

Regression of KL Software Distribution

KL Software Libraries

Wed Jun 11 17:21:34 2014

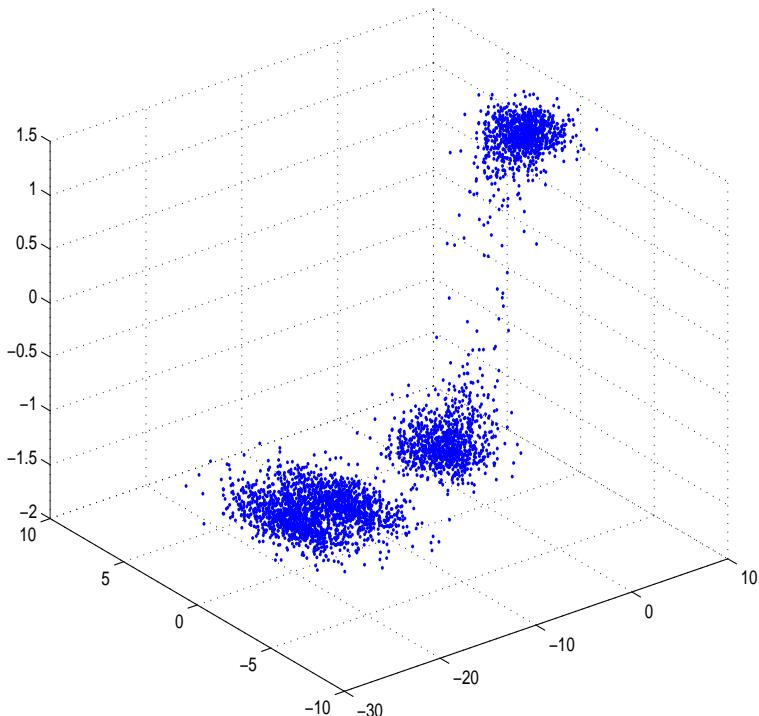
KL Library test output. This LaTex file and the associated diagrams are produced by the KL software libraries.

0.0.1 Matrix Quick Check `double`:

QueryPerformanceCounter = 0.0949109

0.0.2 Linear Regression atan data 3x1

Mixture of four 2d gaussians, reponse is atan of x coord

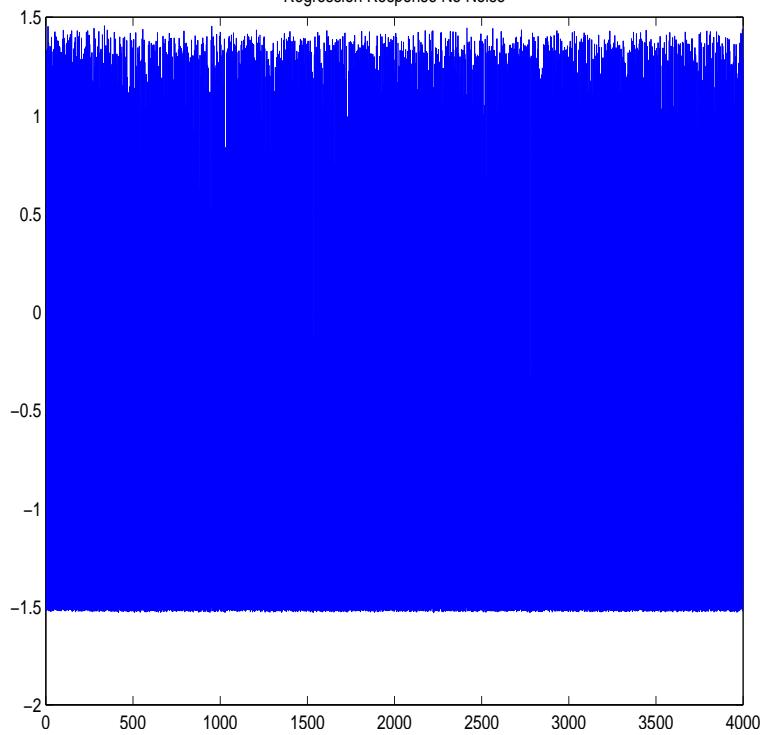


0.0.3 3 x 1 Linear Regression

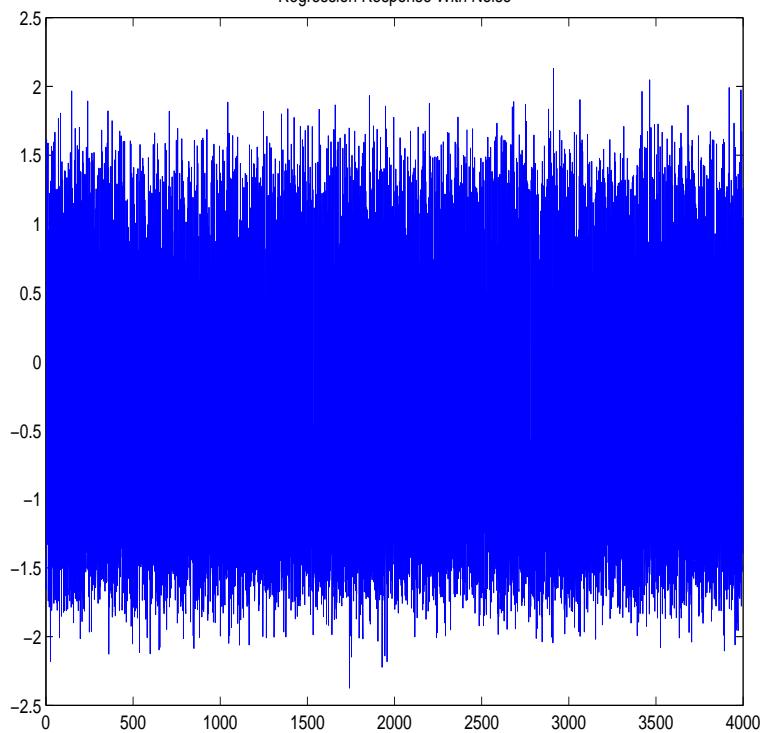
Sample size = 4000

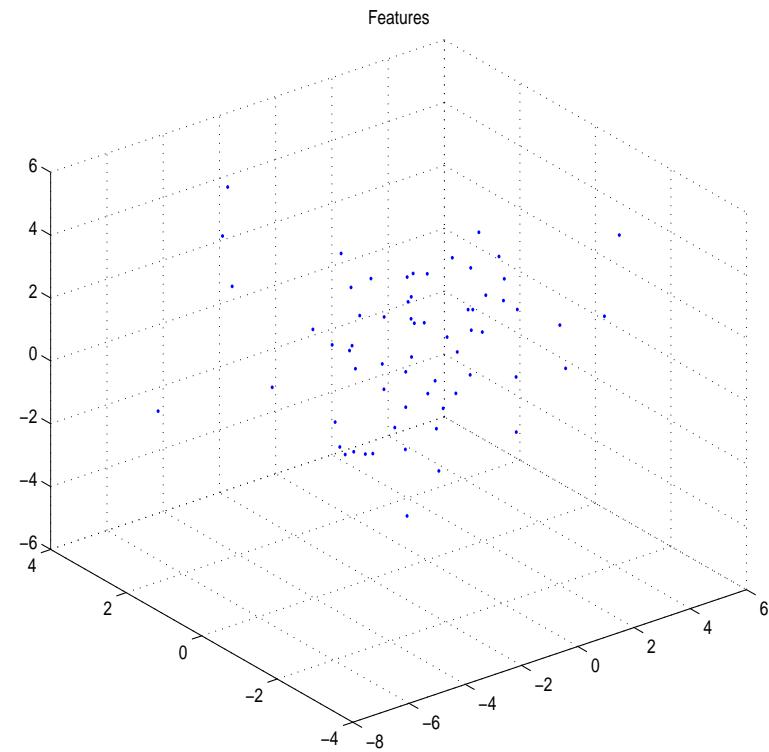
Number of features = 3

Regression Response No Noise



Regression Response With Noise





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 -1.15415 0.952204 -1.17521 -1.36476 -1.78457 1.22304 -1.42609 -1.52062 -1.11813 1.00811 -1.48172 -2.18034 -1.17985 1.31079 -1.45198
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 1.22994 -1.75868 -1.31393 1.16193 -1.51315 -1.04549 -0.396029 1.66721 -1.55155 -1.73297 -0.962694 1.25076 -1.44974 -1.11086 -1.24539
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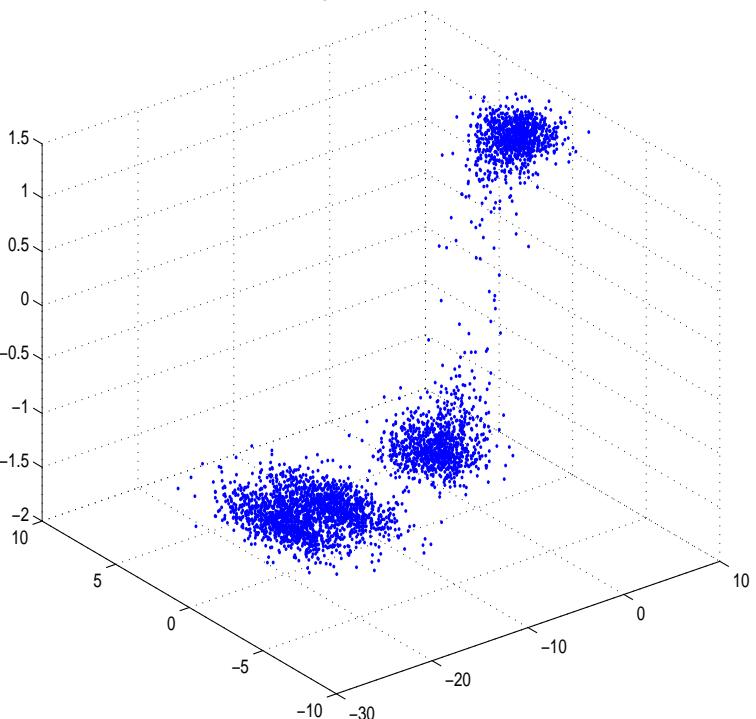
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-1.47741 0.905063 -1.51949 -1.52463 -1.19559 1.43507 -1.63939 -1.66798 -1.16853 1.45091 -1.21482 -1.40142 -1.50426 1.26729 -1.57874 -1.36823 -1.68928 1.06279 -1.391 -0.937474 -1.13887 1.4552 -1.24915 -1.07482 -1.53638 0.651209 -1.5181 -1.50894 -1.39637 1.15578 -1.7398 -1.71256 -1.38184 0.988645 -1.44643 -1.39208 -1.10971 0.960311 -1.53805 -1.55491 -1.54969 1.18112 -1.52249 -2.01894 -1.15611 1.22784 -1.63908 -1.62521 -1.31932 1.41418 -1.20001 -1.40431 -1.59238 1.31588 -1.58626 -1.75972 -1.64831 0.990902 -1.57917 -1.52962 -0.853677 1.38609 -1.67734 -1.41637 -1.16782 1.47548 -1.53101 -1.5243 -1.46128 1.4616 -1.40622 -1.19908 -1.4466 0.753155 -1.2373 -1.69995 -1.0109 1.30156 -1.60618 -1.26321 -1.62251 1.09787 -1.91458 -1.87668 -1.52332 1.39675 -1.77303 -1.7498 -1.28851 1.54281 -1.25017 -1.37815 -0.996041 1.33211 -1.32659 -1.41559 -1.09788 1.30096 -1.49803 -1.59582 -1.18242 1.19559 -1.64257 -1.58394 -1.70432 0.92546 -1.05531 -1.44201 -1.46108 1.11658 -1.2455 -1.32694 -1.13594 1.41225 -1.23196 -1.33555 -1.10145 1.33463 -1.42009 -1.23814 -1.65735 1.36416 -1.55117 -1.42634 -0.124848 1.61372 -1.86889 -1.63872 -0.911589 1.06841 -1.64643 -1.7417 -1.38107 1.38995 -1.79521 -1.50997 -1.71466 1.30671 -1.57079 -1.67859 -1.23461 1.28174 -1.71214 -1.5876 -1.21891 1.15684 -1.67032 -1.35793 -1.32024 1.04082 -1.49838 -1.49041 -0.874913 1.51795 -1.57606 -1.10683 -1.40227 1.44979 -1.47473 -1.10795 -1.32173 1.13822 -1.40803 -1.48314 -1.46321 1.20369 -1.40137 -1.61318 -0.924267 1.27243 -1.7874 -1.72772 -1.52188 1.2956 -1.77559 -1.27139 -1.51661 1.25402 -1.23719 -1.47324 -1.0697 1.29977 -1.8327 -1.356 -1.34816 1.51166 -1.79225 -1.24116 -0.985712 1.31729 -1.341 -1.41286 -1.3706 1.14608 -1.43686 -1.29587 -1.45506 1.47704 -1.50783 -1.8602 -1.21901 1.43983 -1.10665 -1.66794 -1.33132 1.70926 -1.04048 -1.15199 -1.51278 1.05587 -1.62835 -1.36508 -1.42912 1.31929 -1.17418 -1.40979 -1.42821 0.693001 -1.78687 -1.51472 -1.19247 1.20145 -1.52765 -1.66178 -1.20635 1.47679 -1.26346 -1.71825 -1.59339 1.19612 -1.53254 -1.7443 -1.11654 1.32025 -1.822 -1.57888 -0.984779 1.06345 -1.4809 -1.60542 -0.992633 1.01288 -1.70422 -1.51068 -1.50213 1.25293 -1.90183 -1.58803 -1.14731 1.06207 -1.59877 -1.12989 -1.19499 1.30308 -1.49159 -1.34977 -1.65633 1.34437 -1.43 -1.71483 -0.926707 1.35502 -1.51141 -1.34642 -1.26215 0.996357 -1.54352 -1.7837 -1.0198 1.20571 -1.65175 -1.74357 -1.14834 0.767488 -1.17647 -1.19836 -0.853417 1.35009 -1.70613 -1.77519 -1.34589 1.42596 -1.99386 -1.34461 -1.60626 0.881308 -1.67724 -1.12175 -1.61939 1.1385 -1.89442 -1.13315 -1.12558 1.4163 -1.63759 -1.34259 -1.11983 1.54809 -1.83206 -1.44109 -0.854114 1.64428 -1.56209 -1.46521 -1.14783 1.36508 -1.31023 -1.73689 -0.699035 1.96347 -1.0466 -1.50157 -0.884015 1.38306 -1.83273 -1.16477 -1.22997 1.3284 -1.32824 -1.15025 -1.35926 1.21652 -1.69646 -1.51102 -1.09779 1.5579 -1.63054 -1.44146 -1.37987 1.14865 -1.42686 -1.24365 -0.185233 1.55226 -1.26638 -1.92519 -1.02708 1.26346 -1.27077 -1.60892 -1.02749 1.67338 -1.53533 -1.30722 -1.25567 0.734469 -1.36356 -1.32218 -1.48904 0.869207 -1.57764 -1.83541 -1.03712 2.04832 -1.54804 -1.46024 -1.18777 1.3098 -1.30369 -1.21691 -1.24394 1.70131 -1.42767 -1.44358 -1.4282 1.1602 -1.69015 -1.3451 -1.13772 1.33759 -1.66513 -1.59146 -1.47536 0.906408 -1.71367 -1.54744 -1.28337 1.32592 -1.61133 -1.50675 -1.15231 1.72325 -1.31918 -1.81546 -1.19579 0.754347 -1.3837 -1.42326 -1.24691 1.17893 -1.31604 -1.60293 -1.29162 1.5034 -1.17991

-1.65519 -1.14445 1.21557 -1.86479 -1.31428 -1.12867 1.72705 -1.6785 -1.66425 -1.628 0.944428 -1.45149 -1.35921 -1.26467 1.37196 -1.09606 -1.55446 -0.986062 1.56251 -1.35214 -2.07898 -1.42558 0.872438 -1.4552 -1.50197 -1.5258 0.777759 -1.63766 -1.18809 -1.36716
 1.66607 -1.83129 -1.29778 -1.24453 1.36661 -1.32231 -1.65181 -1.28164 1.35484 -1.36952 -1.20625 -1.14194 1.17075 -1.71989 -1.54549
 -1.42162 1.41231 -1.44114 -1.8692 -1.09178 0.852253 -1.48671 -1.22655 -0.93306 1.30727 -1.17038 -1.46652 -1.28087 1.61854 -1.43274
 -1.66401 -1.33149 1.21673 -1.47066 -1.41198 -1.65996 1.4249 -1.64648 -1.32681 -1.36238 1.3911 -1.61502 -1.54533 -0.996945 1.50068 -1.3198 -1.8435 -1.43456 1.30755 -1.61618 -1.64789 -1.63136 1.41789 -1.8662 -1.42854 -1.39703 1.7147 -1.77069 -1.58078 -1.52639 1.27815
 -1.29222 -1.05864 -1.61767 1.0127 -1.59416 -1.26642 -1.1299 0.870424 -1.79321 -1.20178 -1.0689 1.59567 -1.3935 -1.62933 -1.43602
 1.36416 -1.65834 -1.0446 -0.687962 1.25061 -1.86245 -1.30132 -1.23852 1.27992 -1.45861 -1.55771 -1.37621 0.971177 -1.42356 -1.16158
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 -1.59744 -1.6731 1.66075 -1.86453 -1.54824 -1.65688 1.56039 -1.35108 -1.60201 -1.07763 1.32868 -1.75418 -1.68671 -1.11519 1.29009
 -1.73537 -1.6873 -1.47816 1.24378 -1.13506 -1.76798 -1.6403 1.04418 -1.30704 -1.41268 -1.01004 1.21124 -1.41361 -1.4422 -1.40133
 1.42187 -1.61572 -1.77477 -1.04579 1.27379 -1.67674 -1.6583 -1.5242 1.33046 -1.46043 -1.49247 -1.23826 1.86342 -0.960575 -1.60592
 -1.27122 1.57428 -1.24707 -1.97 -1.02268 1.45133 -1.55125 -1.31307 -1.39311 1.32312 -1.56827 -1.68908 -1.28515 1.10828 -1.64023 -1.7666 -0.872034 1.56278 -2.03966 -1.38525 -1.07292 1.60889 -1.20189 -1.55942 -1.1642 0.532046 -1.18414 -1.12756 -1.49287 1.38933
 -1.57712 -1.4511 -1.52861 1.36624 -1.44109 -1.27768 -1.32817 1.27147 -1.38188 -1.41689 -0.962444 1.14416 -1.62259 -1.48775 -0.968078
 1.11054 -1.51735 -1.24087 -1.39546 0.854537 -1.63913 -1.35472 -1.41267 1.03456 -1.57882 -1.27878 -1.14562 1.6414 -1.63094 -1.85192
 -0.996913 1.15151 -1.84268 -1.80336 -1.52417 1.27163 -1.43106 -1.55452 -0.985712 1.45808 -1.3719 -1.6061 -1.10301 1.48252 -1.43208
 -1.72046 -1.17546 1.1863 -1.8251 -1.07028 -1.21126 1.2692 -1.67441 -1.903 -1.61482 1.42773 -1.85957 -1.74094 -1.19867 1.1741 -1.40708
 -1.21052 -1.48034 1.42084 -1.26667 -1.52834 -1.23555 1.05175 -1.51381 -1.71195 -1.2782 0.863682 -1.41995 -1.27495 -0.0971476 1.3539
 -1.68011 -1.34408 -1.39139 1.22574 -1.43957 -1.6037 -1.22561 1.32766 -1.58889 -1.45881 -1.48528 1.19418 -1.57905 -1.43679 -1.06319
 1.57517 -1.67881 -1.5535 -0.406248 1.67155 -1.63633 -1.54532 -1.24799 1.46771 -1.65209 -1.53892 -1.88754 1.33801 -1.80283 -1.63614
 -1.47635 1.61158 -1.41225 -1.50741 -0.881467 1.44355 -1.60903 -1.51572 -0.987592 1.61019 -1.18453 -1.71465 -1.72688 0.755218 -1.58096
 -1.64428 -1.43137 0.929263 -1.11605 -1.76012 -1.19389 0.915804 -1.7781 -1.35244 -0.535522 1.58183 -0.694799 -1.41471 -1.4977 1.26079
 -1.46779 -1.44475 -0.983746 1.02199 -1.39852 -1.38355 -0.977771 1.24388 -1.56798 -1.81144 -1.32078 1.30134 -1.7207 -1.70917 -1.2752
 1.09895 -1.11209 -1.20652 -1.05006 1.25714 -1.44485 -1.8119 -1.42833 1.4041 -1.32275 -1.50627 -1.07743 0.916211 -1.13779 -1.52854
 -1.45042 1.57917 -1.38263 -1.35727 -1.25805 1.61677 -1.98907 -1.49229 -1.54909 1.59253 -2.10398 -1.41441 -1.59264 1.12008 -1.3697
 -1.66223 -1.1947 1.1938 -1.19147 -1.31025 -1.12905 1.23239 -1.14227 -1.41195 -1.72831 1.273 -1.38419 -1.50281 -1.15899 0.798089 -1.65706 -1.46002 -1.0177 1.36527 -1.32254 -1.18454 -1.30933 1.99183 -1.79637 -1.66828 -0.995727 1.15464 -1.59542 -1.39042 -1.62131
 1.08312 -1.58194 -1.50116 -1.57535 1.31803 -1.80581 -1.75309 -1.14167 1.17427 -0.589121 -1.66842 -1.73096 1.20446 -1.6489 -1.12024
 -1.62622 1.15003 -1.30617 -1.51204 -1.13434 1.73278 -1.5461 -1.20907 -1.5389 0.907571 -1.78324 -2.06005 -1.22097 1.55636 -1.35659
 -1.80766 -1.7002 1.34488 -1.95257 -1.56042 -1.21632 1.25346 -1.32922 -1.89247 -1.20382 0.19202 -1.63868 -1.80098 -1.95668 1.36165
 -1.5254 -1.56737 -1.64261 1.23639 -1.70655 -1.19567 -1.25804 1.44601 -1.72262 -1.3587 -1.32884 1.36488 -1.58242 -1.60421 -0.279258
 1.97472 -1.65437 -1.77785 -0.946302 1.6693 -1.41126 -1.52884 0.198092 1.04135 -1.08062 -1.39281 -1.21884 1.25925 Estimate for Beta
 0.000299912 -0.000395098 0.996784 QueryPerformanceCounter = 5.3626

0.0.4 Linear Regression 3x1

Mixture of four 2d gaussians, reponse is atan of x coord

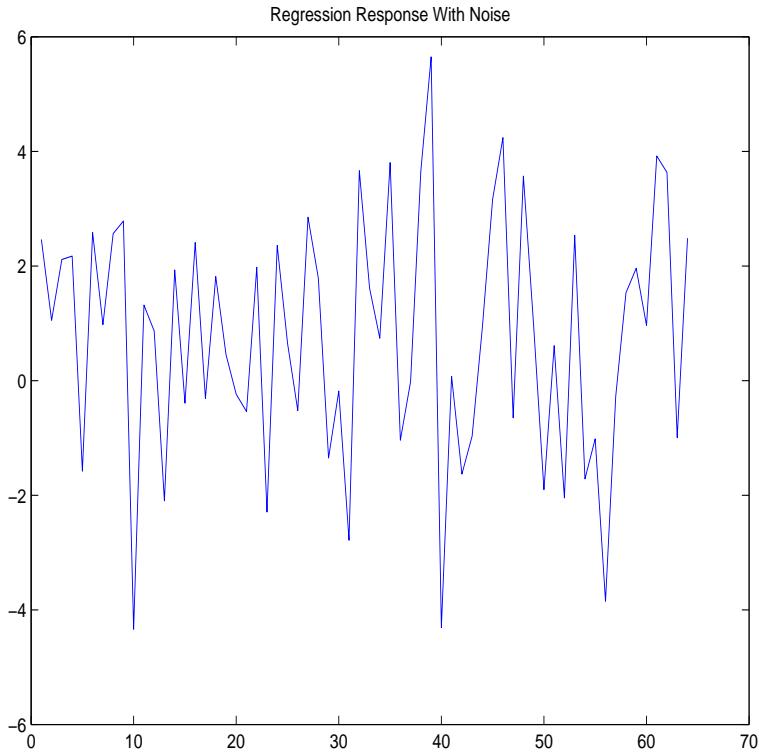
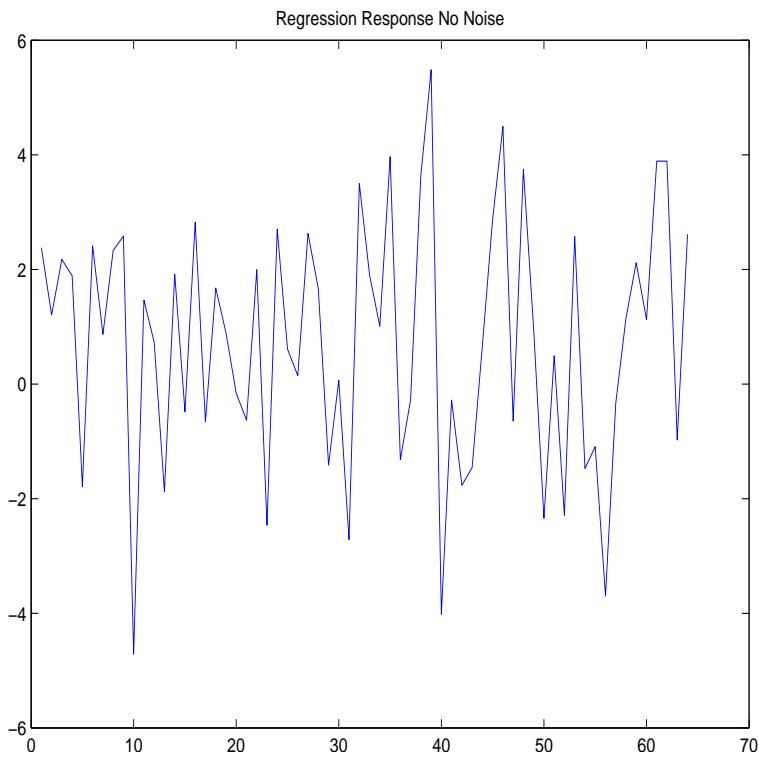


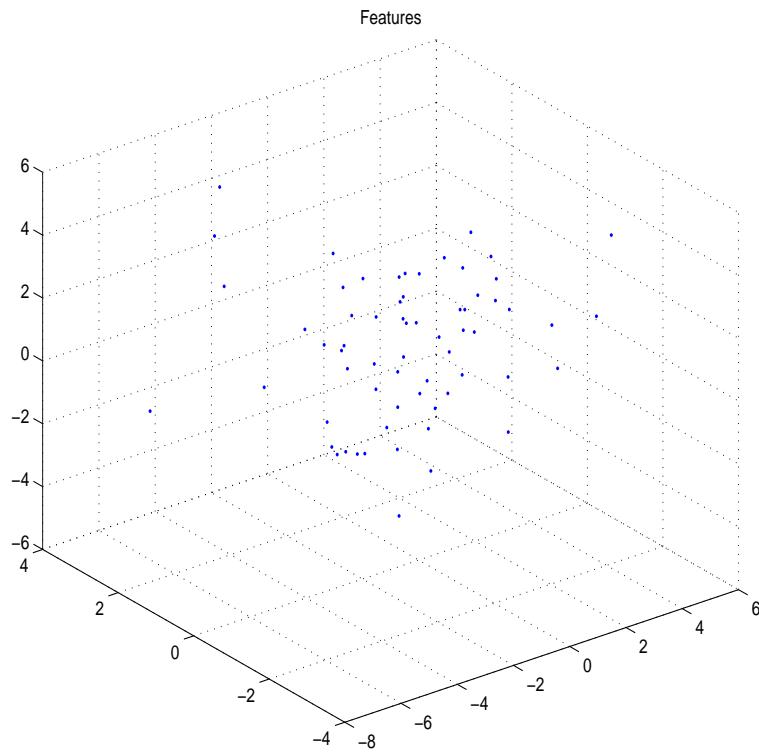
0.0.5 3 x 1 Linear Regression

Sample size = 64

Number of features = 3

$$\sigma = \begin{pmatrix} +3.952 & -0.499 & -0.010 \\ -0.499 & +1.895 & +0.465 \\ -0.010 & +0.465 & +4.477 \end{pmatrix}$$



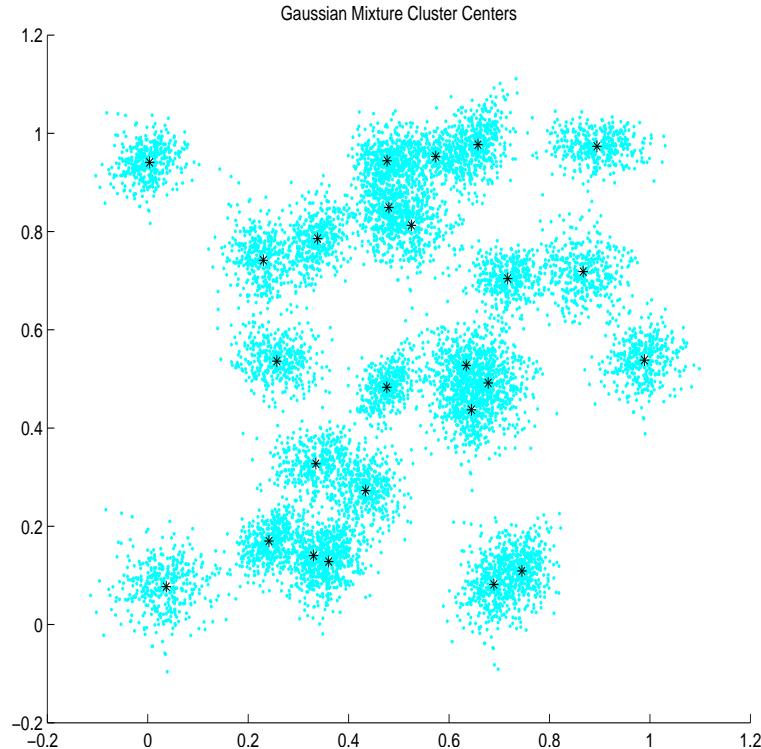


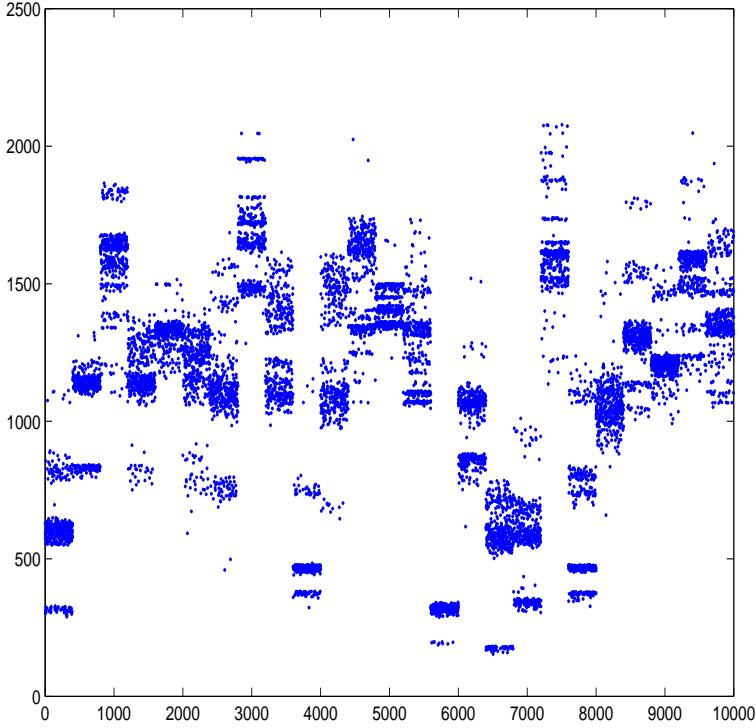
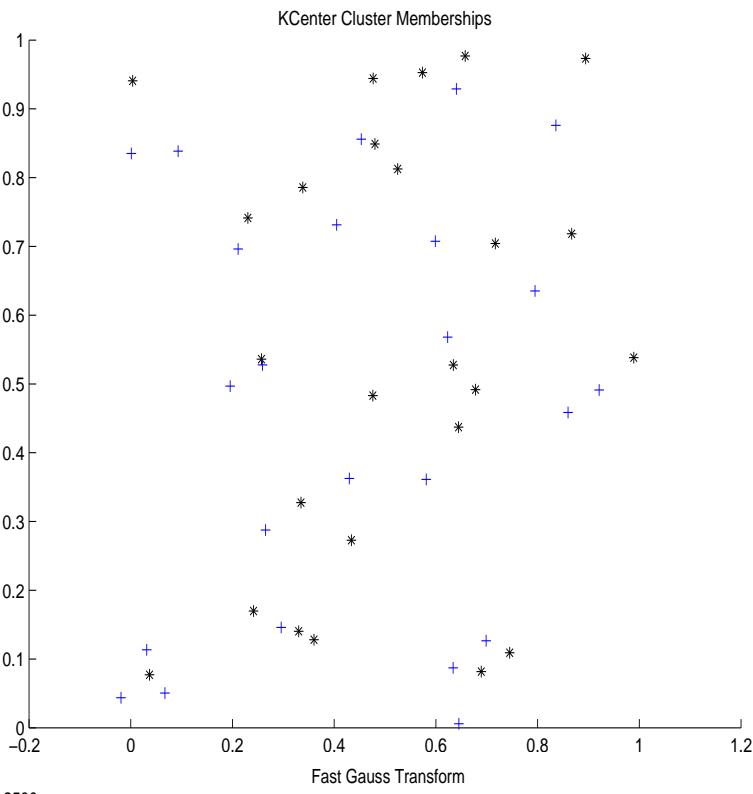
Beta +0.817, +0.999, +0.510

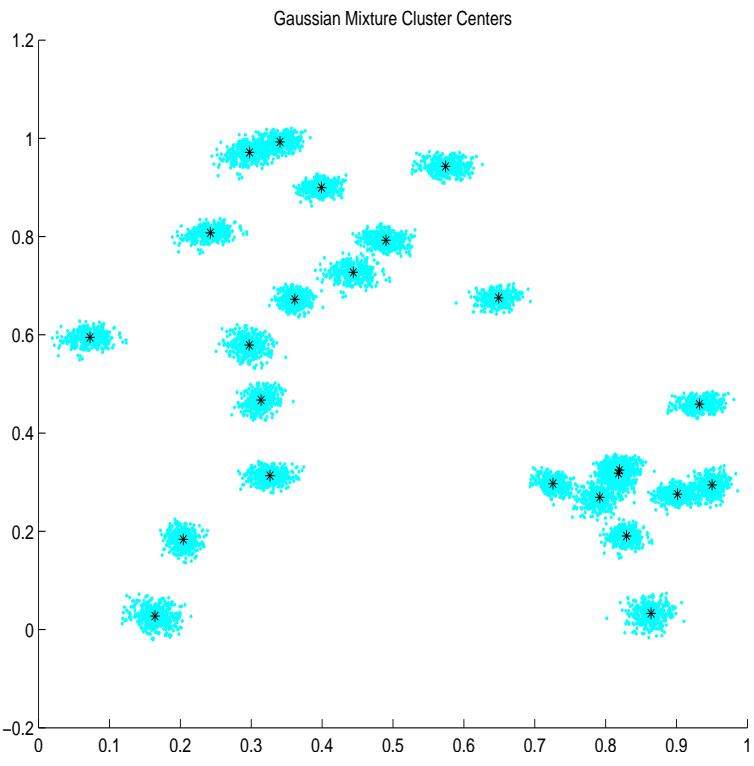
Response +2.463 +1.053 +2.113 +2.172 -1.579 +2.584 +0.976 +2.566 +2.785 -4.338 +1.319 +0.865 -2.100 +1.931 -0.394 +2.410
 -0.311 +1.817 +0.458 -0.239 -0.538 +1.979 -2.294 +2.359 +0.652 -0.523 +2.852 +1.788 -1.352 -0.181 -2.786 +3.664 +1.615 +0.739
 +3.805 -1.041 -0.017 +3.651 +5.649 -4.307 +0.074 -1.631 -0.965 +0.930 +3.163 +4.241 -0.650 +3.570 +0.939 -1.903 +0.610 -2.049
 +2.536 -1.714 -1.016 -3.850 -0.274 +1.533 +1.959 +0.962 +3.920 +3.633 -0.995 +2.486 Estimate for Beta +0.812 +1.004 +0.484 Error:
 -0.005, +0.005, -0.026

QueryPerformanceCounter = +4.194

0.0.6 Fast Gauss Transform

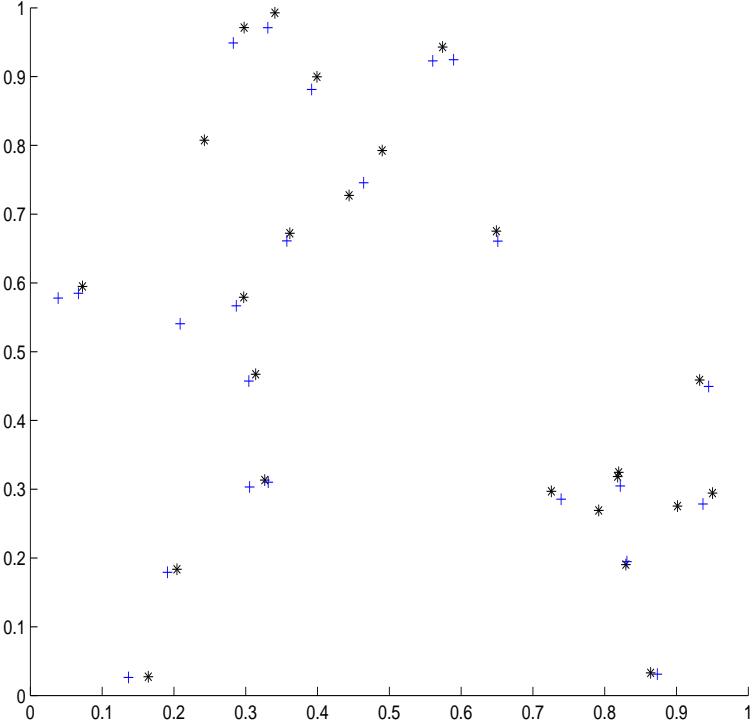




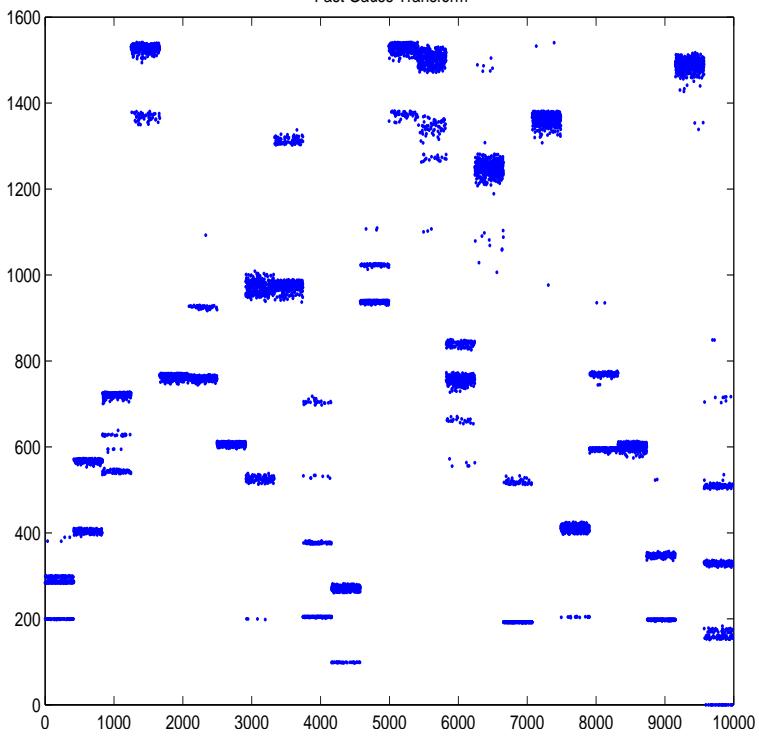


QueryPerformanceCounter = +6.518

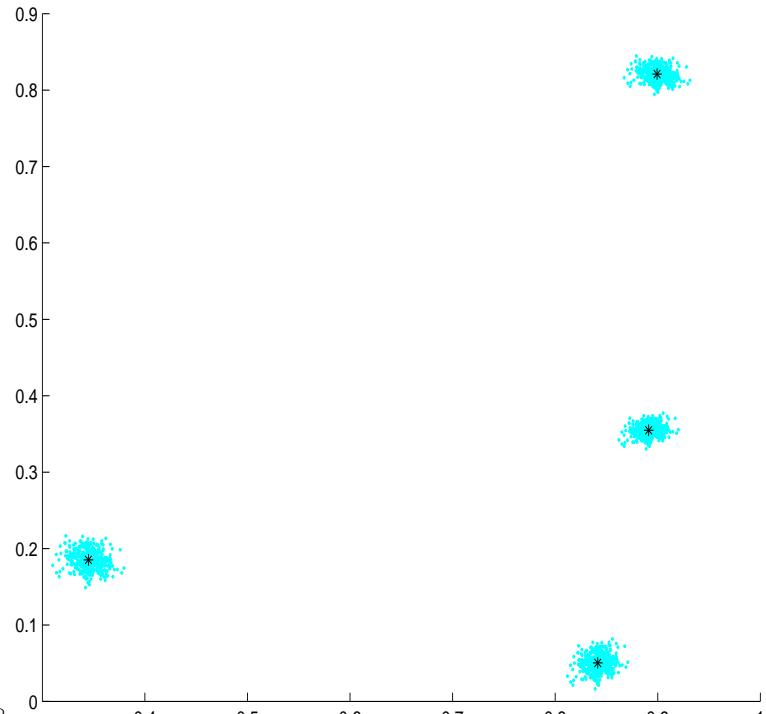
KCenter Cluster Memberships



Fast Gauss Transform

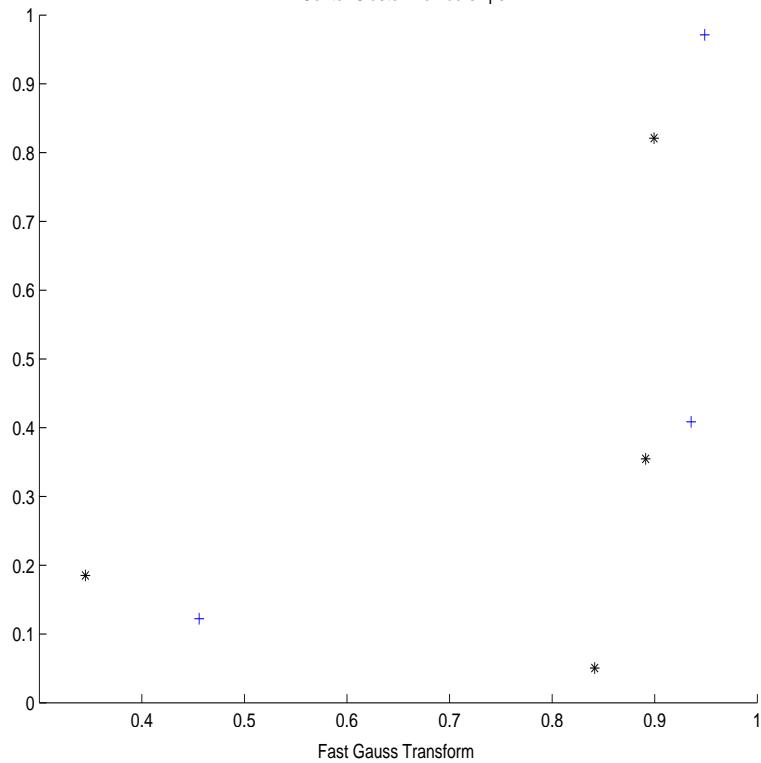


Gaussian Mixture Cluster Centers

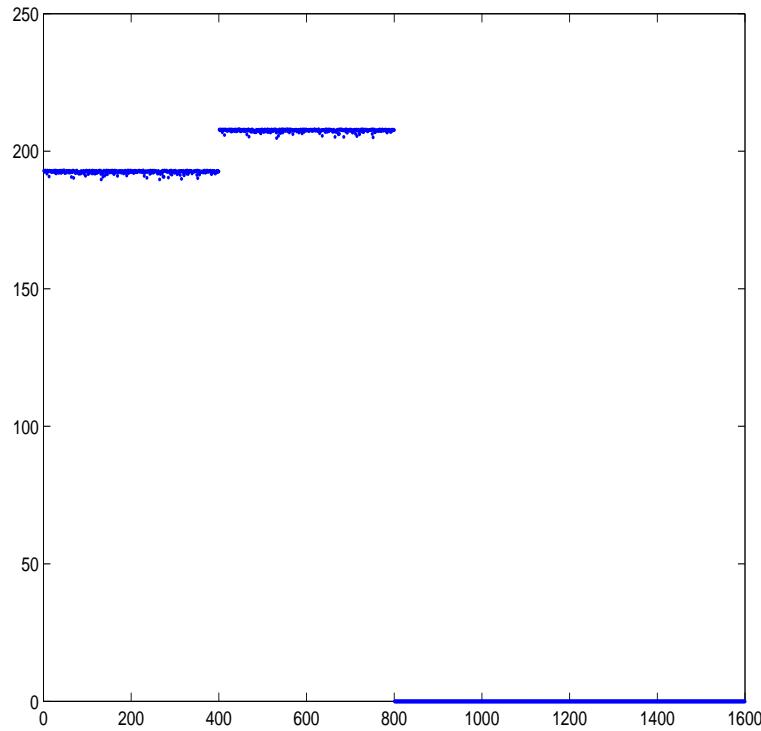


QueryPerformanceCounter = +6.948

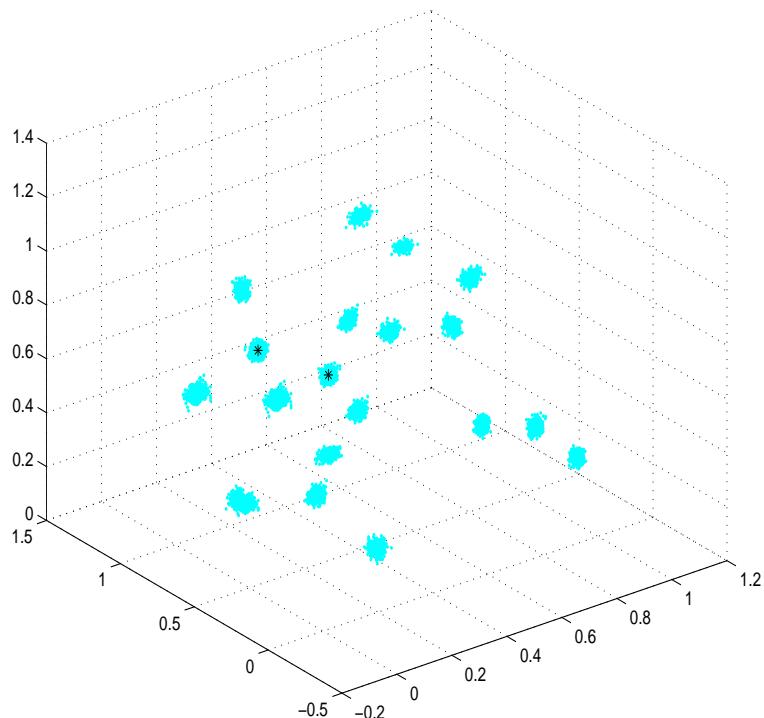
KCenter Cluster Memberships



Fast Gauss Transform

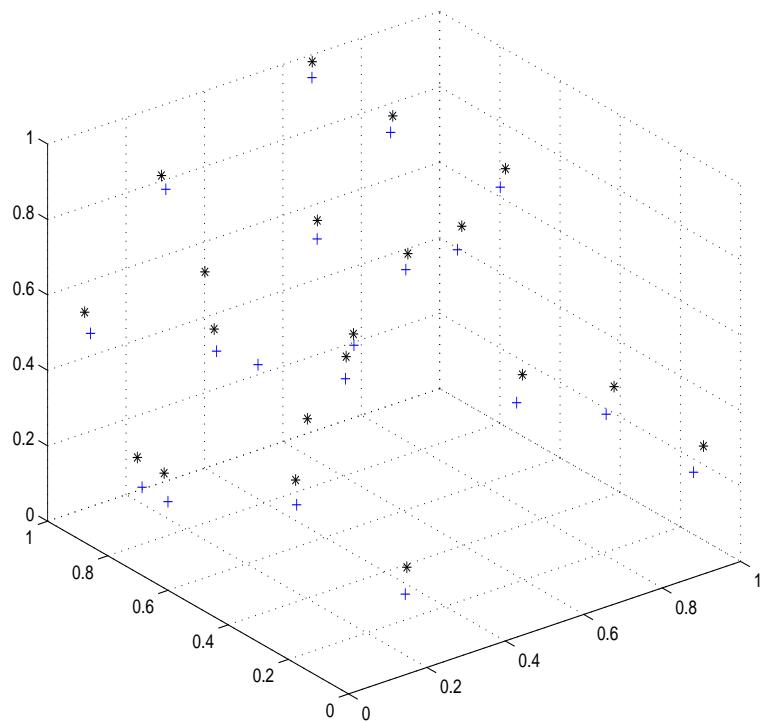


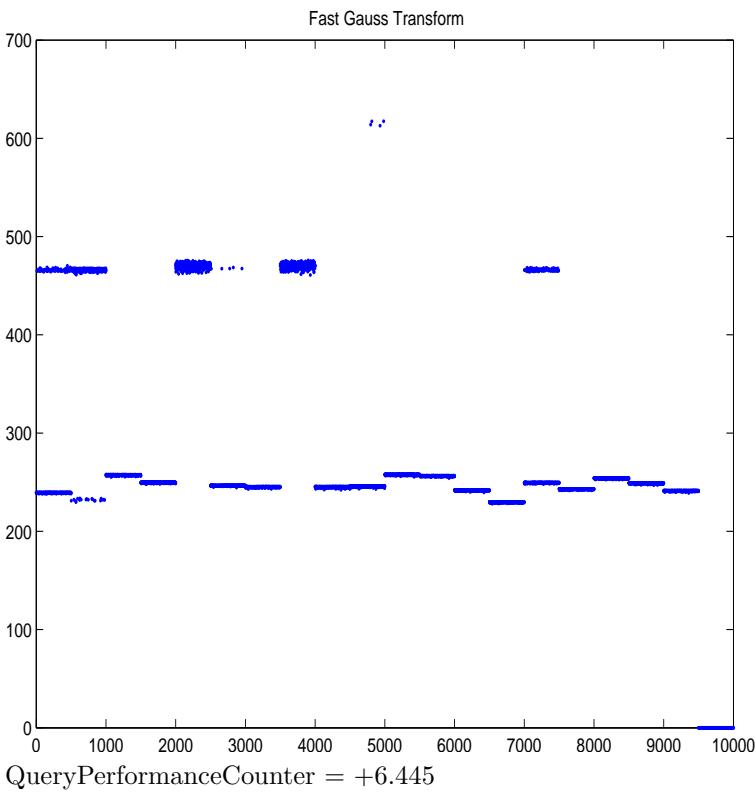
Gaussian Mixture Cluster Centers



QueryPerformanceCounter = +3.846

KCenter Cluster Memberships





0.0.7 Matrix Norms

0.0.8 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: +12

$$A = \begin{pmatrix} +0.167 & -0.040 & -0.084 & -0.017 & +0.402 & -0.371 & +0.167 & -0.088 & +0.587 & +0.093 & -0.434 & +0.296 \\ -0.425 & -0.489 & -0.431 & +0.091 & -0.091 & -0.284 & -0.411 & -0.245 & +0.126 & -0.159 & +0.164 & +0.022 \\ +0.369 & -0.223 & -0.035 & +0.506 & -0.205 & -0.469 & +0.125 & +0.049 & -0.473 & +0.148 & -0.130 & +0.117 \\ -0.207 & -0.179 & +0.343 & +0.376 & -0.493 & +0.209 & +0.143 & -0.159 & +0.392 & +0.134 & -0.310 & -0.257 \\ -0.055 & +0.053 & +0.041 & +0.589 & +0.550 & +0.229 & +0.158 & -0.379 & -0.010 & +0.003 & +0.345 & -0.054 \\ -0.045 & +0.590 & -0.106 & +0.020 & -0.002 & -0.291 & -0.306 & -0.398 & -0.122 & +0.069 & -0.293 & -0.442 \\ +0.144 & -0.054 & +0.212 & +0.196 & +0.126 & +0.227 & -0.409 & +0.049 & -0.108 & -0.669 & -0.413 & +0.160 \\ -0.523 & +0.361 & -0.023 & +0.287 & -0.005 & -0.300 & +0.254 & +0.510 & +0.054 & -0.307 & +0.033 & +0.032 \\ +0.343 & -0.004 & -0.387 & -0.072 & -0.211 & +0.009 & +0.471 & -0.190 & +0.137 & -0.557 & +0.091 & -0.296 \\ -0.159 & +0.169 & +0.384 & -0.183 & -0.244 & -0.209 & +0.176 & -0.529 & -0.101 & -0.201 & +0.162 & +0.524 \\ -0.391 & -0.330 & +0.030 & -0.259 & +0.293 & +0.028 & +0.396 & -0.128 & -0.436 & -0.027 & -0.423 & -0.202 \\ +0.131 & -0.237 & +0.576 & -0.139 & +0.191 & -0.441 & -0.089 & +0.089 & +0.121 & -0.168 & +0.286 & -0.452 \end{pmatrix}$$

$\text{Det}(A) : A \in O(n) = (-1.000, +0.000)$

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.814 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.250 & +0.188 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.707 & -0.041 & -0.105 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.104 & -0.019 & +0.055 & +0.799 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.397 & +0.412 & +0.806 & +0.505 & -0.762 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.748 & +0.767 & +0.561 & -0.489 & +0.224 & +0.007 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.304 & -0.075 & +0.555 & -0.370 & -0.629 & +0.246 & +0.047 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.320 & -0.097 & -0.203 & +0.076 & +0.650 & -0.606 & +0.319 & +0.215 & +1.000 & +0.000 & +0.000 & +0.000 \\ -0.657 & -0.299 & -0.810 & +0.060 & -0.089 & -0.273 & +0.624 & +0.322 & +0.959 & +1.000 & +0.000 & +0.000 \\ -0.275 & -0.058 & +0.281 & +0.397 & +0.200 & +0.225 & -0.614 & -0.440 & -0.682 & +0.553 & +1.000 & +0.000 \\ +0.086 & -0.715 & -0.615 & -0.187 & +0.039 & -0.435 & -0.829 & +0.956 & -0.220 & -0.470 & +0.362 & +1.000 \end{pmatrix}$$

$$U = \begin{pmatrix} -0.523 & +0.361 & -0.023 & +0.287 & -0.005 & -0.300 & +0.254 & +0.510 & +0.054 & -0.307 & +0.033 & +0.032 \\ +0.000 & -0.782 & -0.412 & -0.142 & -0.088 & -0.040 & -0.617 & -0.660 & +0.082 & +0.090 & +0.137 & -0.004 \\ +0.000 & +0.000 & +0.648 & -0.041 & +0.206 & -0.508 & +0.091 & +0.340 & +0.119 & -0.261 & +0.269 & -0.443 \\ +0.000 & +0.000 & +0.000 & +0.699 & -0.191 & -0.737 & +0.288 & +0.417 & -0.419 & -0.093 & -0.073 & +0.093 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.690 & +0.877 & -0.116 & -0.797 & +0.314 & +0.125 & +0.387 & -0.106 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.794 & -0.011 & -1.181 & +0.692 & +0.571 & -0.264 & -0.038 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.796 & +0.197 & -0.885 & +0.203 & -0.824 & +0.095 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.989 & -0.262 & -0.062 & +0.334 & +0.733 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.221 & +0.172 & -0.571 & +0.067 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.042 & +1.292 & -1.021 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.925 & +1.277 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -2.264 \end{pmatrix}$$

$$L * U = \begin{pmatrix} -0.523 & +0.361 & -0.023 & +0.287 & -0.005 & -0.300 & +0.254 & +0.510 & +0.054 & -0.307 & +0.033 & +0.032 \\ -0.425 & -0.489 & -0.431 & +0.091 & -0.091 & -0.284 & -0.411 & -0.245 & +0.126 & -0.159 & +0.164 & +0.022 \\ +0.131 & -0.237 & +0.576 & -0.139 & +0.191 & -0.441 & -0.089 & +0.089 & +0.121 & -0.168 & +0.286 & -0.452 \\ +0.369 & -0.223 & -0.035 & +0.506 & -0.205 & -0.469 & +0.125 & +0.049 & -0.473 & +0.148 & -0.130 & +0.117 \\ -0.055 & +0.053 & +0.041 & +0.589 & +0.550 & +0.229 & +0.158 & -0.379 & -0.010 & +0.003 & +0.345 & -0.054 \\ -0.207 & -0.179 & +0.343 & +0.376 & -0.493 & +0.209 & +0.143 & -0.159 & +0.392 & +0.134 & -0.310 & -0.257 \\ -0.391 & -0.330 & +0.030 & -0.259 & +0.293 & +0.028 & +0.396 & -0.128 & -0.436 & -0.027 & -0.423 & -0.202 \\ -0.159 & +0.169 & +0.384 & -0.183 & -0.244 & -0.209 & +0.176 & -0.529 & -0.101 & -0.201 & +0.162 & +0.524 \\ +0.167 & -0.040 & -0.084 & -0.017 & +0.402 & -0.371 & +0.167 & -0.088 & +0.587 & +0.093 & -0.434 & +0.296 \\ +0.343 & -0.004 & -0.387 & -0.072 & -0.211 & +0.009 & +0.471 & -0.190 & +0.137 & -0.557 & +0.091 & -0.296 \\ +0.144 & -0.054 & +0.212 & +0.196 & +0.126 & +0.227 & -0.409 & +0.049 & -0.108 & -0.669 & -0.413 & +0.160 \\ -0.045 & +0.590 & -0.106 & +0.020 & -0.002 & -0.291 & -0.306 & -0.398 & -0.122 & +0.069 & -0.293 & -0.442 \end{pmatrix}$$

$\text{Det}(L) := (+1.000, +0.000)$ $\text{Det}(U) := (+1.000, +0.000)$ $\text{Det}(LU) := (+1.000, +0.000)$

$$\|A\|_{L_1} = +3.103$$

$$\|A\|_{L_\infty} = +3.202$$

$$\|A^{-1}\|_{L_1} = +3.202$$

$$\|A^{-1}\|_{L_\infty} = +3.103$$

$$\|A\|_{L_\infty} * \|A^{-1}\|_{L_\infty} = +9.933$$

$$\|A\|_{L_1} * \|A^{-1}\|_{L_1} = +9.933$$

$$\text{Frobenius Norm } \|A\|_F \text{ via } \sum_{i,j=0}^n |A_{i,j}| \text{ of } A \in O(n) + 3.464$$

L_1 condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n) + 9.933$

$$A = \begin{pmatrix} +0.167 & -0.040 & -0.084 & -0.017 & +0.402 & -0.371 & +0.167 & -0.088 & +0.587 & +0.093 & -0.434 & +0.296 \\ -0.425 & -0.489 & -0.431 & +0.091 & -0.091 & -0.284 & -0.411 & -0.245 & +0.126 & -0.159 & +0.164 & +0.022 \\ +0.369 & -0.223 & -0.035 & +0.506 & -0.205 & -0.469 & +0.125 & +0.049 & -0.473 & +0.148 & -0.130 & +0.117 \\ -0.207 & -0.179 & +0.343 & +0.376 & -0.493 & +0.209 & +0.143 & -0.159 & +0.392 & +0.134 & -0.310 & -0.257 \\ -0.055 & +0.053 & +0.041 & +0.589 & +0.550 & +0.229 & +0.158 & -0.379 & -0.010 & +0.003 & +0.345 & -0.054 \\ -0.045 & +0.590 & -0.106 & +0.020 & -0.002 & -0.291 & -0.306 & -0.398 & -0.122 & +0.069 & -0.293 & -0.442 \\ +0.144 & -0.054 & +0.212 & +0.196 & +0.126 & +0.227 & -0.409 & +0.049 & -0.108 & -0.669 & -0.413 & +0.160 \\ -0.523 & +0.361 & -0.023 & +0.287 & -0.005 & -0.300 & +0.254 & +0.510 & +0.054 & -0.307 & +0.033 & +0.032 \\ +0.343 & -0.004 & -0.387 & -0.072 & -0.211 & +0.009 & +0.471 & -0.190 & +0.137 & -0.557 & +0.091 & -0.296 \\ -0.159 & +0.169 & +0.384 & -0.183 & -0.244 & -0.209 & +0.176 & -0.529 & -0.101 & -0.201 & +0.162 & +0.524 \\ -0.391 & -0.330 & +0.030 & -0.259 & +0.293 & +0.028 & +0.396 & -0.128 & -0.436 & -0.027 & -0.423 & -0.202 \\ +0.131 & -0.237 & +0.576 & -0.139 & +0.191 & -0.441 & -0.089 & +0.089 & +0.121 & -0.168 & +0.286 & -0.452 \end{pmatrix}$$

L_∞ condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n) + 9.006$

Eigenvalues of $A \in O(n)$

$$(+0.642, +0.767), (+0.642, -0.767), (-0.287, +0.958), (-0.287, -0.958), (-1.000, +0.000), (-0.707, +0.707), (-0.707, -0.707), (-0.876, +0.482), (-0.876, -0.482), (+1.000, +0.000), (+0.948, +0.318), (+0.948, -0.318)$$

$$|\lambda| : \lambda \in \sigma(A), A \in O(n)$$

$$+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000$$

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & +0.000 & +1.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} +0.167 & -0.425 & +0.369 & -0.207 & -0.055 & -0.045 & +0.144 & -0.523 & +0.343 & -0.159 & -0.391 & +0.131 \\ -0.040 & -0.489 & -0.223 & -0.179 & +0.053 & +0.590 & -0.054 & +0.361 & -0.004 & +0.169 & -0.330 & -0.237 \\ -0.084 & -0.431 & -0.035 & +0.343 & +0.041 & -0.106 & +0.212 & -0.023 & -0.387 & +0.384 & +0.030 & +0.576 \\ -0.017 & +0.091 & +0.506 & +0.376 & +0.589 & +0.020 & +0.196 & +0.287 & -0.072 & -0.183 & -0.259 & -0.139 \\ +0.402 & -0.091 & -0.205 & -0.493 & +0.550 & -0.002 & +0.126 & -0.005 & -0.211 & -0.244 & +0.293 & +0.191 \\ -0.371 & -0.284 & -0.469 & +0.209 & +0.229 & -0.291 & +0.227 & -0.300 & +0.009 & -0.209 & +0.028 & -0.441 \\ +0.167 & -0.411 & +0.125 & +0.143 & +0.158 & -0.306 & -0.409 & +0.254 & +0.471 & +0.176 & +0.396 & -0.089 \\ -0.088 & -0.245 & +0.049 & -0.159 & -0.379 & -0.398 & +0.049 & +0.510 & -0.190 & -0.529 & -0.128 & +0.089 \\ +0.587 & +0.126 & -0.473 & +0.392 & -0.010 & -0.122 & -0.108 & +0.054 & +0.137 & -0.101 & -0.436 & +0.121 \\ +0.093 & -0.159 & +0.148 & +0.134 & +0.003 & +0.069 & -0.669 & -0.307 & -0.557 & -0.201 & -0.027 & -0.168 \\ -0.434 & +0.164 & -0.130 & -0.310 & +0.345 & -0.293 & -0.413 & +0.033 & +0.091 & +0.162 & -0.423 & +0.286 \\ +0.296 & +0.022 & +0.117 & -0.257 & -0.054 & -0.442 & +0.160 & +0.032 & -0.296 & +0.524 & -0.202 & -0.452 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & +1.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 \\ -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} -0.254 & +0.661 & +0.123 & +0.090 & +0.076 & -0.316 & -0.419 & -0.219 & -0.070 & +0.373 & +0.040 & +0.023 \\ -0.026 & -0.386 & -0.098 & -0.101 & +0.043 & +0.290 & -0.139 & -0.515 & -0.025 & +0.543 & -0.246 & -0.323 \\ +0.622 & +0.233 & -0.573 & +0.192 & -0.089 & +0.035 & -0.071 & -0.032 & -0.351 & +0.046 & -0.186 & +0.132 \\ -0.417 & -0.037 & -0.188 & +0.239 & -0.685 & -0.195 & +0.146 & -0.031 & +0.050 & -0.039 & -0.444 & +0.048 \\ +0.122 & -0.258 & +0.186 & +0.161 & -0.006 & -0.542 & +0.451 & +0.015 & -0.382 & +0.401 & +0.233 & +0.012 \\ -0.167 & +0.040 & +0.084 & +0.017 & -0.402 & +0.371 & -0.167 & +0.088 & -0.587 & -0.093 & +0.434 & -0.296 \\ -0.059 & +0.020 & +0.176 & -0.095 & +0.242 & -0.018 & -0.090 & +0.588 & -0.320 & +0.092 & -0.586 & -0.299 \\ -0.033 & -0.088 & +0.227 & -0.061 & -0.126 & +0.364 & -0.081 & +0.199 & -0.087 & +0.375 & -0.058 & +0.768 \\ +0.392 & -0.245 & +0.542 & +0.171 & -0.211 & -0.206 & -0.461 & -0.217 & -0.040 & -0.286 & -0.182 & +0.035 \\ -0.328 & -0.122 & -0.122 & -0.257 & +0.315 & -0.128 & +0.009 & -0.378 & -0.511 & -0.389 & -0.157 & +0.319 \\ +0.249 & +0.275 & +0.166 & -0.790 & -0.349 & -0.080 & +0.235 & -0.117 & +0.002 & +0.031 & -0.101 & -0.056 \\ -0.058 & -0.363 & -0.389 & -0.360 & -0.119 & -0.384 & -0.515 & +0.297 & +0.078 & +0.101 & +0.222 & +0.050 \end{pmatrix}$$

$$\begin{aligned}
S = & \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \end{pmatrix} \\
V = & \begin{pmatrix} -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ -0.012 & +0.192 & -0.157 & +0.305 & -0.099 & -0.000 & -0.351 & -0.217 & +0.066 & +0.303 & -0.419 & +0.627 \\ -0.044 & -0.073 & +0.499 & +0.264 & +0.701 & -0.000 & +0.102 & +0.016 & +0.343 & -0.000 & -0.232 & +0.000 \\ -0.114 & +0.326 & -0.357 & +0.581 & -0.074 & +0.000 & -0.051 & +0.010 & +0.236 & -0.455 & -0.083 & -0.372 \\ +0.108 & +0.115 & -0.040 & -0.389 & +0.105 & -0.000 & -0.335 & -0.163 & -0.130 & +0.076 & -0.618 & -0.518 \\ +0.710 & +0.200 & +0.241 & +0.148 & -0.159 & +0.000 & +0.142 & -0.521 & +0.095 & +0.076 & +0.173 & -0.116 \\ -0.185 & -0.231 & +0.122 & -0.031 & -0.526 & -0.000 & +0.269 & +0.051 & +0.598 & +0.303 & -0.250 & -0.179 \\ +0.155 & -0.724 & -0.252 & +0.362 & +0.035 & +0.000 & +0.214 & -0.162 & -0.341 & +0.000 & -0.250 & -0.076 \\ -0.538 & -0.169 & +0.283 & +0.104 & -0.092 & +0.000 & -0.346 & -0.594 & -0.092 & +0.000 & +0.273 & -0.160 \\ +0.223 & -0.219 & -0.230 & +0.150 & +0.167 & +0.000 & -0.523 & +0.250 & +0.251 & +0.455 & +0.368 & -0.240 \\ -0.264 & +0.358 & -0.233 & +0.120 & +0.216 & +0.000 & +0.451 & -0.122 & -0.228 & +0.606 & +0.078 & -0.205 \\ -0.030 & -0.126 & -0.523 & -0.384 & +0.309 & +0.000 & +0.130 & -0.441 & +0.445 & -0.152 & +0.102 & +0.156 \end{pmatrix} \\
USV = & \begin{pmatrix} -0.087 & +0.290 & -0.277 & +0.188 & +0.370 & +0.254 & -0.604 & +0.145 & -0.008 & +0.205 & -0.131 & +0.387 \\ +0.387 & +0.205 & +0.323 & -0.185 & -0.069 & +0.026 & -0.418 & +0.322 & +0.082 & +0.059 & +0.521 & -0.323 \\ +0.268 & +0.152 & -0.516 & -0.041 & -0.377 & -0.622 & -0.111 & +0.124 & -0.008 & -0.154 & +0.026 & +0.236 \\ -0.183 & -0.209 & -0.066 & +0.227 & -0.357 & +0.417 & +0.025 & +0.226 & +0.266 & -0.436 & +0.370 & +0.342 \\ -0.257 & -0.176 & -0.265 & +0.022 & +0.149 & -0.122 & +0.181 & +0.658 & +0.387 & +0.265 & -0.092 & -0.318 \\ +0.448 & +0.322 & +0.002 & +0.383 & +0.084 & +0.167 & +0.564 & +0.245 & -0.207 & +0.217 & +0.103 & +0.188 \\ +0.480 & -0.559 & +0.127 & +0.140 & +0.050 & +0.059 & -0.207 & +0.284 & -0.195 & -0.229 & -0.449 & -0.002 \\ +0.411 & -0.306 & -0.317 & -0.079 & +0.446 & +0.033 & +0.079 & -0.369 & +0.450 & +0.014 & +0.288 & +0.046 \\ -0.154 & +0.167 & +0.284 & +0.073 & +0.564 & -0.392 & +0.089 & +0.176 & +0.005 & -0.569 & +0.101 & +0.133 \\ +0.139 & +0.259 & -0.082 & -0.750 & +0.028 & +0.328 & +0.193 & +0.184 & +0.098 & -0.224 & -0.286 & +0.156 \\ -0.042 & -0.278 & +0.420 & -0.243 & -0.038 & -0.249 & +0.035 & +0.109 & +0.113 & +0.443 & +0.082 & +0.627 \\ -0.162 & -0.307 & -0.313 & -0.279 & +0.193 & +0.058 & +0.044 & +0.143 & -0.684 & +0.033 & +0.415 & +0.007 \end{pmatrix}
\end{aligned}$$

0.0.9 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +56267.800 L_{inf} condition number of Wishart Matrix +56267.800

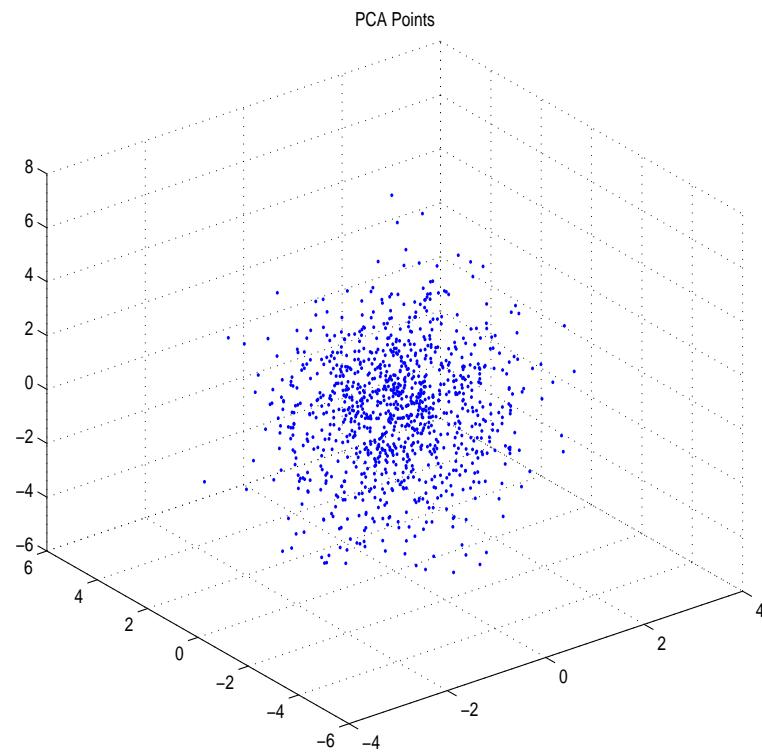
0.0.10 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +470.231 L_∞ condition number of GOE Matrix +470.231

0.0.11 The Identity Matrix $I \in M(n)$

L_1 condition number of $I = +1.000$ L_∞ condition number of $I = +1.000$ QueryPerformanceCounter = +0.387

0.0.12 Principal Components Matlab



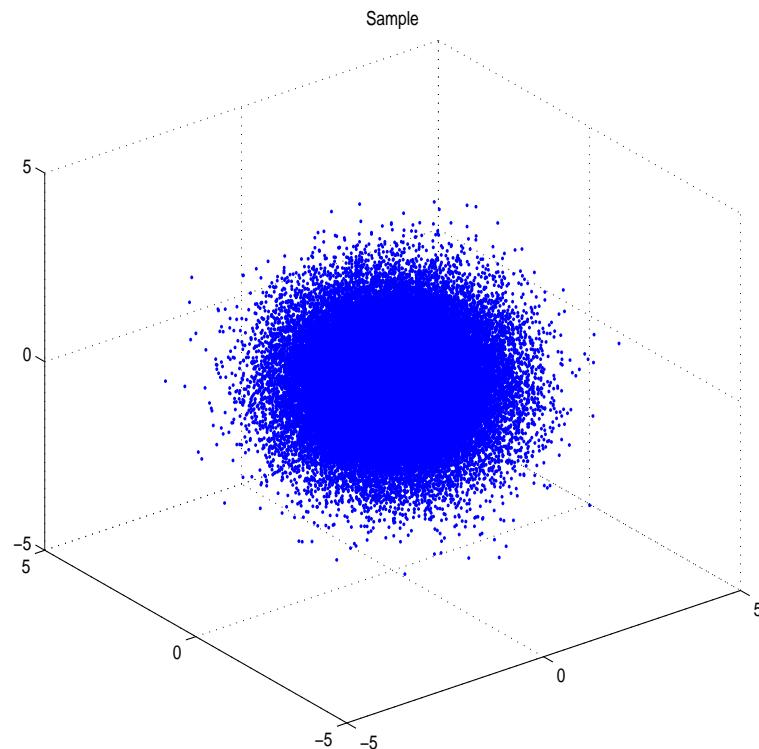
The eigenvectors: +0.121, +0.241, +0.963 +0.199, +0.944, -0.261 -0.972, +0.224, +0.067

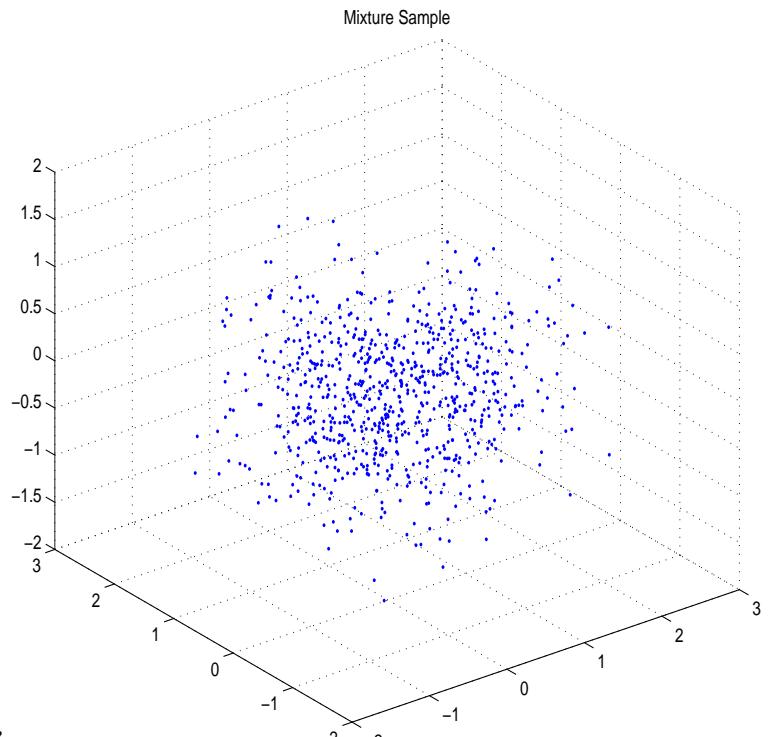
All of the eigenvalues of the covariance matrix: (+0.958,+0.000), (+2.025,+0.000), (+3.017,+0.000)

QueryPerformanceCounter = +1.040

0.0.13 Multi Variate Random Number Generator

Sample from $N(\mu, \Sigma)$ mean= -0.002, variance=+1.004, skewness=+0.006, kurtosis=+3.003 mean= -0.001, variance=+1.017, skewness=-0.005, kurtosis=+2.988 mean= -0.002, variance=+1.006, skewness=-0.016, kurtosis=+3.014 Covariance Matrix +1.004, +0.009, +0.003 +0.009, +1.017, -0.003 +0.003, -0.003, +1.006





Generate a sample from a uniform mixture of three Gaussians in R^3

QueryPerformanceCounter = +16.975

0.0.14 Matrix Multiply

Comparing naive matrix multiply versus Intel MKL dgemm for matrix of size +2048. This is for type double (hence the d in dgemm). Naive type double matrix multiply tic toc = +0.411 dgemm plus row to column major transpose operation tic toc = +0.318 Comparing naive matrix multiply versus Intel MKL sgemm for matrix of size +2048. This is for type float (hence the s in sgemm). Naive type float matrix multiply tic toc = +0.255 sgemm plus row to column major transpose operation tic toc = +0.229 QueryPerformanceCounter = +1.336

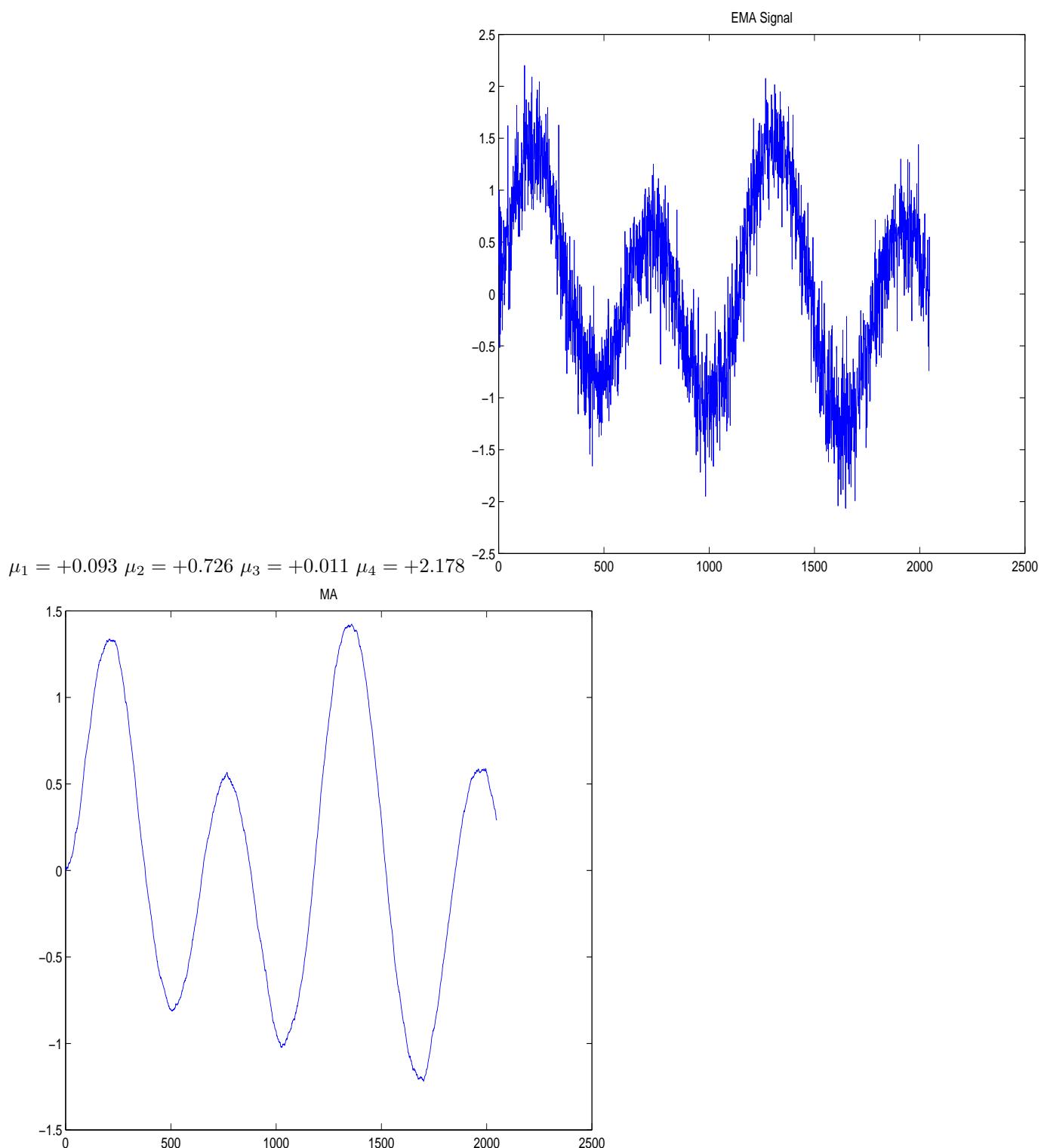
0.0.15 Descriptive Statistics

Mean N(0,1): +0.003 Variance N(0,1): +1.006 Mean N(0,1) [recurrence relation method] :+0.003 Variance [recurrence relation method] :+1.006 Skewness : +0.007 Kurtosis : +2.997 QueryPerformanceCounter = +0.032

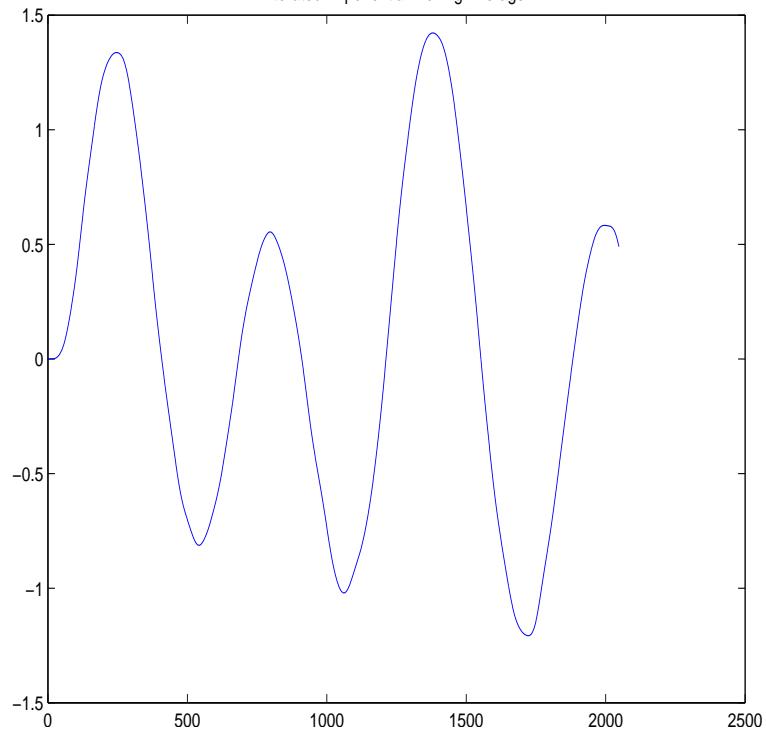
0.0.16 Time Series

+0.093 +0.726 +0.011 +2.178 QueryPerformanceCounter = +0.034 QueryPerformanceCounter = +6.281

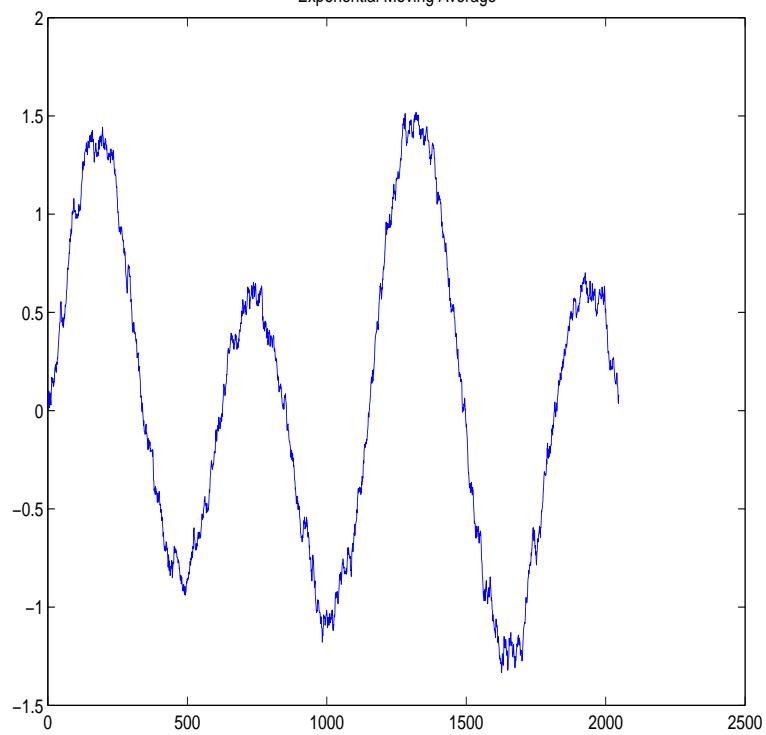
0.0.17 Iterated Exponential Filtering

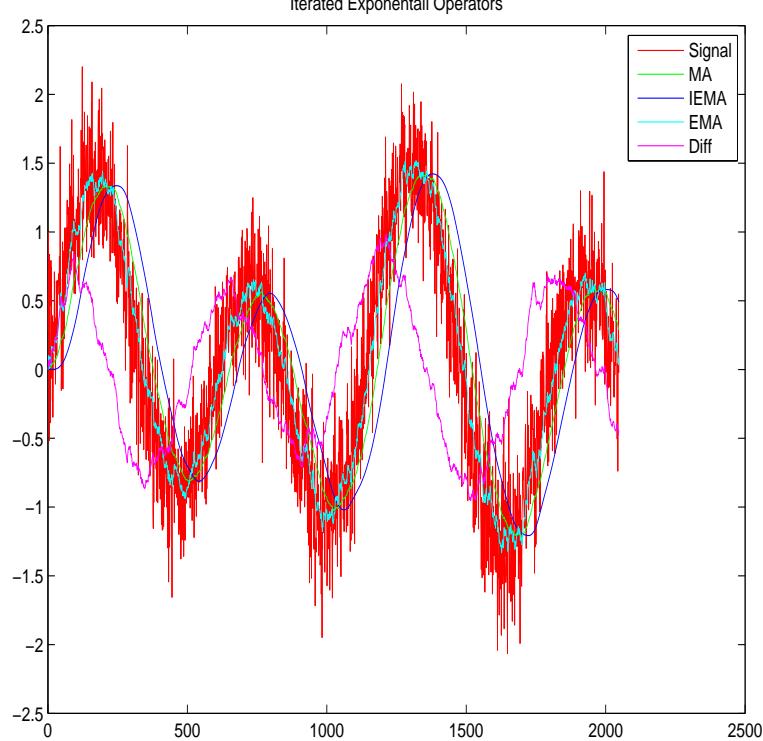
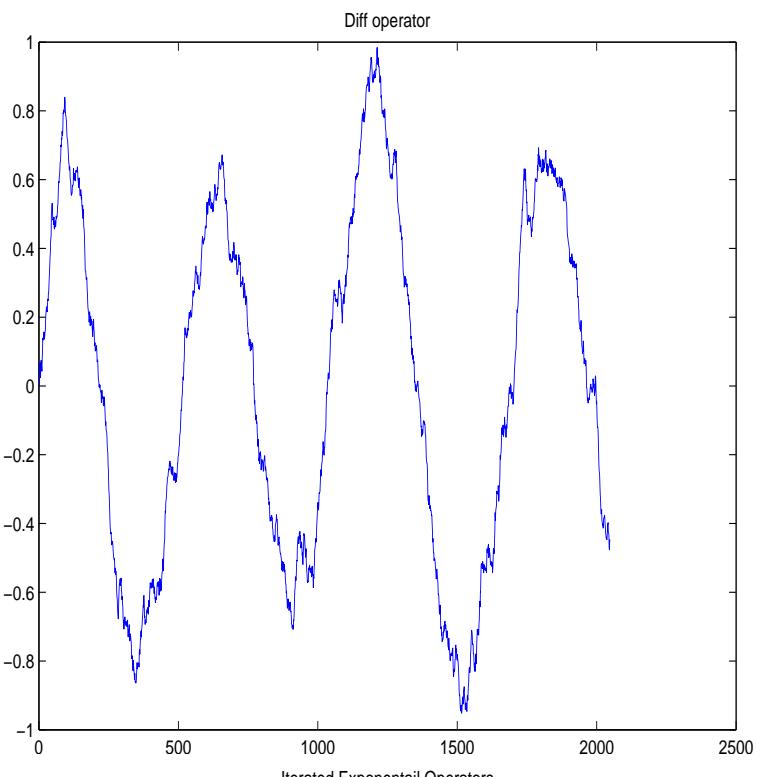


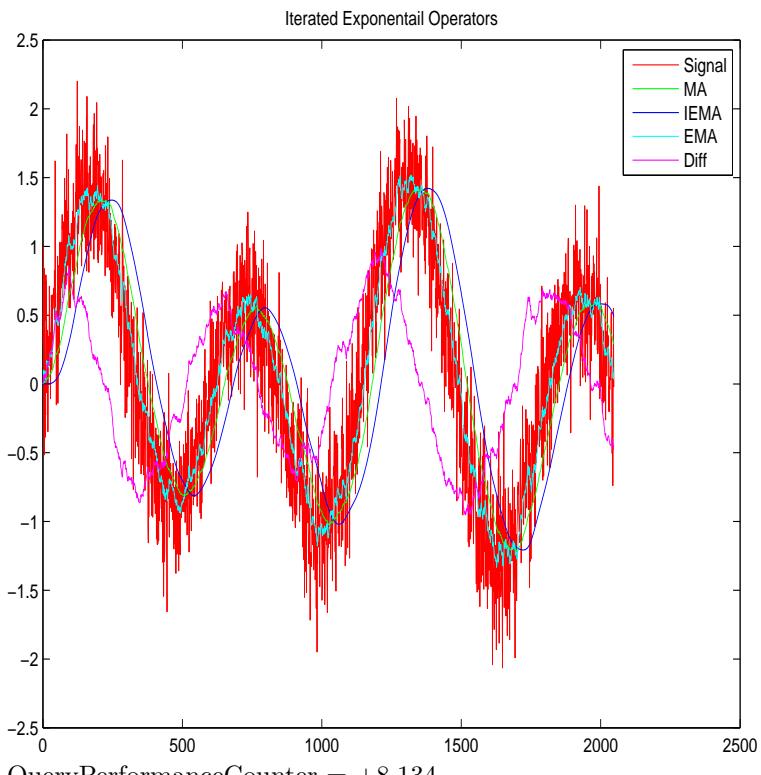
Iterated Exponential Moving Average



Exponential Moving Average



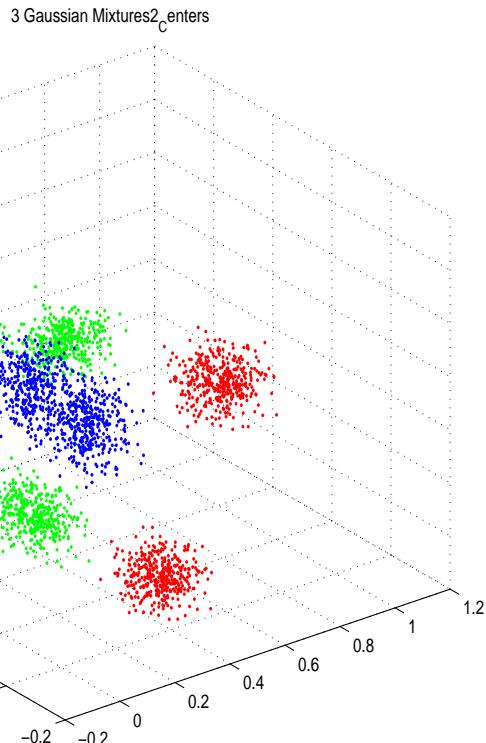


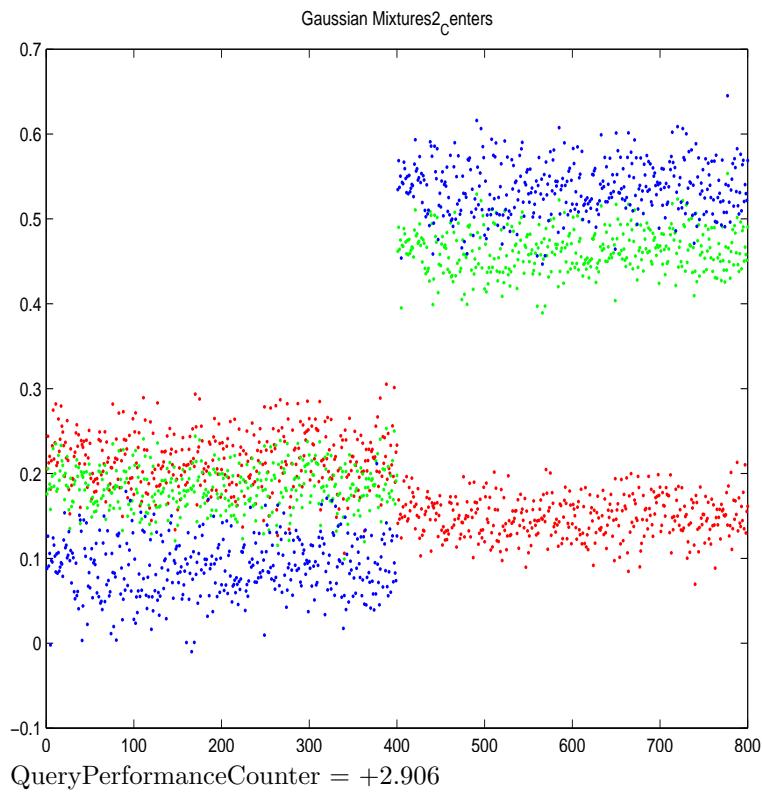


0.0.18 Testing binary writer

Binary writer Speedup 1GB Double Matrix +63.144
 Binary reader Speedup 1GB Double Matrix +195.705
 Binary writer Speedup 1GB Double vector +10.109
 Binary reader Speedup 1GB Double Matrix +201.560
 QueryPerformanceCounter = +1.026

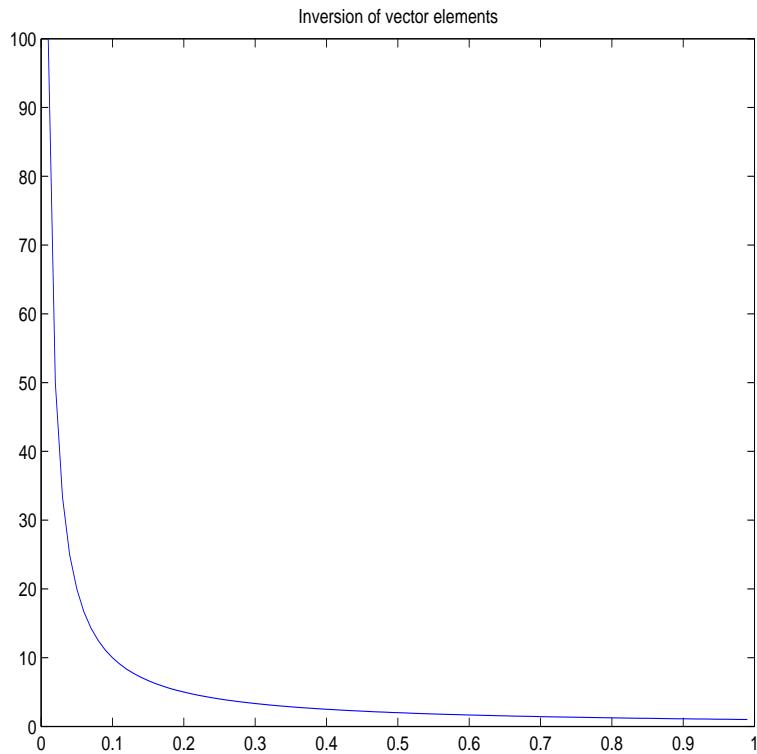
0.0.19 Testing Gaussian Mixture Point Cloud and Latex Plotting Capabilities.



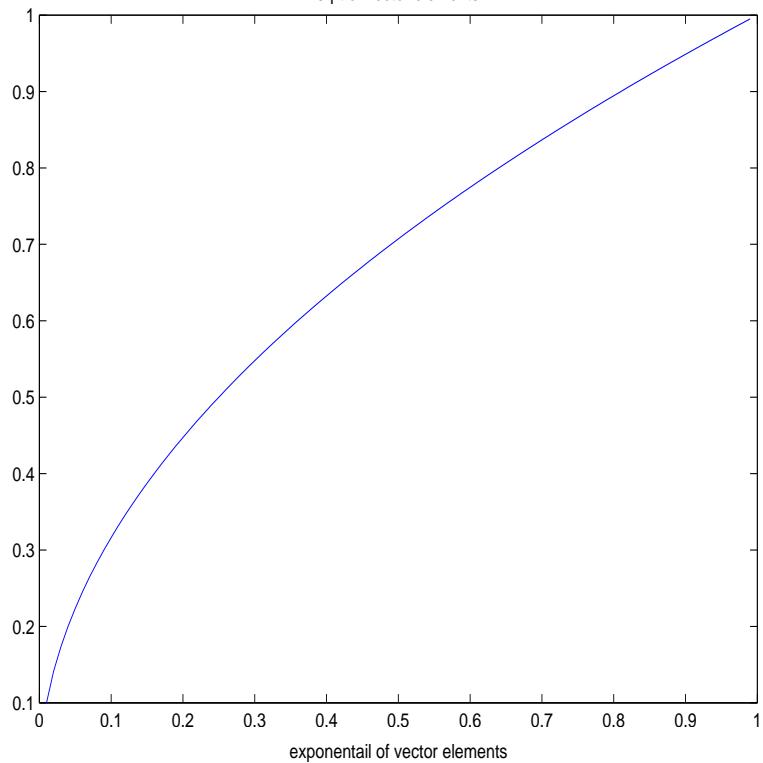


QueryPerformanceCounter = +2.906

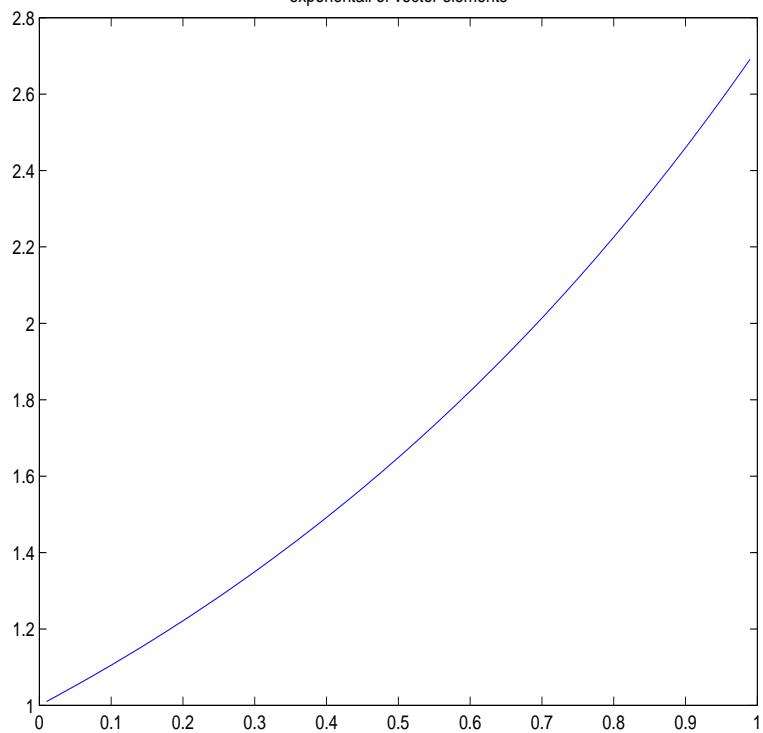
0.0.20 Intel VSL Function Check



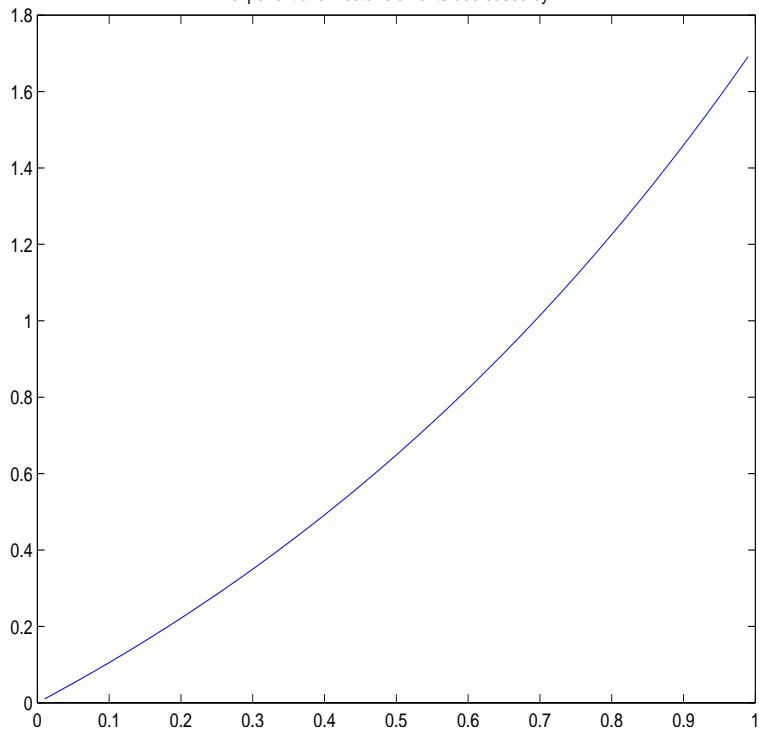
sqrt of vector elements



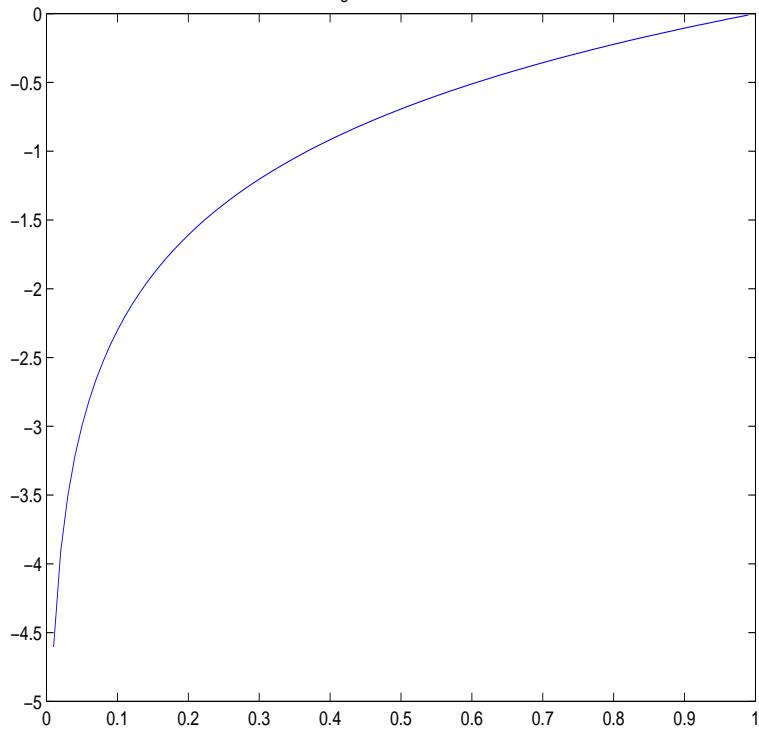
exponentail of vector elements



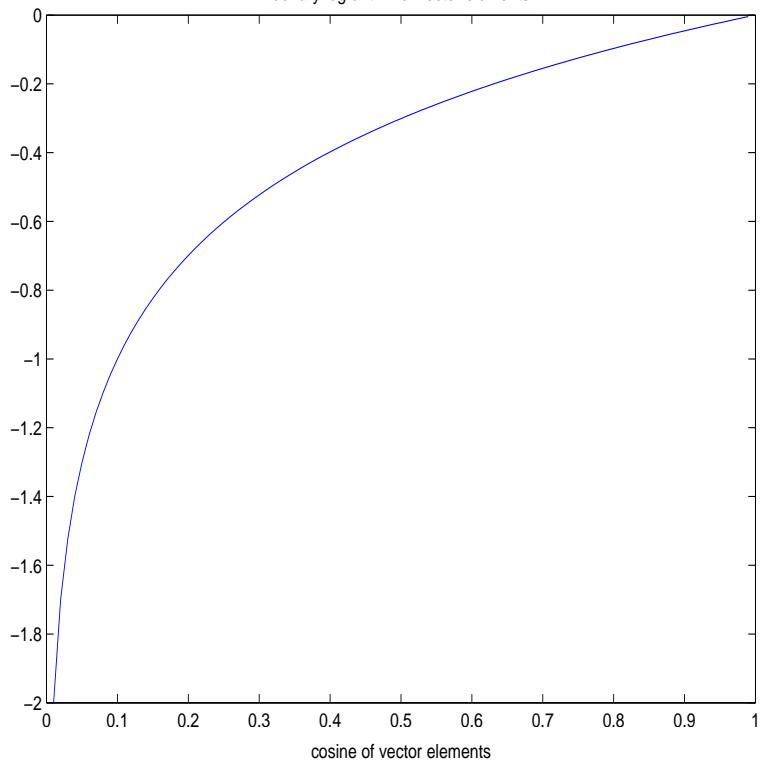
exponential of vector elements decreased by 1



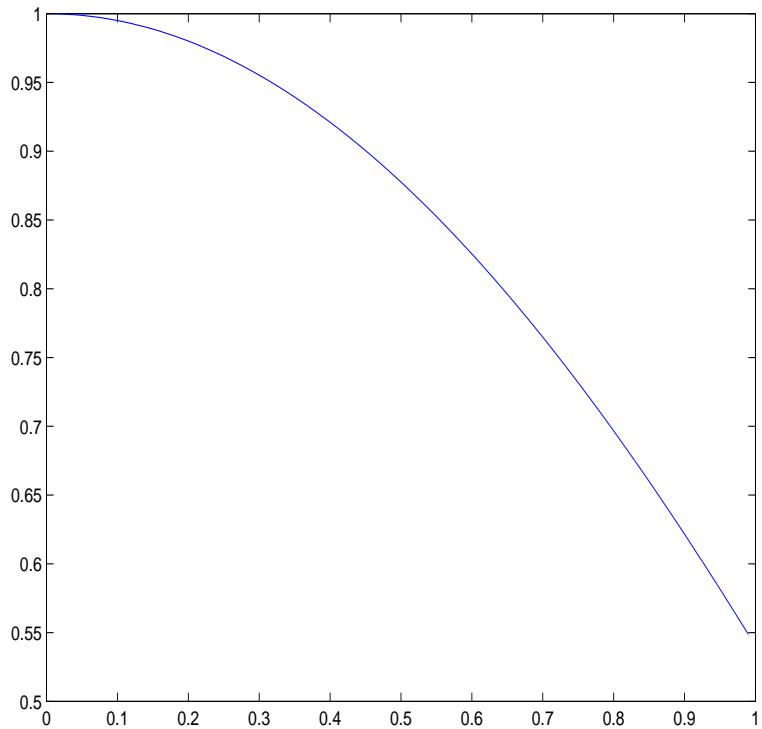
natural logarithm of vector elements



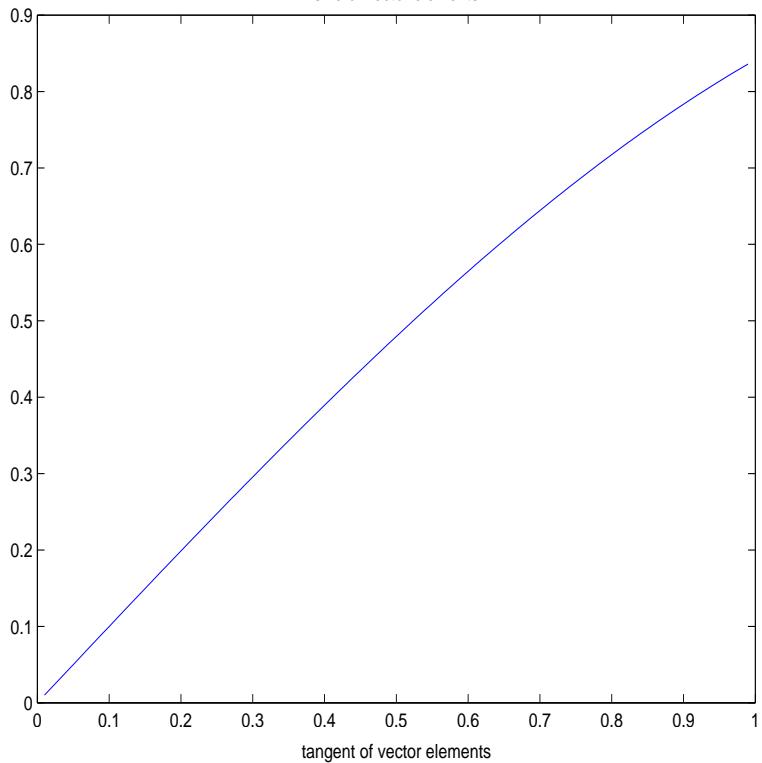
denary logarithm of vector elements



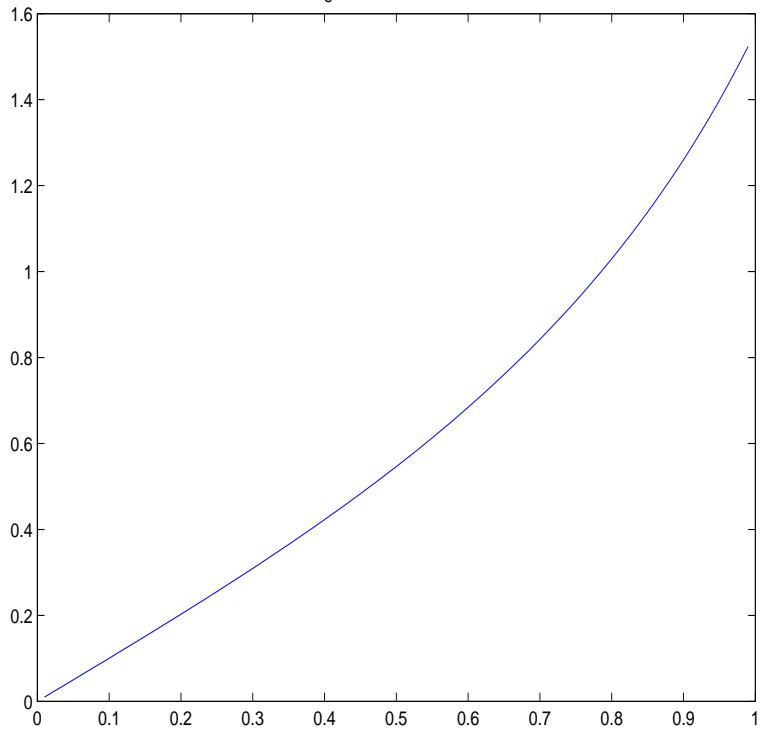
cosine of vector elements



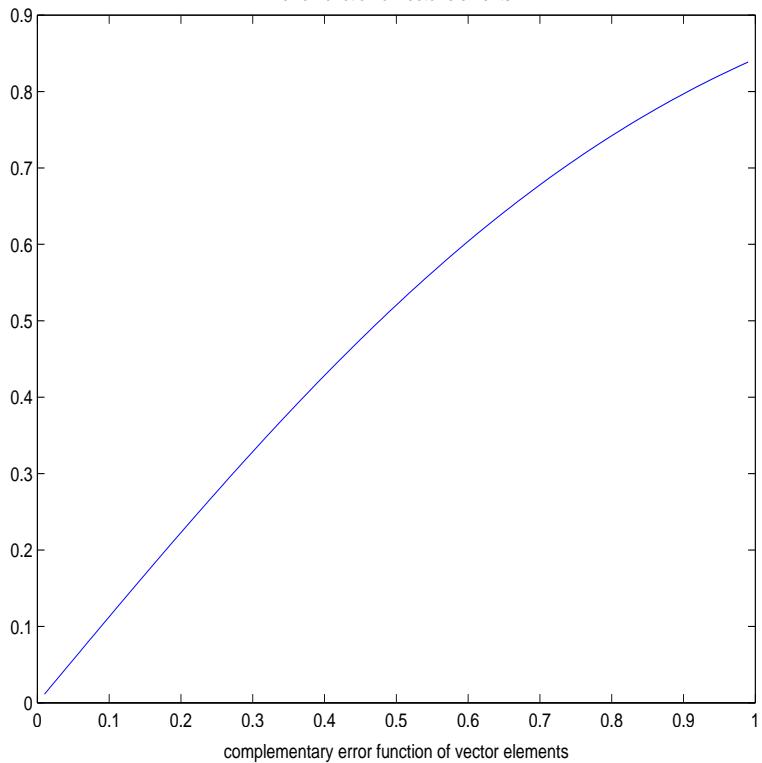
sine of vector elements



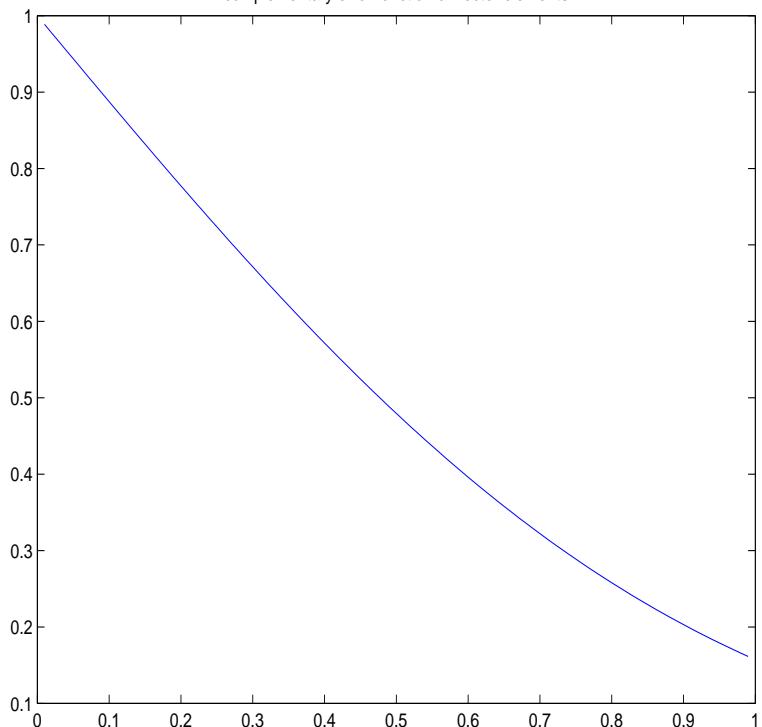
tangent of vector elements



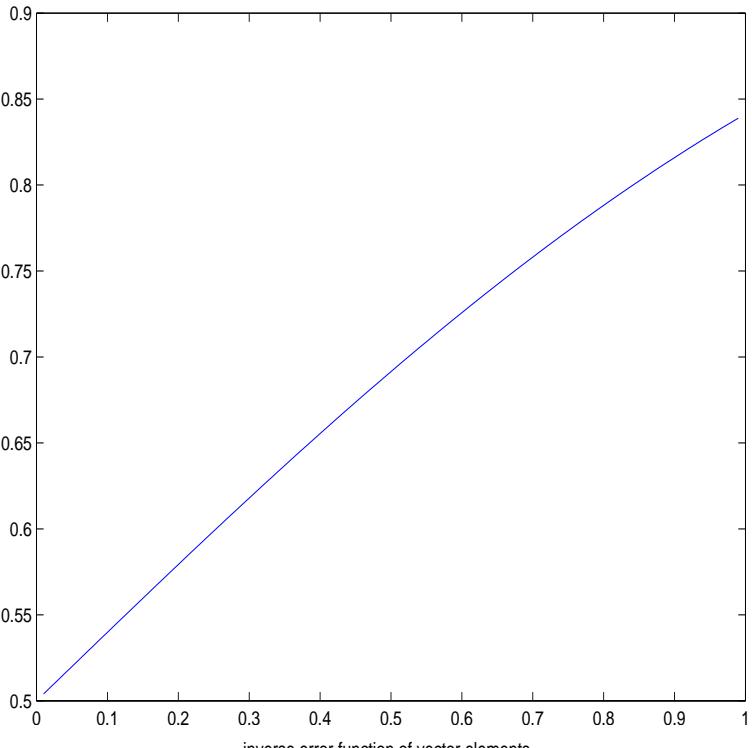
error function of vector elements



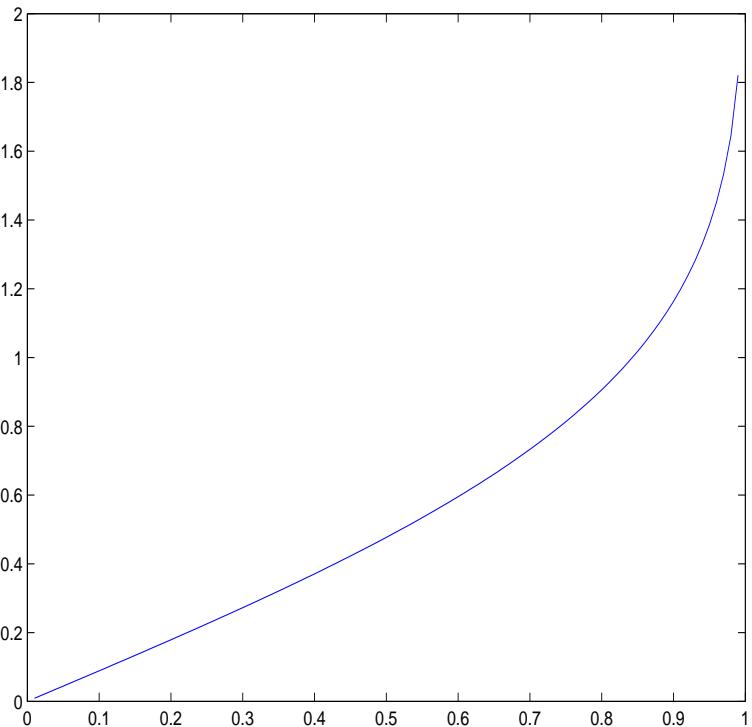
complementary error function of vector elements



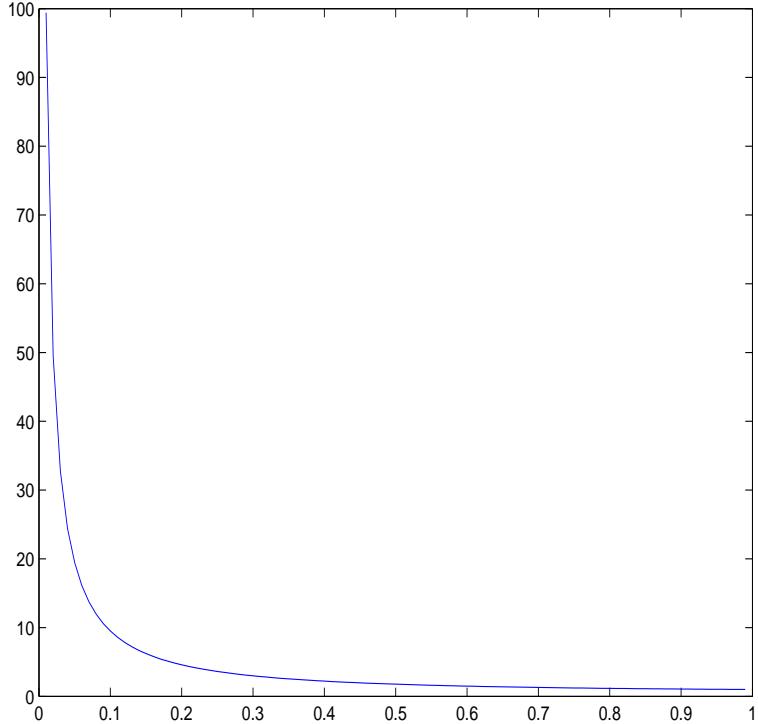
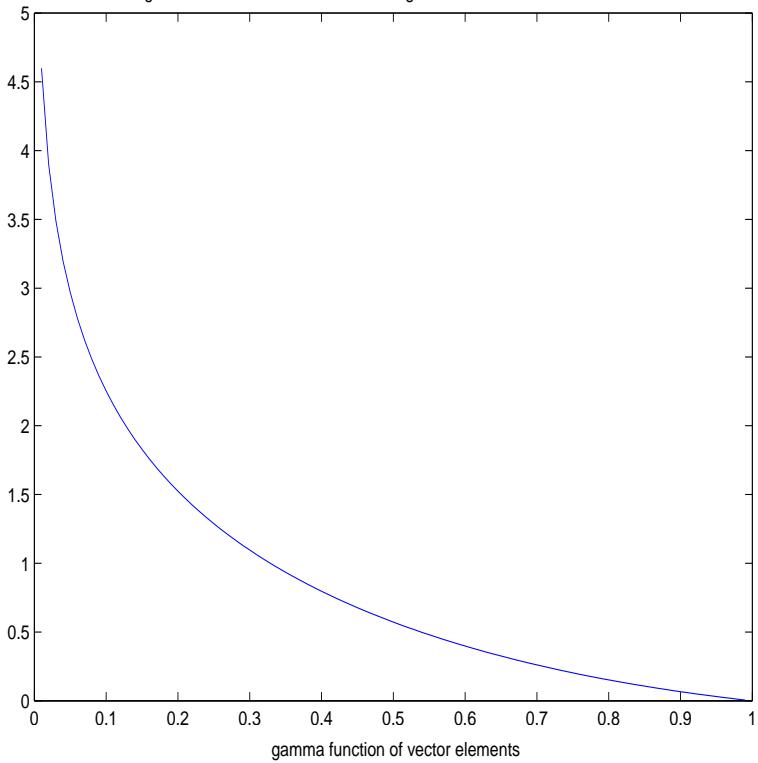
cumulative normal distribution function of vector elements



inverse error function of vector elements



logarithm for the absolute value of the gamma function of vector elements



QueryPerformanceCounter = +15.665

0.0.21 Gram Matrix Consistency Check

Sample Size = 4096 Feature dim = 3

Sigma

$$= \begin{pmatrix} +1.140 & +1.535 & +0.581 \\ +1.535 & +9.988 & +1.605 \\ +0.581 & +1.605 & +0.428 \end{pmatrix}$$

$$\text{SampleCovariance} = \begin{pmatrix} +1.153 & +1.656 & +0.591 \\ +1.656 & +10.198 & +1.681 \\ +0.591 & +1.681 & +0.437 \end{pmatrix}$$

$$\text{SampleMean} = (+1.02688 \quad +1.08012 \quad +1.01670)$$

$$SampleCovariance-\Omega = \begin{pmatrix} +0.013 & +0.121 & +0.010 \\ +0.121 & +0.209 & +0.076 \\ +0.010 & +0.076 & +0.009 \end{pmatrix}$$

SampleCovarianceEigs = ((+10.79008, +0.00000) (+0.95798, +0.00000) (+0.03928, +0.00000))

$$CenteredMean = \begin{pmatrix} +0.00000 & -0.00000 & +0.00000 \end{pmatrix}$$

$$CenteredCovariance = \begin{pmatrix} +1.153 & +1.656 & +0.591 \\ +1.656 & +10.198 & +1.681 \\ +0.591 & +1.681 & +0.437 \end{pmatrix}$$

$$GramMatrixGfNotscaledbysamplesize = \begin{pmatrix} +4723.612 & +6780.059 & +2420.098 \\ +6780.059 & +41769.171 & +6884.815 \\ +2420.098 & +6884.815 & +1788.187 \end{pmatrix}$$

$$GramMatrixGf scaled by sample size = \begin{pmatrix} +1.153 & +1.655 & +0.591 \\ +1.655 & +10.198 & +1.681 \\ +0.591 & +1.681 & +0.437 \end{pmatrix}$$

$$SampleCovariance - ScaledGf = \begin{pmatrix} +0.000 & +0.000 & +0.000 \\ +0.000 & +0.002 & +0.000 \\ +0.000 & +0.000 & +0.000 \end{pmatrix}$$

$$EigenDecompoofSampleCovariance = \begin{pmatrix} -0.177 & -0.970 & -0.168 \\ +0.920 & -0.223 & +0.322 \\ -0.350 & -0.097 & +0.932 \end{pmatrix}$$

$$EigenDecompoGramMatrix = \begin{pmatrix} -0.131 & -0.972 & -0.194 \\ -0.362 & +0.229 & -0.903 \\ +0.923 & -0.048 & -0.382 \end{pmatrix}$$

QueryPerformanceCounter = +1.416

0.0.22 Eigen Solver Checks

0.0.23 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: +8

$$A = \begin{pmatrix} -0.024 & +0.193 & -0.083 & -0.583 & +0.760 & -0.115 & -0.086 & +0.130 \\ +0.409 & -0.023 & -0.263 & -0.536 & -0.324 & -0.149 & +0.428 & -0.407 \\ +0.110 & -0.558 & -0.646 & +0.165 & +0.106 & -0.340 & -0.319 & +0.048 \\ +0.449 & +0.186 & +0.029 & +0.523 & +0.470 & -0.010 & +0.131 & -0.501 \\ +0.322 & +0.590 & -0.023 & +0.085 & -0.245 & -0.558 & -0.272 & +0.306 \\ +0.075 & +0.399 & -0.633 & +0.062 & -0.057 & +0.630 & -0.023 & +0.171 \\ -0.555 & +0.295 & -0.174 & -0.051 & -0.103 & -0.140 & -0.362 & -0.641 \\ -0.448 & +0.144 & -0.271 & +0.243 & +0.103 & -0.347 & +0.696 & +0.175 \end{pmatrix}$$

$$Det(A) : A \in O(n) = (-1.000, -0.000)$$

$L =$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	+1.000	+0.000	+0.000	+0.000	+0.000	+0.000	+0.000	+0.000
	-0.581	+1.000	+0.000	+0.000	+0.000	+0.000	+0.000	+0.000
	-0.198	-0.657	+1.000	+0.000	+0.000	+0.000	+0.000	+0.000
	-0.736	+0.255	+0.472	+1.000	+0.000	+0.000	+0.000	+0.000
	+0.044	+0.237	+0.060	+0.893	+1.000	+0.000	+0.000	+0.000
	-0.135	+0.576	+0.768	+0.182	+0.223	+1.000	+0.000	+0.000
	+0.806	-0.123	+0.190	-0.376	+0.064	-0.029	+1.000	+0.000
	-0.810	+0.558	+0.056	-0.650	+0.359	+0.331	+0.349	+1.000

$$L * U = \begin{pmatrix} -0.555 & +0.295 & -0.174 & -0.051 & -0.103 & -0.140 & -0.362 & -0.641 \\ +0.322 & +0.590 & -0.023 & +0.085 & -0.245 & -0.558 & -0.272 & +0.306 \\ +0.110 & -0.558 & -0.646 & +0.165 & +0.106 & -0.340 & -0.319 & +0.048 \\ +0.409 & -0.023 & -0.263 & -0.536 & -0.324 & -0.149 & +0.428 & -0.407 \\ -0.024 & +0.193 & -0.083 & -0.583 & +0.760 & -0.115 & -0.086 & +0.130 \\ +0.075 & +0.399 & -0.633 & +0.062 & -0.057 & +0.630 & -0.023 & +0.171 \\ -0.448 & +0.144 & -0.271 & +0.243 & +0.103 & -0.347 & +0.696 & +0.175 \\ +0.449 & +0.186 & +0.029 & +0.523 & +0.470 & -0.010 & +0.131 & -0.501 \end{pmatrix}$$

$$\text{Det}(L) := (+1.000, +0.000) \text{Det}(U) := (+1.000, +0.000) \text{Det}(LU) := (+1.000, +0.000)$$

$$\|A\|_{L_1} = +2.393$$

$$\|A\|_{L_\infty} = +2.539$$

$$\|A^{-1}\|_{L_1} = +2.539$$

$$\|A^{-1}\|_{L_\infty} = +2.393$$

$$\|A\|_{L_\infty} * \|A^{-1}\|_{L_\infty} = +6.075$$

$$\|A\|_{L_1} * \|A^{-1}\|_{L_1} = +6.075$$

$$\text{Frobenius Norm } \|A\|_F \text{ via } \sum_{i,j=0}^n |A_{i,j}| \text{ of } A \in O(n) + 2.828$$

$$L_1 \text{ condition number of Haar Distributed Random Orthogonal Matrix } A \in O(n) + 6.075$$

$$A = \begin{pmatrix} -0.024 & +0.193 & -0.083 & -0.583 & +0.760 & -0.115 & -0.086 & +0.130 \\ +0.409 & -0.023 & -0.263 & -0.536 & -0.324 & -0.149 & +0.428 & -0.407 \\ +0.110 & -0.558 & -0.646 & +0.165 & +0.106 & -0.340 & -0.319 & +0.048 \\ +0.449 & +0.186 & +0.029 & +0.523 & +0.470 & -0.010 & +0.131 & -0.501 \\ +0.322 & +0.590 & -0.023 & +0.085 & -0.245 & -0.558 & -0.272 & +0.306 \\ +0.075 & +0.399 & -0.633 & +0.062 & -0.057 & +0.630 & -0.023 & +0.171 \\ -0.555 & +0.295 & -0.174 & -0.051 & -0.103 & -0.140 & -0.362 & -0.641 \\ -0.448 & +0.144 & -0.271 & +0.243 & +0.103 & -0.347 & +0.696 & +0.175 \end{pmatrix}$$

$$L_\infty \text{ condition number of Haar Distributed Random Orthogonal Matrix } A \in O(n) + 6.068$$

$$\text{Eigenvalues of } A \in O(n)$$

$$(+0.625, +0.780), (+0.625, -0.780), (+0.232, +0.973), (+0.232, -0.973), (-0.844, +0.537), (-0.844, -0.537), (-1.000, +0.000), (+1.000, +0.000)$$

$$|\lambda| : \lambda \in \sigma(A), A \in O(n)$$

$$+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000$$

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & -0.000 & -0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} -0.024 & +0.409 & +0.110 & +0.449 & +0.322 & +0.075 & -0.555 & -0.448 \\ +0.193 & -0.023 & -0.558 & +0.186 & +0.590 & +0.399 & +0.295 & +0.144 \\ -0.083 & -0.263 & -0.646 & +0.029 & -0.023 & -0.633 & -0.174 & -0.271 \\ -0.583 & -0.536 & +0.165 & +0.523 & +0.085 & +0.062 & -0.051 & +0.243 \\ +0.760 & -0.324 & +0.106 & +0.470 & -0.245 & -0.057 & -0.103 & +0.103 \\ -0.115 & -0.149 & -0.340 & -0.010 & -0.558 & +0.630 & -0.140 & -0.347 \\ -0.086 & +0.428 & -0.319 & +0.131 & -0.272 & -0.023 & -0.362 & +0.696 \\ +0.130 & -0.407 & +0.048 & -0.501 & +0.306 & +0.171 & -0.641 & +0.175 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\ -0.000 & +0.000 & -0.000 & +1.000 & -0.000 & +0.000 & +0.000 & -0.000 \\ -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +1.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} -0.927 & +0.013 & +0.210 & +0.104 & +0.028 & +0.076 & -0.236 & +0.154 \\ +0.022 & -0.641 & +0.013 & -0.382 & -0.260 & -0.224 & -0.012 & +0.569 \\ -0.106 & +0.290 & -0.615 & -0.366 & -0.453 & -0.015 & -0.414 & -0.123 \\ +0.014 & +0.499 & +0.074 & +0.207 & -0.121 & -0.742 & +0.107 & +0.354 \\ -0.024 & +0.193 & -0.083 & -0.583 & +0.760 & -0.115 & -0.086 & +0.130 \\ -0.139 & +0.228 & -0.376 & +0.005 & -0.037 & +0.422 & +0.645 & +0.437 \\ +0.330 & +0.244 & +0.187 & +0.182 & +0.017 & +0.417 & -0.545 & +0.541 \\ -0.020 & -0.326 & -0.624 & +0.541 & +0.363 & -0.167 & -0.199 & +0.108 \end{pmatrix}$$

$$S = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

$$V = \begin{pmatrix} +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 \\ -0.674 & +0.107 & +0.330 & -0.084 & +0.000 & +0.082 & -0.539 & -0.349 \\ -0.168 & +0.369 & +0.247 & -0.068 & +0.000 & -0.232 & -0.131 & +0.836 \\ -0.449 & -0.716 & -0.409 & -0.003 & +0.000 & -0.185 & -0.043 & +0.288 \\ -0.225 & -0.038 & +0.315 & +0.838 & +0.000 & -0.170 & +0.339 & -0.047 \\ -0.112 & -0.315 & +0.480 & -0.237 & +0.000 & +0.647 & +0.382 & +0.194 \\ +0.449 & -0.486 & +0.548 & -0.034 & +0.000 & -0.359 & -0.362 & -0.015 \\ +0.225 & -0.057 & -0.185 & +0.477 & +0.000 & +0.573 & -0.547 & +0.237 \end{pmatrix}$$

$$USV = \begin{pmatrix} -0.177 & +0.085 & -0.099 & +0.071 & -0.927 & +0.151 & +0.000 & +0.255 \\ +0.808 & +0.264 & -0.353 & +0.161 & +0.022 & +0.245 & -0.120 & +0.228 \\ -0.038 & +0.297 & -0.260 & -0.402 & -0.106 & +0.379 & -0.002 & -0.725 \\ -0.204 & +0.098 & -0.302 & +0.192 & +0.014 & -0.310 & -0.844 & -0.109 \\ -0.022 & +0.449 & +0.394 & +0.721 & -0.024 & +0.045 & +0.106 & -0.330 \\ +0.257 & -0.588 & +0.444 & +0.062 & -0.139 & +0.404 & -0.398 & -0.215 \\ -0.452 & +0.067 & -0.142 & +0.158 & +0.330 & +0.716 & -0.097 & +0.340 \\ -0.047 & -0.523 & -0.578 & +0.471 & -0.020 & -0.018 & +0.306 & -0.273 \end{pmatrix}$$

Calculating first few eigenvectors of $A \in O(n)$ using LAPACK syevx

0.0.24 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +1489.694 L_{∞} condition number of Wishart Matrix +1489.694

0.0.25 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +66.900 L_{∞} condition number of GOE Matrix +66.900

0.0.26 The Identity Matrix $I \in M(n)$

L_1 condition number of $I = +1.000$ L_{∞} condition number of $I = +1.000$ QueryPerformanceCounter = +0.326

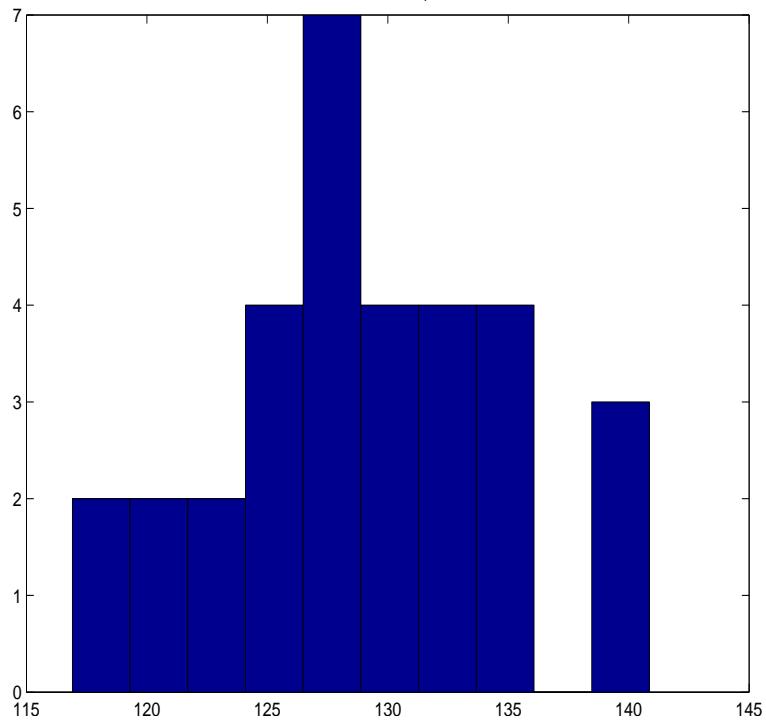
0.0.27 Generate Tracey Widom Sample

0.0.28 Sample from $W_n m$ times and calculate empirical PDF of the first eig

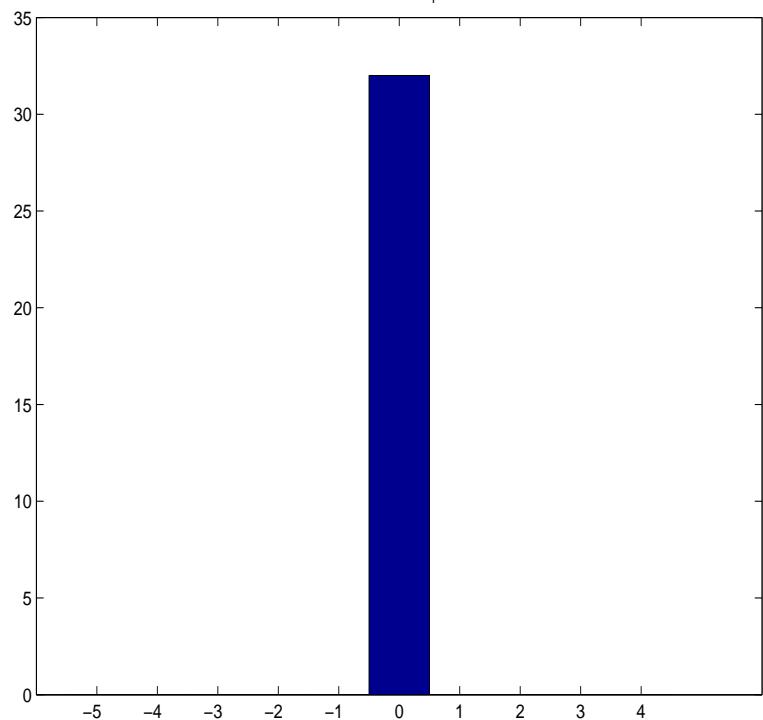
Here we generate histograms of λ_1 for GOE (Gaussian Orthogonal Ensemble), and W (Wishart) distributed of random matrices These should approximate the celebrated Tracy Widom distribution. Dimension $n = +128$

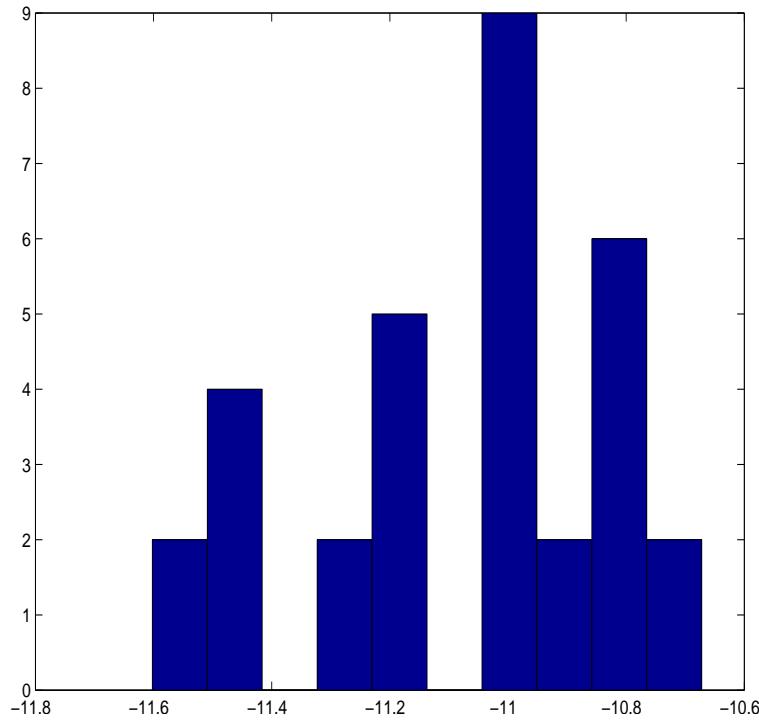
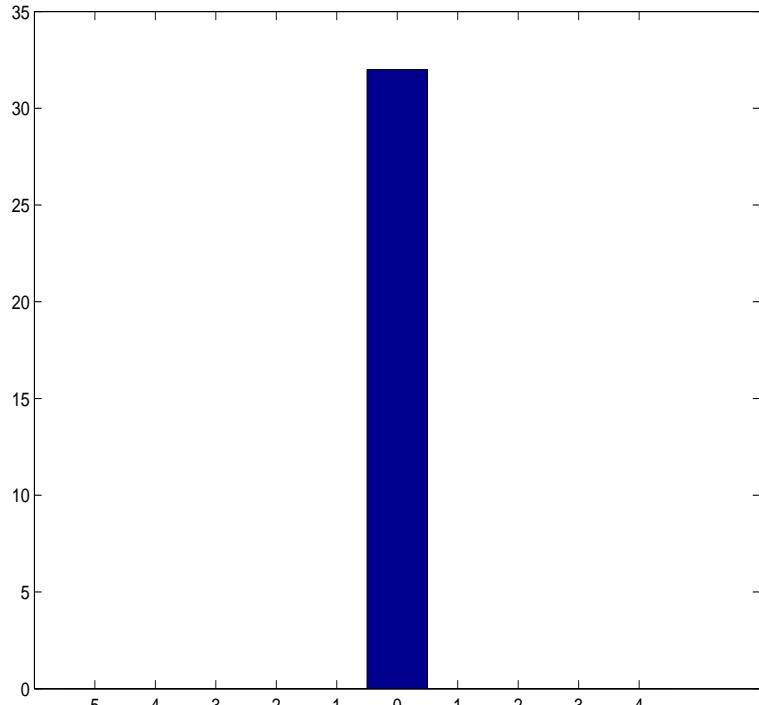
Sample size $m = 32$

Histogram of $\operatorname{Re}(\lambda_1)$ $A \in W$



Histogram of $\operatorname{Im}(\lambda_1)$ $A \in W$



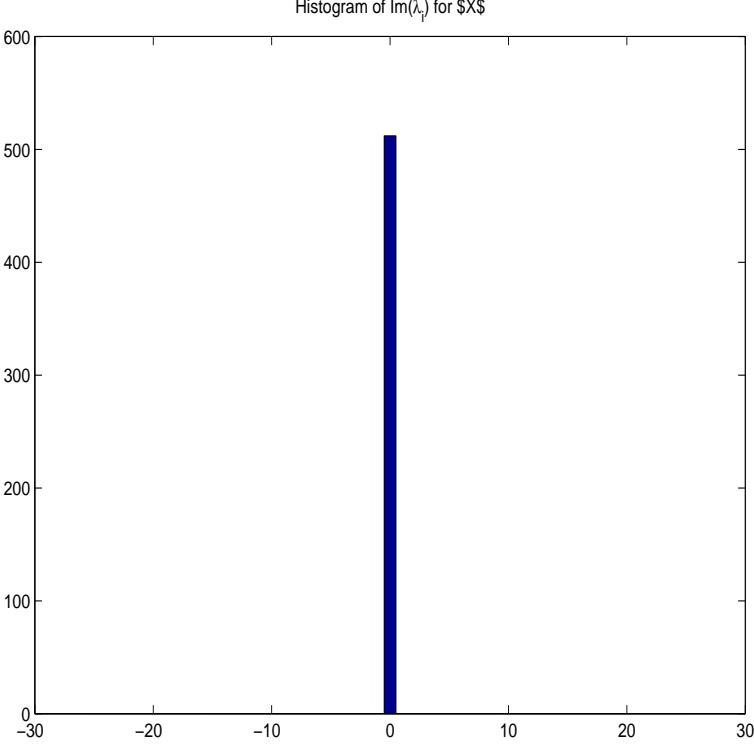
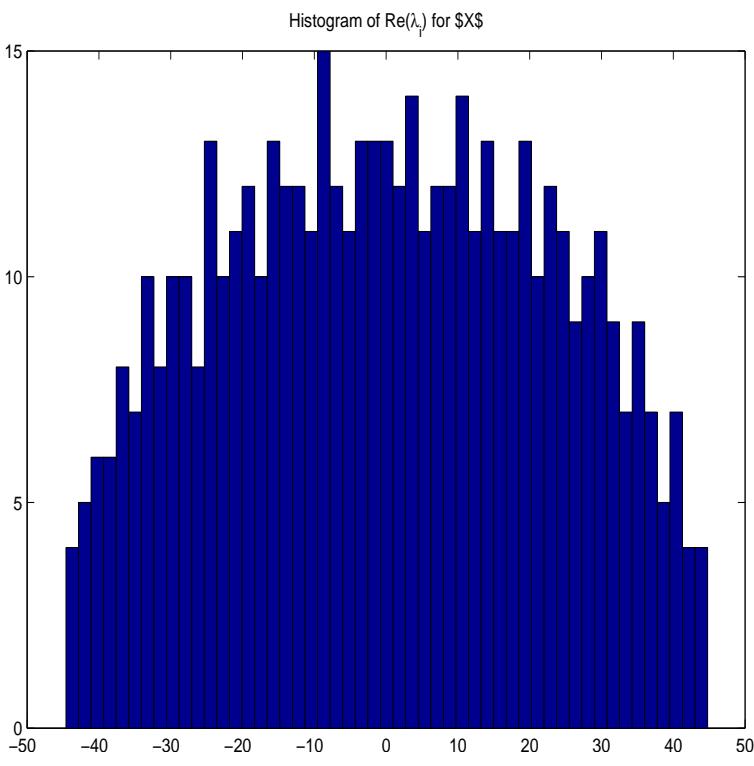
Histogram of $\text{Re}(\lambda_1)$ $A \in \text{GOE}(1024)$ Histogram of $\text{Im}(\lambda_1)$ $A \in \text{GOE}(1024)$ 

QueryPerformanceCounter = +5.590

0.0.29 Approximate Wigner Distribution

0.0.30 Verfy Winger Law.

Let $M_n = [X_{ij}]$ a symmetric $n \times n$ matrix with Random entries such that $X_{i,j} = X_{j,i}$, and $X_{i,j}$ are iid or all $i \neq j$, and X_{jj} are iid for all j ; $E[X_{ij}^2] = 1$, $E[X_{ij}] = 0$ and that all moments exists for each of the entries. The eigenvector of this random matrix; $\lambda_1 \leq \dots \leq \lambda_n$ depends continuously on M_n . Dimension $n = +512$



QueryPerformanceCounter = +2.667

0.0.31 Matrix Exponential

$$SPDMatrix = \begin{pmatrix} +10.539 & -0.499 & -0.010 & +0.368 & +0.465 & -0.492 & -0.126 & +0.437 \\ -0.499 & +7.286 & +0.365 & -0.481 & -0.337 & -0.466 & +0.279 & +0.056 \\ -0.010 & +0.365 & +6.705 & -0.205 & +0.467 & +0.131 & +0.077 & -0.089 \\ +0.368 & -0.481 & -0.205 & +6.496 & -0.402 & -0.209 & +0.043 & -0.041 \\ +0.465 & -0.337 & +0.467 & -0.402 & +4.578 & +0.272 & +0.289 & -0.285 \\ -0.492 & -0.466 & +0.131 & -0.209 & +0.272 & +8.181 & +0.343 & -0.244 \\ -0.126 & +0.279 & +0.077 & +0.043 & +0.289 & +0.343 & +5.938 & -0.212 \\ +0.437 & +0.056 & -0.089 & -0.041 & -0.285 & -0.244 & -0.212 & +9.691 \end{pmatrix}$$

$$SPDEigs = ((+10.93611, +0.00000) \quad (+9.60778, +0.00000) \quad (+4.23666, +0.00000) \quad (+8.36911, +0.00000) \quad (+7.56229, +0.00000))$$

$$exp(SPD) = \begin{pmatrix} +47863.969 & -6460.093 & -1078.770 & +4706.958 & +2535.224 & -8475.398 & -2406.368 & +12977.552 \\ -6460.093 & +2780.574 & +516.920 & -1069.918 & -548.083 & -109.707 & +386.466 & -807.216 \\ -1078.770 & +516.920 & +1015.281 & -385.755 & +176.069 & +458.541 & +212.284 & -859.022 \\ +4706.958 & -1069.918 & -385.755 & +1267.210 & +111.181 & -1018.272 & -287.809 & +1036.628 \\ +2535.224 & -548.083 & +176.069 & +111.181 & +413.265 & +135.193 & +45.490 & -502.411 \\ -8475.398 & -109.707 & +458.541 & -1018.272 & +135.193 & +5613.026 & +968.003 & -4270.737 \\ -2406.368 & +386.466 & +212.284 & -287.809 & +45.490 & +968.003 & +632.432 & -1645.725 \\ +12977.552 & -807.216 & -859.022 & +1036.628 & -502.411 & -4270.737 & -1645.725 & +19362.944 \end{pmatrix}$$

$$exp(SPD)eigs = ((+56168.17045, +0.00000) \quad (+14880.07985, +0.00000) \quad (+4311.77579, +0.00000) \quad (+1924.25027, +0.00000) \quad (+6)$$

$$log(exp(SPD)eigs) = ((+10.93611, +0.00000) \quad (+9.60778, +0.00000) \quad (+8.36911, +0.00000) \quad (+7.56229, +0.00000) \quad (+4.23666, +0.00000) \quad (+2.71828, +0.00000))$$

$$exp(Id) = \begin{pmatrix} +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 \end{pmatrix}$$

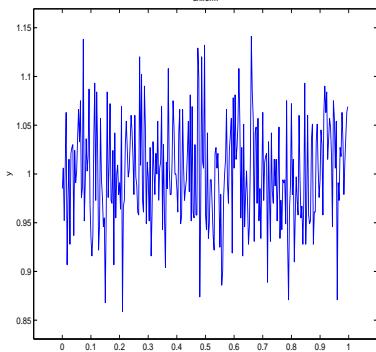
$$exp(Id)eigs = ((+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000))$$

$$log(exp(Id)eigs) = ((+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000))$$

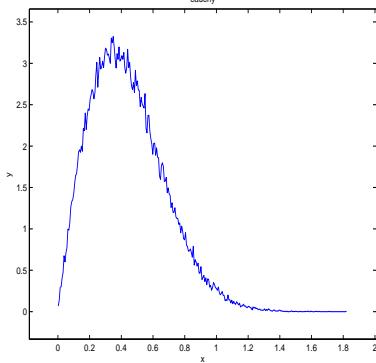
For $n \in \mathbb{Z}[16, 128]$ we calculate $|(SPD(n)Eigs - log(exp(SPD(n))eigs)|_{l^2}$
 $|SPD(n)Eigs - log(exp(SPD(n))eigs)|_{l^2} = ((+5.36543, +0.00000) \quad (+5.36543, +0.00000) \quad (+5.36543, +0.00000) \quad (+5.36543, +0.00000))$

QueryPerformanceCounter = +0.00939 The sample size generated for this run is 100000.

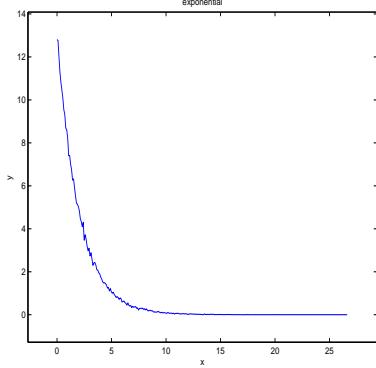
	mean	variance	skewness	kurtosis
uniform	$\mu_1 = +0.50030$	$\mu_2 = +0.08353$	$\mu_3 = +0.00339$	$\mu_4 = +1.80113$



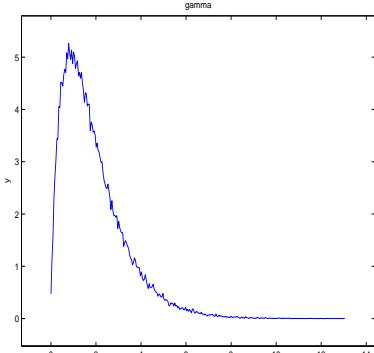
	mean	variance	skewness	kurtosis
cauchy	$\mu_1 = +0.44288$	$\mu_2 = +0.05341$	$\mu_3 = +0.63935$	$\mu_4 = +3.28094$



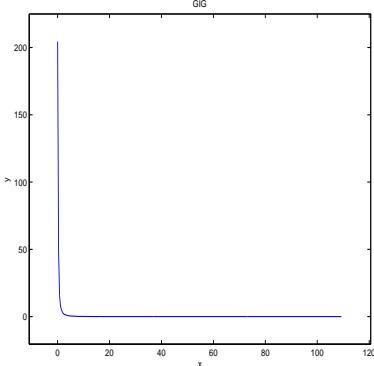
	mean	variance	skewness	kurtosis
exponential	$\mu_1 = +1.99647$	$\mu_2 = +3.99339$	$\mu_3 = +2.03097$	$\mu_4 = +9.30842$



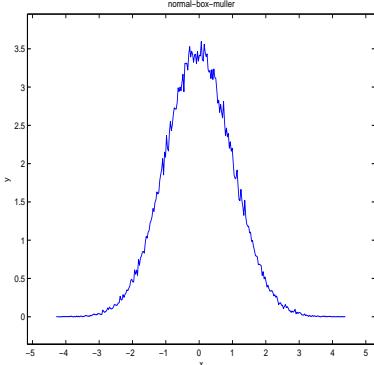
	mean	variance	skewness	kurtosis
gamma	$\mu_1 = +1.90056$	$\mu_2 = +1.91033$	$\mu_3 = +1.44173$	$\mu_4 = +6.04257$



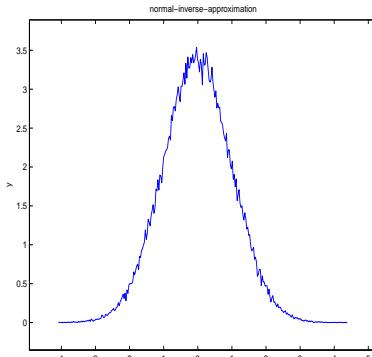
	mean	variance	skewness	kurtosis
GIG	$\mu_1 = +0.80447$	$\mu_2 = +11.40800$	$\mu_3 = +15.21529$	$\mu_4 = +307.90771$



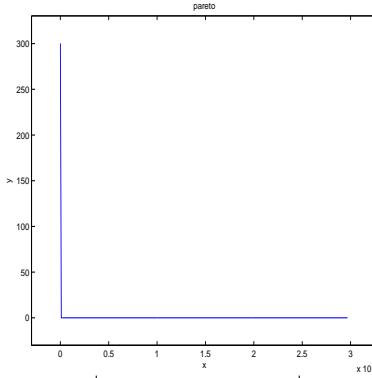
	mean	variance	skewness	kurtosis
normal-box-muller	$\mu_1 = +0.00383$	$\mu_2 = +1.00688$	$\mu_3 = +0.01937$	$\mu_4 = +2.99192$



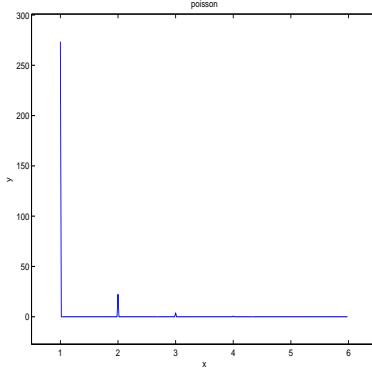
normal-inverse-approximation	mean	variance	skewness	kurtosis
	$\mu_1 = +0.00230$	$\mu_2 = +1.00486$	$\mu_3 = +0.01163$	$\mu_4 = +2.99254$



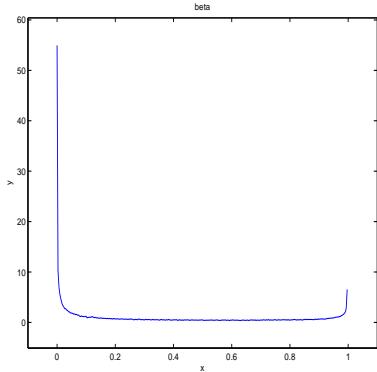
pareto	mean	variance	skewness	kurtosis
	$\mu_1 = +3184578.26493$	$\mu_2 = +888468246174112900.00000$	$\mu_3 = +315.36997$	$\mu_4 = +99629.09819$



poisson	mean	variance	skewness	kurtosis
	$\mu_1 = +1.10590$	$\mu_2 = +0.13329$	$\mu_3 = +3.97097$	$\mu_4 = +21.44867$



beta	mean	variance	skewness	kurtosis
	$\mu_1 = +0.33278$	$\mu_2 = +0.12665$	$\mu_3 = +0.68238$	$\mu_4 = +1.91359$



QueryPerformanceCounter = +10.64632

0.0.32 Multiclass Support Vector Machine

- Number of training points = 1024
- Feature dimension = 3
- Number of classes = 3

The mean vectors of the 3 classes

$$\mu_1 = \begin{pmatrix} +1.90000 & +0.10000 & +0.10000 \end{pmatrix}$$

$$\mu_2 = \begin{pmatrix} +0.10000 & +1.90000 & +0.10000 \end{pmatrix}$$

$$\mu_3 = \begin{pmatrix} +0.00000 & +0.00000 & +1.90000 \end{pmatrix}$$

A random SPD covariance matrix is generated for each of the classes.

$$\rho_1 = \begin{pmatrix} +1.880 & -0.029 & +0.474 \\ -0.029 & +3.581 & +0.171 \\ +0.474 & +0.171 & +4.276 \end{pmatrix}$$

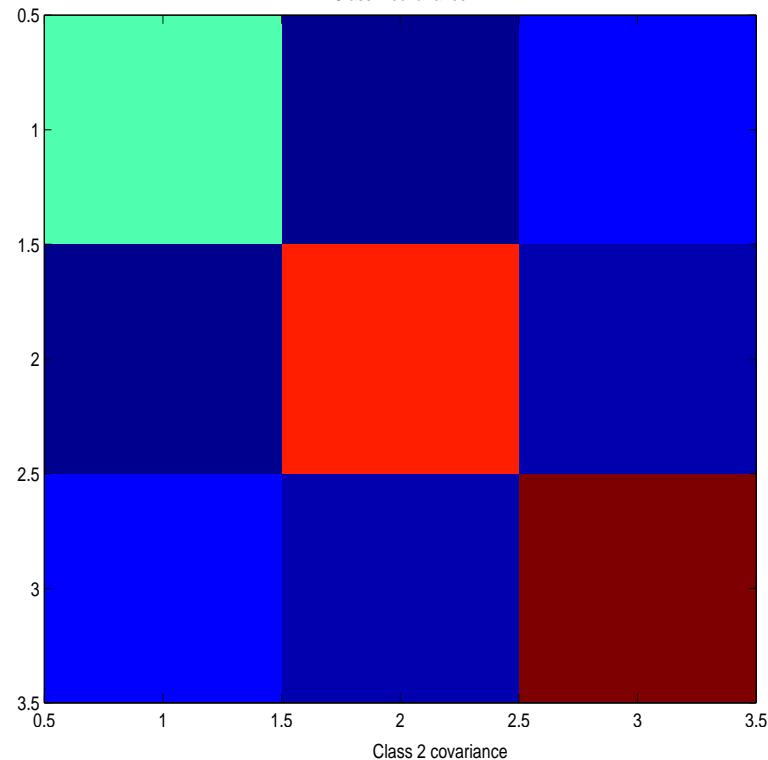
$$\rho_2 = \begin{pmatrix} +1.662 & -0.414 & +0.131 \\ -0.414 & +3.303 & +0.471 \\ +0.131 & +0.471 & +1.612 \end{pmatrix}$$

$$\rho_3 = \begin{pmatrix} +2.671 & +0.444 & -0.470 \\ +0.444 & +2.297 & -0.196 \\ -0.470 & -0.196 & +4.030 \end{pmatrix}$$

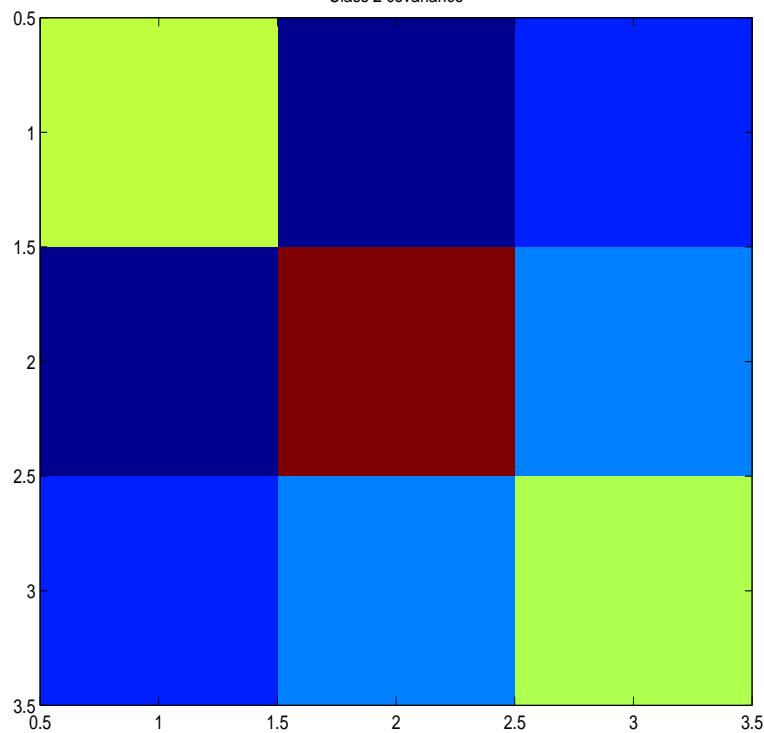
Verify L_1 condition number of covariance. The diagonal entries of the matrix have the form $(0.5 + U(0, 1)) * \dim(\text{Dom}(\text{Cov}))$. The lower-diagonal entries take the form $U(0, 1) - 0.5$. The L_1 condition numbers are :

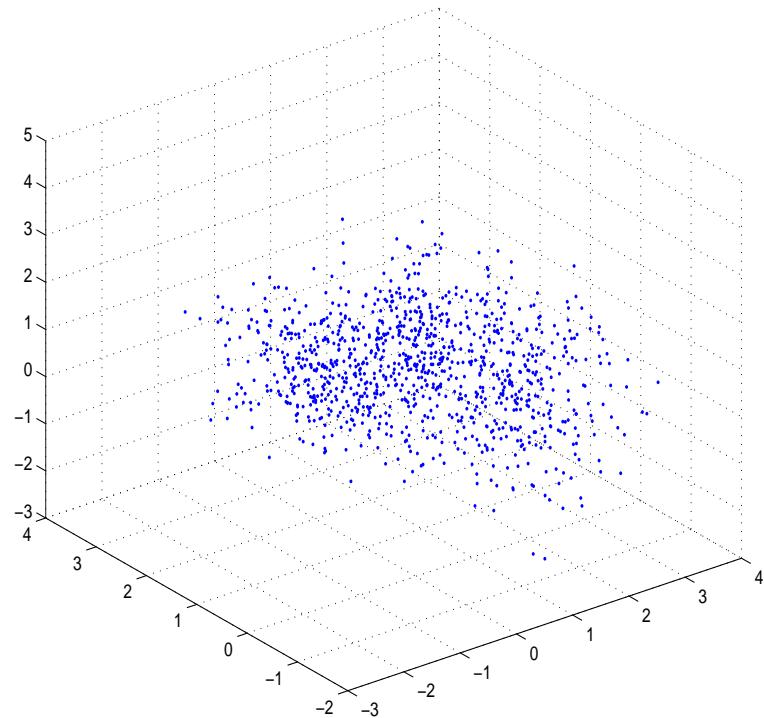
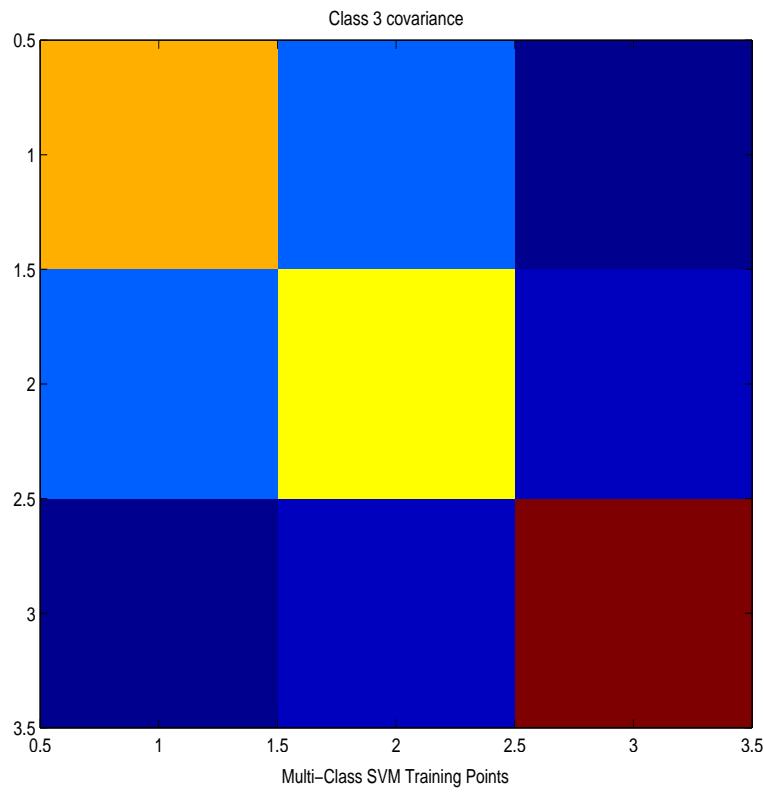
- +3.030
- +3.508
- +2.520

Class 1 covariance



Class 2 covariance

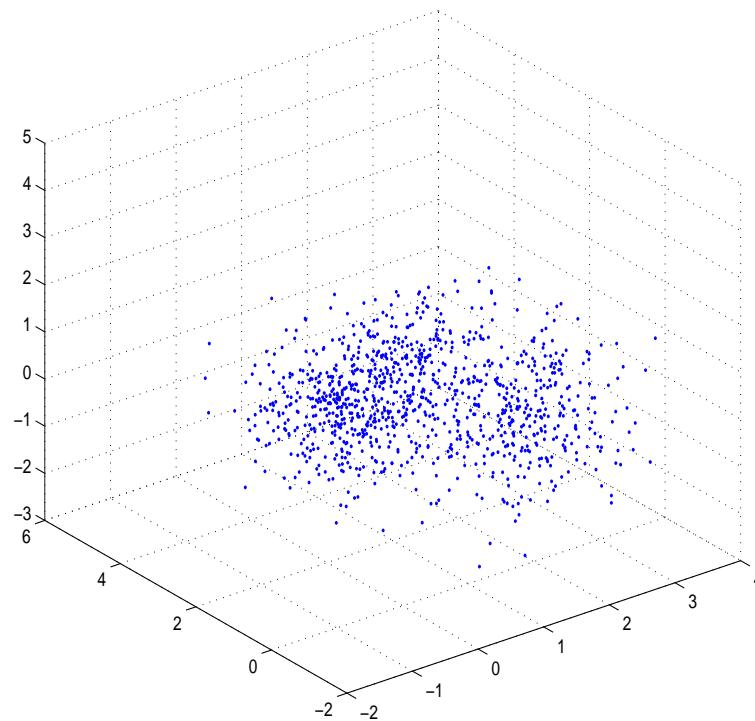




These are the SVM parameters - the RBF kernel is used

- allOutlierFraction=0.05
- mixingCoeff=0.3
- smoThresh=1.0/10000.0
- sigma=1

Multi-Class SVM Test Points



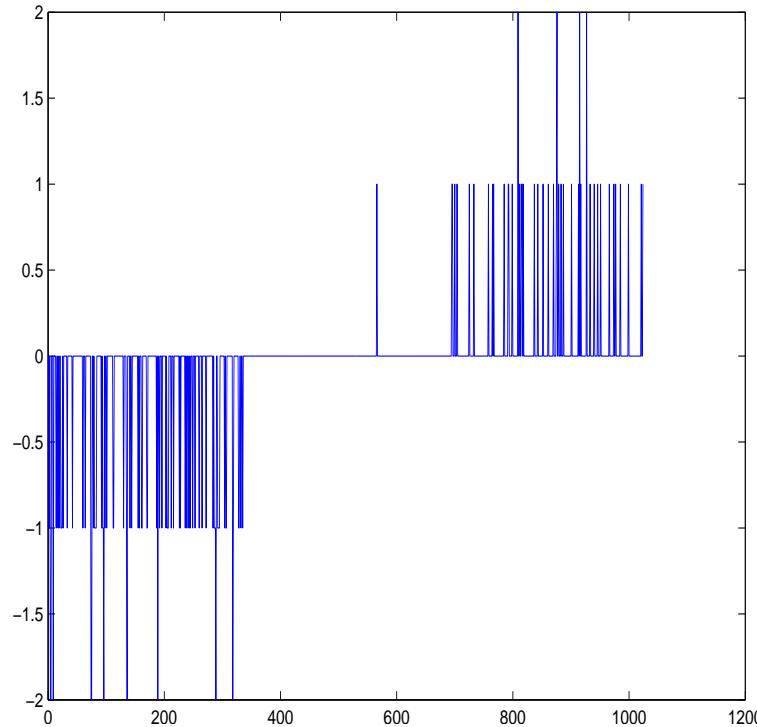
The marginal sample moments (mean var skew kurtosis) for training points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.668	+1.181	+0.252	+2.193
1	+0.704	+1.299	+0.349	+2.401
2	+0.711	+1.384	+0.412	+2.687

The marginal sample moments (mean var skew kurtosis) for test points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.675	+1.113	+0.365	+2.273
1	+0.666	+1.295	+0.415	+2.434
2	+0.726	+1.293	+0.442	+2.596

Class Differences for Test Points



The error rate for this run is +0.123

QueryPerformanceCounter = +6.632

0.0.33 Semidefinite Programming SDPA

QueryPerformanceCounter = +0.006