Regression of KL Software Distribution

KL Software Libraries

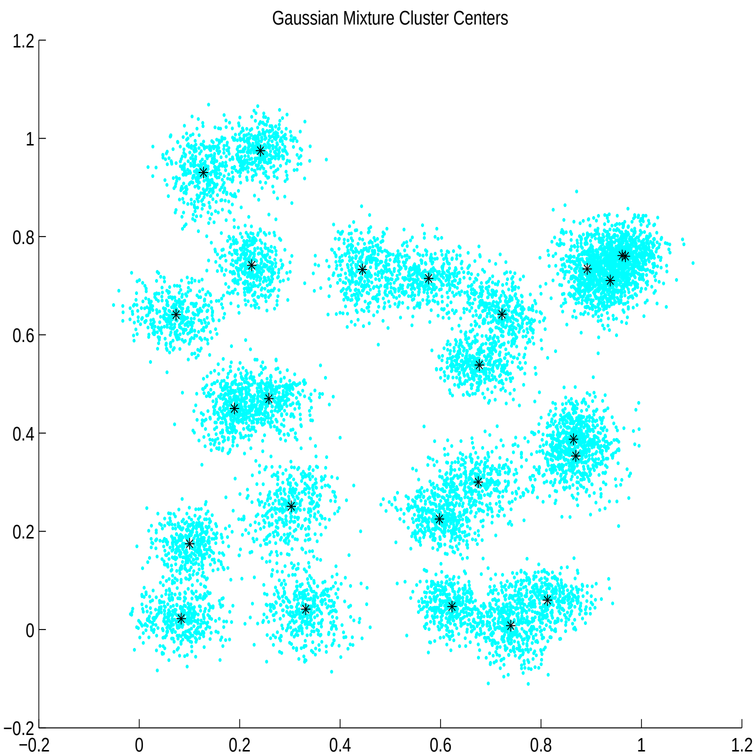
Sun May 11 18:11:14 2014

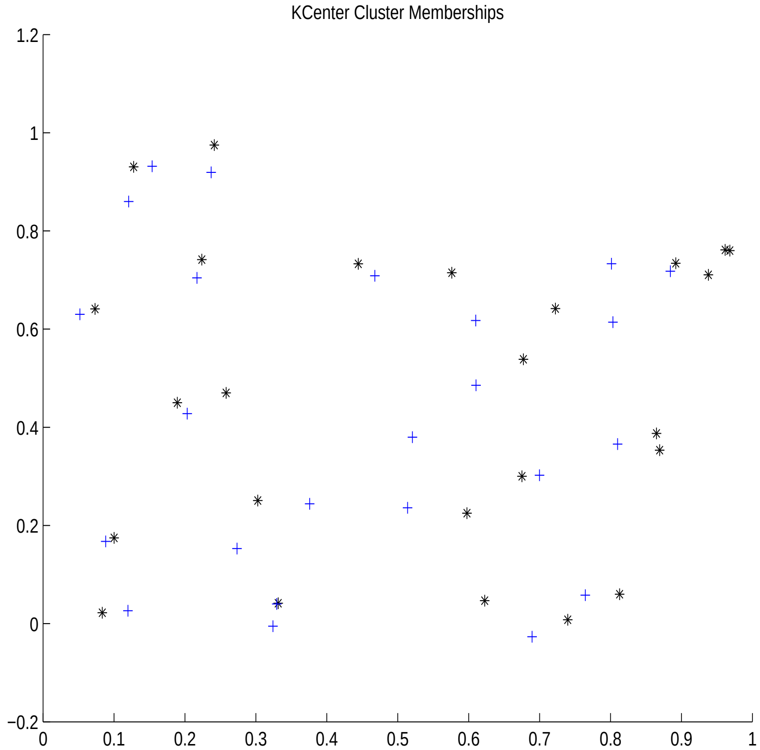
KL Library test output. This LaTex file and the associated diagrams are produced by the KL software libraries.

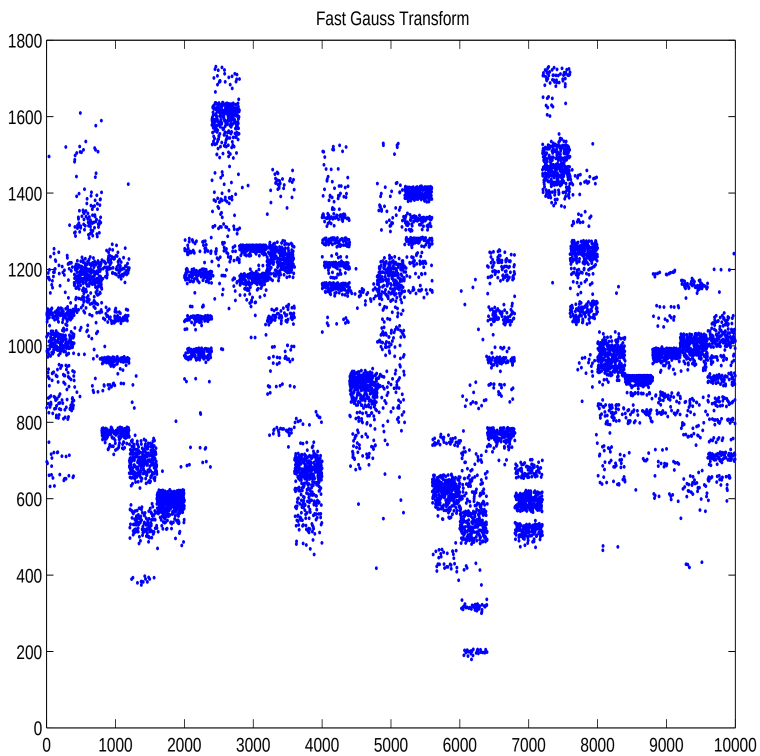
0.0.1 Testing binary writer

Binary writer Speedup 1GB Double Matrix 1.77916 Binary reader Speedup 1GB Double Matrix 441.717 Binary writer Speedup 1GB Double vector 0.368195 Binary reader Speedup 1GB Double Matrix 586.932 QueryPerformanceCounter = 28.9483

0.0.2 Fast Gauss Transform

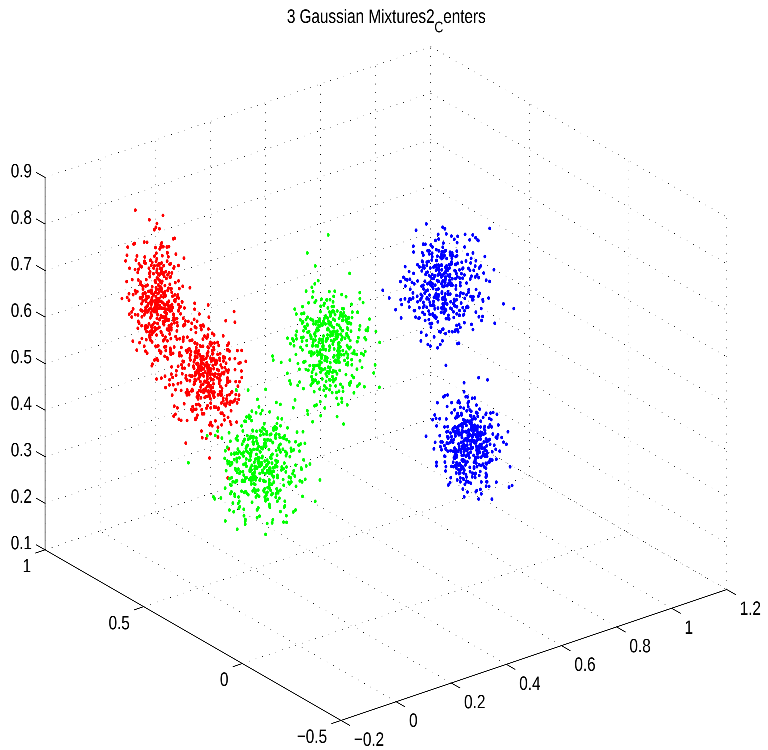


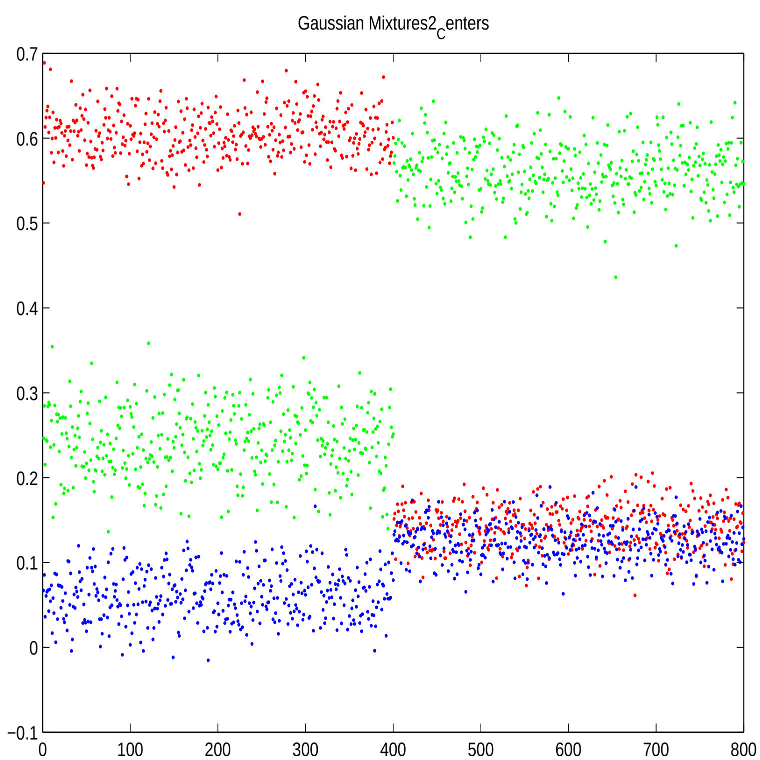




QueryPerformanceCounter = 14.0082

0.0.3 Testing Gaussian Mixture Point Cloud and Latex Plotting Capabilities.





QueryPerformanceCounter = 2.27408

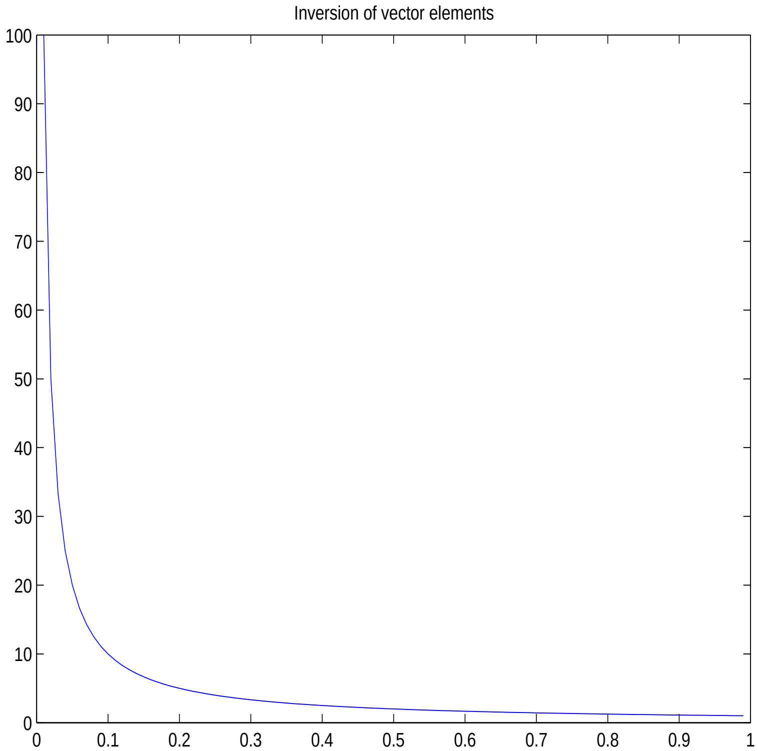
0.0.4 Matrix Quick Check ¡double¿

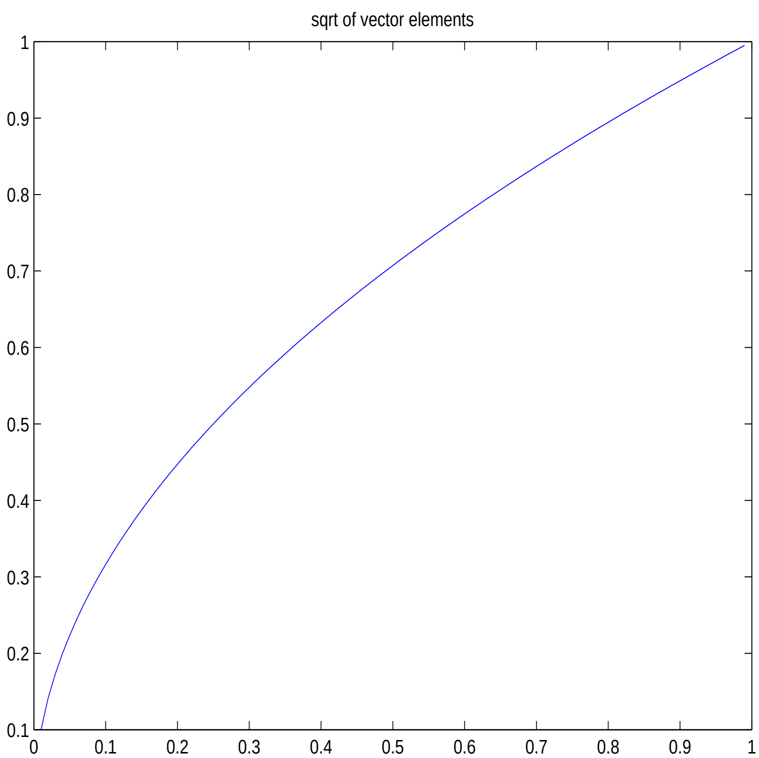
QueryPerformanceCounter = 0.735242

0.0.5 Matrix Quick Check ¡float¿

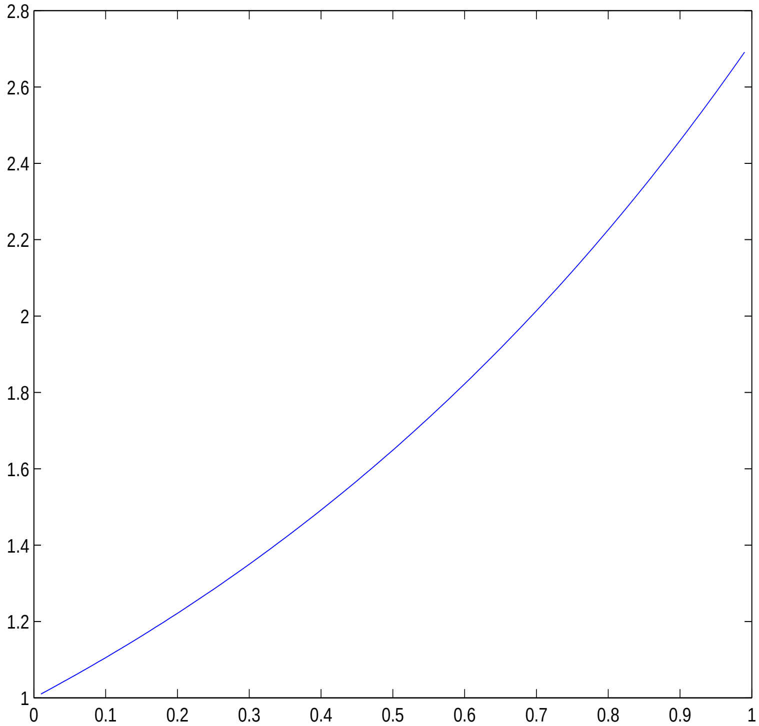
QueryPerformanceCounter = 0.590516

0.0.6 Intel VSL Function Check

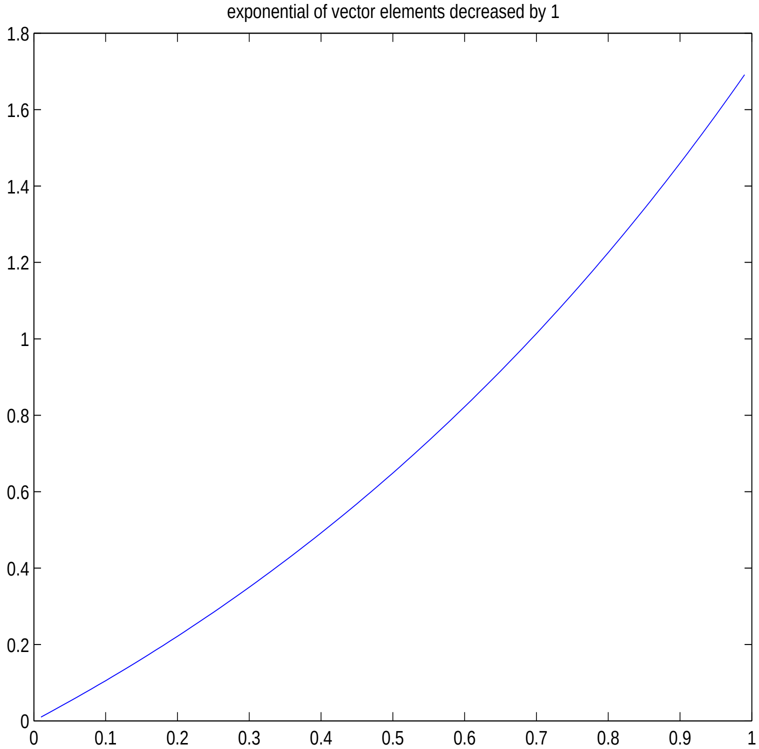


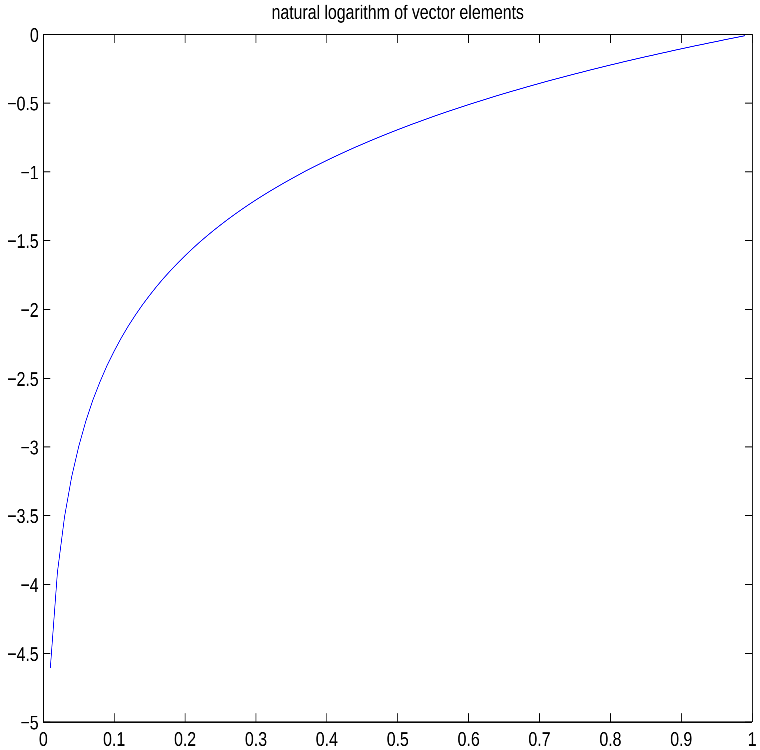


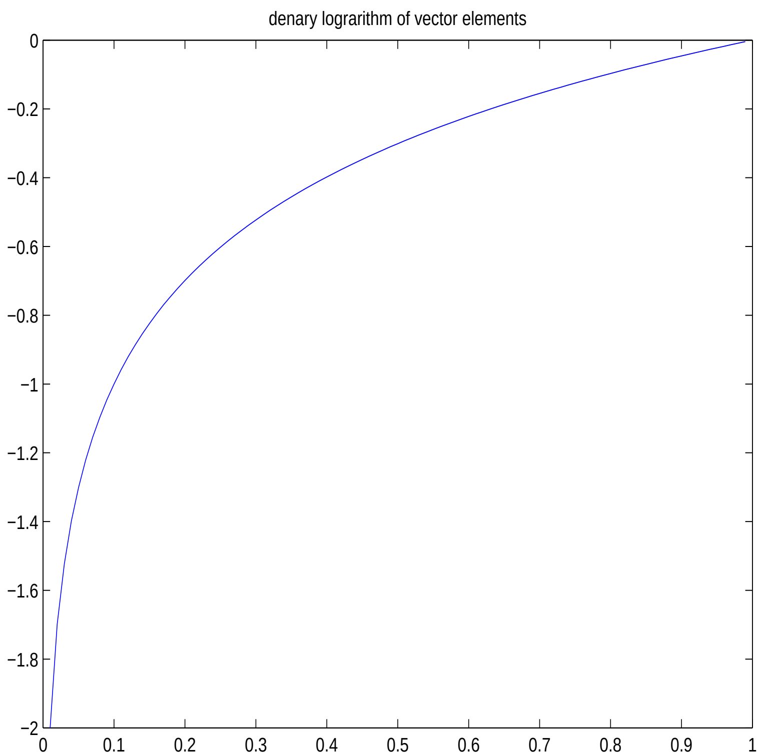
exponentail of vector elements

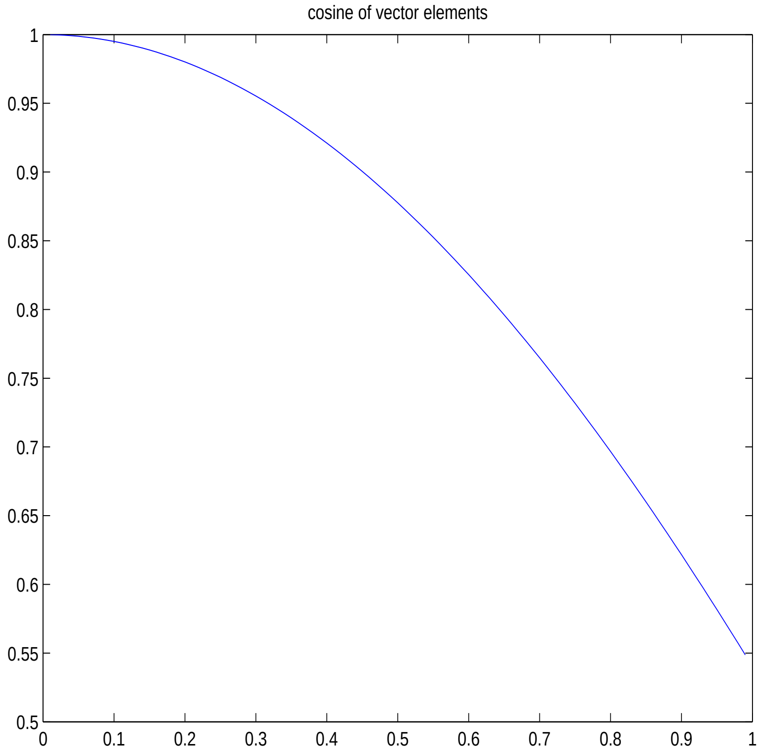


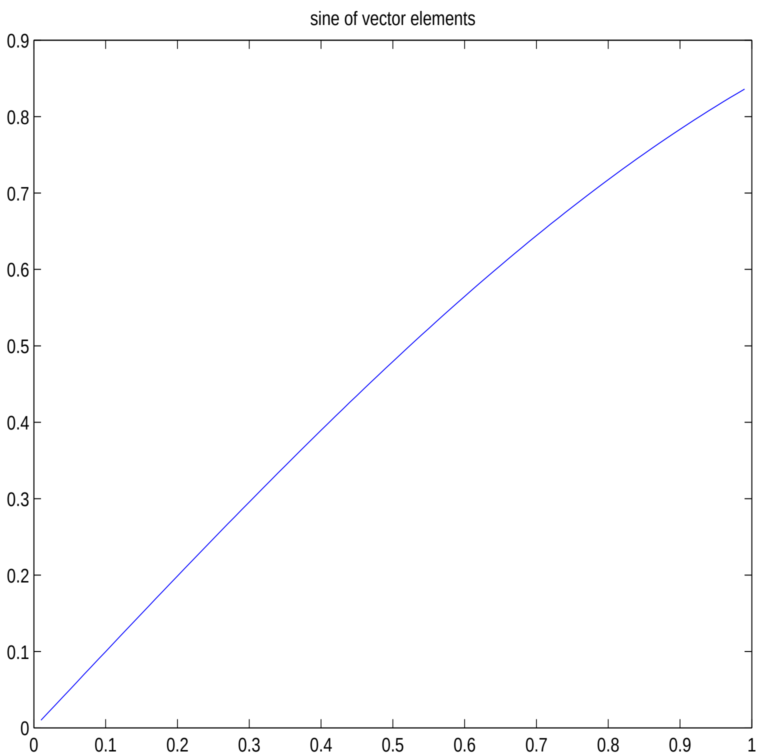
exponential of vector elements decreased by 1

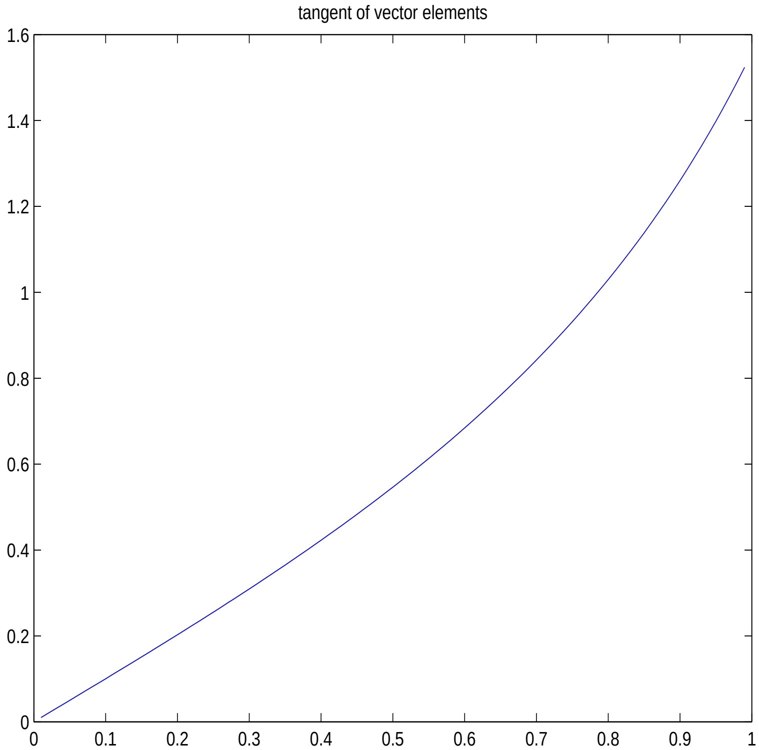


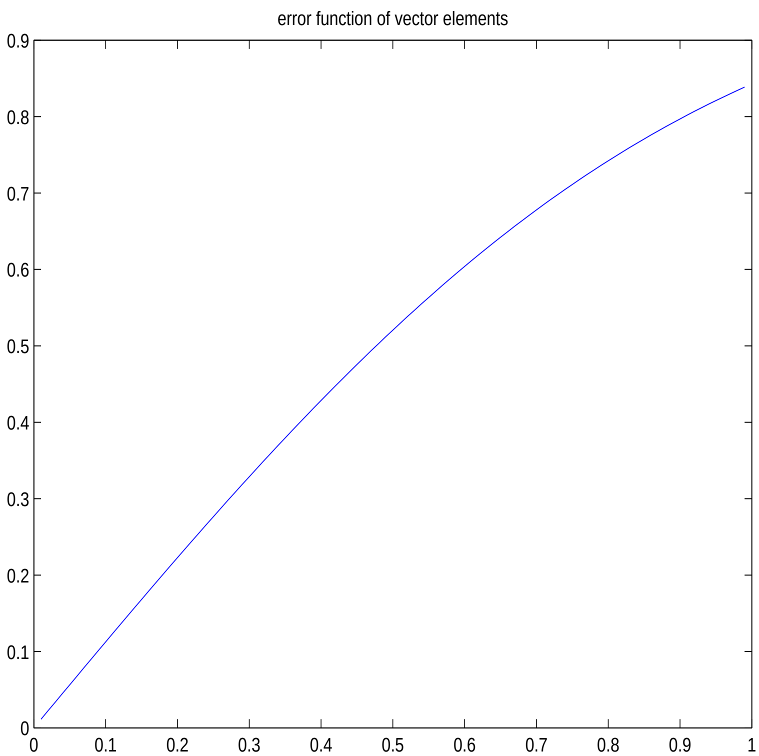

natural logarithm of vector elements
0
−0.5
−1
−1.5
−2
−2.5
−3
−3.5
−4
−4.5
−5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

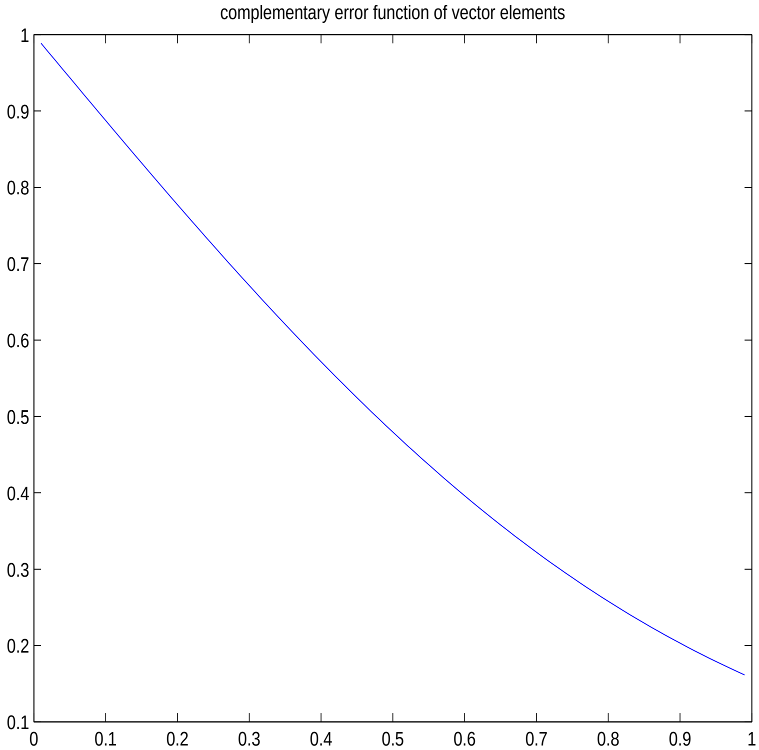

denary lograrithm of vector elements
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
−1.8
−2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

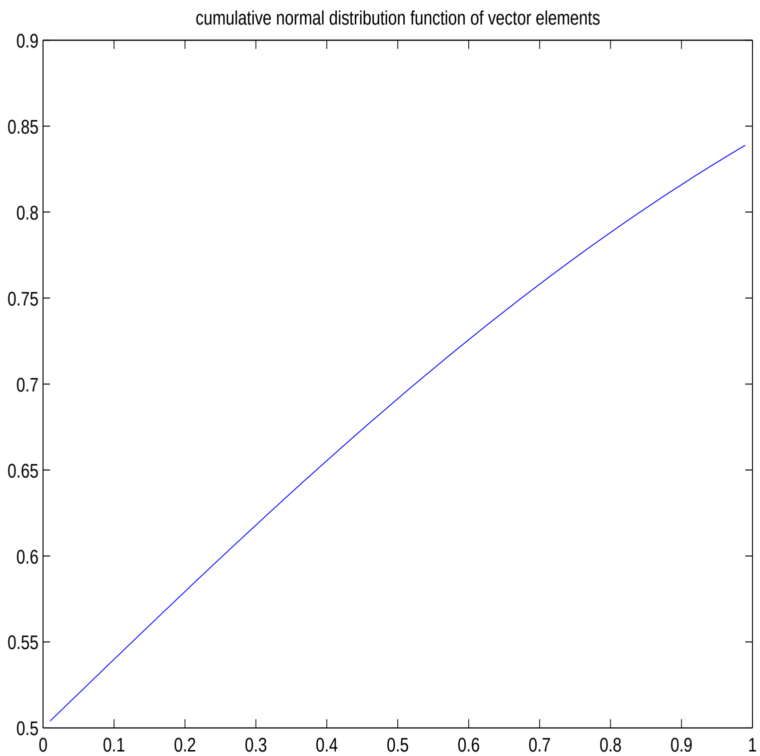

cosine of vector elements
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

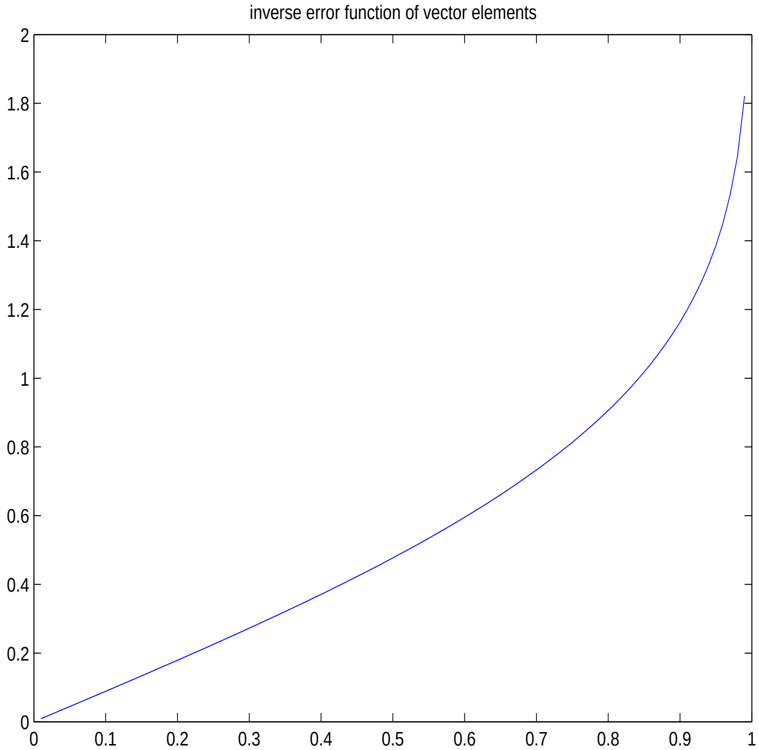

sine of vector elements
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

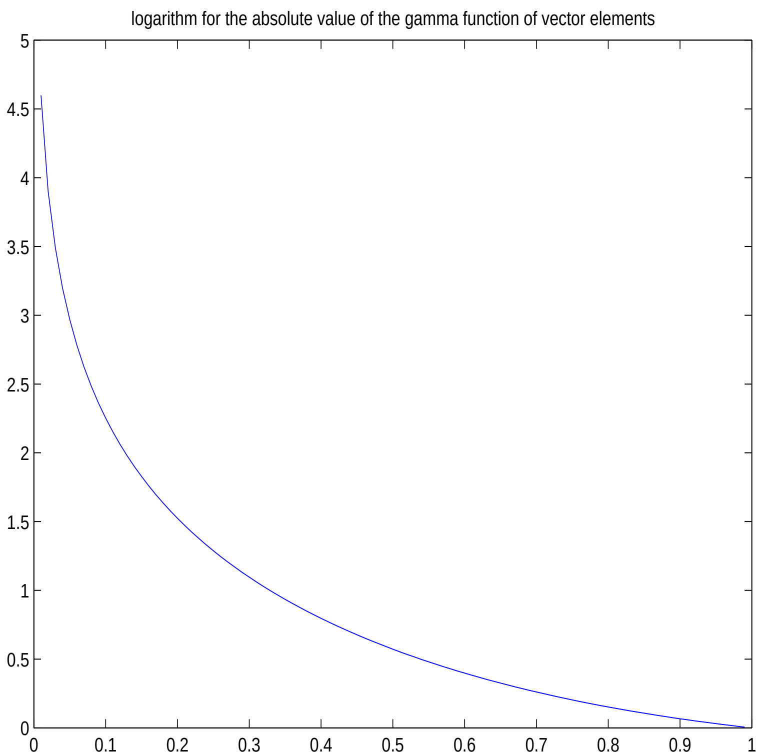

tangent of vector elements
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

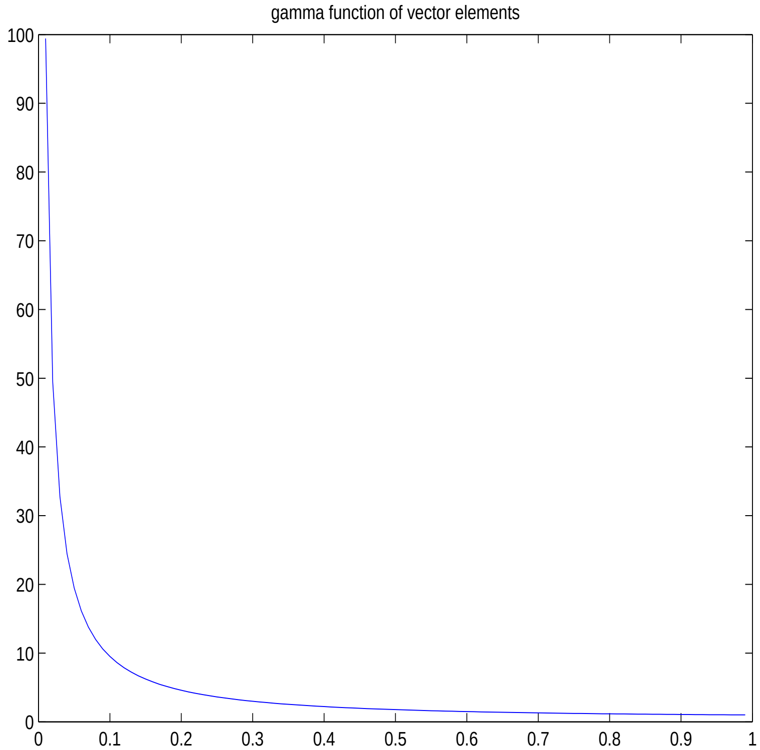

error function of vector elements
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


complementary error function of vector elements
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


cumulative normal distribution function of vector elements
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


inverse error function of vector elements
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


logarithm for the absolute value of the gamma function of vector elements
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1



QueryPerformanceCounter = 9.95268

0.0.7 Gram Matrix Consistency Check

Sample Size = 4096 Feature dim = 3

$$Sigma$$

$$= \begin{pmatrix} +1.140 & +1.535 & +0.581 \\ +1.535 & +9.988 & +1.605 \\ +0.581 & +1.605 & +0.428 \end{pmatrix}$$

$$SampleCovariance = \begin{pmatrix} +1.169 & +1.576 & +0.591 \\ +1.576 & +10.057 & +1.620 \\ +0.591 & +1.620 & +0.432 \end{pmatrix}$$

$$SampleMean = \begin{pmatrix} +1.02107 & +0.97126 & +1.00360 \end{pmatrix}$$

$$SampleCovariance{-}\mathrm{Omega}{=} \begin{pmatrix} +0.029 & +0.041 & +0.010 \\ +0.041 & +0.068 & +0.014 \\ +0.010 & +0.014 & +0.004 \end{pmatrix}$$

$$SampleCovarianceEigs = \begin{pmatrix} (+10.61086, +0.00000) & (+1.00560, +0.00000) & (+0.04130, +0.00000) \end{pmatrix}$$

$$CenteredMean = \begin{pmatrix} -0.00000 & -0.00000 & -0.00000 \end{pmatrix}$$

$$CenteredCovariance = \begin{pmatrix} +1.169 & +1.576 & +0.591 \\ +1.576 & +10.057 & +1.620 \\ +0.591 & +1.620 & +0.432 \end{pmatrix}$$

$$GramMatrixGfNotscaledbysamplesize = \begin{pmatrix} +4789.170 & +6453.516 & +2421.489 \\ +6453.516 & +41191.861 & +6633.169 \\ +2421.489 & +6633.169 & +1769.140 \end{pmatrix}$$

$$GramMatrixGfscaledbysamplesize = \begin{pmatrix} +1.169 & +1.576 & +0.591 \\ +1.576 & +10.057 & +1.619 \\ +0.591 & +1.619 & +0.432 \end{pmatrix}$$

$$SampleCovariance - ScaledGf = \begin{pmatrix} +0.000 & +0.000 & +0.000 \\ +0.000 & +0.002 & +0.000 \\ +0.000 & +0.000 & +0.000 \end{pmatrix}$$

$$EigenDecompofSampleCovariance = \begin{pmatrix} -0.172 & -0.971 & -0.165 \\ +0.919 & -0.219 & +0.329 \\ -0.356 & -0.094 & +0.930 \end{pmatrix}$$

$$EigenDecompofGramMatrix = \begin{pmatrix} -0.118 & -0.975 & -0.187 \\ -0.312 & +0.215 & -0.925 \\ +0.943 & -0.051 & -0.330 \end{pmatrix}$$

QueryPerformanceCounter = +93.152

0.0.8 Eigen Solver Checks

0.0.9 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: +8

$$A = \begin{pmatrix} -0.441 & -0.310 & -0.604 & +0.297 & -0.504 & -0.016 & +0.047 & -0.007 \\ +0.565 & +0.040 & -0.380 & +0.201 & +0.118 & -0.445 & +0.530 & +0.044 \\ +0.215 & -0.652 & +0.090 & -0.113 & +0.043 & -0.041 & -0.058 & -0.708 \\ +0.410 & -0.163 & +0.141 & +0.057 & -0.441 & -0.373 & -0.585 & +0.327 \\ -0.006 & -0.296 & +0.598 & +0.228 & -0.352 & +0.144 & +0.550 & +0.235 \\ -0.159 & +0.518 & +0.271 & +0.353 & -0.278 & -0.370 & -0.014 & -0.542 \\ -0.340 & -0.055 & +0.067 & -0.695 & -0.122 & -0.570 & +0.222 & +0.074 \\ -0.363 & -0.303 & +0.164 & +0.442 & +0.565 & -0.424 & -0.137 & +0.189 \end{pmatrix}$$

$Det(A) : A \in O(n) =$ (-1.000,+0.000)

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.381 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.780 & +0.417 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.603 & +0.046 & +0.173 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.643 & +0.416 & +0.178 & -0.846 & +1.000 & +0.000 & +0.000 & +0.000 \\ -0.010 & +0.443 & -0.492 & -0.877 & -0.729 & +1.000 & +0.000 & +0.000 \\ +0.726 & +0.288 & -0.350 & -0.231 & -0.908 & +0.917 & +1.000 & +0.000 \\ -0.282 & -0.794 & -0.351 & -0.679 & -0.514 & +0.986 & +0.602 & +1.000 \end{pmatrix}$$

$$U = \begin{pmatrix} +0.565 & +0.040 & -0.380 & +0.201 & +0.118 & -0.445 & +0.530 & +0.044 \\ +0.000 & -0.667 & +0.235 & -0.190 & -0.002 & +0.129 & -0.261 & -0.724 \\ +0.000 & +0.000 & -0.998 & +0.533 & -0.411 & -0.416 & +0.570 & +0.329 \\ +0.000 & +0.000 & +0.000 & -0.657 & +0.021 & -0.772 & +0.455 & +0.077 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.733 & -1.341 & +0.595 & +0.525 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.776 & +1.783 & +1.167 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.686 & +0.042 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.845 \end{pmatrix}$$

$$L * U = \begin{pmatrix} +0.565 & +0.040 & -0.380 & +0.201 & +0.118 & -0.445 & +0.530 & +0.044 \\ +0.215 & -0.652 & +0.090 & -0.113 & +0.043 & -0.041 & -0.058 & -0.708 \\ -0.441 & -0.310 & -0.604 & +0.297 & -0.504 & -0.016 & +0.047 & -0.007 \\ -0.340 & -0.055 & +0.067 & -0.695 & -0.122 & -0.570 & +0.222 & +0.074 \\ -0.363 & -0.303 & +0.164 & +0.442 & +0.565 & -0.424 & -0.137 & +0.189 \\ -0.006 & -0.296 & +0.598 & +0.228 & -0.352 & +0.144 & +0.550 & +0.235 \\ +0.410 & -0.163 & +0.141 & +0.057 & -0.441 & -0.373 & -0.585 & +0.327 \\ -0.159 & +0.518 & +0.271 & +0.353 & -0.278 & -0.370 & -0.014 & -0.542 \end{pmatrix}$$

$Det(L) := (+1.000, +0.000) Det(U) := (+1.000, +0.000) Det(LU) := (+1.000, +0.000)$
$||A||_{L_1} = +2.498$
$||A||_{L_\infty} = +2.589$
$||A^{-1}||_{L_1} = +2.589$
$||A^{-1}||_{L_\infty} = +2.498$
$||A||_{L_\infty} * ||A^{-1}||_{L_\infty} = +6.467$
$||A||_{L_1} * ||A^{-1}||_{L_1} = +6.467$
Frobenious Norm $||A||_F$ via $\sum\limits_{i,j=0}^{n} \|A_{i,j}|$ of $A \in O(n)$ +2.828
L_1 condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +6.018

$$A = \begin{pmatrix} -0.441 & -0.310 & -0.604 & +0.297 & -0.504 & -0.016 & +0.047 & -0.007 \\ +0.565 & +0.040 & -0.380 & +0.201 & +0.118 & -0.445 & +0.530 & +0.044 \\ +0.215 & -0.652 & +0.090 & -0.113 & +0.043 & -0.041 & -0.058 & -0.708 \\ +0.410 & -0.163 & +0.141 & +0.057 & -0.441 & -0.373 & -0.585 & +0.327 \\ -0.006 & -0.296 & +0.598 & +0.228 & -0.352 & +0.144 & +0.550 & +0.235 \\ -0.159 & +0.518 & +0.271 & +0.353 & -0.278 & -0.370 & -0.014 & -0.542 \\ -0.340 & -0.055 & +0.067 & -0.695 & -0.122 & -0.570 & +0.222 & +0.074 \\ -0.363 & -0.303 & +0.164 & +0.442 & +0.565 & -0.424 & -0.137 & +0.189 \end{pmatrix}$$

L_∞ condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +6.272
Eigenvalues of $A \in O(n)$

(-0.274,+0.962), (-0.274,-0.962), (-0.761,+0.649), (-0.761,-0.649), (-1.000,+0.000), (+1.000,+0.000), (+0.753,+0.658), (+0.753,-0.658)

$|\lambda| : \lambda \in \sigma(A), A \in O(n)$

+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & +0.000 & +0.000 & +1.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} -0.441 & +0.565 & +0.215 & +0.410 & -0.006 & -0.159 & -0.340 & -0.363 \\ -0.310 & +0.040 & -0.652 & -0.163 & -0.296 & +0.518 & -0.055 & -0.303 \\ -0.604 & -0.380 & +0.090 & +0.141 & +0.598 & +0.271 & +0.067 & +0.164 \\ +0.297 & +0.201 & -0.113 & +0.057 & +0.228 & +0.353 & -0.695 & +0.442 \\ -0.504 & +0.118 & +0.043 & -0.441 & -0.352 & -0.278 & -0.122 & +0.565 \\ -0.016 & -0.445 & -0.041 & -0.373 & +0.144 & -0.370 & -0.570 & -0.424 \\ +0.047 & +0.530 & -0.058 & -0.585 & +0.550 & -0.014 & +0.222 & -0.137 \\ -0.007 & +0.044 & -0.708 & +0.327 & +0.235 & -0.542 & +0.074 & +0.189 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 \\ -0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & +1.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} +0.121 & -0.089 & -0.472 & -0.489 & +0.184 & -0.373 & -0.583 & -0.054 \\ +0.768 & -0.066 & -0.326 & -0.008 & -0.254 & +0.480 & +0.056 & -0.032 \\ +0.274 & +0.105 & +0.113 & -0.074 & -0.426 & -0.641 & +0.327 & -0.444 \\ -0.266 & +0.394 & +0.189 & -0.510 & -0.567 & +0.325 & -0.225 & -0.029 \\ -0.198 & -0.256 & -0.019 & -0.174 & +0.260 & +0.337 & +0.101 & -0.821 \\ -0.441 & -0.310 & -0.604 & +0.297 & -0.504 & -0.016 & +0.047 & -0.007 \\ -0.104 & +0.728 & -0.503 & -0.014 & +0.272 & +0.024 & +0.361 & -0.047 \\ +0.073 & +0.362 & +0.067 & +0.614 & -0.055 & +0.006 & -0.597 & -0.350 \end{pmatrix}$$

$$S = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

$$V = \begin{pmatrix} -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 \\ +0.022 & +0.338 & +0.490 & -0.641 & -0.251 & +0.000 & +0.370 & -0.187 \\ +0.192 & +0.169 & +0.312 & -0.244 & +0.714 & +0.000 & -0.518 & -0.004 \\ +0.351 & +0.169 & +0.197 & +0.076 & -0.620 & +0.000 & -0.588 & +0.272 \\ -0.820 & -0.000 & +0.152 & -0.002 & -0.154 & +0.000 & -0.423 & -0.318 \\ -0.242 & -0.338 & +0.607 & +0.174 & +0.079 & +0.000 & +0.189 & +0.622 \\ +0.329 & -0.507 & +0.416 & +0.268 & -0.061 & +0.000 & +0.001 & -0.621 \\ +0.021 & -0.676 & -0.244 & -0.650 & -0.096 & +0.000 & -0.187 & +0.129 \end{pmatrix}$$

$$USV = \begin{pmatrix} -0.518 & +0.266 & -0.715 & -0.051 & -0.029 & +0.121 & +0.360 & -0.050 \\ +0.043 & -0.248 & +0.148 & +0.241 & -0.135 & +0.768 & +0.353 & +0.352 \\ +0.601 & +0.393 & -0.137 & +0.165 & +0.137 & +0.274 & +0.166 & -0.563 \\ +0.178 & +0.102 & +0.176 & -0.321 & +0.482 & -0.266 & +0.654 & +0.305 \\ -0.349 & +0.271 & +0.321 & +0.774 & +0.217 & -0.198 & +0.124 & -0.041 \\ +0.414 & -0.171 & -0.346 & +0.383 & -0.463 & -0.441 & +0.235 & +0.262 \\ -0.197 & -0.001 & +0.414 & -0.214 & -0.591 & -0.104 & +0.436 & -0.440 \\ +0.076 & +0.775 & +0.151 & -0.133 & -0.345 & +0.073 & -0.172 & +0.446 \end{pmatrix}$$

Calculating first few eigenvectors of $A \in O(n)$ using LAPACK syevx

0.0.10 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +1489.694 L_infty condition number of Wishart Matrix +1489.694

0.0.11 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +66.900 L_∞ condition number of GOE Matrix +66.900

0.0.12 The Identity Matrix $I \in M(n)$

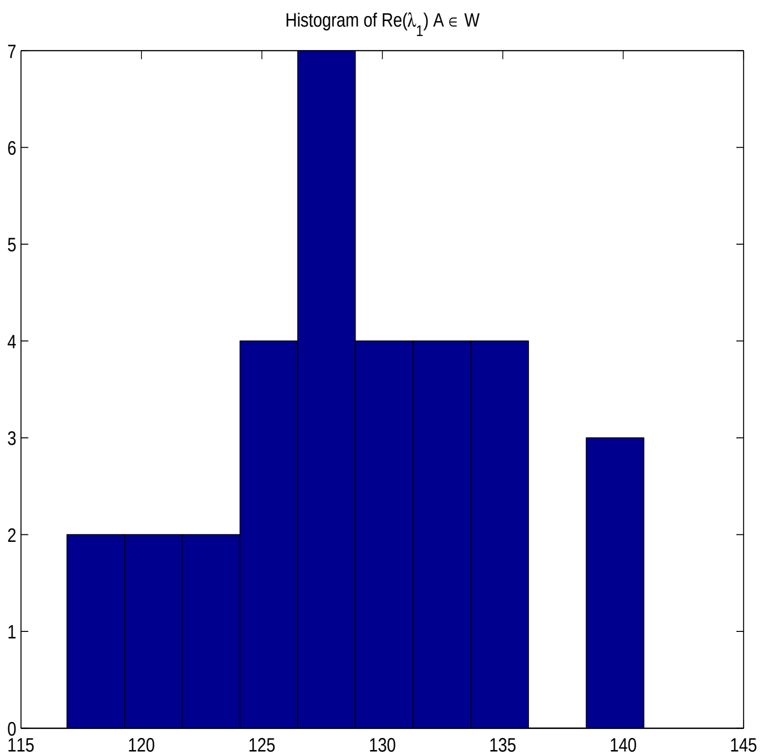
L_1 condition number of $I = +1.000$ L_∞ condition number of $I = +1.000$ QueryPerformanceCounter = +1.554

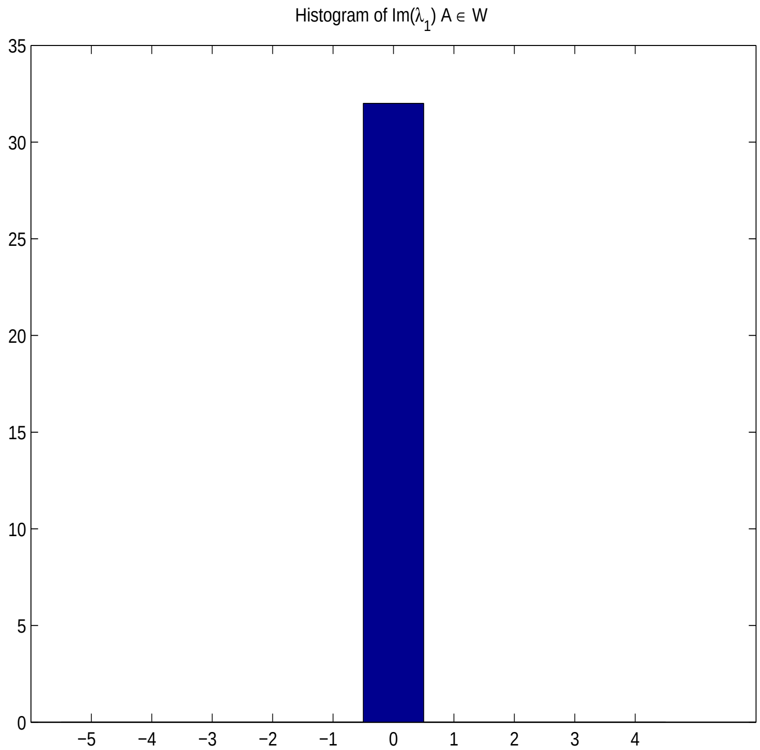
0.0.13 Generate Tracey Widom Sample

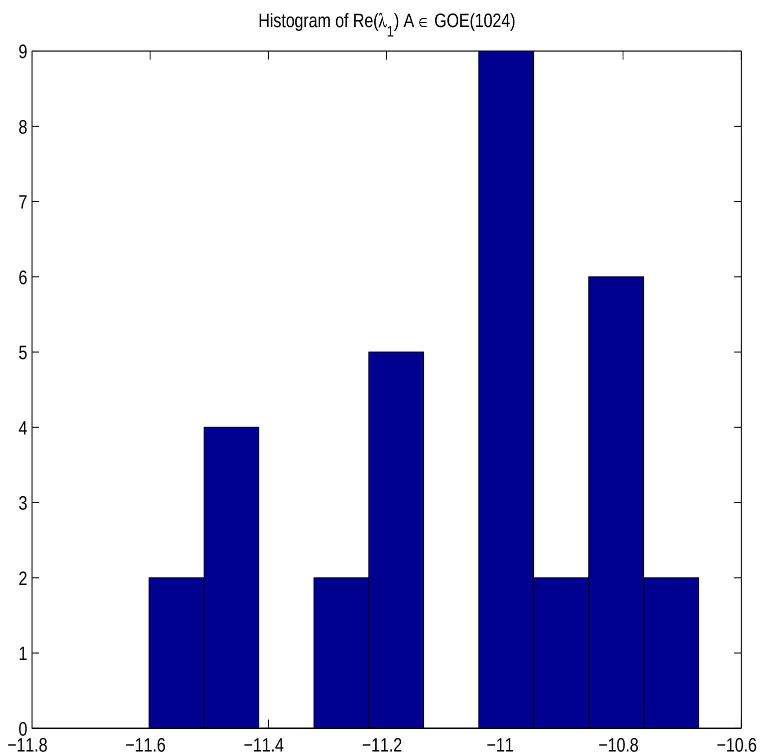
0.0.14 Sample from $W_n m$ times and calculate empirical PDF of the first eig

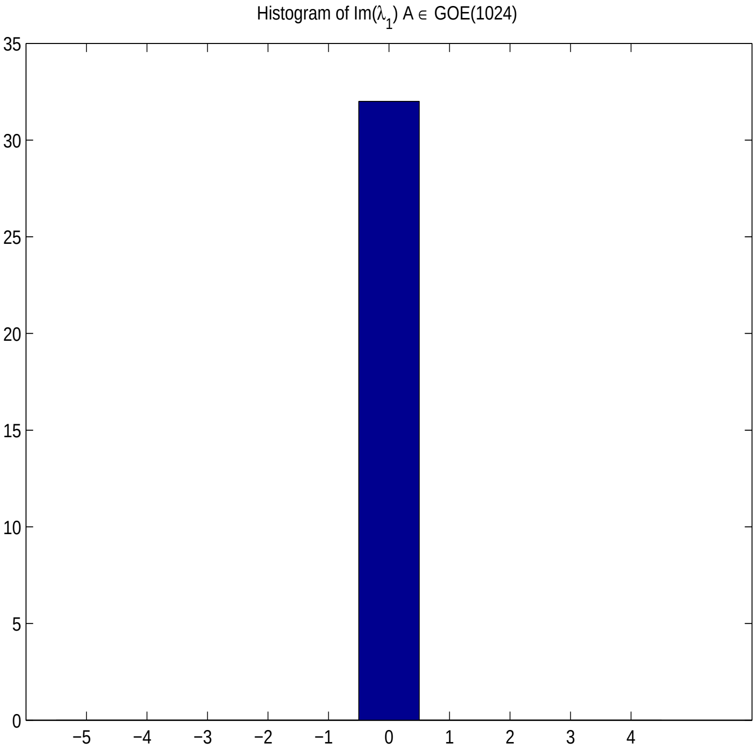
Here we generate histograms of λ_1 for GOE (Gaussian Orthogonal Ensemble), and W (Wishart) distributed of random matrices These should approximate the celebrated Tracy Widom distribution. Dimension $n = +128$

Sample size $m = 32$




Histogram of Im(λ1) A ∈ W
35
30
25
20
15
10
5
0
−5
−4
−3
−2
−1
0
1
2
3
4


Histogram of Re(λ1) A ∈ GOE(1024)
9
8
7
6
5
4
3
2
1
0
−11.8
−11.6
−11.4
−11.2
−11
−10.8
−10.6



QueryPerformanceCounter $= +5.508$

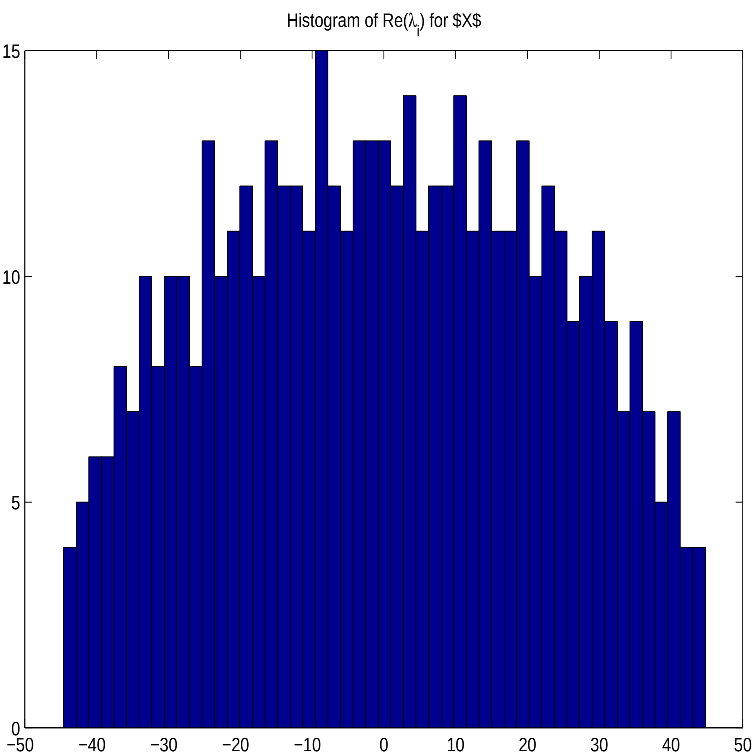
0.0.15 Approximate Winger Distribution

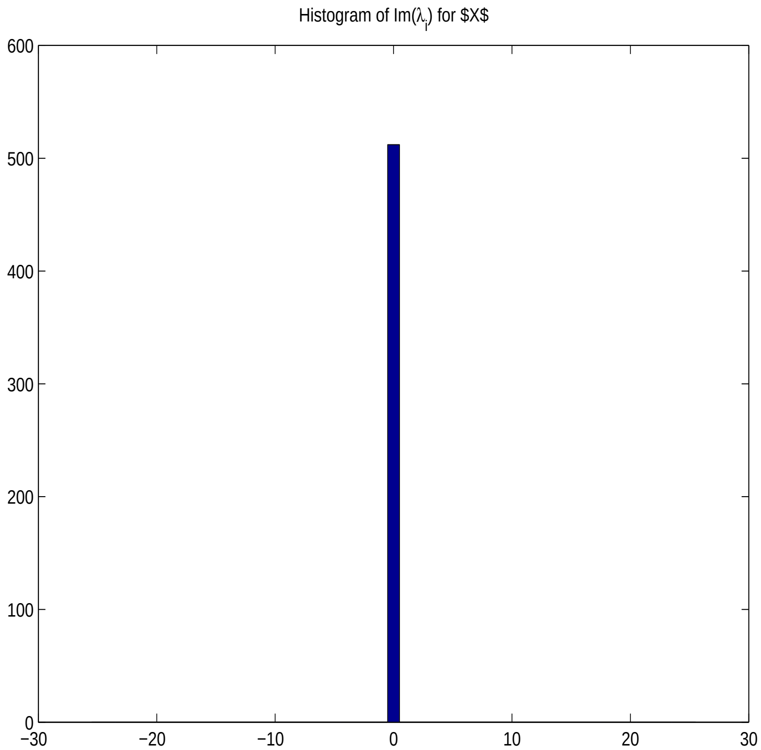
0.0.16 Verfy Winger Law.

Let $M_n = [X_{ij}]$ a symmetric n x n matrix with Random entries such that $X_{i,j} = X_{j,i}$, and $X_{i,j}$ are iid

orall i ¡ j,andXjj$areiid$

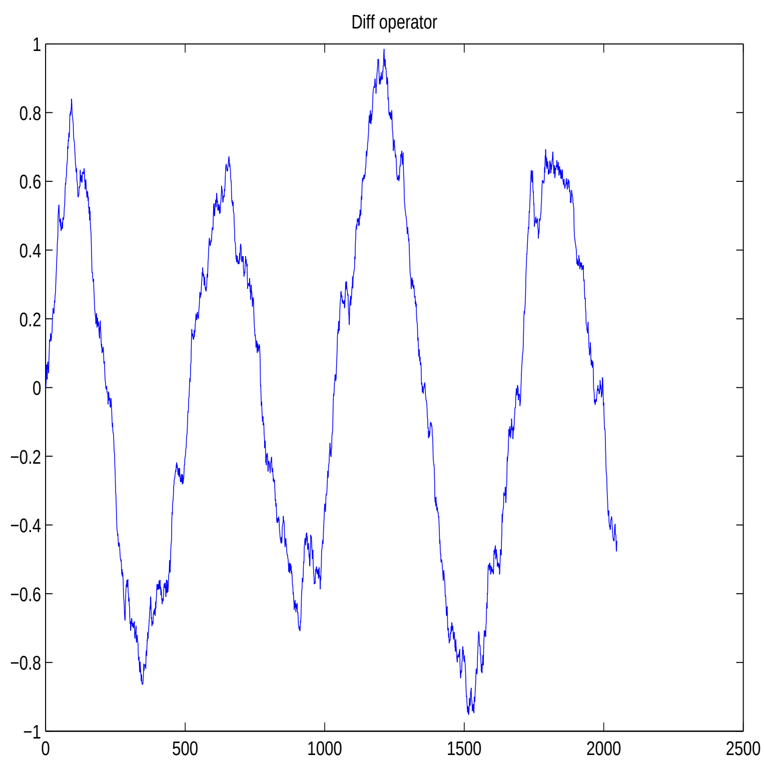
orall j : ; E[X$_{ij}^2$] $= 1, E[X_{ij}] = 0$ and that all moments exists for each of the entries. The eigenvector of this random matrix; $lambda_1 leq ... leq lambda_n$ depends continuously on Mn. Dimension $n = +512$





QueryPerformanceCounter $= +1.905$

0.0.17 Iterated Exponential Filtering



$\mu_1 = +0.093$ $\mu_2 = +0.726$ $\mu_3 = +0.011$ $\mu_4 = +2.178$
QueryPerformanceCounter $= +0.856$

0.0.18 Matrix Exponential

$$SPDMatrix = \begin{pmatrix} +10.539 & -0.499 & -0.010 & +0.368 & +0.465 & -0.492 & -0.126 & +0.437 \\ -0.499 & +7.286 & +0.365 & -0.481 & -0.337 & -0.466 & +0.279 & +0.056 \\ -0.010 & +0.365 & +6.705 & -0.205 & +0.467 & +0.131 & +0.077 & -0.089 \\ +0.368 & -0.481 & -0.205 & +6.496 & -0.402 & -0.209 & +0.043 & -0.041 \\ +0.465 & -0.337 & +0.467 & -0.402 & +4.578 & +0.272 & +0.289 & -0.285 \\ -0.492 & -0.466 & +0.131 & -0.209 & +0.272 & +8.181 & +0.343 & -0.244 \\ -0.126 & +0.279 & +0.077 & +0.043 & +0.289 & +0.343 & +5.938 & -0.212 \\ +0.437 & +0.056 & -0.089 & -0.041 & -0.285 & -0.244 & -0.212 & +9.691 \end{pmatrix}$$

$SPDEigs = \Big(\ (+10.93611, +0.00000) \quad (+9.60778, +0.00000) \quad (+4.23666, +0.00000) \quad (+8.36911, +0.00000) \quad (+7.56229, +0.00000)$

$$exp(SPD) = \begin{pmatrix} +47863.969 & -6460.093 & -1078.770 & +4706.958 & +2535.224 & -8475.398 & -2406.368 & +12977.552 \\ -6460.093 & +2780.574 & +516.920 & -1069.918 & -548.083 & -109.707 & +386.466 & -807.216 \\ -1078.770 & +516.920 & +1015.281 & -385.755 & +176.069 & +458.541 & +212.284 & -859.022 \\ +4706.958 & -1069.918 & -385.755 & +1267.210 & +111.181 & -1018.272 & -287.809 & +1036.628 \\ +2535.224 & -548.083 & +176.069 & +111.181 & +413.265 & +135.193 & +45.490 & -502.411 \\ -8475.398 & -109.707 & +458.541 & -1018.272 & +135.193 & +5613.026 & +968.003 & -4270.737 \\ -2406.368 & +386.466 & +212.284 & -287.809 & +45.490 & +968.003 & +632.432 & -1645.725 \\ +12977.552 & -807.216 & -859.022 & +1036.628 & -502.411 & -4270.737 & -1645.725 & +19362.944 \end{pmatrix}$$

$exp(SPD)eigs = \Big(\ (+56168.17045, +0.00000) \quad (+14880.07985, +0.00000) \quad (+4311.77579, +0.00000) \quad (+1924.25027, +0.00000) \quad (+6$

$log(exp(SPD)eigs) = \Big(\ (+10.93611, +0.00000) \quad (+9.60778, +0.00000) \quad (+8.36911, +0.00000) \quad (+7.56229, +0.00000) \quad (+4.23666, +0$

$$exp(Id) = \begin{pmatrix} +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 \end{pmatrix}$$

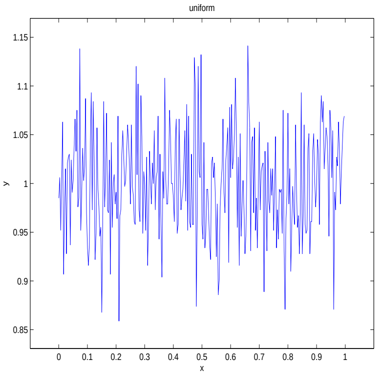
$exp(Id)eigs = \Big(\ (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000) \quad (+2.71828, +0.00000)$

$log(exp(Id)eigs) = \Big(\ (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.00000) \quad (+1.00000, +0.000$

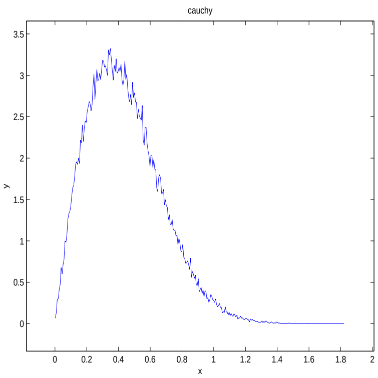
For $n \in \mathbb{Z}[16, 128)$ we calculate $|(SPD(n)Eigs - log(exp(SPD(n))eigs)|_{l^2}$
$|(SPD(n)Eigs - log(exp(SPD(n))eigs)|_{l^2} = \Big(\ (+5.36543, +0.00000) \quad (+5.36543, +0.00000) \quad (+5.36543, +0.00000) \quad (+5.36543, +0.00$

QueryPerformanceCounter = +0.02163 The sample size generated for this run is 100000.

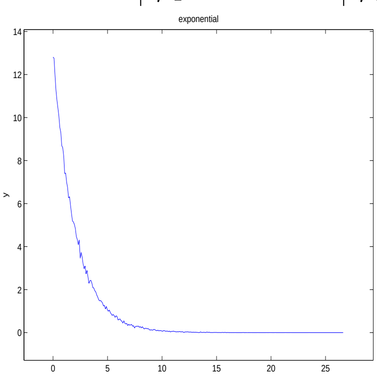
uniform	mean	variance	skewness	kurtosis
	$\mu_1 = +0.50030$	$\mu_2 = +0.08353$	$\mu_3 = +0.00339$	$\mu_4 = +1.80113$



cauchy	mean	variance	skewness	kurtosis
	$\mu_1 = +0.44288$	$\mu_2 = +0.05341$	$\mu_3 = +0.63935$	$\mu_4 = +3.28094$

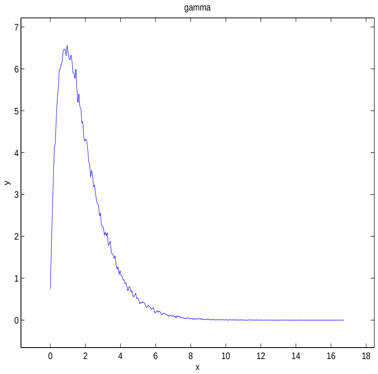


exponential	mean	variance	skewness	kurtosis
	$\mu_1 = +1.99647$	$\mu_2 = +3.99339$	$\mu_3 = +2.03097$	$\mu_4 = +9.30842$



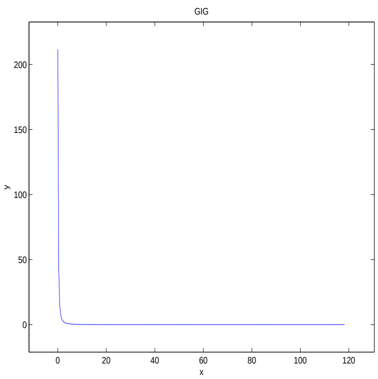
gamma

mean	variance	skewness	kurtosis
$\mu_1 = +1.90706$	$\mu_2 = +1.92833$	$\mu_3 = +1.44894$	$\mu_4 = +6.13430$



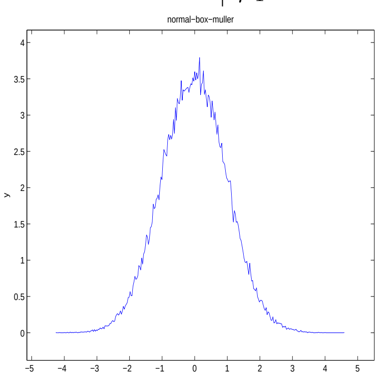
GIG

mean	variance	skewness	kurtosis
$\mu_1 = +0.81043$	$\mu_2 = +11.68253$	$\mu_3 = +15.18063$	$\mu_4 = +303.36105$

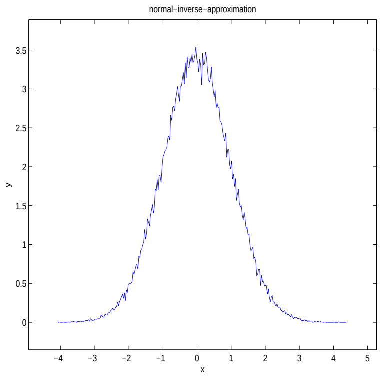


normal-box-muller

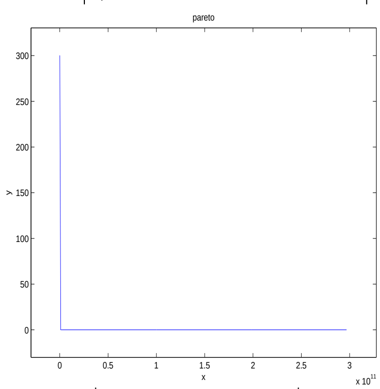
mean	variance	skewness	kurtosis
$\mu_1 = -0.00169$	$\mu_2 = +1.00159$	$\mu_3 = +0.00284$	$\mu_4 = +2.98019$



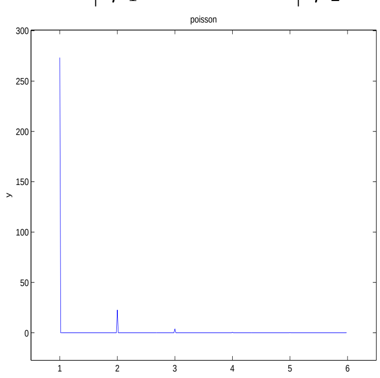
normal-inverse-approximation	mean	variance	skewness	kurtosis
	$\mu_1 = +0.00230$	$\mu_2 = +1.00486$	$\mu_3 = +0.01163$	$\mu_4 = +2.99254$



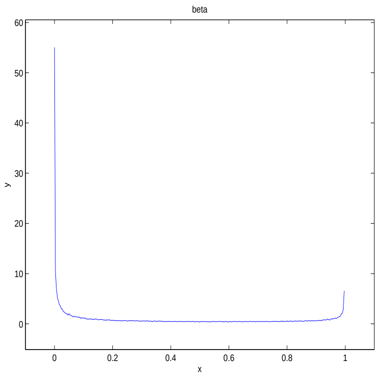
pareto	mean	variance	skewness	kurtosis
	$\mu_1 = +3184578.26493$	$\mu_2 = +888468246174112900.00000$	$\mu_3 = +315.36997$	$\mu_4 = +99629.09819$



poisson	mean	variance	skewness	kurtosis
	$\mu_1 = +1.10595$	$\mu_2 = +0.13160$	$\mu_3 = +3.94092$	$\mu_4 = +21.35260$



beta	mean	variance	skewness	kurtosis
	$\mu_1 = +0.33358$	$\mu_2 = +0.12720$	$\mu_3 = +0.68014$	$\mu_4 = +1.90728$



QueryPerformanceCounter = +16.94875

0.0.19 Multiclass Support Vector Machine

- Number or training points = 1024
- Feature dimension = 3
- Number or classes = 3

The mean vectors of the 3 classes

$\mu_1 = \begin{pmatrix} +1.90000 & +0.10000 & +0.10000 \end{pmatrix}$

$\mu_2 = \begin{pmatrix} +0.10000 & +1.90000 & +0.10000 \end{pmatrix}$

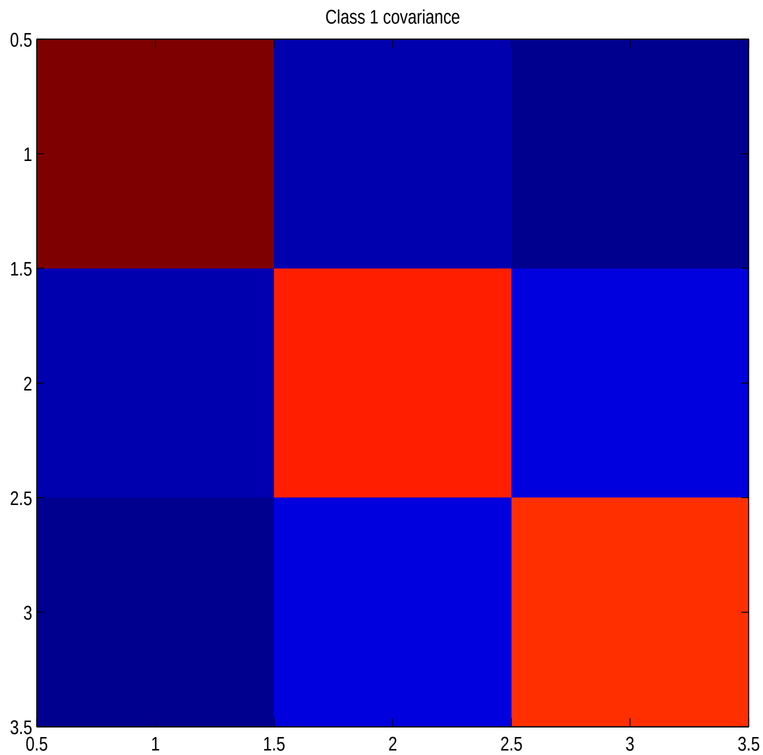
$\mu_3 = \begin{pmatrix} +0.00000 & +0.00000 & +1.90000 \end{pmatrix}$

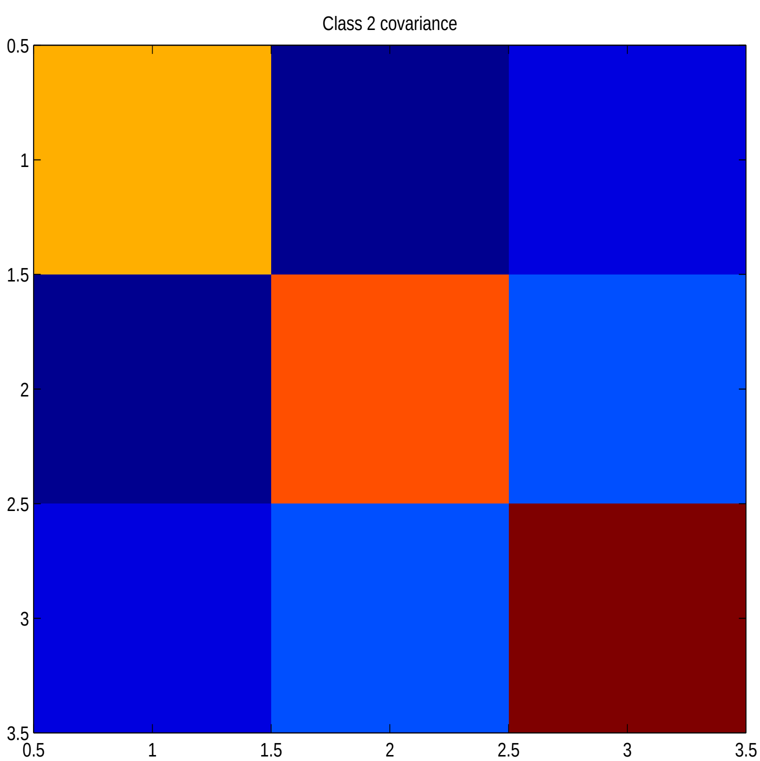
A random SPD covairance matrix is generated for each of the classes.

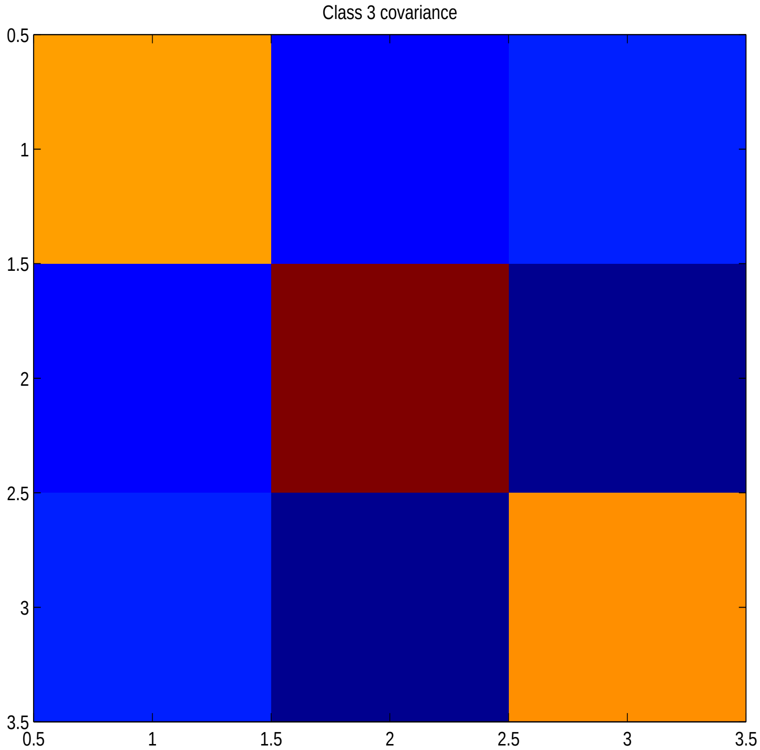
$$\rho_1 = \begin{pmatrix} +3.484 & -0.077 & -0.245 \\ -0.077 & +2.854 & +0.065 \\ -0.245 & +0.065 & +2.804 \end{pmatrix}$$
$$\rho_2 = \begin{pmatrix} +2.779 & -0.475 & -0.094 \\ -0.475 & +3.239 & +0.433 \\ -0.094 & +0.433 & +4.199 \end{pmatrix}$$
$$\rho_3 = \begin{pmatrix} +1.828 & +0.200 & +0.300 \\ +0.200 & +2.617 & -0.102 \\ +0.300 & -0.102 & +1.891 \end{pmatrix}$$

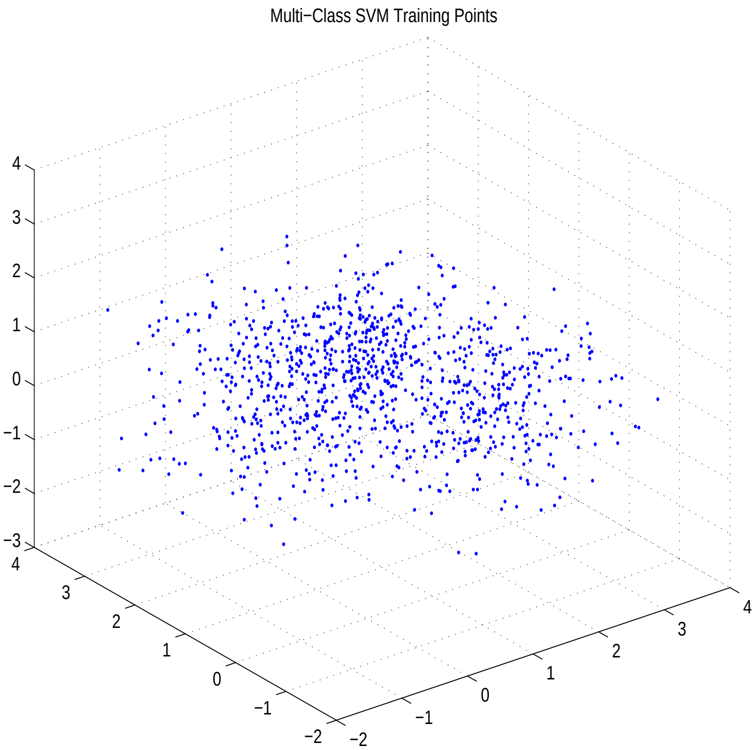
Verify L_1 condition number of covariance. The diagonal entries of the matrix have the form $(0.5 + U(0,1)) * dim(Dom(Cov))$ The lower-diagonal entries take the form $U(0,1) - 0.5$. The L_1 condition numbers are :

- +1.490
- +2.011
- +2.064


Class 1 covariance
0.5
1
1.5
2
2.5
3
3.5
0.5
1
1.5
2
2.5
3
3.5

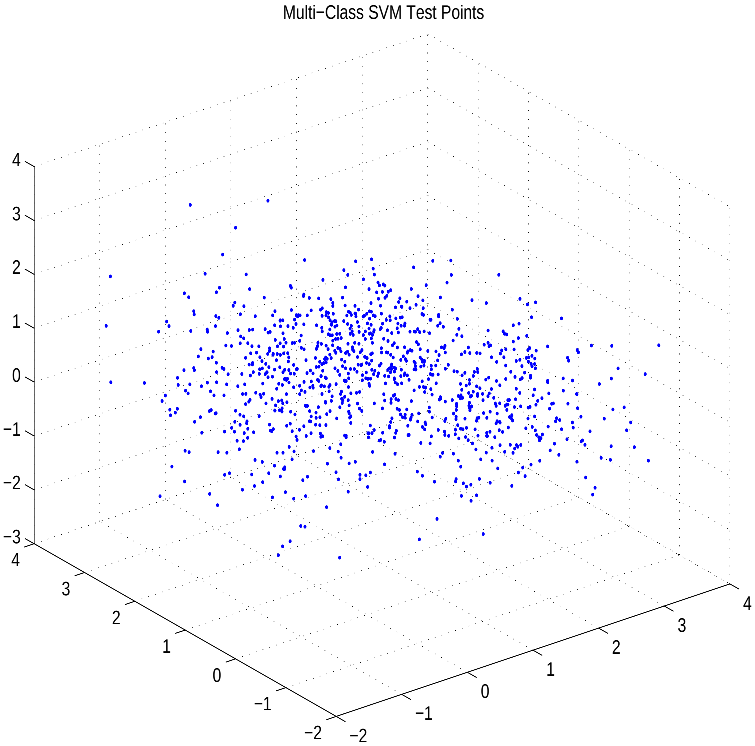

Class 2 covariance
0.5
1
1.5
2
2.5
3
3.5
0.5
1
1.5
2
2.5
3
3.5





These are the SVM parameters - the RBF kernel is used

- allOutlierFraction=0.05
- mixingCoeff=0.3
- smoThresh=1.0/10000.0
- sigma=1

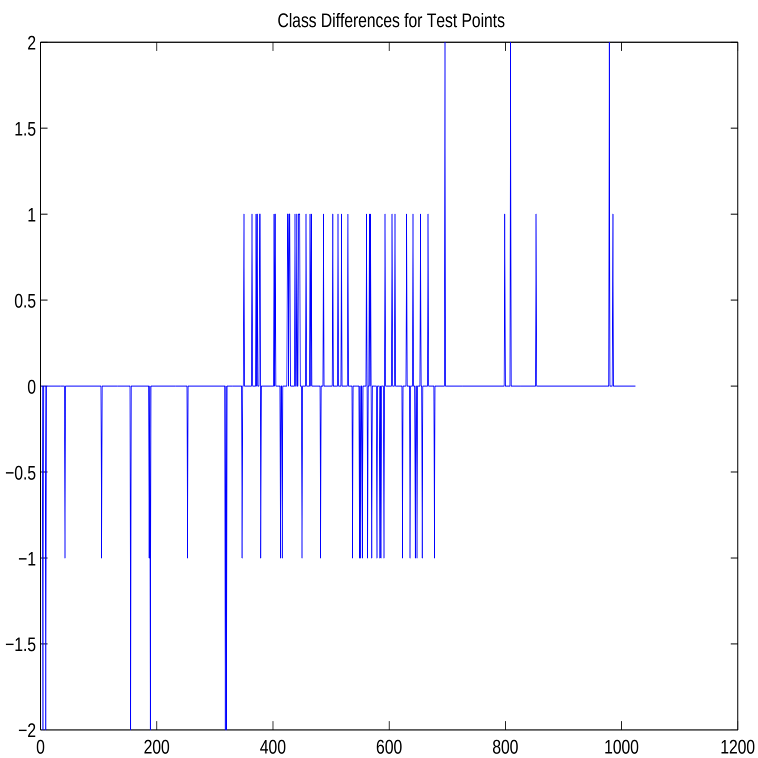


The marginal sample moments (mean var skew kurtosis) for training points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.670	+1.296	+0.431	+2.469
1	+0.704	+1.268	+0.389	+2.408
2	+0.697	+1.291	+0.075	+2.268

The marginal sample moments (mean var skew kurtosis) for test points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.678	+1.216	+0.587	+2.609
1	+0.668	+1.266	+0.444	+2.436
2	+0.721	+1.245	+0.044	+2.256



The error rate for this run is +0.071

QueryPerformanceCounter = +4.962

0.0.20 Semidefinite Programming SDPA

QueryPerformanceCounter = +0.077

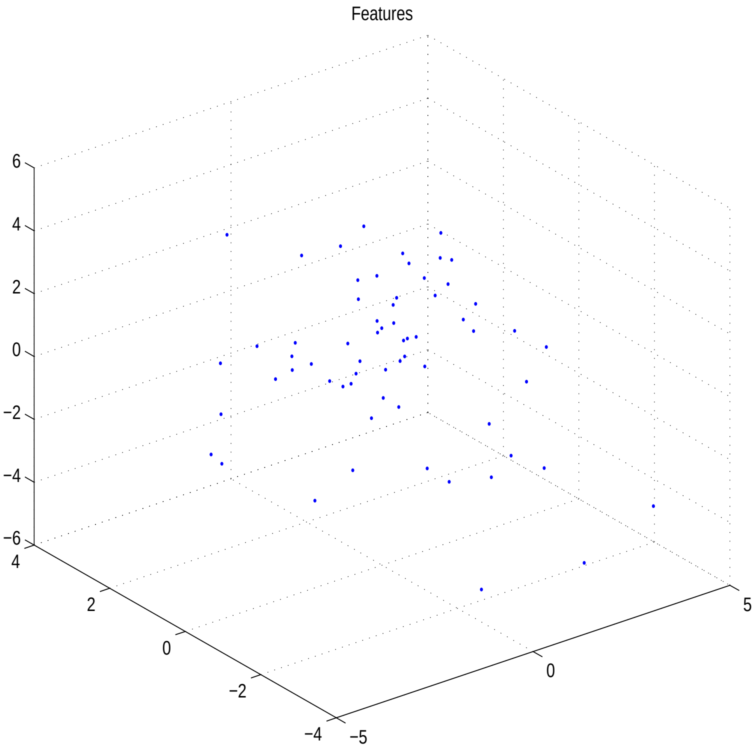
0.0.21 Linear Regression 3x1

0.0.22 3 x 1 Linear Regression

Sample size = 64

Number of features = 3

$$\sigma = \begin{pmatrix} +3.952 & -0.499 & -0.010 \\ -0.499 & +1.895 & +0.465 \\ -0.010 & +0.465 & +4.477 \end{pmatrix}$$



Beta +0.817, +0.999, +0.510

Response -0.381 +1.408 -1.713 +0.259 -1.306 +2.272 +1.266 +3.538 -0.637 +1.549 +0.578 +0.732 +1.308 +3.660 +2.464 +1.218 -1.373 +1.314 +0.704 +4.726 -0.599 +3.010 +0.758 +0.625 -3.797 +3.632 -0.636 -1.660 +1.251 +2.213 +4.689 +2.470 -2.245 +2.095 +2.124 +3.392 +3.957 +3.971 +2.550 +0.851 -5.725 -0.439 +3.625 +3.857 +4.110 -3.120 +2.110 +1.781 -1.776 -0.745 -3.564 +3.439 -3.030 -1.633 +0.022 -2.021 +4.097 -2.208 +0.514 -3.470 -0.888 -0.101 +0.895 +0.274 Estimate for Beta -6277438562204192200000000000000000 -627743856220419220000000000000000000000000000000000000000000000000000000.000 -62774385622041922000000000000000000000000000000000000

Error: +0.000, +0.000, +0.000

QueryPerformanceCounter = +0.721

0.0.23 Matrix Norms

0.0.24 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: +12

$$A = \begin{pmatrix} +0.036 & -0.263 & +0.538 & +0.353 & -0.100 & -0.152 & -0.256 & -0.313 & +0.128 & +0.052 & -0.185 & +0.516 \\ +0.116 & +0.111 & -0.028 & +0.205 & +0.323 & +0.460 & -0.187 & -0.037 & +0.530 & -0.216 & -0.437 & -0.246 \\ -0.547 & -0.032 & -0.027 & +0.383 & -0.062 & +0.144 & -0.217 & -0.150 & -0.306 & +0.440 & -0.127 & -0.394 \\ +0.219 & +0.062 & -0.297 & -0.273 & +0.166 & -0.209 & -0.469 & -0.257 & -0.461 & -0.124 & -0.443 & +0.063 \\ -0.269 & -0.556 & +0.105 & -0.138 & -0.166 & +0.018 & -0.018 & -0.167 & -0.066 & -0.653 & +0.056 & -0.314 \\ +0.018 & -0.335 & +0.079 & +0.203 & +0.525 & -0.329 & -0.230 & +0.623 & -0.061 & +0.005 & +0.069 & -0.086 \\ +0.055 & -0.111 & +0.062 & +0.184 & +0.402 & -0.073 & +0.741 & -0.224 & -0.273 & -0.020 & -0.324 & -0.010 \\ +0.292 & -0.460 & -0.224 & -0.160 & +0.005 & -0.266 & +0.015 & -0.327 & +0.386 & +0.454 & +0.057 & -0.301 \\ +0.417 & -0.027 & +0.549 & -0.204 & -0.301 & +0.154 & +0.026 & +0.270 & -0.228 & +0.172 & -0.273 & -0.371 \\ +0.447 & +0.035 & +0.081 & +0.289 & +0.230 & +0.285 & -0.150 & -0.309 & -0.289 & -0.084 & +0.589 & -0.138 \\ +0.005 & +0.498 & +0.203 & +0.202 & -0.057 & -0.638 & -0.010 & -0.159 & +0.157 & -0.219 & +0.036 & -0.400 \\ -0.321 & +0.139 & +0.448 & -0.571 & +0.493 & +0.061 & -0.079 & -0.213 & +0.073 & +0.154 & +0.154 & -0.016 \end{pmatrix}$$

$Det(A) : A \in O(n) =$ (-1.000,+0.000)

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.491 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.764 & +0.095 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.588 & -0.292 & +0.964 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.033 & +0.622 & +0.009 & -0.415 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.010 & -0.922 & +0.603 & +0.167 & -0.123 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.100 & +0.212 & +0.066 & -0.282 & +0.714 & -0.176 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.818 & -0.017 & +0.117 & -0.579 & +0.726 & -0.589 & +0.023 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.400 & -0.091 & -0.573 & +0.080 & -0.135 & +0.041 & -0.730 & +0.584 & +1.000 & +0.000 & +0.000 & +0.000 \\ -0.535 & +0.883 & -0.663 & -0.410 & +0.213 & -0.003 & -0.155 & +0.503 & -0.046 & +1.000 & +0.000 & +0.000 \\ -0.213 & -0.192 & -0.021 & -0.225 & +0.485 & -0.733 & -0.001 & +0.511 & -0.797 & -0.097 & +1.000 & +0.000 \\ -0.066 & +0.492 & +0.923 & -0.426 & +0.653 & +0.257 & +0.024 & +0.735 & -0.529 & +0.538 & +0.478 & +1.000 \end{pmatrix}$$

$$U = \begin{pmatrix} -0.547 & -0.032 & -0.027 & +0.383 & -0.062 & +0.144 & -0.217 & -0.150 & -0.306 & +0.440 & -0.127 & -0.394 \\ +0.000 & -0.540 & +0.119 & -0.327 & -0.136 & -0.052 & +0.089 & -0.093 & +0.085 & -0.870 & +0.118 & -0.120 \\ +0.000 & +0.000 & +0.517 & +0.120 & -0.335 & +0.269 & -0.148 & +0.165 & -0.469 & +0.590 & -0.381 & -0.660 \\ +0.000 & +0.000 & +0.000 & -1.007 & +0.812 & -0.298 & +0.217 & -0.310 & +0.730 & -0.927 & +0.630 & +0.817 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.947 & -0.418 & -0.201 & +0.545 & +0.183 & +0.170 & +0.256 & +0.321 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.849 & +0.098 & -0.227 & +0.416 & -1.197 & +0.300 & -0.213 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.933 & -0.747 & -0.143 & -0.423 & -0.289 & -0.017 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.145 & +0.055 & -1.164 & +0.894 & -0.271 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.032 & +0.827 & -1.463 & -0.352 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.988 & -0.726 & -0.460 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.905 & -0.683 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.938 \end{pmatrix}$$

$$L * U = \begin{pmatrix} -0.547 & -0.032 & -0.027 & +0.383 & -0.062 & +0.144 & -0.217 & -0.150 & -0.306 & +0.440 & -0.127 & -0.394 \\ -0.269 & -0.556 & +0.105 & -0.138 & -0.166 & +0.018 & -0.018 & -0.167 & -0.066 & -0.653 & +0.056 & -0.314 \\ +0.417 & -0.027 & +0.549 & -0.204 & -0.301 & +0.154 & +0.026 & +0.270 & -0.228 & +0.172 & -0.273 & -0.371 \\ -0.321 & +0.139 & +0.448 & -0.571 & +0.493 & +0.061 & -0.079 & -0.213 & +0.073 & +0.154 & +0.154 & -0.016 \\ +0.018 & -0.335 & +0.079 & +0.203 & +0.525 & -0.329 & -0.230 & +0.623 & -0.061 & +0.005 & +0.069 & -0.086 \\ +0.005 & +0.498 & +0.203 & +0.202 & -0.057 & -0.638 & -0.010 & -0.159 & +0.157 & -0.219 & +0.036 & -0.400 \\ +0.055 & -0.111 & +0.062 & +0.184 & +0.402 & -0.073 & +0.741 & -0.224 & -0.273 & -0.020 & -0.324 & -0.010 \\ +0.447 & +0.035 & +0.081 & +0.289 & +0.230 & +0.285 & -0.150 & -0.309 & -0.289 & -0.084 & +0.589 & -0.138 \\ +0.219 & +0.062 & -0.297 & -0.273 & +0.166 & -0.209 & -0.469 & -0.257 & -0.461 & -0.124 & -0.443 & +0.063 \\ +0.292 & -0.460 & -0.224 & -0.160 & +0.005 & -0.266 & +0.015 & -0.327 & +0.386 & +0.454 & +0.057 & -0.301 \\ +0.116 & +0.111 & -0.028 & +0.205 & +0.323 & +0.460 & -0.187 & -0.037 & +0.530 & -0.216 & -0.437 & -0.246 \\ +0.036 & -0.263 & +0.538 & +0.353 & -0.100 & -0.152 & -0.256 & -0.313 & +0.128 & +0.052 & -0.185 & +0.516 \end{pmatrix}$$

$Det(L) := (+1.000, +0.000) Det(U) := (-1.000, +0.000) Det(LU) := (-1.000, -0.000)$

$||A||_{L_1} = +3.166$

$||A||_{L_\infty} = +3.043$

$||A^{-1}||_{L_1} = +3.043$

$||A^{-1}||_{L_\infty} = +3.166$

$||A||_{L_\infty} * ||A^{-1}||_{L_\infty} = +9.634$

$||A||_{L_1} * ||A^{-1}||_{L_1} = +9.634$

Frobenious Norm $||A||_F$ via $\sum_{i,j=0}^{n} \|A_{i,j}|$ of $A \in O(n)$ +3.464

L_1 condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +9.634

$$A = \begin{pmatrix} +0.036 & -0.263 & +0.538 & +0.353 & -0.100 & -0.152 & -0.256 & -0.313 & +0.128 & +0.052 & -0.185 & +0.516 \\ +0.116 & +0.111 & -0.028 & +0.205 & +0.323 & +0.460 & -0.187 & -0.037 & +0.530 & -0.216 & -0.437 & -0.246 \\ -0.547 & -0.032 & -0.027 & +0.383 & -0.062 & +0.144 & -0.217 & -0.150 & -0.306 & +0.440 & -0.127 & -0.394 \\ +0.219 & +0.062 & -0.297 & -0.273 & +0.166 & -0.209 & -0.469 & -0.257 & -0.461 & -0.124 & -0.443 & +0.063 \\ -0.269 & -0.556 & +0.105 & -0.138 & -0.166 & +0.018 & -0.018 & -0.167 & -0.066 & -0.653 & +0.056 & -0.314 \\ +0.018 & -0.335 & +0.079 & +0.203 & +0.525 & -0.329 & -0.230 & +0.623 & -0.061 & +0.005 & +0.069 & -0.086 \\ +0.055 & -0.111 & +0.062 & +0.184 & +0.402 & -0.073 & +0.741 & -0.224 & -0.273 & -0.020 & -0.324 & -0.010 \\ +0.292 & -0.460 & -0.224 & -0.160 & +0.005 & -0.266 & +0.015 & -0.327 & +0.386 & +0.454 & +0.057 & -0.301 \\ +0.417 & -0.027 & +0.549 & -0.204 & -0.301 & +0.154 & +0.026 & +0.270 & -0.228 & +0.172 & -0.273 & -0.371 \\ +0.447 & +0.035 & +0.081 & +0.289 & +0.230 & +0.285 & -0.150 & -0.309 & -0.289 & -0.084 & +0.589 & -0.138 \\ +0.005 & +0.498 & +0.203 & +0.202 & -0.057 & -0.638 & -0.010 & -0.159 & +0.157 & -0.219 & +0.036 & -0.400 \\ -0.321 & +0.139 & +0.448 & -0.571 & +0.493 & +0.061 & -0.079 & -0.213 & +0.073 & +0.154 & +0.154 & -0.016 \end{pmatrix}$$

L_∞ condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +8.035

Eigenvalues of $A \in O(n)$

(-1.000,+0.000), (-0.835,+0.550), (-0.835,-0.550), (-0.269,+0.963), (-0.269,-0.963), (+0.410,+0.912), (+0.410,-0.912), (+0.136,+0.991), (+0.136,-0.991), (+0.295,+0.956), (+0.295,-0.956), (+1.000,+0.000)

$|\lambda| : \lambda \in \sigma(A), A \in O(n)$

+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix}
+1.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 \\
+0.000 & +1.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\
-0.000 & -0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\
+0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 \\
+0.000 & +0.000 & -0.000 & +0.000 & +1.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\
-0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +1.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\
-0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +1.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 \\
+0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000 & -0.000 & +0.000 & -0.000 & -0.000 \\
+0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\
+0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\
-0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & -0.000 \\
+0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +1.000
\end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix}
+0.036 & +0.116 & -0.547 & +0.219 & -0.269 & +0.018 & +0.055 & +0.292 & +0.417 & +0.447 & +0.005 & -0.321 \\
-0.263 & +0.111 & -0.032 & +0.062 & -0.556 & -0.335 & -0.111 & -0.460 & -0.027 & +0.035 & +0.498 & +0.139 \\
+0.538 & -0.028 & -0.027 & -0.297 & +0.105 & +0.079 & +0.062 & -0.224 & +0.549 & +0.081 & +0.203 & +0.448 \\
+0.353 & +0.205 & +0.383 & -0.273 & -0.138 & +0.203 & +0.184 & -0.160 & -0.204 & +0.289 & +0.202 & -0.571 \\
-0.100 & +0.323 & -0.062 & +0.166 & -0.166 & +0.525 & +0.402 & +0.005 & -0.301 & +0.230 & -0.057 & +0.493 \\
-0.152 & +0.460 & +0.144 & -0.209 & +0.018 & -0.329 & -0.073 & -0.266 & +0.154 & +0.285 & -0.638 & +0.061 \\
-0.256 & -0.187 & -0.217 & -0.469 & -0.018 & -0.230 & +0.741 & +0.015 & +0.026 & -0.150 & -0.010 & -0.079 \\
-0.313 & -0.037 & -0.150 & -0.257 & -0.167 & +0.623 & -0.224 & -0.327 & +0.270 & -0.309 & -0.159 & -0.213 \\
+0.128 & +0.530 & -0.306 & -0.461 & -0.066 & -0.061 & -0.273 & +0.386 & -0.228 & -0.289 & +0.157 & +0.073 \\
+0.052 & -0.216 & +0.440 & -0.124 & -0.653 & +0.005 & -0.020 & +0.454 & +0.172 & -0.084 & -0.219 & +0.154 \\
-0.185 & -0.437 & -0.127 & -0.443 & +0.056 & +0.069 & -0.324 & +0.057 & -0.273 & +0.589 & +0.036 & +0.154 \\
+0.516 & -0.246 & -0.394 & +0.063 & -0.314 & -0.086 & -0.010 & -0.301 & -0.371 & -0.138 & -0.400 & -0.016
\end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix}
+1.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\
-0.000 & +1.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\
-0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 \\
+0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 \\
+0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\
-0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\
+0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 \\
+0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +1.000 & -0.000 & +0.000 & +0.000 & -0.000 \\
-0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & -0.000 & -0.000 & +0.000 \\
-0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 \\
+0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +1.000 & -0.000 \\
-0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +1.000
\end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix}
+0.074 & -0.201 & -0.009 & -0.215 & -0.308 & +0.518 & -0.173 & -0.152 & -0.534 & +0.254 & +0.326 & +0.188 \\
-0.315 & -0.278 & +0.385 & -0.559 & +0.184 & +0.128 & +0.266 & +0.005 & +0.097 & -0.456 & +0.098 & +0.126 \\
-0.135 & +0.242 & +0.065 & +0.164 & +0.381 & +0.599 & +0.071 & -0.140 & +0.479 & +0.317 & +0.172 & +0.064 \\
-0.295 & +0.269 & +0.251 & -0.252 & +0.452 & -0.146 & -0.287 & +0.004 & -0.410 & +0.357 & -0.324 & -0.076 \\
-0.346 & -0.131 & -0.297 & -0.122 & -0.347 & +0.319 & -0.094 & -0.014 & +0.182 & +0.007 & -0.703 & +0.025 \\
-0.495 & -0.314 & -0.347 & +0.149 & +0.174 & -0.163 & -0.432 & -0.332 & +0.044 & -0.140 & +0.331 & -0.174 \\
+0.044 & -0.329 & -0.178 & +0.095 & +0.229 & -0.094 & +0.641 & -0.486 & -0.216 & +0.239 & -0.187 & -0.048 \\
-0.031 & -0.060 & -0.444 & +0.030 & +0.335 & -0.056 & +0.029 & +0.369 & -0.101 & -0.031 & +0.006 & +0.731 \\
-0.036 & +0.263 & -0.538 & -0.353 & +0.100 & +0.152 & +0.256 & +0.313 & -0.128 & -0.052 & +0.185 & -0.516 \\
+0.632 & -0.276 & -0.103 & -0.174 & +0.411 & +0.201 & -0.367 & -0.112 & +0.076 & -0.210 & -0.234 & -0.151 \\
+0.150 & +0.191 & -0.192 & -0.585 & -0.162 & -0.341 & -0.075 & -0.358 & +0.368 & +0.293 & +0.121 & +0.224 \\
+0.011 & +0.585 & -0.105 & +0.085 & -0.002 & +0.121 & +0.017 & -0.487 & -0.241 & -0.538 & -0.083 & +0.181
\end{pmatrix}$$

$$S = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

$$V = \begin{pmatrix} +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -1.000 & +0.000 & +0.000 & -0.000 \\ -0.407 & -0.048 & +0.528 & -0.065 & +0.377 & -0.025 & -0.150 & -0.158 & -0.000 & +0.533 & -0.235 & +0.137 \\ +0.197 & -0.387 & +0.141 & +0.416 & +0.271 & +0.408 & +0.148 & -0.320 & -0.000 & -0.327 & -0.364 & -0.126 \\ +0.108 & -0.193 & -0.470 & +0.469 & +0.189 & +0.125 & +0.048 & +0.166 & +0.000 & +0.511 & +0.186 & +0.364 \\ +0.037 & +0.580 & -0.393 & -0.283 & +0.155 & +0.443 & +0.031 & -0.330 & +0.000 & +0.124 & -0.283 & +0.039 \\ -0.322 & -0.012 & -0.167 & +0.239 & -0.347 & +0.171 & -0.177 & +0.351 & +0.000 & +0.216 & -0.406 & -0.544 \\ -0.236 & +0.145 & +0.000 & +0.113 & -0.096 & -0.265 & +0.844 & +0.080 & +0.000 & -0.008 & -0.296 & +0.158 \\ -0.071 & -0.193 & +0.017 & -0.182 & -0.013 & +0.224 & +0.413 & -0.264 & +0.000 & +0.308 & +0.513 & -0.525 \\ +0.209 & +0.000 & -0.244 & +0.165 & +0.011 & -0.658 & -0.129 & -0.525 & +0.000 & +0.183 & -0.181 & -0.279 \\ +0.412 & -0.193 & +0.219 & -0.162 & -0.660 & +0.161 & +0.040 & -0.166 & +0.000 & +0.334 & -0.214 & +0.265 \\ -0.635 & -0.203 & -0.230 & +0.078 & -0.341 & +0.071 & -0.137 & -0.458 & +0.000 & -0.211 & +0.132 & +0.292 \\ +0.040 & +0.580 & +0.370 & +0.600 & -0.206 & +0.091 & -0.041 & -0.156 & +0.000 & +0.019 & +0.294 & -0.026 \end{pmatrix}$$

$$USV = \begin{pmatrix} -0.277 & -0.132 & +0.207 & +0.136 & -0.651 & +0.371 & -0.265 & +0.368 & -0.074 & -0.264 & +0.002 & -0.039 \\ -0.193 & +0.257 & -0.023 & +0.097 & +0.095 & +0.009 & +0.232 & -0.207 & +0.315 & -0.663 & -0.229 & -0.436 \\ -0.130 & +0.124 & -0.256 & +0.237 & -0.289 & +0.016 & -0.186 & -0.297 & +0.135 & +0.484 & -0.606 & -0.143 \\ +0.308 & +0.113 & +0.367 & -0.424 & +0.156 & +0.614 & -0.101 & -0.073 & +0.295 & +0.072 & -0.251 & +0.079 \\ +0.378 & +0.084 & +0.157 & -0.017 & -0.076 & -0.376 & -0.110 & +0.540 & +0.346 & +0.172 & -0.015 & -0.474 \\ -0.005 & +0.085 & -0.513 & -0.179 & -0.039 & -0.069 & -0.514 & -0.001 & +0.495 & -0.202 & +0.217 & +0.308 \\ +0.201 & +0.351 & -0.196 & -0.053 & -0.228 & -0.083 & +0.456 & +0.362 & -0.044 & -0.136 & -0.357 & +0.498 \\ -0.070 & +0.726 & +0.062 & +0.089 & -0.207 & +0.167 & +0.123 & -0.128 & +0.031 & +0.255 & +0.530 & -0.084 \\ -0.565 & -0.031 & -0.043 & -0.734 & -0.103 & -0.156 & +0.195 & +0.101 & +0.036 & +0.185 & -0.030 & -0.138 \\ +0.190 & +0.292 & -0.342 & -0.258 & +0.116 & +0.118 & -0.341 & +0.108 & -0.632 & -0.149 & -0.121 & -0.323 \\ +0.098 & +0.195 & +0.493 & -0.188 & -0.262 & -0.512 & -0.302 & -0.376 & -0.150 & -0.193 & -0.108 & +0.197 \\ -0.470 & +0.315 & +0.254 & +0.228 & +0.514 & -0.058 & -0.294 & +0.354 & -0.011 & +0.062 & -0.186 & +0.212 \end{pmatrix}$$

0.0.25 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +56267.800 L_infty condition number of Wishart Matrix +56267.800

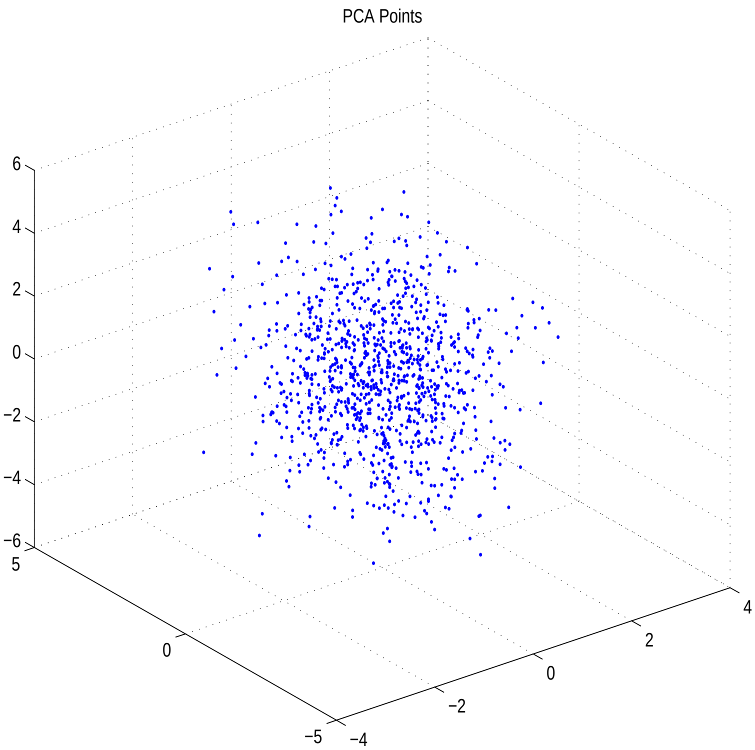
0.0.26 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +470.231 L_∞ condition number of GOE Matrix +470.231

0.0.27 The Identity Matrix $I \in M(n)$

L_1 condition number of $I = +1.000$ L_∞ condition number of $I = +1.000$ QueryPerformanceCounter = +1.436

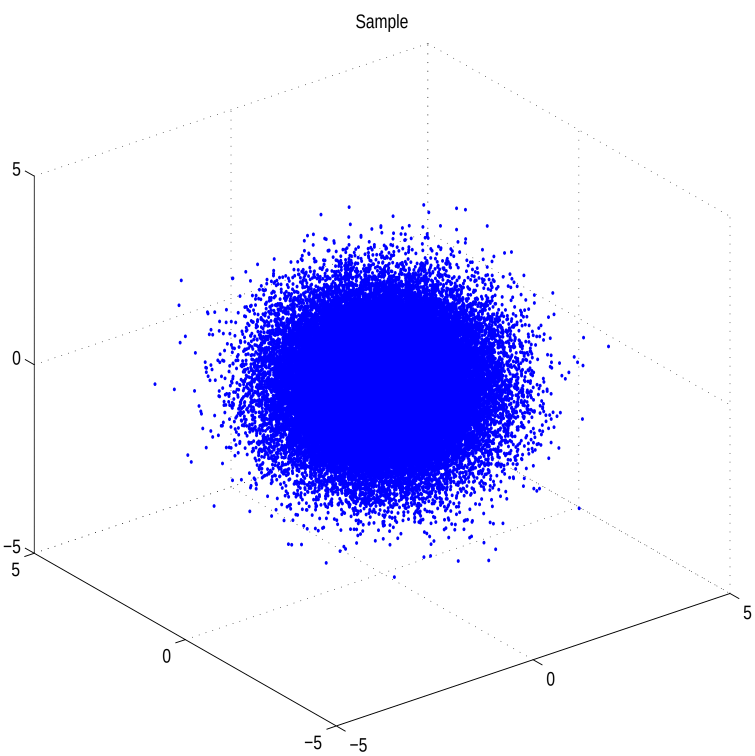
0.0.28 Principal Components Matlab

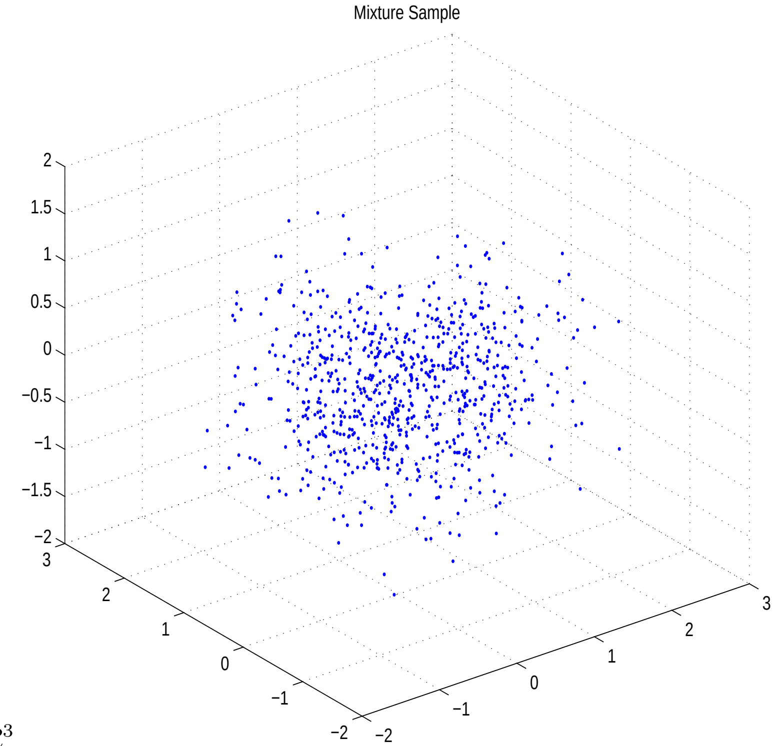


The eigenvectors: +0.067, +0.331, +0.941 +0.132, +0.932, -0.338 -0.989, +0.147, +0.019
All of the eigenvalues of the covariance matrix: (+0.958,+0.000), (+2.025,+0.000), (+3.017,+0.000)
QueryPerformanceCounter = +0.743

0.0.29 Multi Variate Random Number Generator

Sample from $N(\mu, \Sigma)$ mean= -0.002, variance=+1.004, skewness=+0.006, kurtosis=+3.003 mean= -0.001, variance=+1.017, skewness=-0.005, kurtosis=+2.988 mean= -0.002, variance=+1.006, skewness=-0.016, kurtosis=+3.014 Covariance Matrix +1.004, +0.009, +0.003 +0.009, +1.017, -0.003 +0.003, -0.003, +1.006





Generate a sample from a unifom mixture of three Gaussians in R^3
QueryPerformanceCounter = +15.113

0.0.30 Matrix Multiply

Comparing naive matrix multiply verus Intel MKL dgemm for matrix of size +2048. This is for type double (hence the d in dgemm). Naive type double matrix multiply tic toc = +2.165 dgemm plus row to column major transpose operation tic toc = +1.649 Comparing naive matrix multiply verus Intel MKL sgemm for matrix of size +2048. This is for type float (hence the s in dgemm). Naive type float matrix multiply tic toc = +1.805 sgemm plus row to column major transpose operation tic toc = +1.357 QueryPerformanceCounter = +7.835

0.0.31 Descriptive Statistics

Mean N(0,1): +0.003 Variance N(0,1): +1.006 Mean N(0,1) [recurrence relation method] :+0.003 Variance [recurrence relation method] :+1.006 Skewness : +0.007 Kurtosis : +2.997 QueryPerformanceCounter = +0.034

0.0.32 Time Series

+0.093 +0.726 +0.011 +2.178 QueryPerformanceCounter = +0.128