

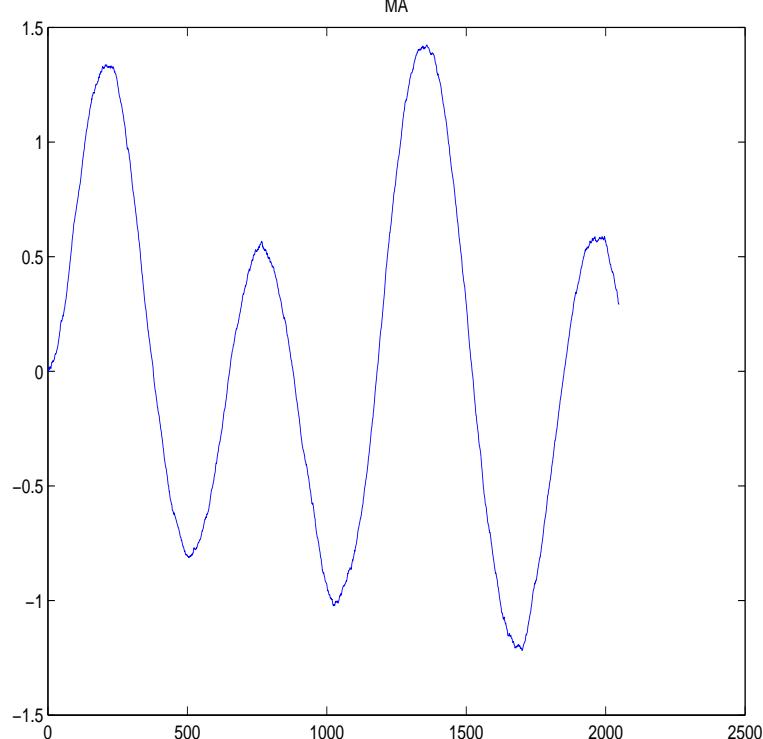
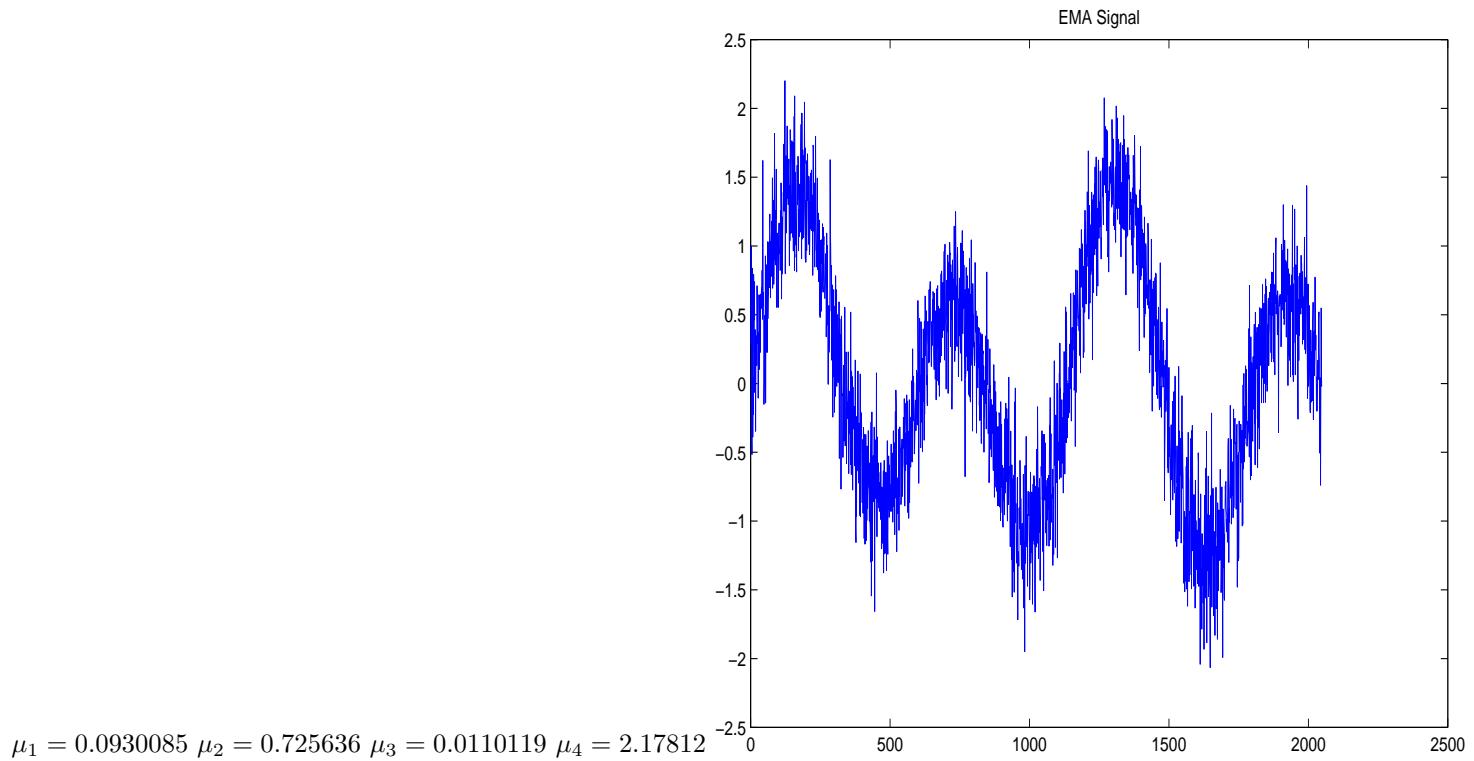
Regression of KL Software Distribution

KL Software Libraries

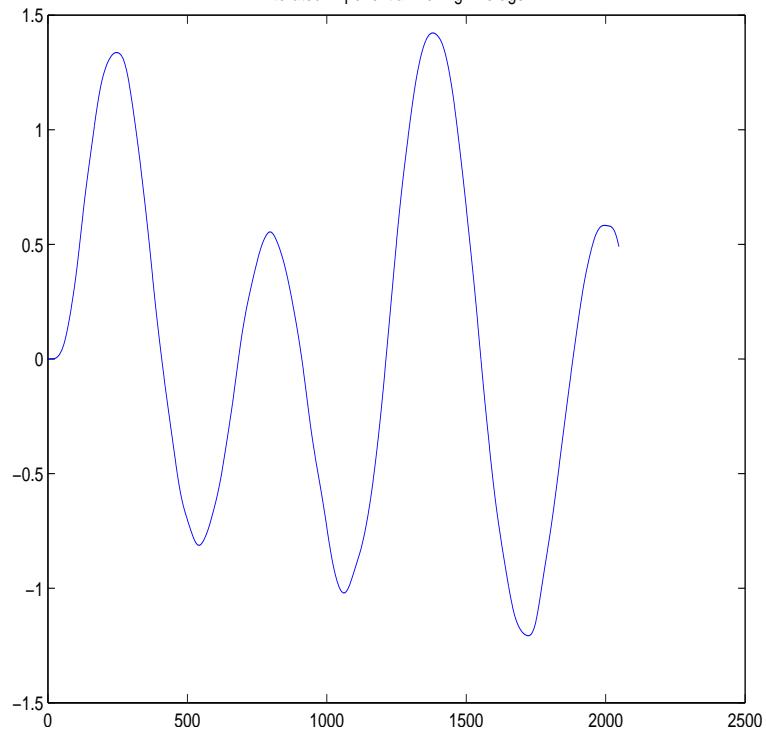
Mon May 12 15:15:05 2014

KL Library test output. This LaTex file and the associated diagrams are produced by the KL software libraries.
QueryPerformanceCounter = 1.6651

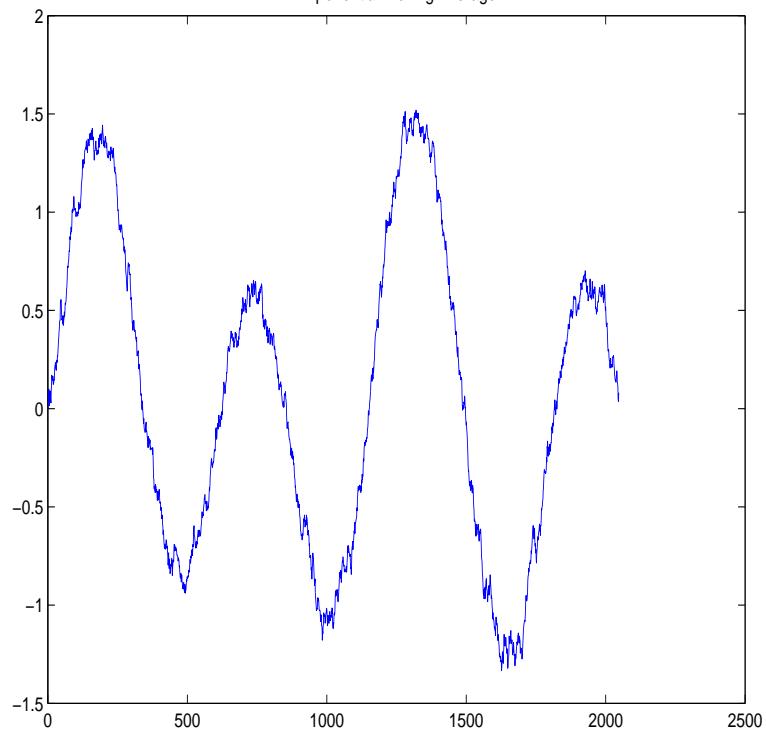
0.0.1 Iterated Exponential Filtering

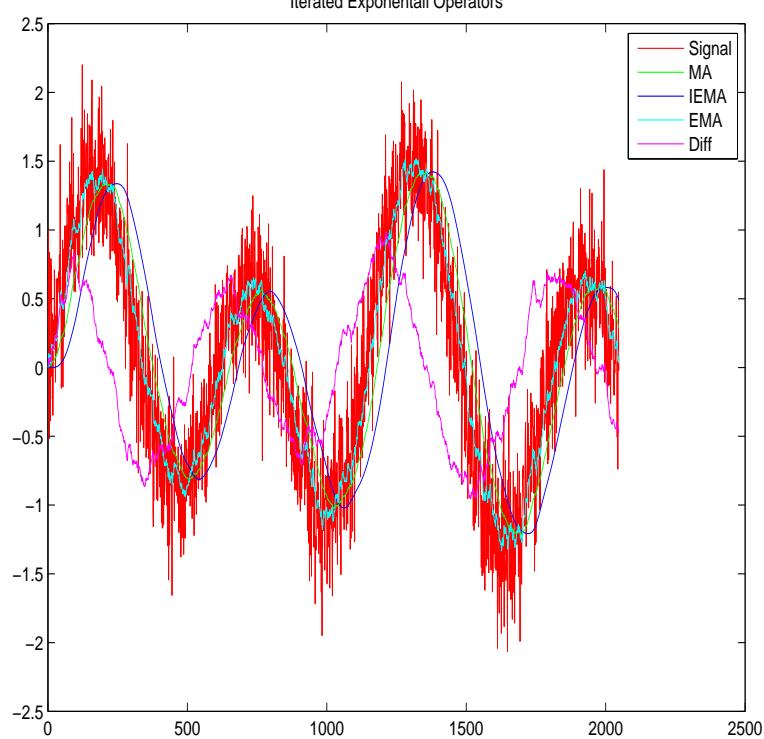
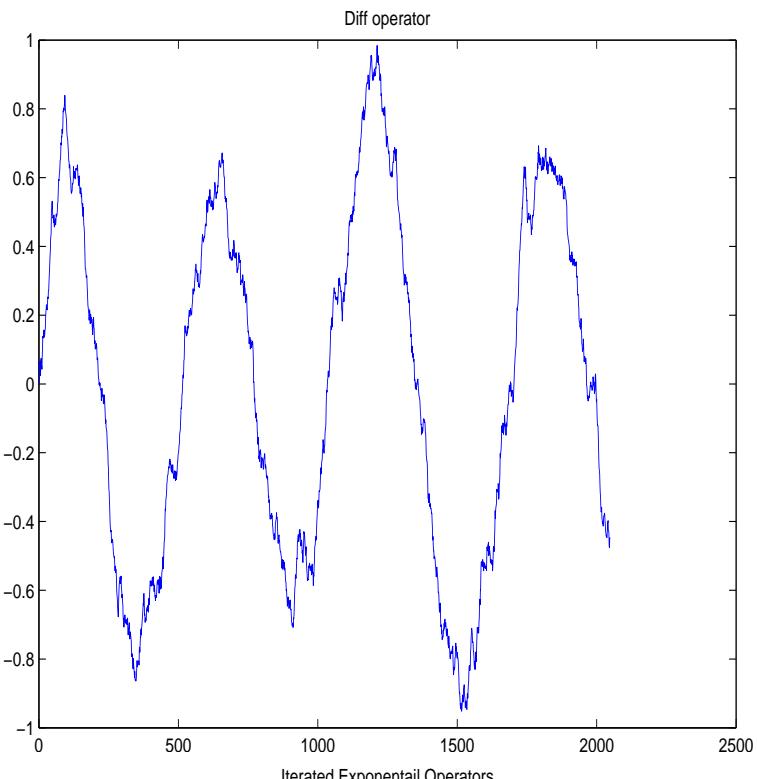


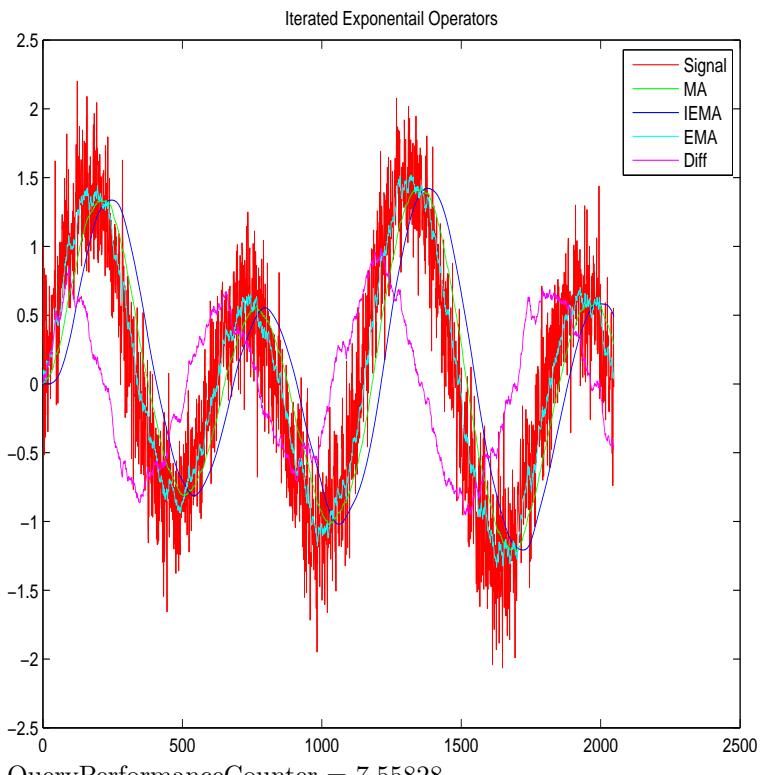
Iterated Exponential Moving Average



Exponential Moving Average



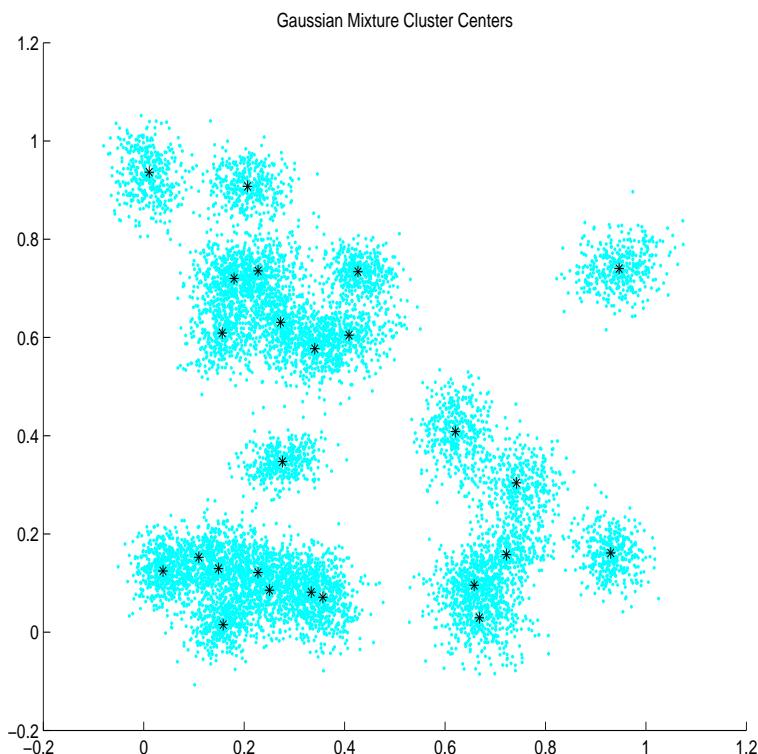




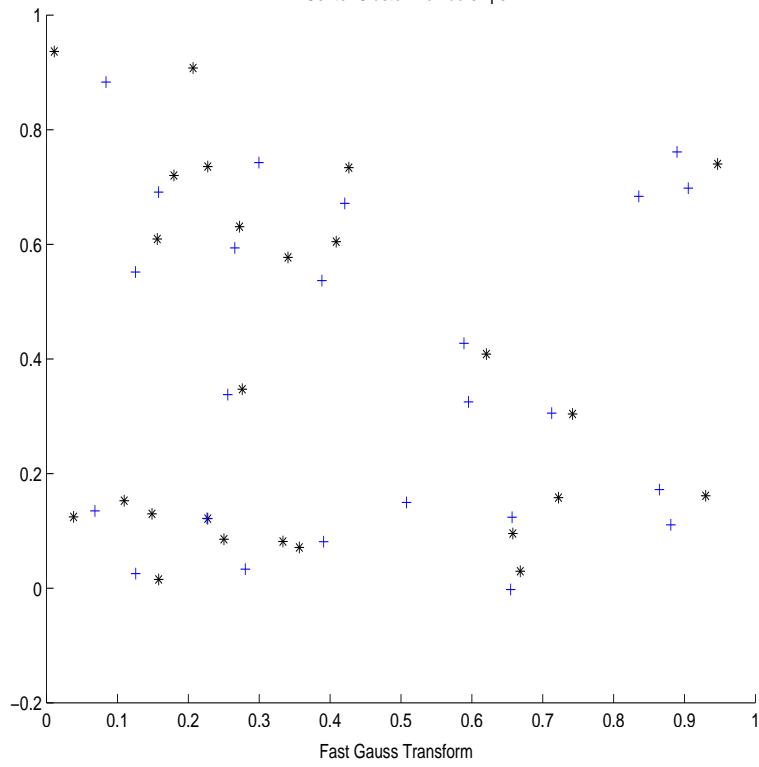
0.0.2 Testing binary writer

Binary writer Speedup 1GB Double Matrix 43.0998 Binary reader Speedup 1GB Double Matrix 342.492 Binary writer Speedup 1GB Double vector 45.7839 Binary reader Speedup 1GB Double Matrix 265.353 QueryPerformanceCounter = 4.48981

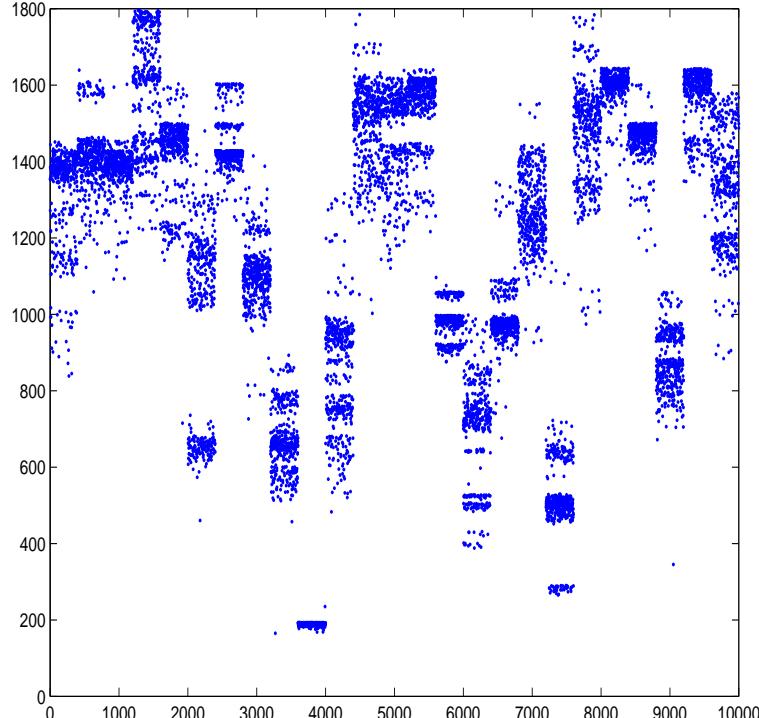
0.0.3 Fast Gauss Transform



KCenter Cluster Memberships

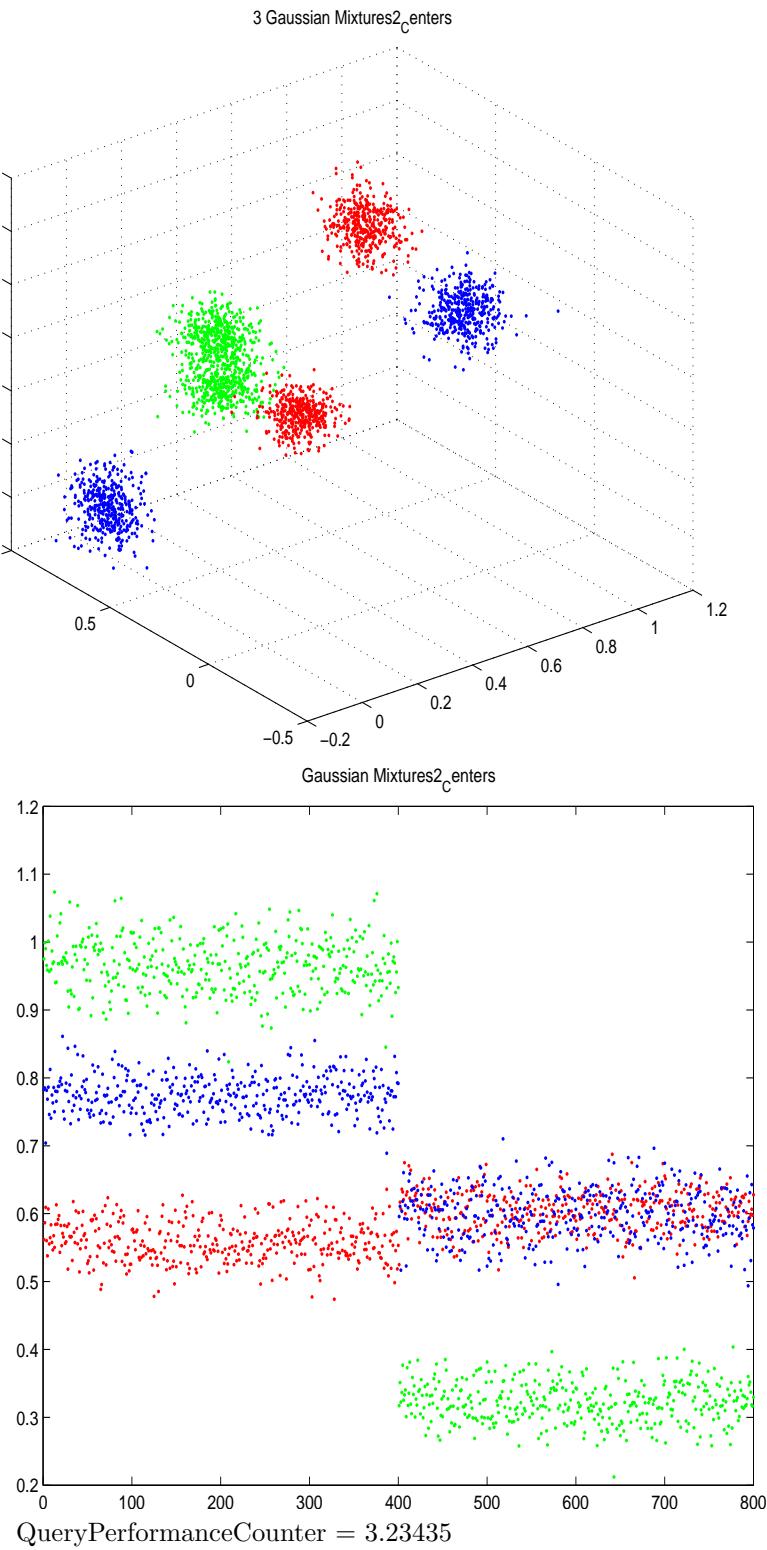


Fast Gauss Transform



QueryPerformanceCounter = 13.3746

0.0.4 Testing Gaussian Mixture Point Cloud and Latex Plotting Capabilities.



0.0.5 Matrix Quick Check `|double|`

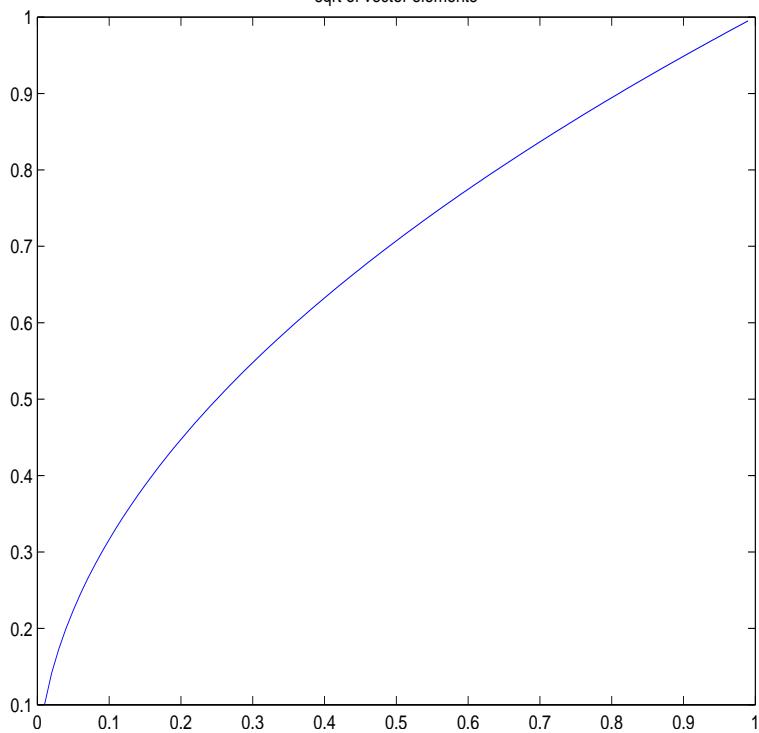
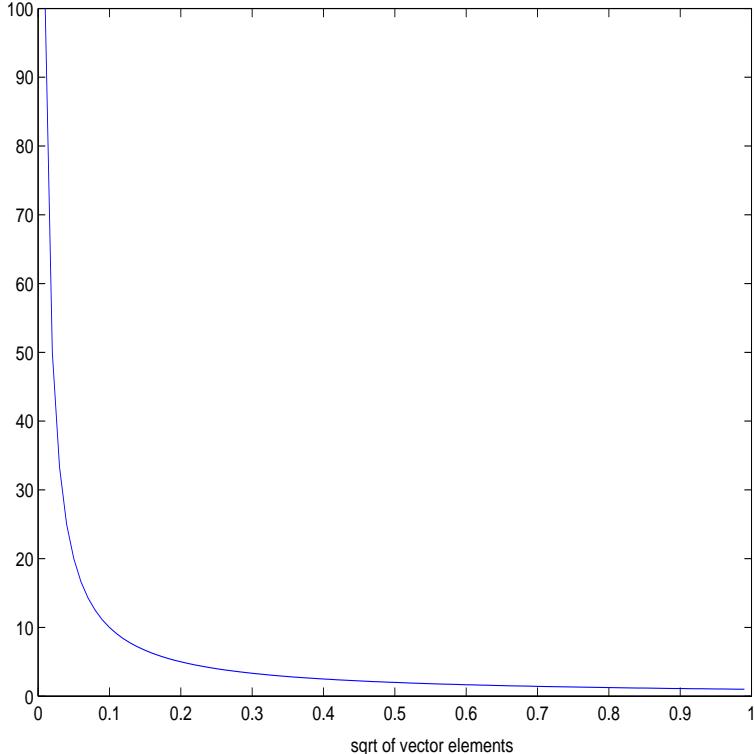
QueryPerformanceCounter = 1.40197

0.0.6 Matrix Quick Check `|float|`

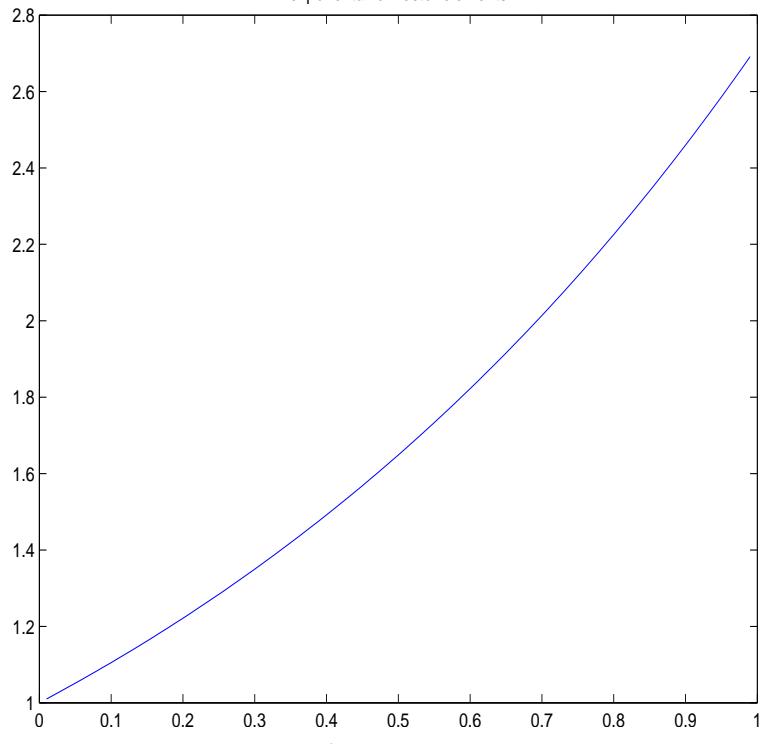
QueryPerformanceCounter = 1.41952

0.0.7 Intel VSL Function Check

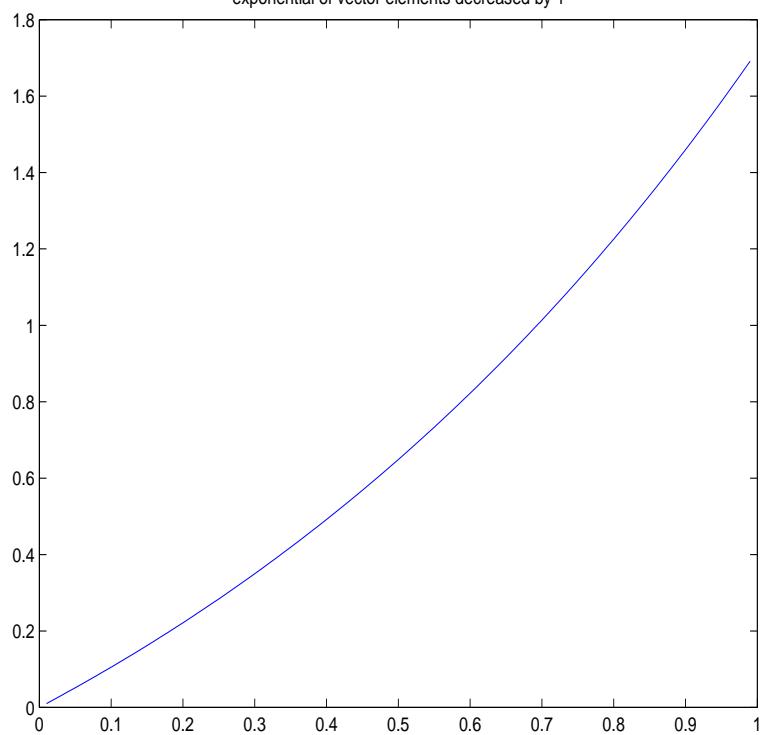
Inversion of vector elements



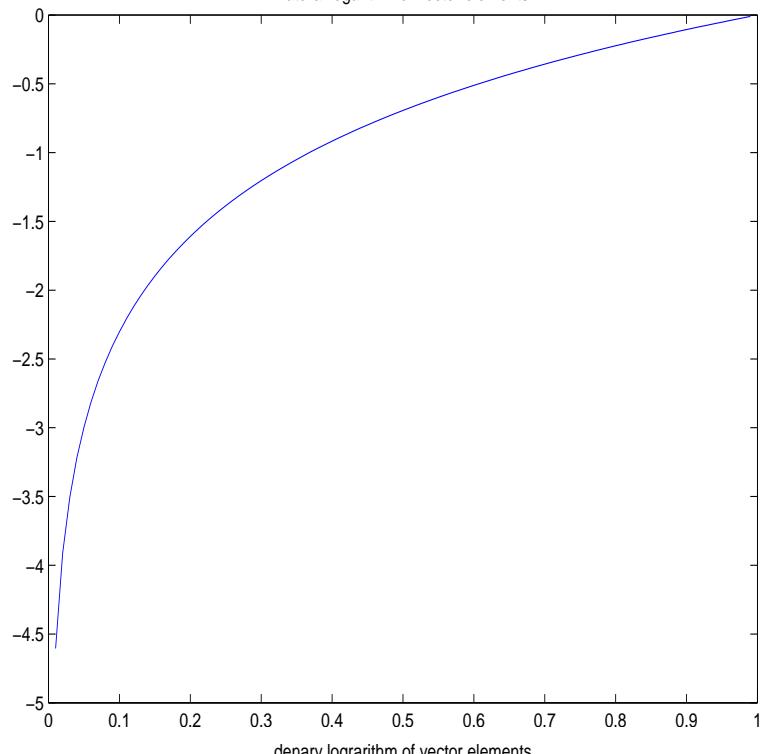
exponentail of vector elements



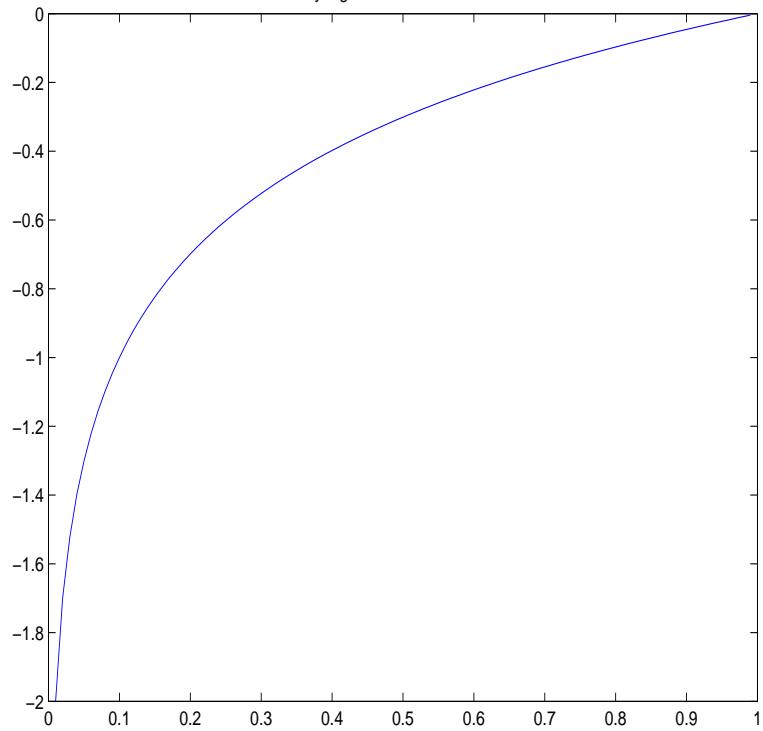
exponentail of vector elements decreased by 1



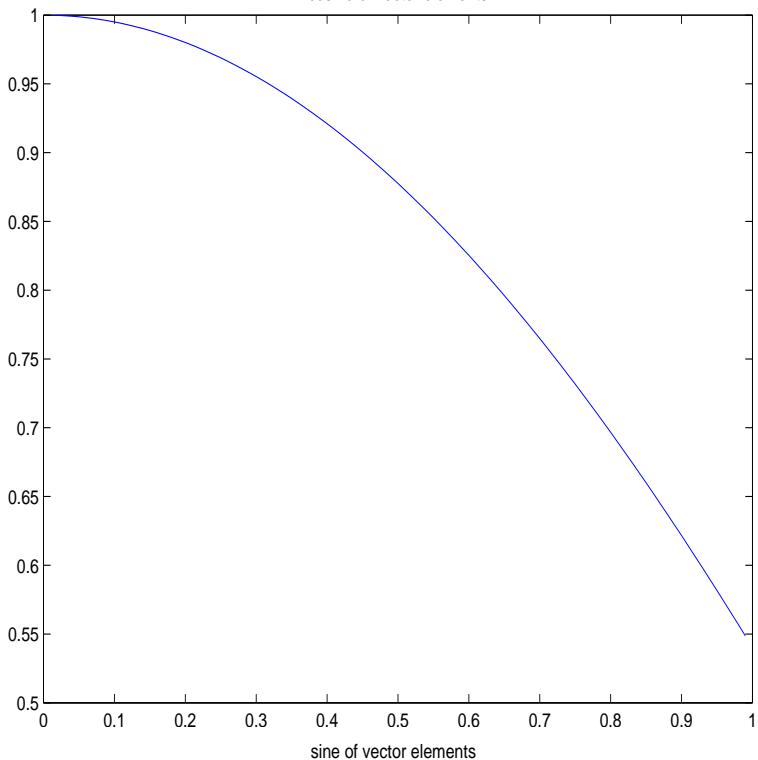
natural logarithm of vector elements



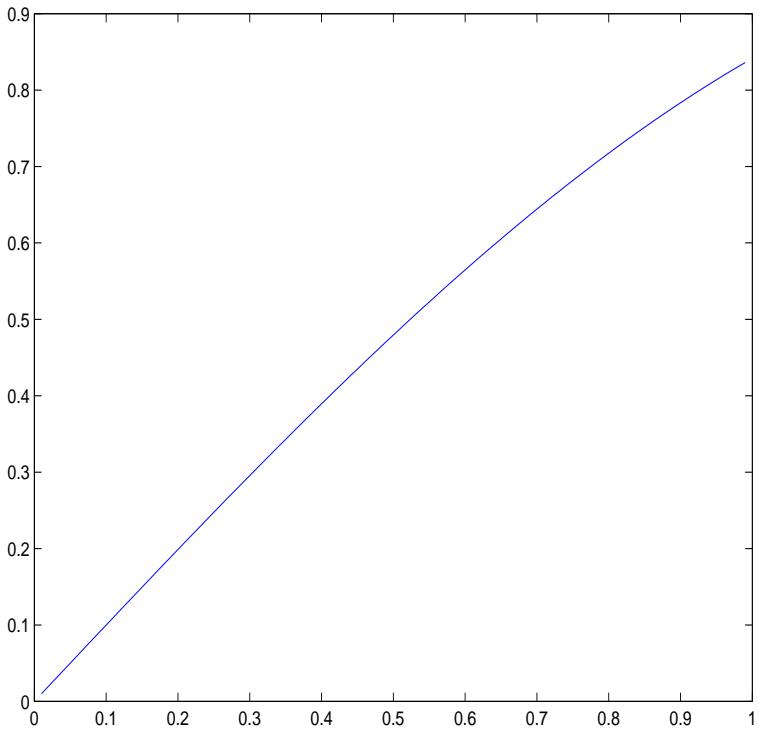
denary logarithm of vector elements



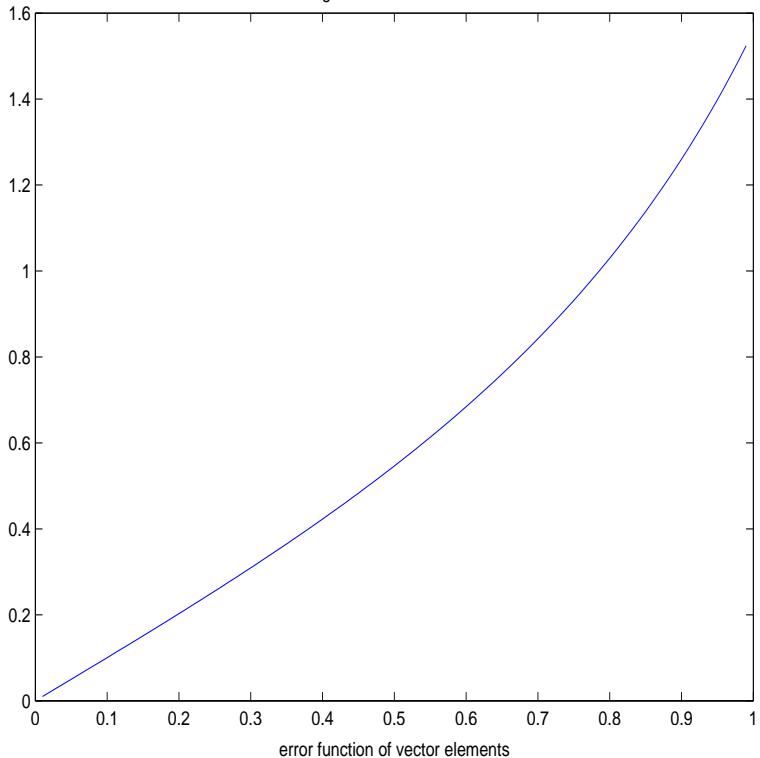
cosine of vector elements



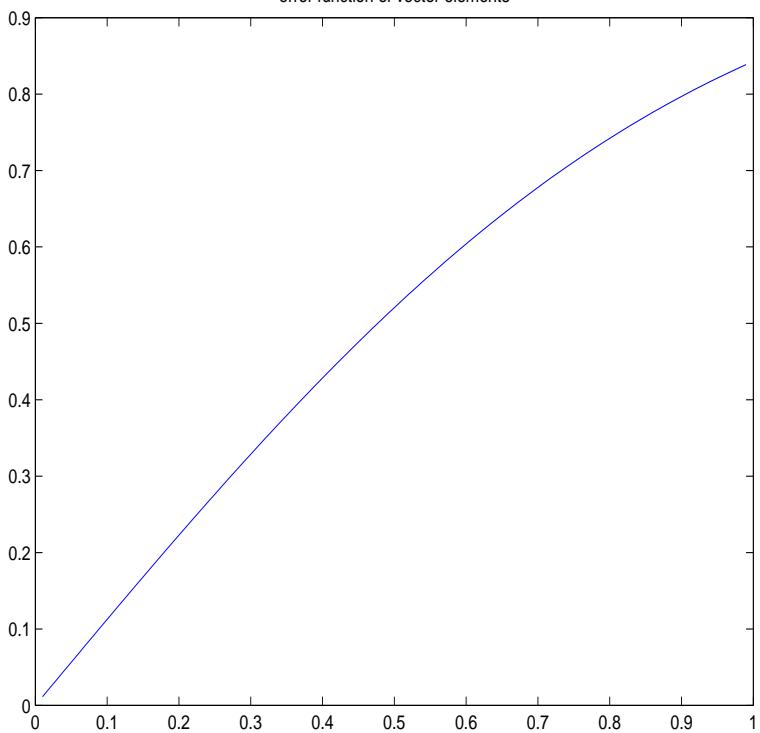
sine of vector elements



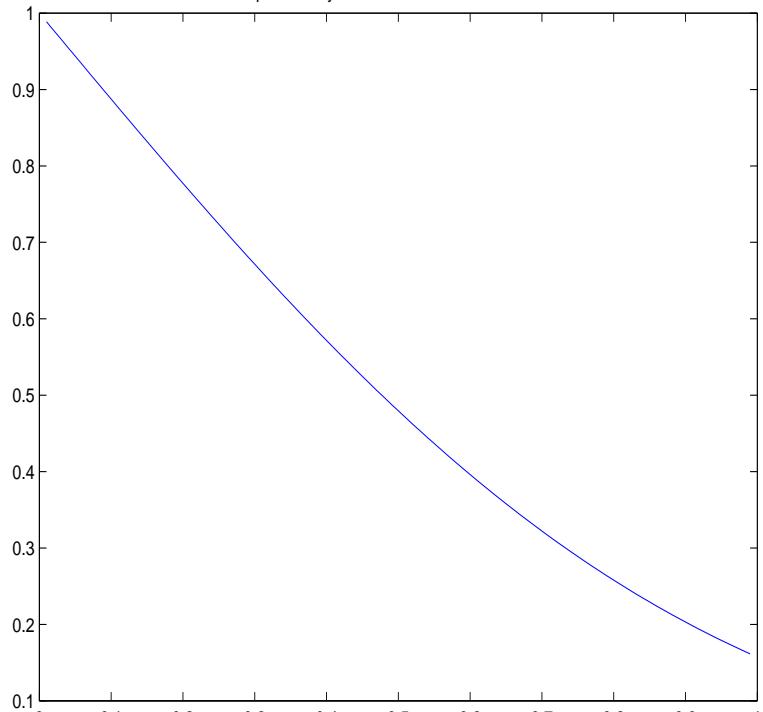
tangent of vector elements



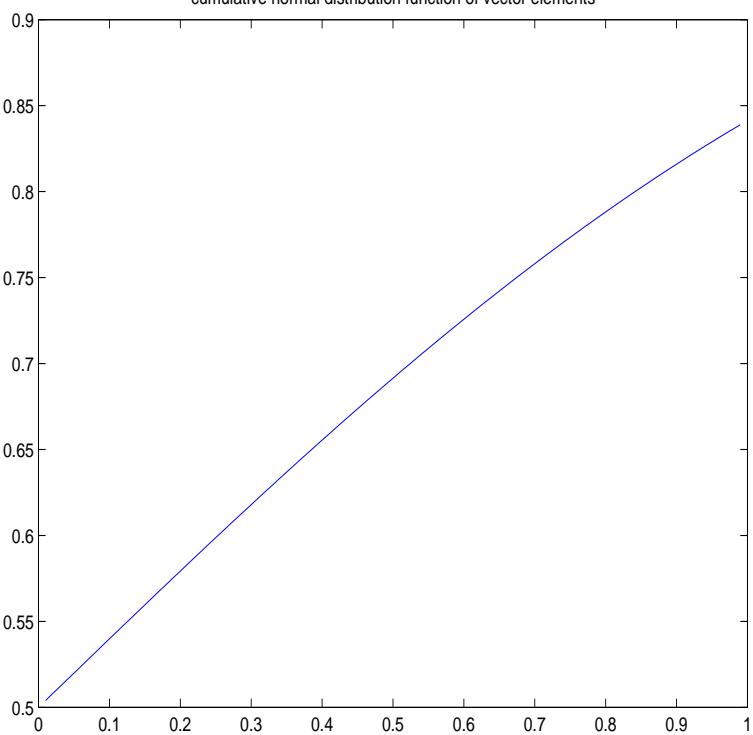
error function of vector elements



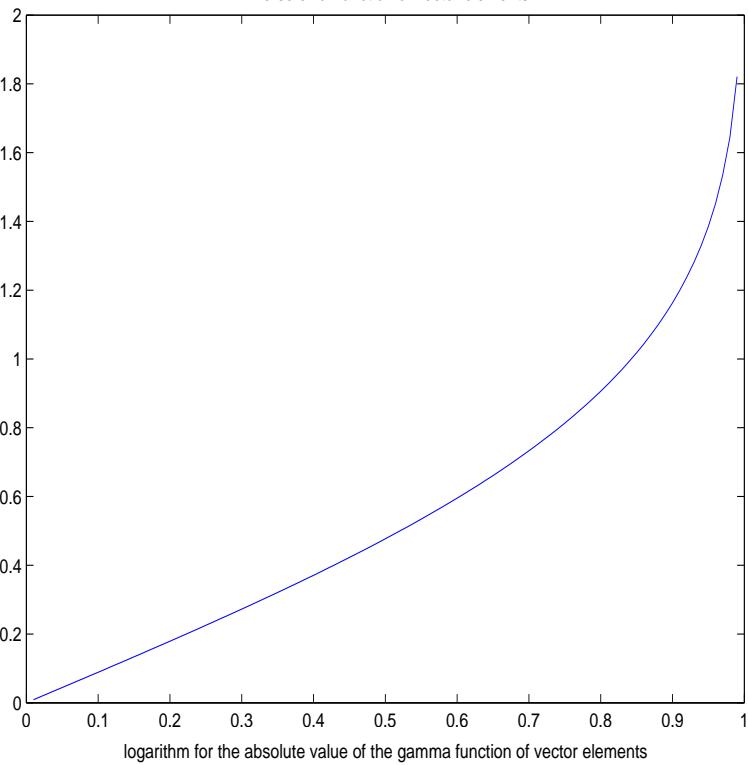
complementary error function of vector elements



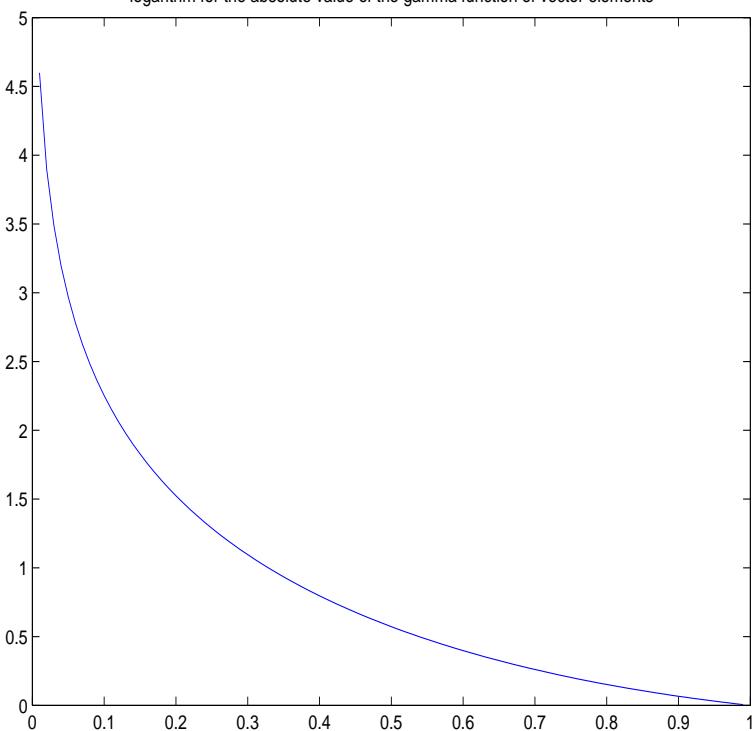
cumulative normal distribution function of vector elements

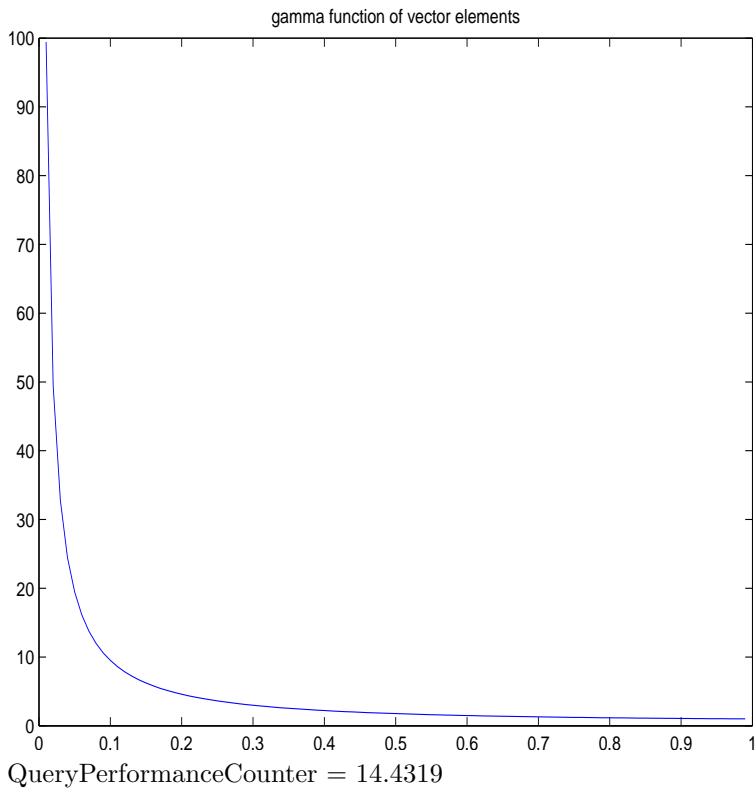


inverse error function of vector elements



logarithm for the absolute value of the gamma function of vector elements





0.0.8 Gram Matrix Consistency Check

Sample Size = 4096 Feature dim = 3

Sigma

$$= \begin{pmatrix} +1.140 & +1.535 & +0.581 \\ +1.535 & +9.988 & +1.605 \\ +0.581 & +1.605 & +0.428 \end{pmatrix}$$

$$\text{SampleCovariance} = \begin{pmatrix} +1.166 & +1.619 & +0.601 \\ +1.619 & +10.343 & +1.672 \\ +0.601 & +1.672 & +0.443 \end{pmatrix}$$

$$\text{SampleMean} = (+1.00523 \quad +1.02214 \quad +1.00933)$$

$$\text{SampleCovariance-Omega} = \begin{pmatrix} +0.026 & +0.084 & +0.019 \\ +0.084 & +0.355 & +0.067 \\ +0.019 & +0.067 & +0.015 \end{pmatrix}$$

$$\text{SampleCovarianceEigs} = ((+10.91314, +0.00000) \quad (+0.99837, +0.00000) \quad (+0.04043, +0.00000))$$

$$\text{CenteredMean} = (-0.00000 \quad -0.00000 \quad -0.00000)$$

$$\text{CenteredCovariance} = \begin{pmatrix} +1.166 & +1.619 & +0.601 \\ +1.619 & +10.343 & +1.672 \\ +0.601 & +1.672 & +0.443 \end{pmatrix}$$

$$\text{GramMatrixGfNotscaledbysamplesize} = \begin{pmatrix} +4774.238 & +6630.177 & +2460.532 \\ +6630.177 & +42366.044 & +6847.206 \\ +2460.532 & +6847.206 & +1814.850 \end{pmatrix}$$

$$\text{GramMatrixGfscaledbysamplesize} = \begin{pmatrix} +1.166 & +1.619 & +0.601 \\ +1.619 & +10.343 & +1.672 \\ +0.601 & +1.672 & +0.443 \end{pmatrix}$$

$$\text{SampleCovariance - ScaledGf} = \begin{pmatrix} +0.000 & +0.000 & +0.000 \\ +0.000 & +0.003 & +0.000 \\ +0.000 & +0.000 & +0.000 \end{pmatrix}$$

$$\text{EigenDecompoofSampleCovariance} = \begin{pmatrix} -0.171 & -0.971 & -0.165 \\ +0.917 & -0.219 & +0.334 \\ -0.360 & -0.094 & +0.928 \end{pmatrix}$$

$$EigenDecompoofGramMatrix = \begin{pmatrix} -0.121 & -0.975 & -0.188 \\ -0.332 & +0.219 & -0.918 \\ +0.935 & -0.049 & -0.350 \end{pmatrix}$$

QueryPerformanceCounter = +76.679

0.0.9 Eigen Solver Checks

0.0.10 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: +8

$$A = \begin{pmatrix} +0.338 & +0.009 & -0.455 & +0.241 & -0.369 & -0.500 & -0.365 & +0.318 \\ +0.039 & +0.036 & +0.255 & -0.833 & +0.042 & -0.321 & -0.353 & +0.093 \\ -0.011 & +0.056 & +0.156 & -0.140 & -0.844 & -0.055 & +0.295 & -0.389 \\ +0.475 & +0.764 & -0.223 & -0.116 & +0.154 & +0.162 & +0.043 & -0.276 \\ +0.453 & -0.487 & -0.059 & -0.064 & -0.081 & +0.502 & -0.440 & -0.313 \\ +0.188 & -0.204 & +0.161 & +0.203 & +0.326 & -0.603 & +0.061 & -0.619 \\ -0.116 & -0.290 & -0.748 & -0.404 & +0.109 & -0.042 & +0.386 & -0.130 \\ -0.636 & +0.222 & -0.254 & +0.080 & -0.045 & +0.023 & -0.554 & -0.407 \end{pmatrix}$$

$\text{Det}(A) : A \in O(n) = (+1.000, +0.000)$

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.747 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.182 & -0.356 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.062 & +0.054 & -0.308 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.018 & +0.056 & -0.216 & +0.243 & +1.000 & +0.000 & +0.000 & +0.000 \\ -0.531 & +0.137 & +0.628 & -0.590 & +0.553 & +1.000 & +0.000 & +0.000 \\ -0.712 & -0.353 & +0.455 & -0.180 & +0.153 & -0.708 & +1.000 & +0.000 \\ -0.296 & -0.149 & -0.028 & -0.214 & -0.422 & +0.872 & -0.506 & +1.000 \end{pmatrix}$$

$$U = \begin{pmatrix} -0.636 & +0.222 & -0.254 & +0.080 & -0.045 & +0.023 & -0.554 & -0.407 \\ +0.000 & +0.930 & -0.413 & -0.057 & +0.121 & +0.178 & -0.370 & -0.580 \\ +0.000 & +0.000 & -0.849 & -0.438 & +0.160 & +0.017 & +0.355 & -0.262 \\ +0.000 & +0.000 & +0.000 & -0.960 & +0.082 & -0.323 & -0.258 & +0.018 \\ +0.000 & +0.000 & +0.000 & +0.000 & -0.835 & +0.017 & +0.465 & -0.410 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.724 & -1.241 & +0.583 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -2.123 & -0.210 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.616 \end{pmatrix}$$

$$L * U = \begin{pmatrix} -0.636 & +0.222 & -0.254 & +0.080 & -0.045 & +0.023 & -0.554 & -0.407 \\ +0.475 & +0.764 & -0.223 & -0.116 & +0.154 & +0.162 & +0.043 & -0.276 \\ -0.116 & -0.290 & -0.748 & -0.404 & +0.109 & -0.042 & +0.386 & -0.130 \\ +0.039 & +0.036 & +0.255 & -0.833 & +0.042 & -0.321 & -0.353 & +0.093 \\ -0.011 & +0.056 & +0.156 & -0.140 & -0.844 & -0.055 & +0.295 & -0.389 \\ +0.338 & +0.009 & -0.455 & +0.241 & -0.369 & -0.500 & -0.365 & +0.318 \\ +0.453 & -0.487 & -0.059 & -0.064 & -0.081 & +0.502 & -0.440 & -0.313 \\ +0.188 & -0.204 & +0.161 & +0.203 & +0.326 & -0.603 & +0.061 & -0.619 \end{pmatrix}$$

$\text{Det}(L) := (+1.000, +0.000) \text{Det}(U) := (-1.000, +0.000) \text{Det}(LU) := (-1.000, -0.000)$

$$\|A\|_{L_1} = +2.544$$

$$\|A\|_{L_\infty} = +2.595$$

$$\|A^{-1}\|_{L_1} = +2.595$$

$$\|A^{-1}\|_{L_\infty} = +2.544$$

$$\|A\|_{L_\infty} * \|A^{-1}\|_{L_\infty} = +6.602$$

$$\|A\|_{L_1} * \|A^{-1}\|_{L_1} = +6.602$$

$$\text{Frobenious Norm } \|A\|_F \text{ via } \sum_{i,j=0}^n |A_{i,j}| \text{ of } A \in O(n) +2.828$$

L_1 condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n) +5.630$

$$A = \begin{pmatrix} +0.338 & +0.009 & -0.455 & +0.241 & -0.369 & -0.500 & -0.365 & +0.318 \\ +0.039 & +0.036 & +0.255 & -0.833 & +0.042 & -0.321 & -0.353 & +0.093 \\ -0.011 & +0.056 & +0.156 & -0.140 & -0.844 & -0.055 & +0.295 & -0.389 \\ +0.475 & +0.764 & -0.223 & -0.116 & +0.154 & +0.162 & +0.043 & -0.276 \\ +0.453 & -0.487 & -0.059 & -0.064 & -0.081 & +0.502 & -0.440 & -0.313 \\ +0.188 & -0.204 & +0.161 & +0.203 & +0.326 & -0.603 & +0.061 & -0.619 \\ -0.116 & -0.290 & -0.748 & -0.404 & +0.109 & -0.042 & +0.386 & -0.130 \\ -0.636 & +0.222 & -0.254 & +0.080 & -0.045 & +0.023 & -0.554 & -0.407 \end{pmatrix}$$

L_∞ condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n) +5.369$

Eigenvalues of $A \in O(n)$

$(+0.269, +0.963), (+0.269, -0.963), (-0.292, +0.956), (-0.292, -0.956), (-0.991, +0.137), (-0.991, -0.137), (+0.867, +0.497), (+0.867, -0.497)$

$|\lambda| : \lambda \in \sigma(A), A \in O(n)$

$+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000$

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 \\ -0.000 & +1.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & -0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & +1.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & +1.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} +0.338 & +0.039 & -0.011 & +0.475 & +0.453 & +0.188 & -0.116 & -0.636 \\ +0.009 & +0.036 & +0.056 & +0.764 & -0.487 & -0.204 & -0.290 & +0.222 \\ -0.455 & +0.255 & +0.156 & -0.223 & -0.059 & +0.161 & -0.748 & -0.254 \\ +0.241 & -0.833 & -0.140 & -0.116 & -0.064 & +0.203 & -0.404 & +0.080 \\ -0.369 & +0.042 & -0.844 & +0.154 & -0.081 & +0.326 & +0.109 & -0.045 \\ -0.500 & -0.321 & -0.055 & +0.162 & +0.502 & -0.603 & -0.042 & +0.023 \\ -0.365 & -0.353 & +0.295 & +0.043 & -0.440 & +0.061 & +0.386 & -0.554 \\ +0.318 & +0.093 & -0.389 & -0.276 & -0.313 & -0.619 & -0.130 & -0.407 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 \\ -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & +1.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & -0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & -0.000 & -0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} +0.434 & -0.706 & -0.108 & +0.001 & +0.279 & +0.353 & -0.308 & -0.068 \\ -0.107 & -0.385 & +0.335 & +0.487 & -0.636 & +0.176 & +0.233 & +0.040 \\ -0.047 & -0.071 & +0.208 & +0.436 & +0.257 & -0.494 & -0.165 & -0.650 \\ -0.403 & -0.345 & -0.495 & +0.246 & +0.344 & -0.168 & +0.468 & +0.218 \\ -0.284 & +0.277 & +0.079 & +0.539 & +0.280 & +0.356 & -0.475 & +0.339 \\ +0.201 & -0.150 & +0.607 & -0.052 & +0.302 & -0.396 & +0.110 & +0.553 \\ -0.633 & -0.360 & +0.083 & -0.400 & -0.166 & -0.197 & -0.483 & +0.049 \\ -0.338 & -0.009 & +0.455 & -0.241 & +0.369 & +0.500 & +0.365 & -0.318 \end{pmatrix}$$

$$S = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

$$V = \begin{pmatrix} +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -1.000 \\ -0.036 & -0.500 & -0.148 & -0.436 & -0.333 & +0.353 & +0.548 & -0.000 \\ -0.380 & +0.543 & -0.018 & -0.354 & -0.573 & -0.324 & +0.046 & +0.000 \\ -0.204 & -0.547 & -0.042 & -0.387 & -0.017 & -0.314 & -0.639 & +0.000 \\ +0.858 & +0.113 & -0.020 & -0.387 & -0.044 & -0.314 & +0.022 & +0.000 \\ +0.110 & -0.112 & +0.901 & +0.072 & -0.350 & +0.157 & -0.109 & +0.000 \\ +0.141 & -0.315 & -0.236 & +0.615 & -0.536 & -0.393 & +0.074 & +0.000 \\ -0.212 & -0.176 & +0.329 & -0.042 & +0.386 & -0.628 & +0.520 & +0.000 \end{pmatrix}$$

$$USV = \begin{pmatrix} +0.315 & +0.395 & +0.469 & +0.077 & +0.300 & -0.083 & -0.483 & -0.434 \\ -0.714 & -0.064 & +0.160 & +0.261 & -0.215 & -0.286 & -0.502 & +0.107 \\ +0.115 & +0.161 & -0.636 & -0.420 & -0.104 & +0.085 & -0.599 & +0.047 \\ +0.447 & -0.359 & -0.147 & +0.364 & +0.271 & -0.494 & -0.195 & +0.403 \\ -0.009 & -0.309 & +0.474 & -0.747 & +0.102 & -0.155 & -0.081 & +0.284 \\ -0.101 & +0.380 & -0.193 & -0.230 & -0.017 & -0.781 & +0.325 & -0.201 \\ -0.180 & +0.592 & +0.025 & +0.034 & +0.433 & +0.152 & +0.070 & +0.633 \\ +0.367 & +0.310 & +0.256 & +0.067 & -0.763 & -0.056 & -0.015 & +0.338 \end{pmatrix}$$

Calculating first few eigenvectors of $A \in O(n)$ using LAPACK syevx

0.0.11 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +1489.694 L_{inf} condition number of Wishart Matrix +1489.694

0.0.12 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +66.900 L_∞ condition number of GOE Matrix +66.900

0.0.13 The Identity Matrix $I \in M(n)$

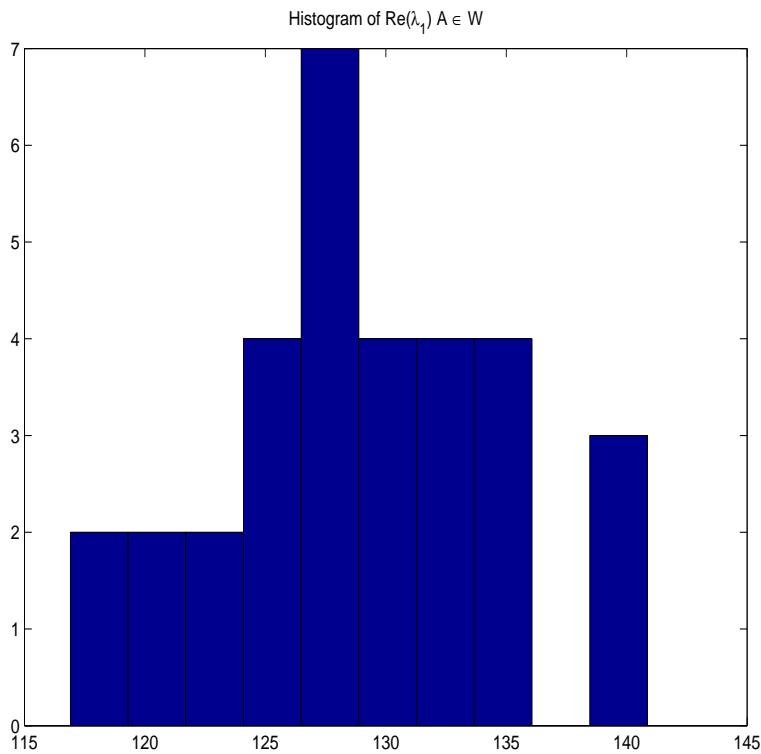
L_1 condition number of $I = +1.000$ L_∞ condition number of $I = +1.000$ QueryPerformanceCounter = +2.127

0.0.14 Generate Tracey Widom Sample

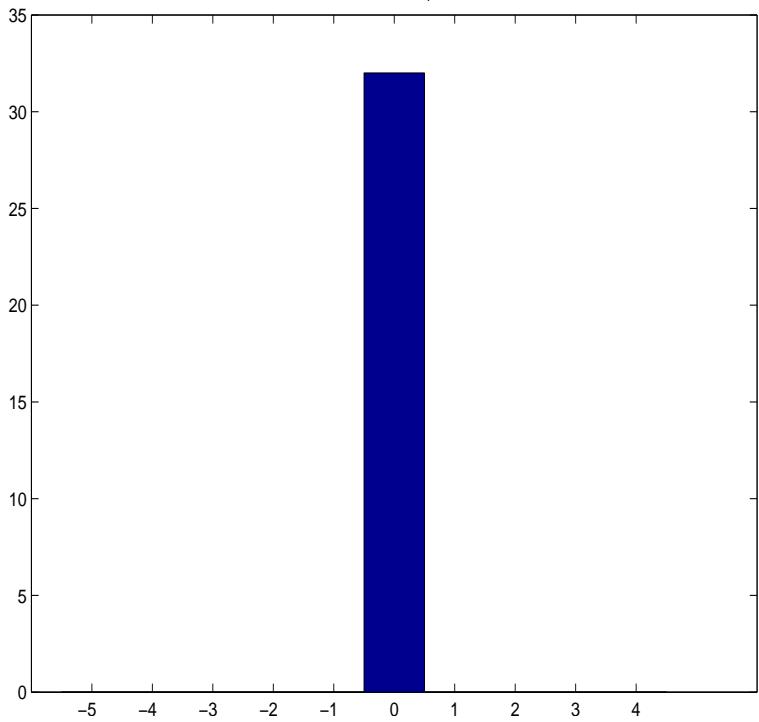
0.0.15 Sample from $W_n m$ times and calculate empirical PDF of the first eig

Here we generate histograms of λ_1 for GOE (Gaussian Orthogonal Ensemble), and W (Wishart) distributed of random matrices These should approximate the celebrated Tracy Widom distribution. Dimension $n = +128$

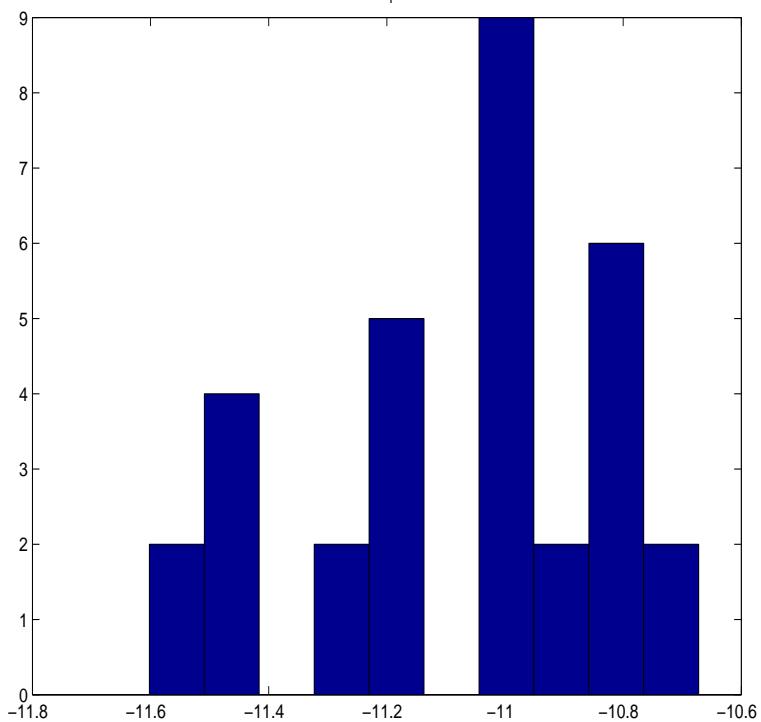
Sample size $m = 32$



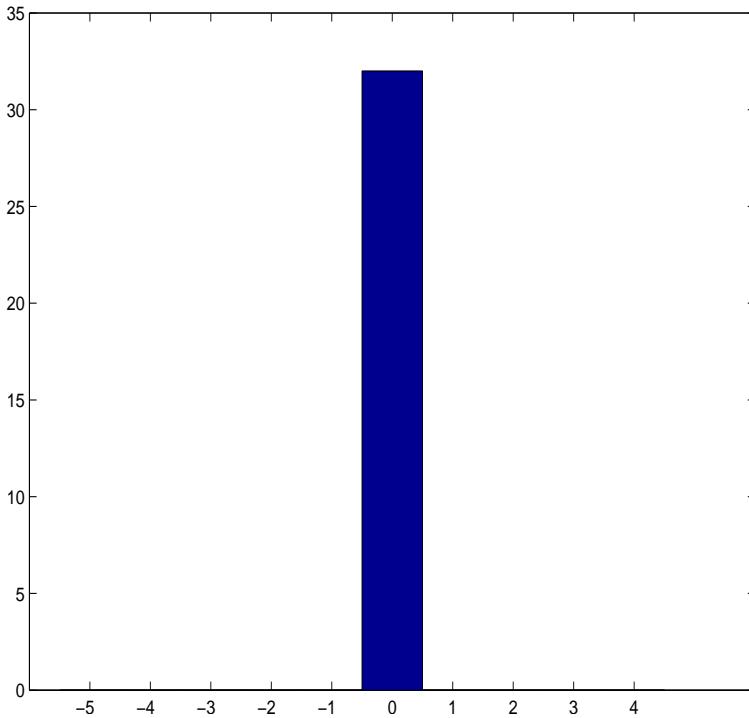
Histogram of $\text{Im}(\lambda_1)$ $A \in W$



Histogram of $\text{Re}(\lambda_1)$ $A \in \text{GOE}(1024)$



Histogram of $\text{Im}(\lambda_i)$ $A \in \text{GOE}(1024)$



QueryPerformanceCounter = +7.926

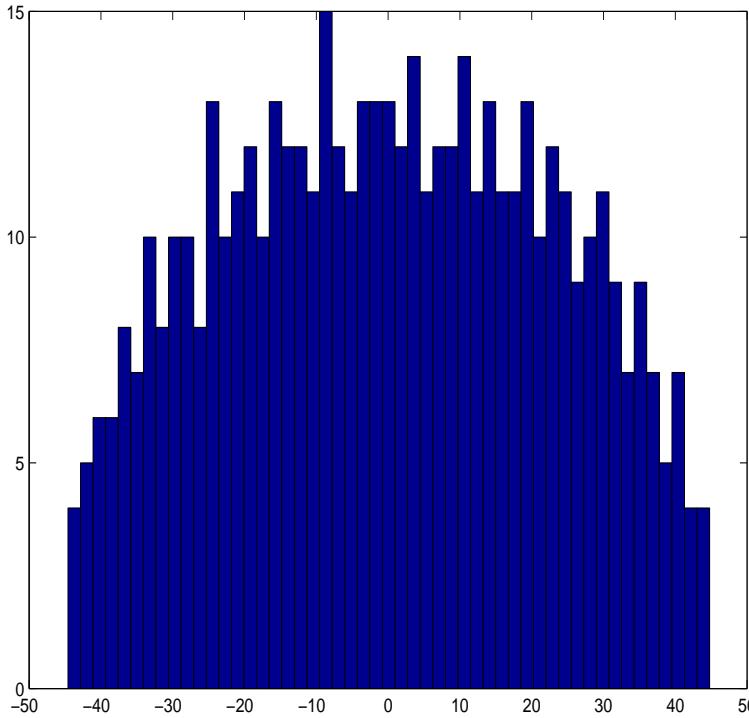
0.0.16 Approximate Winger Distribution

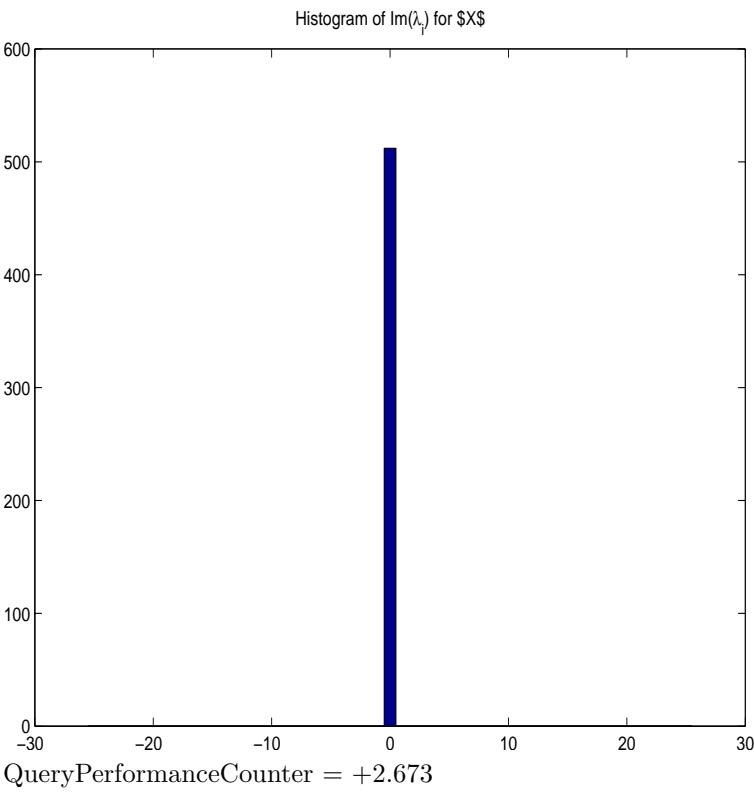
0.0.17 Verify Winger Law.

Let $M_n = [X_{ij}]$ a symmetric $n \times n$ matrix with Random entries such that $X_{i,j} = X_{j,i}$, and $X_{i,j}$ are iid
or all $i \neq j$ and $X_{i,j}$ are iid

or all $j : E[X_{ij}^2] = 1, E[X_{ij}] = 0$ and that all moments exists for each of the entries. The eigenvector of this random matrix;
 $\lambda_1 \leq \dots \leq \lambda_n$ depends continuously on M_n . Dimension $n = +512$

Histogram of $\text{Re}(\lambda_i)$ for X





0.0.18 Matrix Exponential

$$SPDMatrix = \begin{pmatrix} +10.539 & -0.499 & -0.010 & +0.368 & +0.465 & -0.492 & -0.126 & +0.437 \\ -0.499 & +7.286 & +0.365 & -0.481 & -0.337 & -0.466 & +0.279 & +0.056 \\ -0.010 & +0.365 & +6.705 & -0.205 & +0.467 & +0.131 & +0.077 & -0.089 \\ +0.368 & -0.481 & -0.205 & +6.496 & -0.402 & -0.209 & +0.043 & -0.041 \\ +0.465 & -0.337 & +0.467 & -0.402 & +4.578 & +0.272 & +0.289 & -0.285 \\ -0.492 & -0.466 & +0.131 & -0.209 & +0.272 & +8.181 & +0.343 & -0.244 \\ -0.126 & +0.279 & +0.077 & +0.043 & +0.289 & +0.343 & +5.938 & -0.212 \\ +0.437 & +0.056 & -0.089 & -0.041 & -0.285 & -0.244 & -0.212 & +9.691 \end{pmatrix}$$

$$SPDEigs = ((+10.93611, +0.00000) \quad (+9.60778, +0.00000) \quad (+4.23666, +0.00000) \quad (+8.36911, +0.00000) \quad (+7.56229, +0.00000))$$

$$\exp(SPD) = \begin{pmatrix} +47863.969 & -6460.093 & -1078.770 & +4706.958 & +2535.224 & -8475.398 & -2406.368 & +12977.552 \\ -6460.093 & +2780.574 & +516.920 & -1069.918 & -548.083 & -109.707 & +386.466 & -807.216 \\ -1078.770 & +516.920 & +1015.281 & -385.755 & +176.069 & +458.541 & +212.284 & -859.022 \\ +4706.958 & -1069.918 & -385.755 & +1267.210 & +111.181 & -1018.272 & -287.809 & +1036.628 \\ +2535.224 & -548.083 & +176.069 & +111.181 & +413.265 & +135.193 & +45.490 & -502.411 \\ -8475.398 & -109.707 & +458.541 & -1018.272 & +135.193 & +5613.026 & +968.003 & -4270.737 \\ -2406.368 & +386.466 & +212.284 & -287.809 & +45.490 & +968.003 & +632.432 & -1645.725 \\ +12977.552 & -807.216 & -859.022 & +1036.628 & -502.411 & -4270.737 & -1645.725 & +19362.944 \end{pmatrix}$$

$$\exp(SPD)eigs = ((+56168.17045, +0.00000) \quad (+14880.07985, +0.00000) \quad (+4311.77579, +0.00000) \quad (+1924.25027, +0.00000) \quad (+6)$$

$$\log(\exp(SPD)eigs) = ((+10.93611, +0.00000) \quad (+9.60778, +0.00000) \quad (+8.36911, +0.00000) \quad (+7.56229, +0.00000) \quad (+4.23666, +0.00000))$$

$$\exp(Id) = \begin{pmatrix} +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 \end{pmatrix}$$

$$\exp(Id)eigs = ((+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000))$$

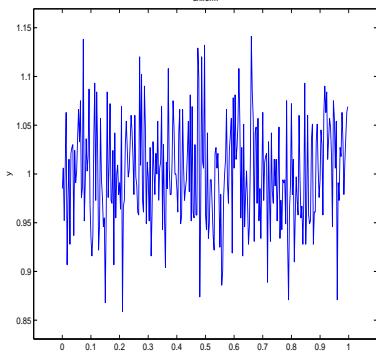
$$\log(\exp(Id)eigs) = ((+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000))$$

For $n \in \mathbb{Z}[16, 128]$ we calculate $\|(SPD(n)Eigs - \log(\exp(SPD(n))eigs)\|_{l^2}$

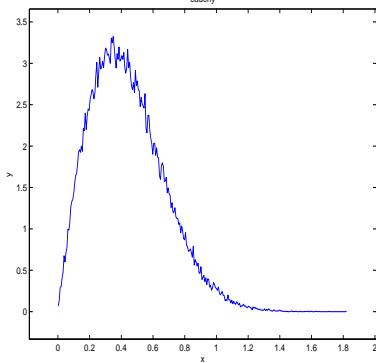
$|(\text{SPD}(n)\text{Eigs} - \log(\exp(\text{SPD}(n))\text{eigs}))|_{l^2} = ((+5.36543, +0.00000) \quad (+5.36543, +0.00000) \quad (+5.36543, +0.00000) \quad (+5.36543, +0.00000)$

QueryPerformanceCounter = +0.04407 The sample size generated for this run is 100000.

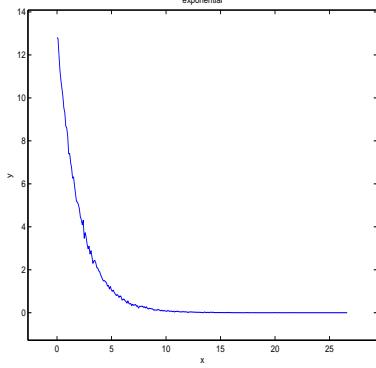
	mean	variance	skewness	kurtosis
uniform	$\mu_1 = +0.50030$	$\mu_2 = +0.08353$	$\mu_3 = +0.00339$	$\mu_4 = +1.80113$



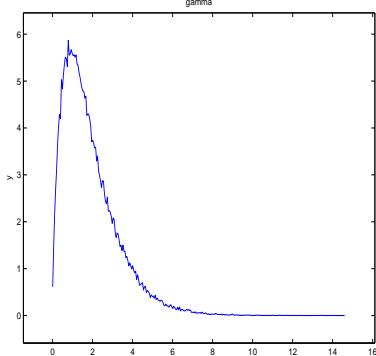
	mean	variance	skewness	kurtosis
cauchy	$\mu_1 = +0.44288$	$\mu_2 = +0.05341$	$\mu_3 = +0.63935$	$\mu_4 = +3.28094$



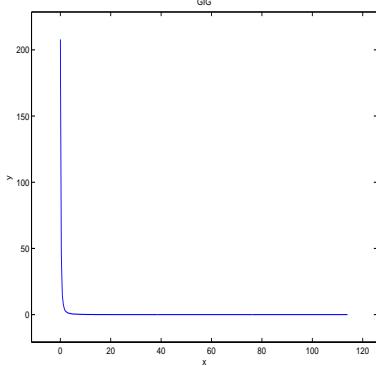
	mean	variance	skewness	kurtosis
exponential	$\mu_1 = +1.99647$	$\mu_2 = +3.99339$	$\mu_3 = +2.03097$	$\mu_4 = +9.30842$



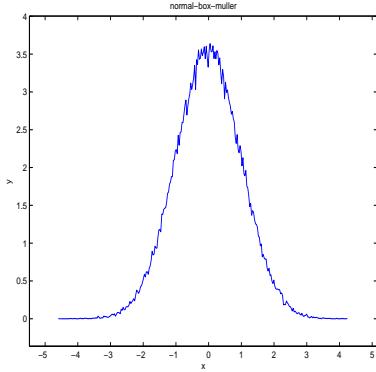
	mean	variance	skewness	kurtosis
gamma	$\mu_1 = +1.89991$	$\mu_2 = +1.89577$	$\mu_3 = +1.45616$	$\mu_4 = +6.28140$



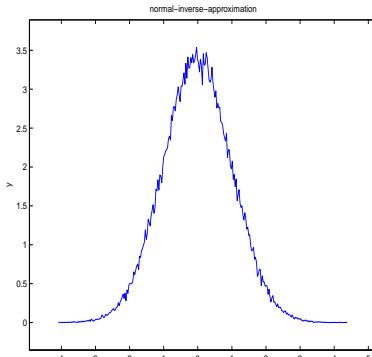
	mean	variance	skewness	kurtosis
GIG	$\mu_1 = +0.82571$	$\mu_2 = +12.30331$	$\mu_3 = +14.83339$	$\mu_4 = +291.47640$



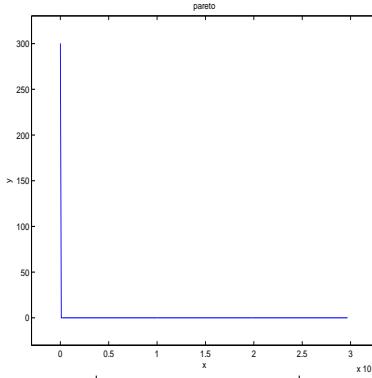
	mean	variance	skewness	kurtosis
normal-box-muller	$\mu_1 = +0.00262$	$\mu_2 = +0.99471$	$\mu_3 = -0.00751$	$\mu_4 = +2.97905$



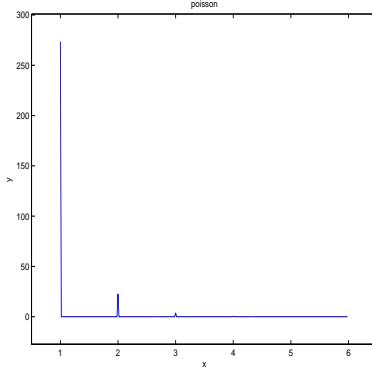
normal-inverse-approximation	mean	variance	skewness	kurtosis
	$\mu_1 = +0.00230$	$\mu_2 = +1.00486$	$\mu_3 = +0.01163$	$\mu_4 = +2.99254$



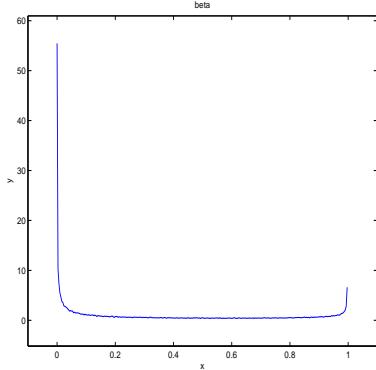
pareto	mean	variance	skewness	kurtosis
	$\mu_1 = +3184578.26493$	$\mu_2 = +888468246174112900.00000$	$\mu_3 = +315.36997$	$\mu_4 = +99629.09819$



poisson	mean	variance	skewness	kurtosis
	$\mu_1 = +1.10472$	$\mu_2 = +0.12949$	$\mu_3 = +3.95698$	$\mu_4 = +21.59641$



beta	mean	variance	skewness	kurtosis
	$\mu_1 = +0.33160$	$\mu_2 = +0.12686$	$\mu_3 = +0.68923$	$\mu_4 = +1.91928$



QueryPerformanceCounter = +21.38910

0.0.19 Multiclass Support Vector Machine

- Number of training points = 1024
- Feature dimension = 3
- Number of classes = 3

The mean vectors of the 3 classes

$$\mu_1 = \begin{pmatrix} +1.90000 & +0.10000 & +0.10000 \end{pmatrix}$$

$$\mu_2 = \begin{pmatrix} +0.10000 & +1.90000 & +0.10000 \end{pmatrix}$$

$$\mu_3 = \begin{pmatrix} +0.00000 & +0.00000 & +1.90000 \end{pmatrix}$$

A random SPD covariance matrix is generated for each of the classes.

$$\rho_1 = \begin{pmatrix} +1.767 & -0.184 & -0.435 \\ -0.184 & +3.573 & +0.289 \\ -0.435 & +0.289 & +2.231 \end{pmatrix}$$

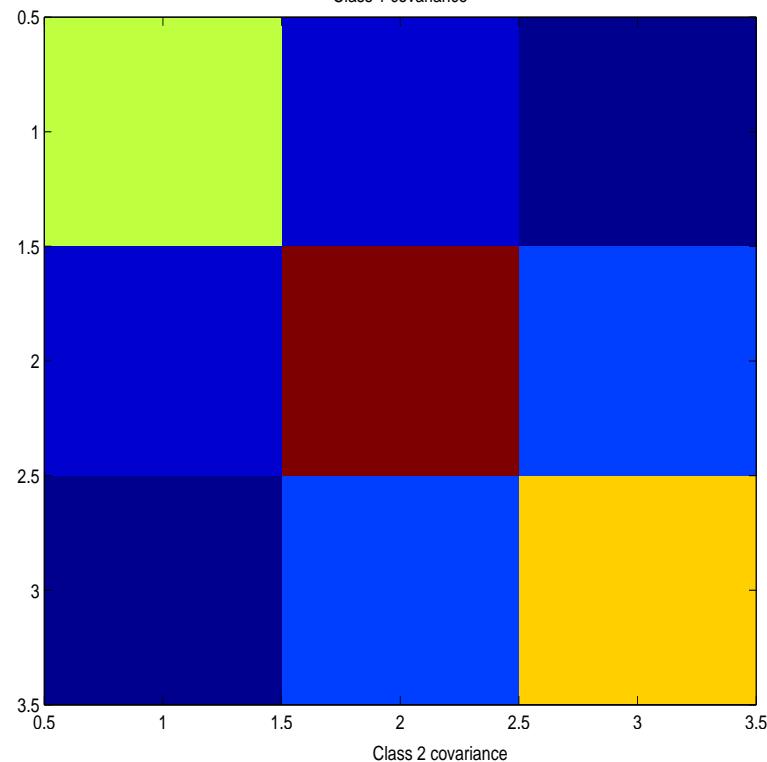
$$\rho_2 = \begin{pmatrix} +3.230 & +0.087 & -0.015 \\ +0.087 & +2.316 & -0.377 \\ -0.015 & -0.377 & +2.230 \end{pmatrix}$$

$$\rho_3 = \begin{pmatrix} +2.357 & +0.205 & -0.491 \\ +0.205 & +1.804 & -0.395 \\ -0.491 & -0.395 & +1.764 \end{pmatrix}$$

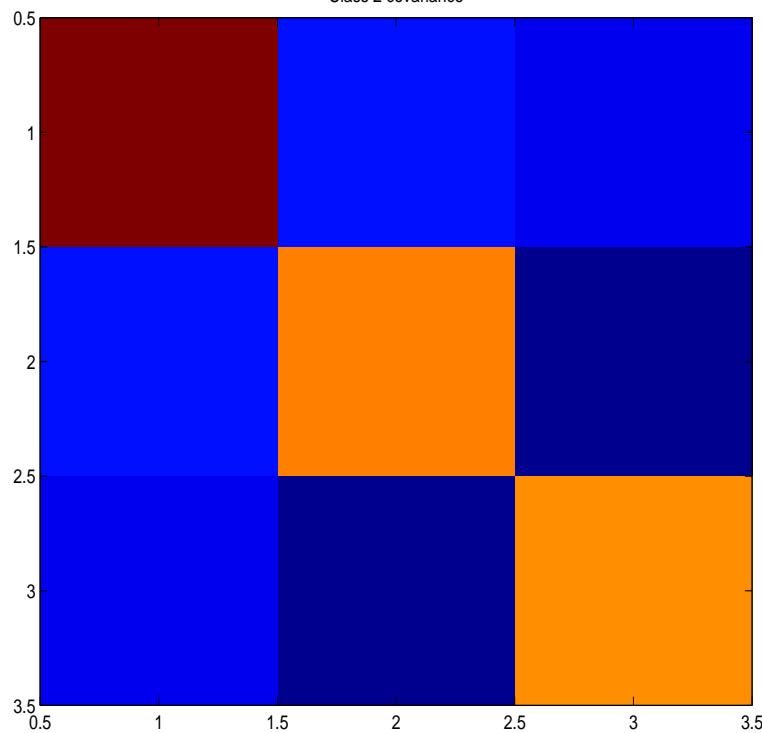
Verify L_1 condition number of covariance. The diagonal entries of the matrix have the form $(0.5 + U(0, 1)) * \dim(\text{Dom}(\text{Cov}))$. The lower-diagonal entries take the form $U(0, 1) - 0.5$. The L_1 condition numbers are :

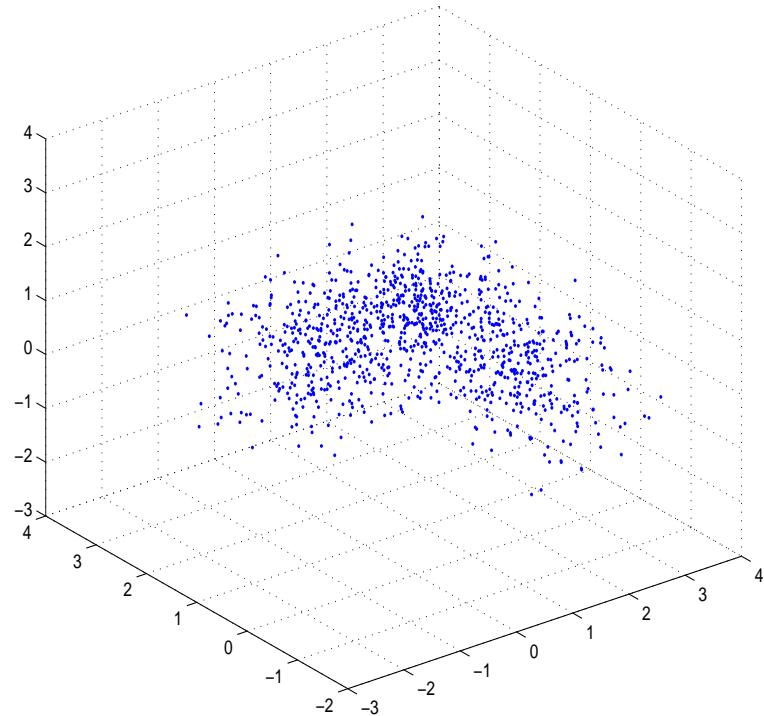
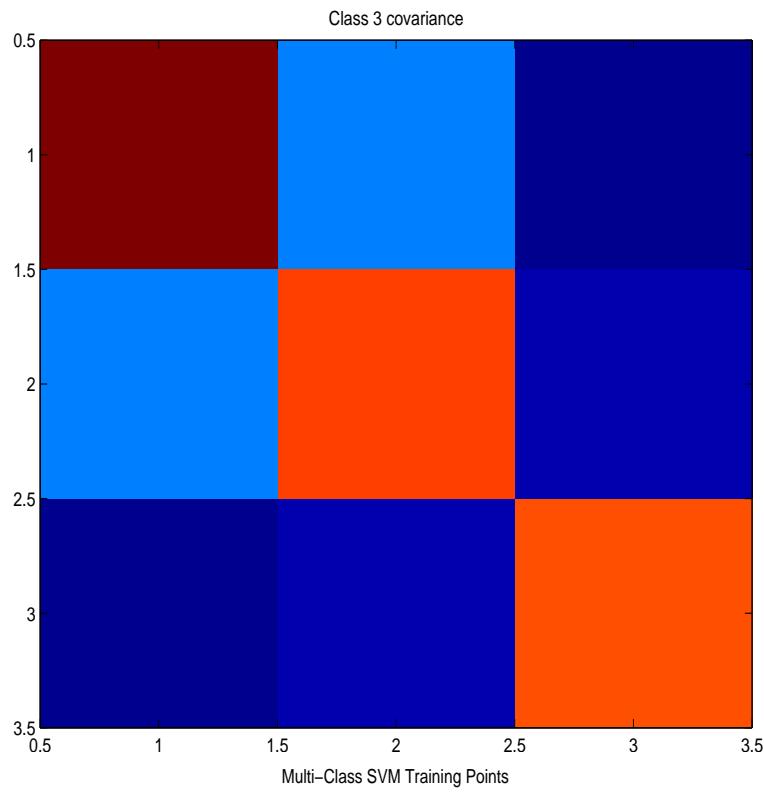
- +2.958
- +1.786
- +2.663

Class 1 covariance



Class 2 covariance

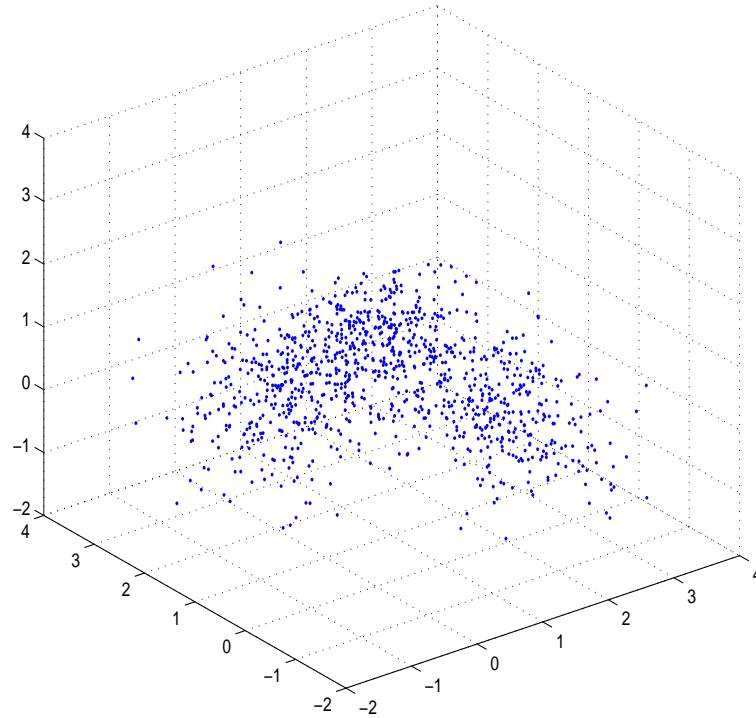




These are the SVM parameters - the RBF kernel is used

- allOutlierFraction=0.05
- mixingCoeff=0.3
- smoThresh=1.0/10000.0
- sigma=1

Multi-Class SVM Test Points



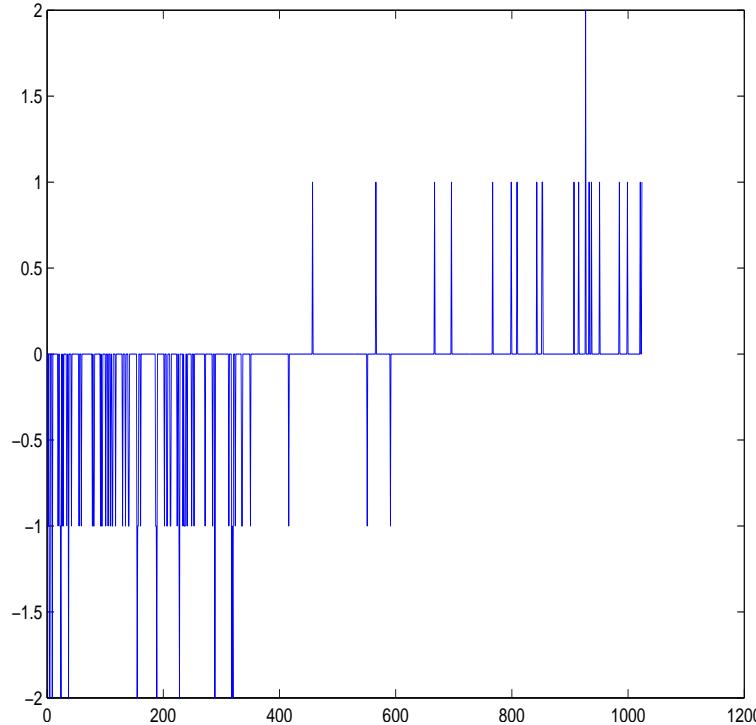
The marginal sample moments (mean var skew kurtosis) for training points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.668	+1.259	+0.115	+2.162
1	+0.699	+1.208	+0.309	+2.265
2	+0.695	+1.124	+0.327	+2.256

The marginal sample moments (mean var skew kurtosis) for test points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.676	+1.165	+0.258	+2.165
1	+0.665	+1.209	+0.345	+2.269
2	+0.714	+1.084	+0.313	+2.185

Class Differences for Test Points



The error rate for this run is +0.085

QueryPerformanceCounter = +6.495

0.0.20 Semidefinite Programming SDPA

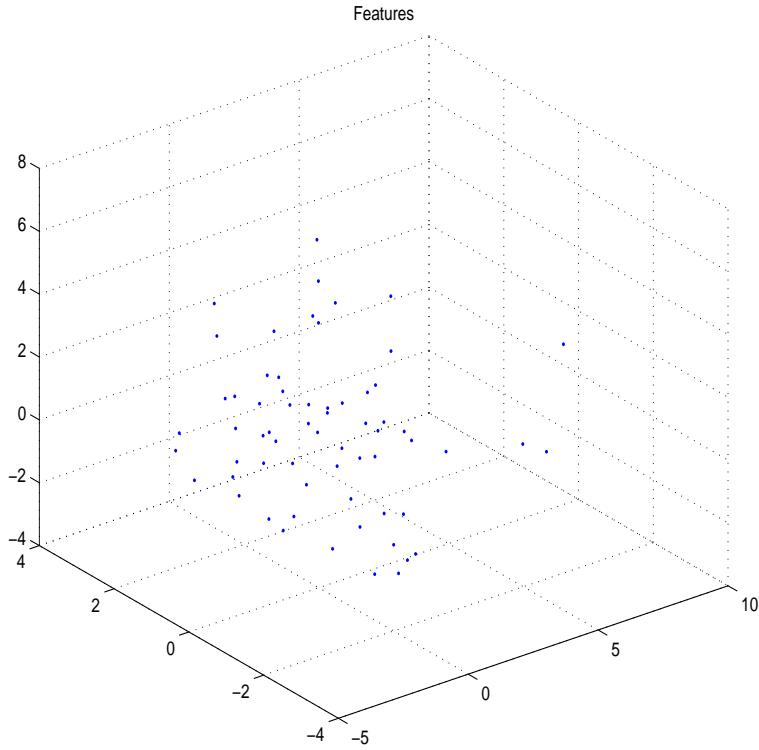
QueryPerformanceCounter = +0.038

0.0.21 Linear Regression 3x1

0.0.22 3 x 1 Linear Regression

Sample size = 64

$$\sigma = \begin{pmatrix} +3.952 & -0.499 & -0.010 \\ -0.499 & +1.895 & +0.465 \\ -0.010 & +0.465 & +4.477 \end{pmatrix}$$



Beta +0.817, +0.999, +0.510

Error: -0.000, -0.000, -0.000

QueryPerformanceCounter = +1.063

0.0.23 Matrix Norms

0.0.24 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: +12

$$A = \begin{pmatrix} +0.007 & -0.301 & -0.301 & +0.094 & -0.058 & +0.403 & -0.158 & -0.082 & +0.416 & +0.279 & +0.491 & +0.346 \\ +0.782 & +0.199 & -0.018 & -0.272 & +0.155 & -0.224 & +0.060 & -0.178 & +0.030 & -0.098 & +0.392 & -0.026 \\ -0.304 & -0.029 & -0.036 & -0.144 & +0.833 & -0.017 & +0.165 & -0.328 & -0.132 & +0.078 & +0.047 & +0.172 \\ -0.038 & +0.100 & -0.176 & +0.132 & -0.222 & -0.570 & -0.280 & -0.262 & -0.274 & +0.379 & -0.060 & +0.444 \\ +0.141 & +0.308 & +0.272 & +0.602 & +0.236 & +0.248 & -0.198 & +0.276 & -0.369 & +0.111 & +0.255 & +0.058 \\ -0.052 & +0.357 & +0.012 & -0.561 & +0.109 & +0.076 & -0.218 & +0.579 & +0.090 & +0.235 & -0.099 & +0.285 \\ -0.179 & +0.059 & +0.073 & -0.121 & -0.225 & +0.089 & +0.162 & -0.005 & -0.266 & -0.643 & +0.258 & +0.556 \\ +0.258 & +0.381 & -0.397 & +0.267 & +0.068 & +0.263 & +0.195 & -0.132 & +0.212 & -0.128 & -0.537 & +0.283 \\ +0.340 & -0.516 & +0.547 & +0.025 & +0.112 & +0.006 & -0.055 & +0.038 & +0.094 & +0.022 & -0.366 & +0.397 \\ +0.155 & -0.362 & -0.400 & +0.069 & +0.039 & -0.125 & +0.536 & +0.484 & -0.333 & +0.164 & +0.026 & +0.057 \\ -0.119 & +0.299 & +0.424 & +0.040 & -0.207 & -0.034 & +0.652 & -0.101 & +0.231 & +0.390 & +0.137 & +0.112 \\ +0.137 & -0.033 & +0.012 & -0.332 & -0.222 & +0.549 & +0.017 & -0.331 & -0.548 & +0.294 & -0.129 & -0.074 \end{pmatrix}$$

$$Det(A) : A \in O(n) = (+1.000, +0.000)$$

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.435 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.198 & +0.667 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.181 & -0.452 & -0.685 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.389 & -0.080 & -0.001 & -0.324 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.176 & +0.113 & +0.062 & -0.411 & -0.171 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.152 & -0.546 & -0.944 & +0.140 & -0.217 & -0.280 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.066 & -0.615 & -0.459 & -0.652 & +0.285 & +0.315 & +0.007 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.009 & +0.503 & +0.756 & +0.005 & -0.068 & +0.683 & -0.490 & -0.241 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.049 & -0.183 & +0.099 & +0.194 & -0.254 & -0.880 & -0.204 & -0.378 & -0.957 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.330 & -0.524 & +0.131 & +0.583 & -0.084 & +0.403 & -0.036 & -0.315 & +0.876 & -0.398 & +1.000 & +0.000 & +0.000 \\ -0.229 & -0.173 & -0.215 & -0.208 & -0.147 & +0.100 & +0.283 & +0.101 & -0.304 & -0.596 & -0.509 & +1.000 & +0.000 \end{pmatrix}$$

$$U = \begin{pmatrix} +0.782 & +0.199 & -0.018 & -0.272 & +0.155 & -0.224 & +0.060 & -0.178 & +0.030 & -0.098 & +0.392 & -0.026 \\ +0.000 & -0.602 & +0.555 & +0.143 & +0.044 & +0.104 & -0.081 & +0.116 & +0.081 & +0.064 & -0.536 & +0.409 \\ +0.000 & +0.000 & -0.767 & +0.027 & -0.022 & -0.150 & +0.578 & +0.442 & -0.393 & +0.140 & +0.306 & -0.210 \\ +0.000 & +0.000 & +0.000 & +0.734 & +0.213 & +0.232 & +0.151 & +0.664 & -0.607 & +0.254 & +0.152 & +0.103 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.966 & -0.021 & +0.231 & -0.172 & -0.311 & +0.127 & +0.206 & +0.228 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.678 & +0.082 & -0.097 & -0.841 & +0.421 & -0.058 & -0.021 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.214 & +0.195 & -0.310 & +0.653 & +0.200 & +0.161 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.353 & -0.078 & +0.324 & -0.206 & +0.446 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.058 & +0.260 & +0.627 & +0.516 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.230 & +0.366 & +1.253 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.496 & +0.694 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.799 \end{pmatrix}$$

$$L * U = \begin{pmatrix} +0.782 & +0.199 & -0.018 & -0.272 & +0.155 & -0.224 & +0.060 & -0.178 & +0.030 & -0.098 & +0.392 & -0.026 \\ +0.340 & -0.516 & +0.547 & +0.025 & +0.112 & +0.006 & -0.055 & +0.038 & +0.094 & +0.022 & -0.366 & +0.397 \\ +0.155 & -0.362 & -0.400 & +0.069 & +0.039 & -0.125 & +0.536 & +0.484 & -0.333 & +0.164 & +0.026 & +0.057 \\ +0.141 & +0.308 & +0.272 & +0.602 & +0.236 & +0.248 & -0.198 & +0.276 & -0.369 & +0.111 & +0.255 & +0.058 \\ -0.304 & -0.029 & -0.036 & -0.144 & +0.833 & -0.017 & +0.165 & -0.328 & -0.132 & +0.078 & +0.047 & +0.172 \\ +0.137 & -0.033 & +0.012 & -0.332 & -0.222 & +0.549 & +0.017 & -0.331 & -0.548 & +0.294 & -0.129 & -0.074 \\ -0.119 & +0.299 & +0.424 & +0.040 & -0.207 & -0.034 & +0.652 & -0.101 & +0.231 & +0.390 & +0.137 & +0.112 \\ -0.052 & +0.357 & +0.012 & -0.561 & +0.109 & +0.076 & -0.218 & +0.579 & +0.090 & +0.235 & -0.099 & +0.285 \\ +0.007 & -0.301 & -0.301 & +0.094 & -0.058 & +0.403 & -0.158 & -0.082 & +0.416 & +0.279 & +0.491 & +0.346 \\ -0.038 & +0.100 & -0.176 & +0.132 & -0.222 & -0.570 & -0.280 & -0.262 & -0.274 & +0.379 & -0.060 & +0.444 \\ +0.258 & +0.381 & -0.397 & +0.267 & +0.068 & +0.263 & +0.195 & -0.132 & +0.212 & -0.128 & -0.537 & +0.283 \\ -0.179 & +0.059 & +0.073 & -0.121 & -0.225 & +0.089 & +0.162 & -0.005 & -0.266 & -0.643 & +0.258 & +0.556 \end{pmatrix}$$

$$\text{Det}(L) := (+1.000, +0.000) \text{Det}(U) := (-1.000, +0.000) \text{Det}(LU) := (-1.000, -0.000)$$

$$\|A\|_{L_1} = +2.995$$

$$\|A\|_{L_\infty} = +3.122$$

$$\|A^{-1}\|_{L_1} = +3.122$$

$$\|A^{-1}\|_{L_\infty} = +2.995$$

$$\|A\|_{L_\infty} * \|A^{-1}\|_{L_\infty} = +9.350$$

$$\|A\|_{L_1} * \|A^{-1}\|_{L_1} = +9.350$$

$$\text{Frobenius Norm } \|A\|_F \text{ via } \sum_{i,j=0}^n |A_{i,j}| \text{ of } A \in O(n) + 3.464$$

$$L_1 \text{ condition number of Haar Distributed Random Orthogonal Matrix } A \in O(n) + 8.023$$

$$A = \begin{pmatrix} +0.007 & -0.301 & -0.301 & +0.094 & -0.058 & +0.403 & -0.158 & -0.082 & +0.416 & +0.279 & +0.491 & +0.346 \\ +0.782 & +0.199 & -0.018 & -0.272 & +0.155 & -0.224 & +0.060 & -0.178 & +0.030 & -0.098 & +0.392 & -0.026 \\ -0.304 & -0.029 & -0.036 & -0.144 & +0.833 & -0.017 & +0.165 & -0.328 & -0.132 & +0.078 & +0.047 & +0.172 \\ -0.038 & +0.100 & -0.176 & +0.132 & -0.222 & -0.570 & -0.280 & -0.262 & -0.274 & +0.379 & -0.060 & +0.444 \\ +0.141 & +0.308 & +0.272 & +0.602 & +0.236 & +0.248 & -0.198 & +0.276 & -0.369 & +0.111 & +0.255 & +0.058 \\ -0.052 & +0.357 & +0.012 & -0.561 & +0.109 & +0.076 & -0.218 & +0.579 & +0.090 & +0.235 & -0.099 & +0.285 \\ -0.179 & +0.059 & +0.073 & -0.121 & -0.225 & +0.089 & +0.162 & -0.005 & -0.266 & -0.643 & +0.258 & +0.556 \\ +0.258 & +0.381 & -0.397 & +0.267 & +0.068 & +0.263 & +0.195 & -0.132 & +0.212 & -0.128 & -0.537 & +0.283 \\ +0.340 & -0.516 & +0.547 & +0.025 & +0.112 & +0.006 & -0.055 & +0.038 & +0.094 & +0.022 & -0.366 & +0.397 \\ +0.155 & -0.362 & -0.400 & +0.069 & +0.039 & -0.125 & +0.536 & +0.484 & -0.333 & +0.164 & +0.026 & +0.057 \\ -0.119 & +0.299 & +0.424 & +0.040 & -0.207 & -0.034 & +0.652 & -0.101 & +0.231 & +0.390 & +0.137 & +0.112 \\ +0.137 & -0.033 & +0.012 & -0.332 & -0.222 & +0.549 & +0.017 & -0.331 & -0.548 & +0.294 & -0.129 & -0.074 \end{pmatrix}$$

$$L_\infty \text{ condition number of Haar Distributed Random Orthogonal Matrix } A \in O(n) + 7.760$$

$$\text{Eigenvalues of } A \in O(n)$$

$$(-1.000, +0.029), (-1.000, -0.029), (-0.602, +0.799), (-0.602, -0.799), (-0.072, +0.997), (-0.072, -0.997), (+0.415, +0.910), (+0.415, -0.910), (+0.743, +0.670), (+0.743, -0.670), (+0.998, +0.068), (+0.998, -0.068)$$

$|\lambda| : \lambda \in \sigma(A), A \in O(n)$

+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 \\ -0.000 & +1.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 \\ -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +1.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} +0.007 & +0.782 & -0.304 & -0.038 & +0.141 & -0.052 & -0.179 & +0.258 & +0.340 & +0.155 & -0.119 & +0.137 \\ -0.301 & +0.199 & -0.029 & +0.100 & +0.308 & +0.357 & +0.059 & +0.381 & -0.516 & -0.362 & +0.299 & -0.033 \\ -0.301 & -0.018 & -0.036 & -0.176 & +0.272 & +0.012 & +0.073 & -0.397 & +0.547 & -0.400 & +0.424 & +0.012 \\ +0.094 & -0.272 & -0.144 & +0.132 & +0.602 & -0.561 & -0.121 & +0.267 & +0.025 & +0.069 & +0.040 & -0.332 \\ -0.058 & +0.155 & +0.833 & -0.222 & +0.236 & +0.109 & -0.225 & +0.068 & +0.112 & +0.039 & -0.207 & -0.222 \\ +0.403 & -0.224 & -0.017 & -0.570 & +0.248 & +0.076 & +0.089 & +0.263 & +0.006 & -0.125 & -0.034 & +0.549 \\ -0.158 & +0.060 & +0.165 & -0.280 & -0.198 & -0.218 & +0.162 & +0.195 & -0.055 & +0.536 & +0.652 & +0.017 \\ -0.082 & -0.178 & -0.328 & -0.262 & +0.276 & +0.579 & -0.005 & -0.132 & +0.038 & +0.484 & -0.101 & -0.331 \\ +0.416 & +0.030 & -0.132 & -0.274 & -0.369 & +0.090 & -0.266 & +0.212 & +0.094 & -0.333 & +0.231 & -0.548 \\ +0.279 & -0.098 & +0.078 & +0.379 & +0.111 & +0.235 & -0.643 & -0.128 & +0.022 & +0.164 & +0.390 & +0.294 \\ +0.491 & +0.392 & +0.047 & -0.060 & +0.255 & -0.099 & +0.258 & -0.537 & -0.366 & +0.026 & +0.137 & -0.129 \\ +0.346 & -0.026 & +0.172 & +0.444 & +0.058 & +0.285 & +0.556 & +0.283 & +0.397 & +0.057 & +0.112 & -0.074 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & +1.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & +1.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 & -0.000 & +0.000 & -0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +1.000 & +0.000 & -0.000 & -0.000 \\ +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +1.000 & +0.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} +0.049 & +0.445 & -0.337 & -0.171 & +0.043 & -0.375 & +0.179 & +0.053 & +0.189 & -0.104 & -0.084 & +0.653 \\ -0.007 & +0.301 & +0.301 & -0.094 & +0.058 & -0.403 & +0.158 & +0.082 & -0.416 & -0.279 & -0.491 & -0.346 \\ +0.252 & +0.333 & +0.609 & -0.015 & -0.065 & +0.150 & -0.484 & +0.327 & +0.093 & -0.052 & +0.126 & +0.241 \\ +0.131 & +0.024 & +0.372 & -0.253 & -0.548 & -0.121 & +0.290 & -0.576 & +0.062 & +0.058 & +0.199 & +0.050 \\ +0.192 & -0.176 & -0.383 & -0.357 & -0.479 & -0.152 & -0.399 & +0.188 & +0.063 & -0.394 & +0.024 & -0.225 \\ +0.243 & -0.487 & +0.165 & -0.145 & +0.236 & +0.221 & +0.243 & +0.004 & -0.206 & -0.556 & -0.021 & +0.374 \\ +0.202 & -0.281 & +0.035 & +0.508 & +0.060 & -0.475 & -0.430 & -0.384 & +0.001 & -0.019 & -0.170 & +0.175 \\ +0.380 & -0.095 & +0.004 & -0.568 & +0.400 & +0.028 & -0.159 & -0.194 & +0.199 & +0.380 & -0.326 & -0.101 \\ -0.227 & -0.437 & +0.277 & -0.008 & -0.215 & -0.252 & +0.205 & +0.437 & +0.444 & +0.179 & -0.316 & +0.093 \\ -0.546 & +0.069 & +0.013 & -0.110 & -0.141 & +0.384 & -0.298 & -0.326 & +0.063 & -0.177 & -0.497 & +0.199 \\ +0.107 & +0.189 & +0.051 & +0.199 & +0.207 & +0.070 & +0.146 & -0.161 & +0.700 & -0.462 & -0.000 & -0.335 \\ -0.523 & -0.108 & +0.179 & -0.342 & +0.365 & -0.382 & -0.210 & -0.083 & +0.040 & -0.138 & +0.462 & -0.037 \end{pmatrix}$$

$$\begin{aligned}
S = & \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ -0.000 & -1.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 \\ +0.237 & -0.000 & +0.176 & +0.304 & +0.177 & +0.161 & +0.053 & +0.358 & -0.440 & -0.632 & +0.195 & +0.000 & +0.000 \\ +0.139 & -0.000 & -0.319 & -0.221 & -0.588 & +0.235 & +0.006 & +0.309 & -0.255 & +0.102 & -0.006 & +0.507 & +0.000 \\ +0.423 & -0.000 & -0.052 & +0.174 & -0.018 & -0.341 & +0.584 & -0.045 & -0.059 & +0.026 & -0.568 & +0.042 & +0.000 \\ -0.219 & +0.000 & +0.509 & -0.364 & -0.439 & -0.064 & +0.097 & -0.357 & -0.348 & -0.246 & -0.129 & -0.169 & +0.000 \\ +0.404 & +0.000 & +0.473 & -0.305 & +0.102 & -0.152 & -0.491 & +0.344 & +0.169 & +0.074 & -0.286 & +0.106 & +0.000 \\ +0.357 & +0.000 & +0.144 & +0.224 & +0.117 & +0.578 & -0.169 & -0.555 & -0.170 & +0.215 & -0.120 & +0.169 & +0.000 \\ +0.486 & +0.000 & -0.129 & -0.196 & -0.149 & +0.039 & +0.056 & +0.078 & -0.168 & +0.287 & +0.328 & -0.677 & +0.000 \\ -0.101 & +0.000 & +0.337 & +0.084 & -0.189 & +0.560 & +0.389 & +0.318 & +0.477 & +0.036 & -0.097 & -0.169 & +0.000 \\ +0.094 & +0.000 & -0.471 & -0.287 & +0.002 & +0.250 & -0.199 & -0.133 & +0.235 & -0.545 & -0.387 & -0.254 & +0.000 \\ +0.157 & +0.000 & +0.032 & +0.573 & -0.588 & -0.234 & -0.321 & -0.125 & +0.296 & -0.158 & +0.083 & -0.085 & +0.000 \\ -0.357 & +0.000 & -0.084 & +0.310 & -0.037 & +0.096 & -0.282 & +0.284 & -0.400 & +0.267 & -0.504 & -0.338 & +0.000 \\ -0.360 & -0.049 & +0.114 & +0.508 & +0.225 & +0.373 & +0.020 & +0.094 & -0.438 & -0.054 & -0.030 & -0.450 & +0.000 \\ +0.057 & +0.007 & -0.183 & -0.214 & +0.197 & +0.082 & +0.293 & -0.167 & -0.604 & -0.026 & +0.508 & +0.367 & +0.000 \\ +0.138 & -0.252 & -0.169 & -0.060 & -0.461 & -0.052 & -0.029 & +0.767 & -0.253 & -0.056 & +0.100 & -0.063 & +0.000 \\ -0.143 & -0.131 & -0.325 & +0.413 & +0.008 & +0.383 & -0.284 & +0.085 & +0.210 & -0.008 & -0.010 & +0.638 & +0.000 \\ -0.213 & -0.192 & -0.060 & +0.126 & +0.302 & -0.257 & +0.044 & +0.195 & +0.408 & +0.291 & +0.648 & -0.176 & +0.000 \\ -0.196 & -0.243 & +0.289 & -0.063 & -0.198 & -0.112 & -0.304 & -0.143 & -0.290 & +0.731 & -0.090 & +0.150 & +0.000 \\ -0.483 & -0.202 & -0.305 & +0.059 & -0.054 & -0.353 & +0.582 & -0.015 & -0.003 & +0.026 & -0.384 & +0.126 & +0.000 \\ -0.489 & -0.380 & +0.117 & -0.591 & -0.013 & +0.323 & -0.160 & -0.043 & +0.128 & -0.316 & +0.057 & -0.051 & +0.000 \\ +0.052 & +0.227 & -0.373 & -0.247 & -0.113 & +0.562 & +0.279 & +0.023 & +0.169 & +0.501 & -0.049 & -0.236 & +0.000 \\ -0.279 & +0.546 & +0.196 & -0.236 & +0.393 & -0.056 & -0.067 & +0.518 & -0.046 & +0.091 & -0.167 & +0.246 & +0.000 \\ +0.098 & -0.107 & +0.669 & +0.136 & -0.163 & +0.273 & +0.537 & +0.121 & +0.193 & +0.018 & +0.086 & +0.252 & +0.000 \\ -0.426 & +0.523 & +0.024 & +0.117 & -0.608 & -0.072 & -0.046 & -0.157 & +0.018 & -0.116 & +0.339 & -0.003 & +0.000 \end{pmatrix} \\
V = & \begin{pmatrix} -0.000 & -1.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 \\ +0.237 & -0.000 & +0.176 & +0.304 & +0.177 & +0.161 & +0.053 & +0.358 & -0.440 & -0.632 & +0.195 & +0.000 & +0.000 \\ +0.139 & -0.000 & -0.319 & -0.221 & -0.588 & +0.235 & +0.006 & +0.309 & -0.255 & +0.102 & -0.006 & +0.507 & +0.000 \\ +0.423 & -0.000 & -0.052 & +0.174 & -0.018 & -0.341 & +0.584 & -0.045 & -0.059 & +0.026 & -0.568 & +0.042 & +0.000 \\ -0.219 & +0.000 & +0.509 & -0.364 & -0.439 & -0.064 & +0.097 & -0.357 & -0.348 & -0.246 & -0.129 & -0.169 & +0.000 \\ +0.404 & +0.000 & +0.473 & -0.305 & +0.102 & -0.152 & -0.491 & +0.344 & +0.169 & +0.074 & -0.286 & +0.106 & +0.000 \\ +0.357 & +0.000 & +0.144 & +0.224 & +0.117 & +0.578 & -0.169 & -0.555 & -0.170 & +0.215 & -0.120 & +0.169 & +0.000 \\ +0.486 & +0.000 & -0.129 & -0.196 & -0.149 & +0.039 & +0.056 & +0.078 & -0.168 & +0.287 & +0.328 & -0.677 & +0.000 \\ -0.101 & +0.000 & +0.337 & +0.084 & -0.189 & +0.560 & +0.389 & +0.318 & +0.477 & +0.036 & -0.097 & -0.169 & +0.000 \\ +0.094 & +0.000 & -0.471 & -0.287 & +0.002 & +0.250 & -0.199 & -0.133 & +0.235 & -0.545 & -0.387 & -0.254 & +0.000 \\ +0.157 & +0.000 & +0.032 & +0.573 & -0.588 & -0.234 & -0.321 & -0.125 & +0.296 & -0.158 & +0.083 & -0.085 & +0.000 \\ -0.357 & +0.000 & -0.084 & +0.310 & -0.037 & +0.096 & -0.282 & +0.284 & -0.400 & +0.267 & -0.504 & -0.338 & +0.000 \end{pmatrix} \\
USV = & \begin{pmatrix} -0.360 & -0.049 & +0.114 & +0.508 & +0.225 & +0.373 & +0.020 & +0.094 & -0.438 & -0.054 & -0.030 & -0.450 & +0.000 \\ +0.057 & +0.007 & -0.183 & -0.214 & +0.197 & +0.082 & +0.293 & -0.167 & -0.604 & -0.026 & +0.508 & +0.367 & +0.000 \\ +0.138 & -0.252 & -0.169 & -0.060 & -0.461 & -0.052 & -0.029 & +0.767 & -0.253 & -0.056 & +0.100 & -0.063 & +0.000 \\ -0.143 & -0.131 & -0.325 & +0.413 & +0.008 & +0.383 & -0.284 & +0.085 & +0.210 & -0.008 & -0.010 & +0.638 & +0.000 \\ -0.213 & -0.192 & -0.060 & +0.126 & +0.302 & -0.257 & +0.044 & +0.195 & +0.408 & +0.291 & +0.648 & -0.176 & +0.000 \\ -0.196 & -0.243 & +0.289 & -0.063 & -0.198 & -0.112 & -0.304 & -0.143 & -0.290 & +0.731 & -0.090 & +0.150 & +0.000 \\ -0.483 & -0.202 & -0.305 & +0.059 & -0.054 & -0.353 & +0.582 & -0.015 & -0.003 & +0.026 & -0.384 & +0.126 & +0.000 \\ -0.489 & -0.380 & +0.117 & -0.591 & -0.013 & +0.323 & -0.160 & -0.043 & +0.128 & -0.316 & +0.057 & -0.051 & +0.000 \\ +0.052 & +0.227 & -0.373 & -0.247 & -0.113 & +0.562 & +0.279 & +0.023 & +0.169 & +0.501 & -0.049 & -0.236 & +0.000 \\ -0.279 & +0.546 & +0.196 & -0.236 & +0.393 & -0.056 & -0.067 & +0.518 & -0.046 & +0.091 & -0.167 & +0.246 & +0.000 \\ +0.098 & -0.107 & +0.669 & +0.136 & -0.163 & +0.273 & +0.537 & +0.121 & +0.193 & +0.018 & +0.086 & +0.252 & +0.000 \\ -0.426 & +0.523 & +0.024 & +0.117 & -0.608 & -0.072 & -0.046 & -0.157 & +0.018 & -0.116 & +0.339 & -0.003 & +0.000 \end{pmatrix} \end{aligned}$$

0.0.25 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +56267.800 L_{inf} condition number of Wishart Matrix +56267.800

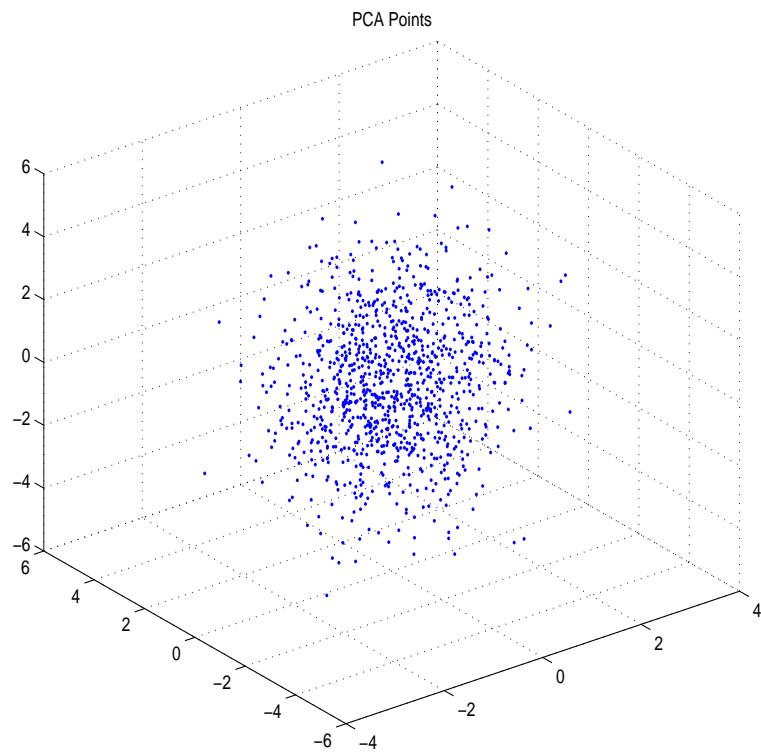
0.0.26 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +470.231 L_∞ condition number of GOE Matrix +470.231

0.0.27 The Identity Matrix $I \in M(n)$

L_1 condition number of $I = +1.000$ L_∞ condition number of $I = +1.000$ QueryPerformanceCounter = +1.828

0.0.28 Principal Components Matlab



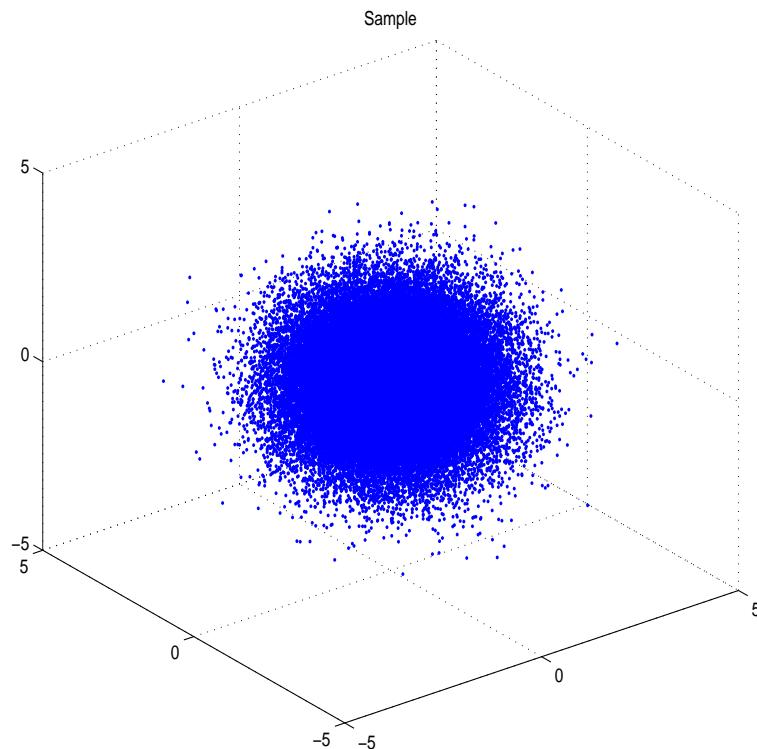
The eigenvectors: +0.132, +0.207, +0.969 +0.166, +0.959, -0.228 -0.977, +0.191, +0.092

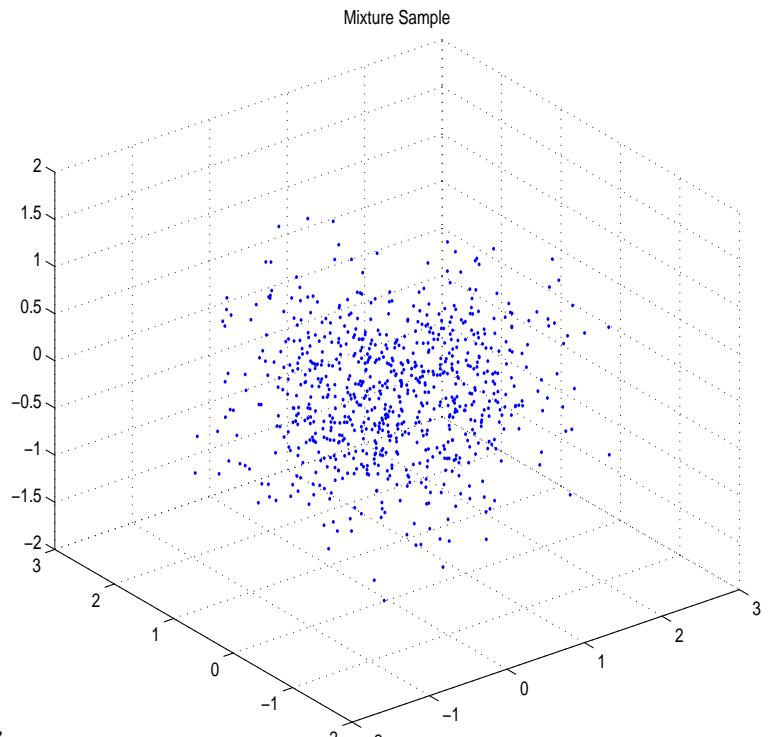
All of the eigenvalues of the covariance matrix: (+0.958,+0.000), (+2.025,+0.000), (+3.017,+0.000)

QueryPerformanceCounter = +1.102

0.0.29 Multi Variate Random Number Generator

Sample from $N(\mu, \Sigma)$ mean= -0.002, variance= +1.004, skewness= +0.006, kurtosis= +3.003 mean= -0.001, variance= +1.017, skewness= -0.005, kurtosis= +2.988 mean= -0.002, variance= +1.006, skewness= -0.016, kurtosis= +3.014 Covariance Matrix +1.004, +0.009, +0.003 +0.009, +1.017, -0.003 +0.003, -0.003, +1.006





Generate a sample from a uniform mixture of three Gaussians in R^3

QueryPerformanceCounter = +20.573

0.0.30 Matrix Multiply

Comparing naive matrix multiply versus Intel MKL dgemm for matrix of size +2048. This is for type double (hence the d in dgemm). Naive type double matrix multiply tic toc = +2.304 dgemm plus row to column major transpose operation tic toc = +1.662 Comparing naive matrix multiply versus Intel MKL sgemm for matrix of size +2048. This is for type float (hence the s in sgemm). Naive type float matrix multiply tic toc = +1.984 sgemm plus row to column major transpose operation tic toc = +1.603 QueryPerformanceCounter = +8.675

0.0.31 Descriptive Statistics

Mean N(0,1): +0.003 Variance N(0,1): +1.006 Mean N(0,1) [recurrence relation method] :+0.003 Variance [recurrence relation method] :+1.006 Skewness : +0.007 Kurtosis : +2.997 QueryPerformanceCounter = +0.067

0.0.32 Time Series

+0.093 +0.726 +0.011 +2.178 QueryPerformanceCounter = +0.213