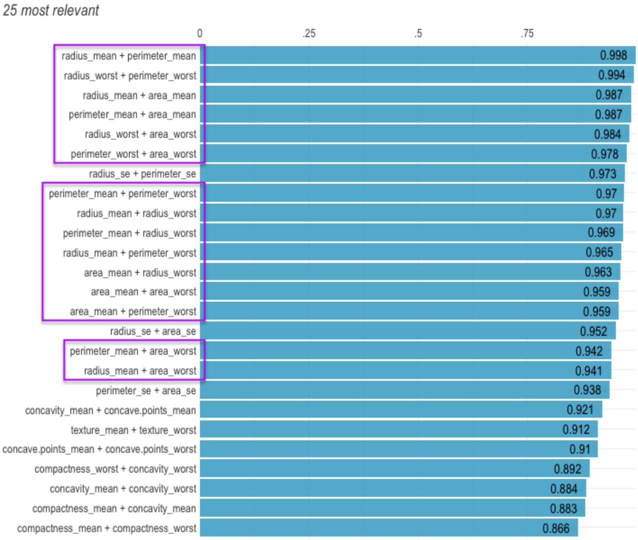
Correlation Heat Plot of all Covariates



Ranking Cross-Correlations

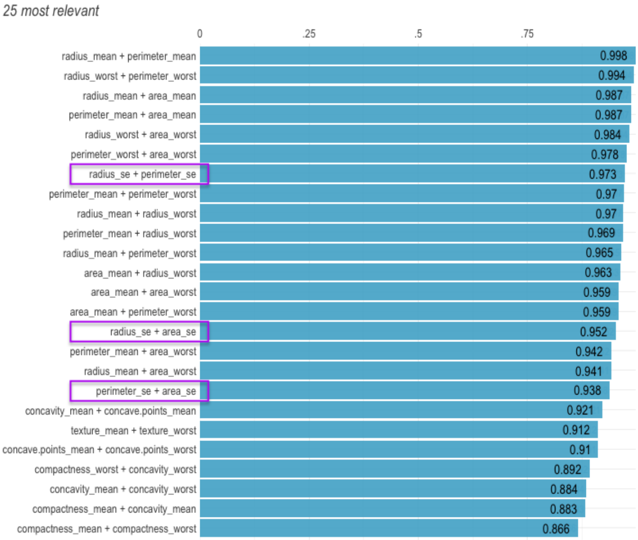


Identifying Equivalence Groups



Best Proxy: `radius_worst`

Identifying Equivalence Groups



Best Proxy: `radius_se`

Finally Selected Variables

Variable	Diagnosis Received		
	B, N = 357[1]	**M**, N = 212[1]	**p-value**[2]
texture_mean	17.91 (4.00)	21.60 (3.78)	<0.001
smoothness_mean	0.09 (0.01)	0.10 (0.01)	<0.001
compactness_mean	0.08 (0.03)	0.15 (0.05)	<0.001
concave points_mean	0.03 (0.02)	0.09 (0.03)	<0.001
symmetry_mean	0.17 (0.02)	0.19 (0.03)	<0.001
fractal_dimension_mean	0.06 (0.01)	0.06 (0.01)	0.5
radius_se	0.28 (0.11)	0.61 (0.35)	<0.001
texture_se	1.22 (0.59)	1.21 (0.48)	0.6
smoothness_se	0.01 (0.00)	0.01 (0.00)	0.2
compactness_se	0.02 (0.02)	0.03 (0.02)	<0.001
concavity_se	0.03 (0.03)	0.04 (0.02)	<0.001
concave points_se	0.01 (0.01)	0.02 (0.01)	<0.001
symmetry_se	0.02 (0.01)	0.02 (0.01)	0.028
fractal_dimension_se	0.00 (0.00)	0.00 (0.00)	<0.001
radius_worst	13.38 (1.98)	21.13 (4.28)	<0.001
smoothness_worst	0.12 (0.02)	0.14 (0.02)	<0.001
compactness_worst	0.18 (0.09)	0.37 (0.17)	<0.001
concavity_worst	0.17 (0.14)	0.45 (0.18)	<0.001
symmetry_worst	0.27 (0.04)	0.32 (0.07)	<0.001
fractal_dimension_worst	0.08 (0.01)	0.09 (0.02)	<0.001

[1] Statistics presented: Mean (SD)

[2] Statistical tests performed: Wilcoxon rank-sum test

Full Model (Newton-Raphson)

Consider the following log-likelihood, gradient, and hessian matrix. First, let

$$\pi_i = P(Y_i = 1 | x_{i,1}, \dots x_{i,p}) = \frac{e^{\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j}}}{1 + e^{\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j}}}.$$

The log-likelihood:

$$l(\mathbf{X}|\vec{\beta}) = \sum_{i=1}^{n} \left[y_i \left(\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j} \right) - \log \left(1 + \exp \left(\beta_0 + \sum_{j=1}^{p} \beta_j x_{i,j} \right) \right) \right]$$

The gradient:

$$\nabla l(\mathbf{X}|\vec{\beta}) = \left[\sum^n y_i - \pi_i \quad \sum^n x_{i,1}(y_i - \pi_i) \quad \dots \quad \sum^n x_{i,p}(y_i - \pi_i) \right]^T_{1 \times (p+1)}$$

The hessian: produces a matrix $(p + 1 \times p + 1)$

$$\nabla^2 l(\mathbf{X}|\vec{\beta}) = -\sum_{i=1}^{n} \begin{pmatrix} 1 \\ X \end{pmatrix} \begin{pmatrix} 1 & X \end{pmatrix} \pi_i (1 - \pi_i)$$

Optimal Model (Logistic LASSO)

Lemma 1. Consider the optimization problem

$$\min_{x \in \mathbb{R}} \left\{ \frac{1}{2}(x-b)^2 + c|x| \right\}$$

for $b \in \mathbb{R}$ and $c \in \mathbb{R}_{++}$. It follows that the minimizer is given by

$$\hat{x} = S(b, c),$$

where S is the soft-thresholding operator.

Lemma 2. Consider the optimization problem

$$\min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^{n} w_i \left(z_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right\}$$

for some $k \in \{1, \ldots, p\}$. It follows that the minimizer is given by

$$\hat{\beta}_k = \left(\sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \sum_{i=1}^{n} w_i x_{ik} \left(z_i - \sum_{j \neq k} \beta_j x_{ij} \right).$$

Optimal Model (Logistic LASSO)

Lemma 3. With $\hat{\beta}_k$ defined as above,

$$\min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^{n} w_i \left(z_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}$$

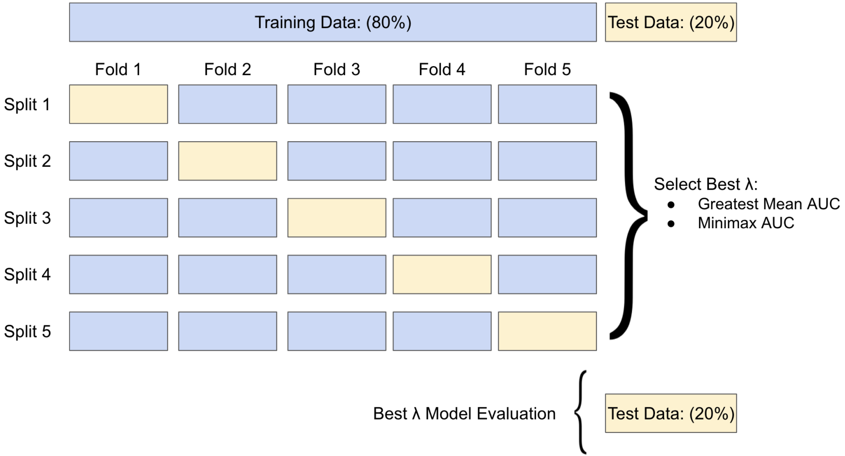
$$= \min_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2} (\beta_k - \hat{\beta}_k)^2 + \left(\frac{1}{n} \sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \lambda |\beta_k| \right\}.$$

By Lemma 1 and Lemma 3,

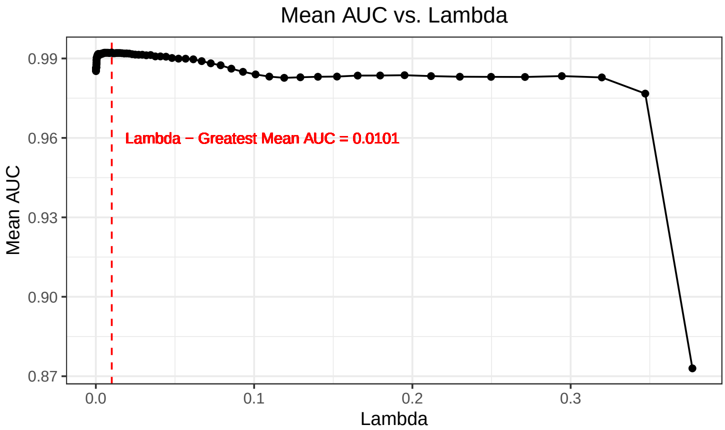
$$\operatorname*{arg\,min}_{\beta_k \in \mathbb{R}} \left\{ \frac{1}{2n} \sum_{i=1}^{n} w_i \left(z_i - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}$$

$$= S \left(\hat{\beta}_k, \left(\frac{1}{n} \sum_{i=1}^{n} w_i x_{ik}^2 \right)^{-1} \lambda \right)$$

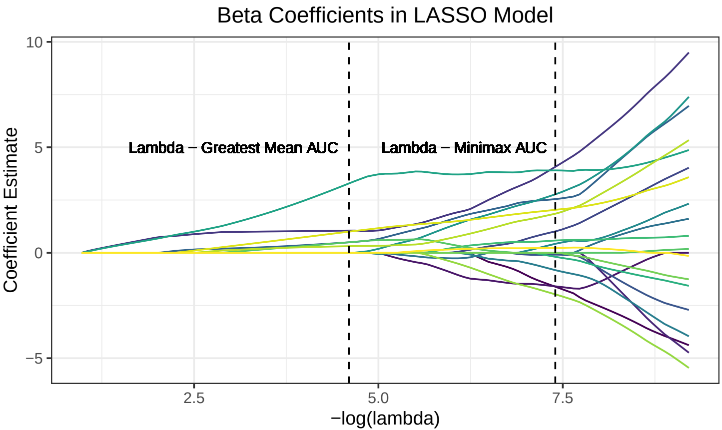
5-Fold Cross Validation



Cross Validation Results: Selecting Best Lambda



Cross Validation Results: LASSO Coefficients



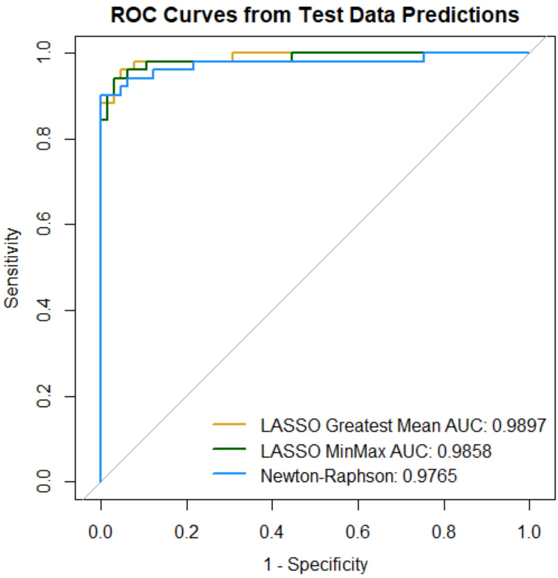
Coefficients Comparison: All Estimates

	NewtonRaphson	LASSO_GreatestMeanAUC	LASSO_MinimaxAUC
intercept	-0.0881	-0.9302	-1.2291
texture_mean	8.2691	0.9994	2.0355
smoothness_mean	-2.5447	0.0000	-0.1919
compactness_mean	-10.9297	0.0000	-1.6093
concave points_mean	20.9342	1.0512	4.0737
symmetry_mean	-1.9716	0.0000	0.0000
fractal_dimension_mean	3.9221	0.0000	0.0000
radius_se	18.5308	0.0000	2.7696
texture_se	-2.0504	0.0000	0.2269
smoothness_se	1.2753	0.0000	0.5802
compactness_se	4.7358	0.0000	-1.5925
concavity_se	-6.4914	0.0000	-0.0893
concave points_se	8.9092	0.0000	1.0126
symmetry_se	-13.8273	0.0000	-1.9717
fractal_dimension_se	-12.2152	0.0000	-0.8241
radius_worst	9.2654	3.2848	3.9047
smoothness_worst	0.6705	0.4955	0.0000
compactness_worst	-14.3912	0.0000	0.0000
concavity_worst	14.9611	0.4943	2.5436
symmetry_worst	12.3102	0.3112	1.8506
fractal_dimension_worst	7.7655	0.0000	0.4242

Coefficients Comparison: Model Performance

Measures	NewtonRaphson	LASSO_GreatestMeanAUC	LASSO_MinimaxAUC
Specificities	0.2462	0.6923	0.5538
AUC	0.9765	0.9897	0.9858
Selected Lambda	0	0.0101	0.0006
Number of Variables (w/o Intercept)	20	6	16

ROC Plot


ROC Curves from Test Data Predictions
Sensitivity
0.0
0.2
0.4
0.6
0.8
1.0
1 - Specificity
0.0
0.2
0.4
0.6
0.8
1.0
LASSO Greatest Mean AUC: 0.9897
LASSO MinMax AUC: 0.9858
Newton-Raphson: 0.9765

Discussion

- Ideal performance is to accurately classify every patient
- Balancing sensitivity and specificity
  - False positives vs false negatives
  - Decision boundaries
- Future work

Resources

Duncan, J. R. (2017, September 1). Information overload: When less is more in medical imaging. De Gruyter. Retrieved March 27, 2022, from https://www.degruyter.com/document/doi/10.1515/dx-2017-0008/html?lang=en

Cancer Stat Facts: Female Breast Cancer. *National Cancer Institute - NIH* https://seer.cancer.gov/statfacts/html/breast.html

American Cancer Society. (2019). Breast cancer facts & figures 2019–2020. Am Cancer Soc, 1-44.