Regression of KL Software Distribution

KL Software Libraries

Tue Apr 22 22:00:00 2014

KL Library test output. This LaTex file and the associated diagrams are produced by the KL software libraries.

0.0.1 Gram Matrix Consistency Check

Sample Size = 4096 Feature dim = 3

$$Sigma$$

$$= \begin{pmatrix} +1.140 & +1.535 & +0.581 \\ +1.535 & +9.988 & +1.605 \\ +0.581 & +1.605 & +0.428 \end{pmatrix}$$

$$SampleCovariance = \begin{pmatrix} +1.139 & +1.522 & +0.578 \\ +1.522 & +9.722 & +1.575 \\ +0.578 & +1.575 & +0.424 \end{pmatrix}$$

$$SampleMean = \begin{pmatrix} +1.01386 & +0.99111 & +1.00460 \end{pmatrix}$$

$$SampleCovariance - \text{Omega} = \begin{pmatrix} -0.001 & -0.013 & -0.003 \\ -0.013 & -0.267 & -0.031 \\ -0.003 & -0.031 & -0.004 \end{pmatrix}$$

$$SampleCovarianceEigs = \begin{pmatrix} (+10.26072, +0.00000) & (+0.98334, +0.00000) & (+0.04047, +0.00000) \end{pmatrix}$$

$$CenteredMean = \begin{pmatrix} +0.00000 & +0.00000 & +0.00000 \end{pmatrix}$$

$$CenteredCovariance = \begin{pmatrix} +1.139 & +1.522 & +0.578 \\ +1.522 & +9.722 & +1.575 \\ +0.578 & +1.575 & +0.424 \end{pmatrix}$$

$$GramMatrixGfNotscaledbysamplesize = \begin{pmatrix} +4665.379 & +6232.575 & +2367.225 \\ +6232.575 & +39820.325 & +6448.871 \\ +2367.225 & +6448.871 & +1735.706 \end{pmatrix}$$

$$GramMatrixGfscaledbysamplesize = \begin{pmatrix} +1.139 & +1.522 & +0.578 \\ +1.522 & +9.722 & +1.574 \\ +0.578 & +1.574 & +0.424 \end{pmatrix}$$

$$SampleCovariance - ScaledGf = \begin{pmatrix} +0.000 & +0.000 & +0.000 \\ +0.000 & +0.002 & +0.000 \\ +0.000 & +0.000 & +0.000 \end{pmatrix}$$

$$EigenDecompofSampleCovariance = \begin{pmatrix} -0.173 & -0.971 & -0.166 \\ +0.918 & -0.219 & +0.331 \\ -0.357 & -0.095 & +0.929 \end{pmatrix}$$

$$EigenDecompofGramMatrix = \begin{pmatrix} -0.117 & -0.975 & -0.188 \\ -0.303 & +0.215 & -0.928 \\ +0.946 & -0.051 & -0.321 \end{pmatrix}$$

QueryPerformanceCounter = +64.019

0.0.2 Solver

0.0.3 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: 8

$$A = \begin{pmatrix} +0.450 & +0.200 & -0.489 & -0.392 & +0.200 & -0.220 & +0.216 & +0.480 \\ +0.267 & +0.436 & +0.434 & +0.218 & +0.142 & -0.347 & +0.537 & -0.271 \\ +0.447 & -0.285 & +0.297 & -0.415 & +0.473 & +0.072 & -0.321 & -0.356 \\ +0.254 & +0.436 & +0.187 & -0.318 & -0.673 & -0.051 & -0.387 & -0.058 \\ +0.396 & -0.165 & -0.576 & +0.358 & -0.213 & -0.112 & -0.032 & -0.544 \\ -0.116 & -0.567 & +0.140 & -0.236 & -0.296 & -0.692 & +0.148 & +0.034 \\ +0.509 & -0.386 & +0.284 & +0.235 & -0.301 & +0.410 & +0.261 & +0.360 \\ +0.179 & +0.037 & +0.129 & +0.535 & +0.199 & -0.404 & -0.567 & +0.373 \end{pmatrix}$$

$Det(A) : A \in O(n) =$ (-1.000,+0.000)

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.227 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.777 & -0.205 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.884 & -0.827 & +0.756 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.500 & -0.960 & -0.320 & +0.662 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.524 & -0.975 & -0.642 & -0.007 & +0.086 & +1.000 & +0.000 & +0.000 \\ +0.878 & -0.082 & -0.085 & +0.730 & -0.539 & +0.145 & +1.000 & +0.000 \\ +0.352 & -0.264 & -0.110 & -0.491 & -0.296 & +0.865 & +0.955 & +1.000 \end{pmatrix}$$

$$U = \begin{pmatrix} +0.509 & -0.386 & +0.284 & +0.235 & -0.301 & +0.410 & +0.261 & +0.360 \\ +0.000 & -0.654 & +0.204 & -0.182 & -0.364 & -0.599 & +0.208 & +0.116 \\ +0.000 & +0.000 & -0.755 & +0.138 & -0.054 & -0.553 & -0.192 & -0.800 \\ +0.000 & +0.000 & +0.000 & -0.855 & +0.205 & -0.659 & +0.302 & +0.862 \\ +0.000 & +0.000 & +0.000 & +0.000 & -1.026 & -0.572 & -0.580 & -0.954 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.457 & +0.532 & -0.772 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.160 & -1.762 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.681 \end{pmatrix}$$

$$L * U = \begin{pmatrix} +0.509 & -0.386 & +0.284 & +0.235 & -0.301 & +0.410 & +0.261 & +0.360 \\ -0.116 & -0.567 & +0.140 & -0.236 & -0.296 & -0.692 & +0.148 & +0.034 \\ +0.396 & -0.165 & -0.576 & +0.358 & -0.213 & -0.112 & -0.032 & -0.544 \\ +0.450 & +0.200 & -0.489 & -0.392 & +0.200 & -0.220 & +0.216 & +0.480 \\ +0.254 & +0.436 & +0.187 & -0.318 & -0.673 & -0.051 & -0.387 & -0.058 \\ +0.267 & +0.436 & +0.434 & +0.218 & +0.142 & -0.347 & +0.537 & -0.271 \\ +0.447 & -0.285 & +0.297 & -0.415 & +0.473 & +0.072 & -0.321 & -0.356 \\ +0.179 & +0.037 & +0.129 & +0.535 & +0.199 & -0.404 & -0.567 & +0.373 \end{pmatrix}$$

$Det(L) := (+1.000, +0.000) Det(U) := (+1.000, +0.000) Det(LU) := (+1.000, +0.000)$

$||A||_{L_1} = +2.707$

$||A||_{L_\infty} = +2.745$

$||A^{-1}||_{L_1} = +2.745$

$||A^{-1}||_{L_\infty} = +2.707$

$||A||_{L_\infty} * ||A^{-1}||_{L_\infty} = +7.431$

$||A||_{L_1} * ||A^{-1}||_{L_1} = +7.431$

Frobenious Norm $||A||_F$ via $\sum_{i,j=0}^{n} \|A_{i,j}|$ of $A \in O(n)$ +2.828

L_1 condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +6.033

$$A = \begin{pmatrix} +0.450 & +0.200 & -0.489 & -0.392 & +0.200 & -0.220 & +0.216 & +0.480 \\ +0.267 & +0.436 & +0.434 & +0.218 & +0.142 & -0.347 & +0.537 & -0.271 \\ +0.447 & -0.285 & +0.297 & -0.415 & +0.473 & +0.072 & -0.321 & -0.356 \\ +0.254 & +0.436 & +0.187 & -0.318 & -0.673 & -0.051 & -0.387 & -0.058 \\ +0.396 & -0.165 & -0.576 & +0.358 & -0.213 & -0.112 & -0.032 & -0.544 \\ -0.116 & -0.567 & +0.140 & -0.236 & -0.296 & -0.692 & +0.148 & +0.034 \\ +0.509 & -0.386 & +0.284 & +0.235 & -0.301 & +0.410 & +0.261 & +0.360 \\ +0.179 & +0.037 & +0.129 & +0.535 & +0.199 & -0.404 & -0.567 & +0.373 \end{pmatrix}$$

L_∞ condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +6.798

Eigenvalues of $A \in O(n)$

(+0.616,+0.788), (+0.616,-0.788), (+0.261,+0.965), (+0.261,-0.965), (-0.580,+0.814), (-0.580,-0.814), (-1.000,+0.000), (+1.000,+0.000)

$|\lambda| : \lambda \in \sigma(A), A \in O(n)$

+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000

Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & -0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & +0.000 & +1.000 & -0.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +1.000 & +0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} +0.450 & +0.267 & +0.447 & +0.254 & +0.396 & -0.116 & +0.509 & +0.179 \\ +0.200 & +0.436 & -0.285 & +0.436 & -0.165 & -0.567 & -0.386 & +0.037 \\ -0.489 & +0.434 & +0.297 & +0.187 & -0.576 & +0.140 & +0.284 & +0.129 \\ -0.392 & +0.218 & -0.415 & -0.318 & +0.358 & -0.236 & +0.235 & +0.535 \\ +0.200 & +0.142 & +0.473 & -0.673 & -0.213 & -0.296 & -0.301 & +0.199 \\ -0.220 & -0.347 & +0.072 & -0.051 & -0.112 & -0.692 & +0.410 & -0.404 \\ +0.216 & +0.537 & -0.321 & -0.387 & -0.032 & +0.148 & +0.261 & -0.567 \\ +0.480 & -0.271 & -0.356 & -0.058 & -0.544 & +0.034 & +0.360 & +0.373 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & +1.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & +1.000 & -0.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & -0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 \\ -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +1.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} -0.450 & -0.200 & +0.489 & +0.392 & -0.200 & +0.220 & -0.216 & -0.480 \\ -0.261 & -0.446 & +0.242 & +0.048 & +0.043 & -0.538 & -0.249 & +0.564 \\ -0.292 & +0.545 & +0.238 & +0.272 & -0.082 & +0.304 & +0.243 & +0.574 \\ -0.265 & +0.509 & +0.024 & -0.404 & -0.481 & -0.446 & -0.211 & -0.178 \\ +0.631 & +0.040 & +0.692 & -0.230 & -0.145 & +0.112 & -0.161 & +0.093 \\ +0.115 & -0.369 & -0.213 & +0.066 & -0.825 & +0.132 & +0.279 & +0.160 \\ -0.386 & -0.241 & +0.033 & -0.720 & +0.076 & +0.499 & -0.048 & +0.126 \\ +0.117 & +0.099 & -0.346 & +0.176 & -0.116 & +0.303 & -0.825 & +0.199 \end{pmatrix}$$

$$S = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

$$V = \begin{pmatrix} -1.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 \\ -0.000 & -0.243 & +0.180 & +0.095 & +0.265 & -0.265 & -0.573 & -0.656 \\ -0.000 & -0.076 & -0.627 & -0.217 & +0.530 & -0.461 & +0.247 & +0.009 \\ +0.000 & -0.169 & +0.033 & +0.726 & +0.530 & +0.239 & -0.033 & +0.323 \\ +0.000 & -0.400 & -0.582 & -0.092 & -0.265 & +0.318 & -0.529 & +0.202 \\ +0.000 & +0.648 & -0.437 & +0.255 & +0.000 & +0.350 & -0.015 & -0.450 \\ +0.000 & +0.023 & +0.192 & -0.577 & +0.530 & +0.589 & -0.049 & -0.020 \\ +0.000 & +0.572 & +0.081 & -0.104 & +0.132 & -0.301 & -0.572 & +0.470 \end{pmatrix}$$

$$USV = \begin{pmatrix} +0.450 & -0.112 & -0.390 & +0.409 & +0.288 & -0.048 & +0.609 & -0.098 \\ +0.261 & +0.032 & -0.023 & -0.116 & -0.034 & -0.473 & -0.011 & +0.832 \\ +0.292 & +0.368 & -0.034 & +0.082 & +0.641 & -0.139 & -0.564 & -0.156 \\ +0.265 & -0.260 & +0.483 & -0.180 & -0.075 & -0.622 & +0.101 & -0.440 \\ -0.631 & +0.156 & -0.422 & -0.188 & +0.221 & -0.514 & +0.186 & -0.127 \\ -0.115 & +0.608 & +0.559 & -0.009 & +0.212 & +0.112 & +0.486 & +0.105 \\ +0.386 & +0.541 & -0.349 & -0.418 & -0.457 & +0.009 & +0.053 & -0.223 \\ -0.117 & +0.310 & +0.034 & +0.755 & -0.444 & -0.301 & -0.164 & -0.061 \end{pmatrix}$$

Calculating first few eigenvectors of $A \in O(n)$ using LAPACK syevx

0.0.4 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +1489.694 L_infty condition number of Wishart Matrix +1489.694

0.0.5 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +66.900 L_∞ condition number of GOE Matrix +66.900

0.0.6 The Identity Matrix $I \in M(n)$

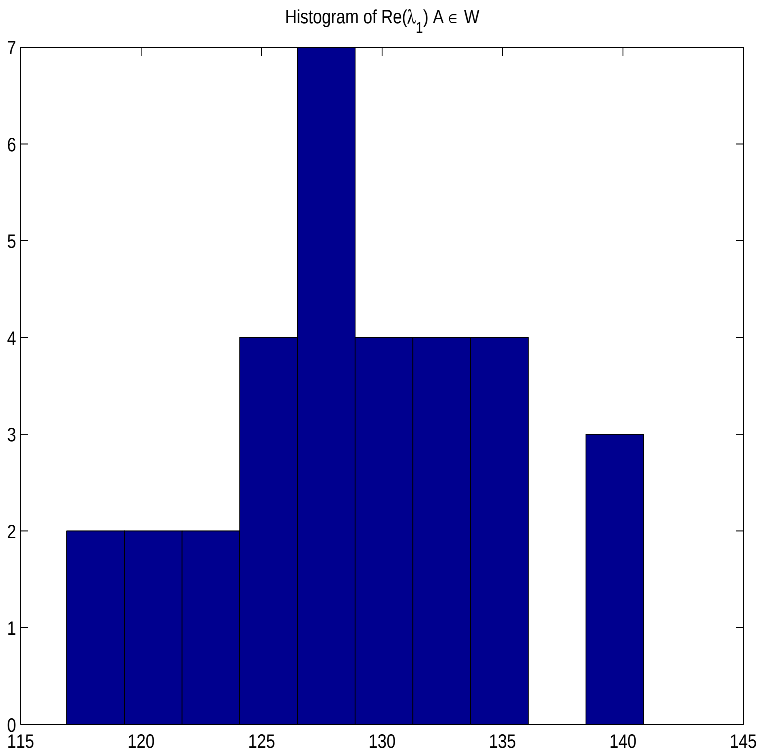
L_1 condition number of I = +1.000 L_∞ condition number of I = +1.000 QueryPerformanceCounter = +2.038

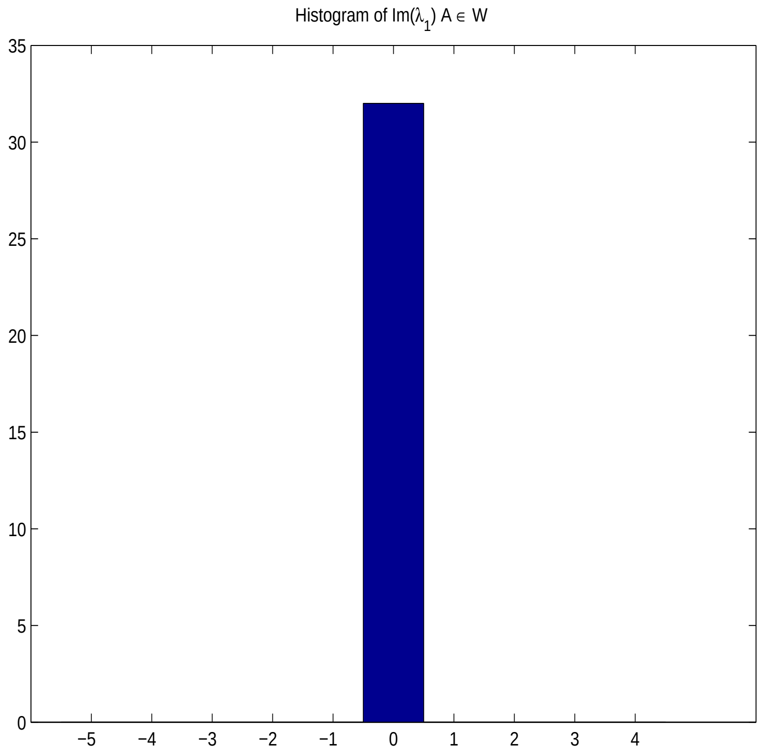
0.0.7 Generate Tracey Widom Sample

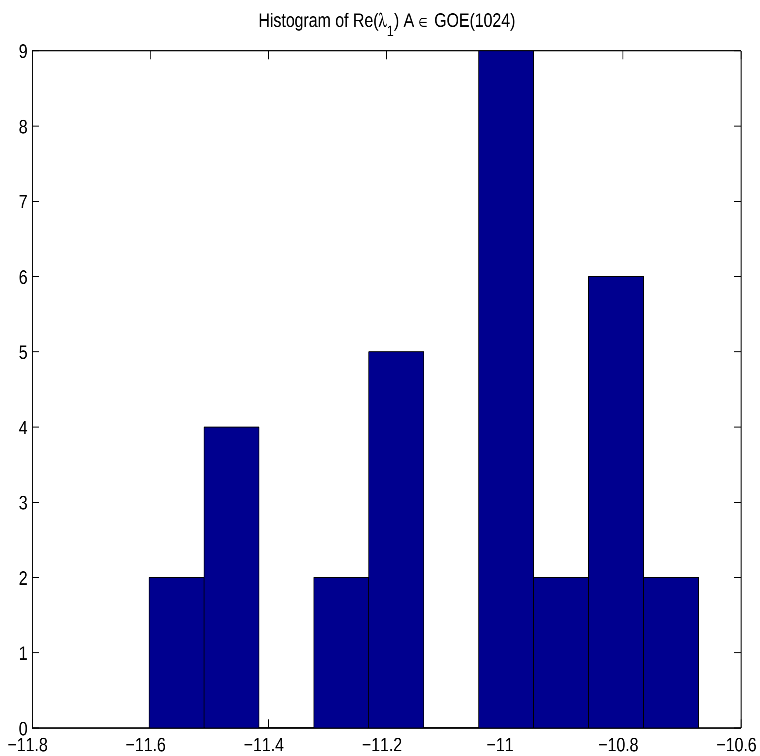
0.0.8 Sample from $W_n m$ times and calculate empirical PDF of the first eig

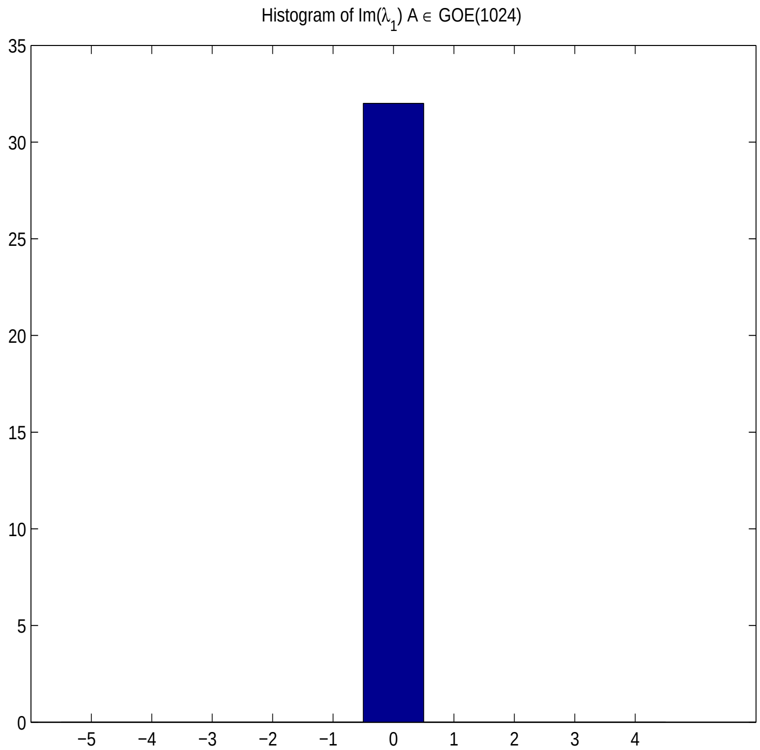
Here we generate histograms of λ_1 for GOE (Gaussian Orthogonal Ensemble), and W (Wishart) distributed of random matrices These should approximate the celebrated Tracy Widom distribution. Dimension $n = 128$

Sample size $m = 32$




Histogram of Im(λ_1) A ∈ W
35
30
25
20
15
10
5
0
−5
−4
−3
−2
−1
0
1
2
3
4


Histogram of Re(λ_1) A ∈ GOE(1024)
9
8
7
6
5
4
3
2
1
0
−11.8
−11.6
−11.4
−11.2
−11
−10.8
−10.6

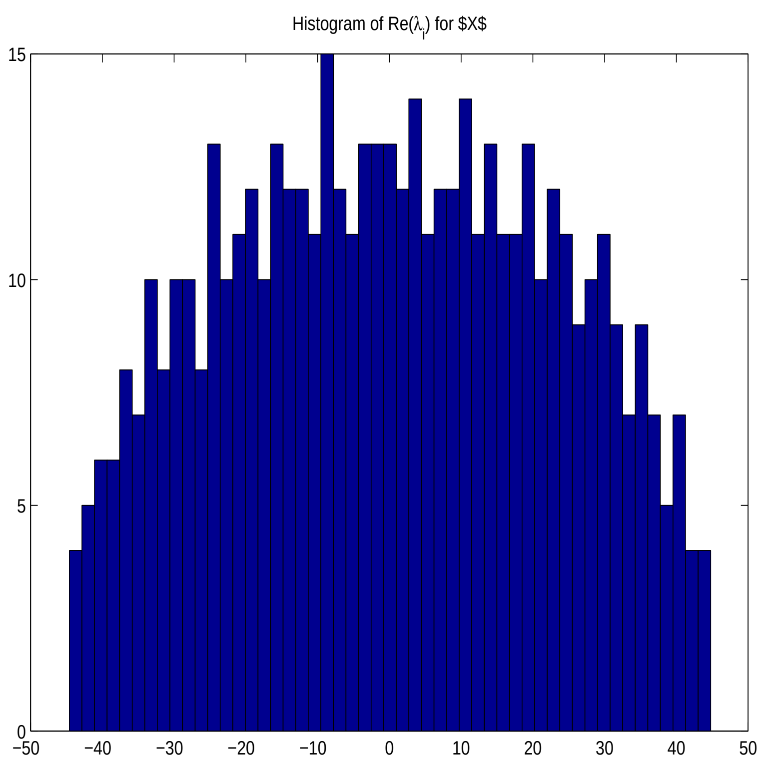


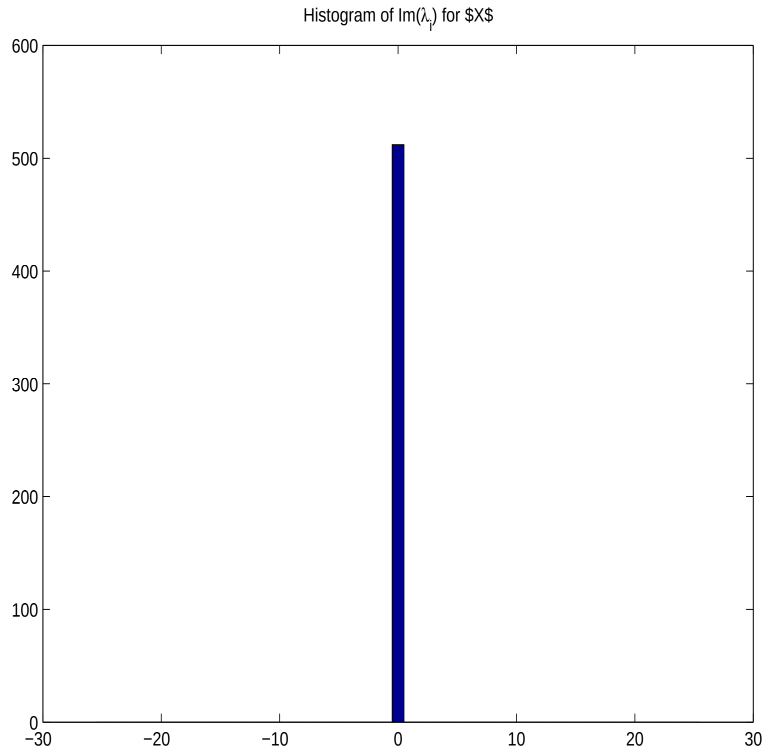
QueryPerformanceCounter = +12.031

0.0.9 Approximate Winger Distribution

0.0.10 Verfy Winger Law.

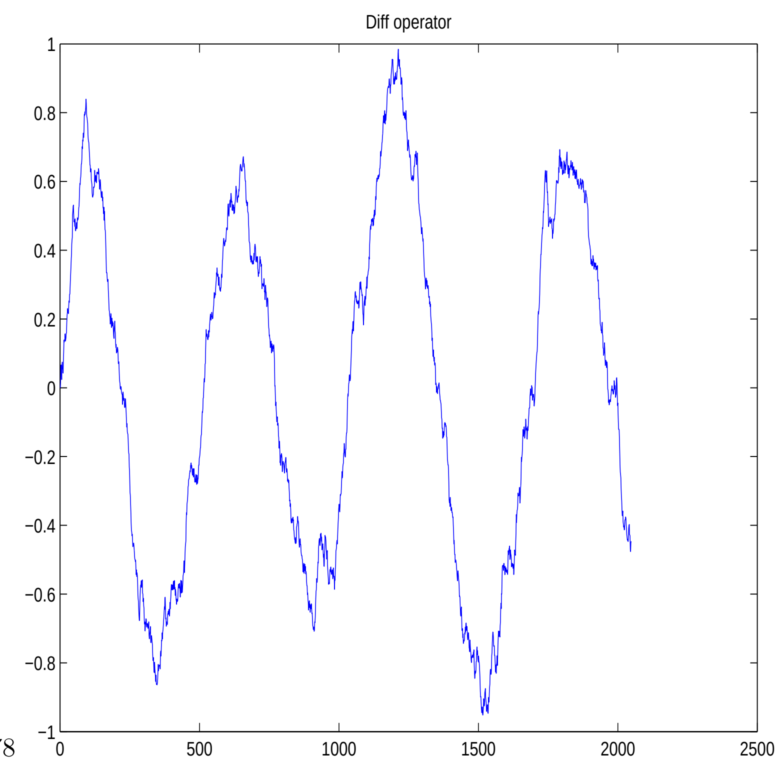
Let $M_n = [X_{ij}]$ a symmetric n x n matrix with Random entries such that $X_{i,j} = X_{j,i}$, and $X_{i,j}$ are iid
orall i ¡ j,andXjj$areiid$
orall j : ; E[X$^2_{ij}$] = $1, E[X_{ij}] = 0$ and that all moments exists for each of the entries. The eigenvector of this random matrix; $lambda_1 leq...leqlambda_n$ depends continuously on Mn. Dimension $n = 512$





QueryPerformanceCounter = +2.693

0.0.11 Iterated Exponential Filtering



$\mu_1 = +0.093$ $\mu_2 = +0.726$ $\mu_3 = +0.011$ $\mu_4 = +2.178$

QueryPerformanceCounter = +1.601

0.0.12 Matrix Exponential

$$SPDMatrix = \begin{pmatrix} +10.539 & -0.499 & -0.010 & +0.368 & +0.465 & -0.492 & -0.126 & +0.437 \\ -0.499 & +7.286 & +0.365 & -0.481 & -0.337 & -0.466 & +0.279 & +0.056 \\ -0.010 & +0.365 & +6.705 & -0.205 & +0.467 & +0.131 & +0.077 & -0.089 \\ +0.368 & -0.481 & -0.205 & +6.496 & -0.402 & -0.209 & +0.043 & -0.041 \\ +0.465 & -0.337 & +0.467 & -0.402 & +4.578 & +0.272 & +0.289 & -0.285 \\ -0.492 & -0.466 & +0.131 & -0.209 & +0.272 & +8.181 & +0.343 & -0.244 \\ -0.126 & +0.279 & +0.077 & +0.043 & +0.289 & +0.343 & +5.938 & -0.212 \\ +0.437 & +0.056 & -0.089 & -0.041 & -0.285 & -0.244 & -0.212 & +9.691 \end{pmatrix}$$

$$SPDEigs = \begin{pmatrix} (+10.93611, +0.00000) & (+9.60778, +0.00000) & (+4.23666, +0.00000) & (+8.36911, +0.00000) & (+7.56229, +0.00000)$$

$$exp(SPD) = \begin{pmatrix} +47863.969 & -6460.093 & -1078.770 & +4706.958 & +2535.224 & -8475.398 & -2406.368 & +12977.552 \\ -6460.093 & +2780.574 & +516.920 & -1069.918 & -548.083 & -109.707 & +386.466 & -807.216 \\ -1078.770 & +516.920 & +1015.281 & -385.755 & +176.069 & +458.541 & +212.284 & -859.022 \\ +4706.958 & -1069.918 & -385.755 & +1267.210 & +111.181 & -1018.272 & -287.809 & +1036.628 \\ +2535.224 & -548.083 & +176.069 & +111.181 & +413.265 & +135.193 & +45.490 & -502.411 \\ -8475.398 & -109.707 & +458.541 & -1018.272 & +135.193 & +5613.026 & +968.003 & -4270.737 \\ -2406.368 & +386.466 & +212.284 & -287.809 & +45.490 & +968.003 & +632.432 & -1645.725 \\ +12977.552 & -807.216 & -859.022 & +1036.628 & -502.411 & -4270.737 & -1645.725 & +19362.944 \end{pmatrix}$$

$$exp(SPD)eigs = \begin{pmatrix} (+56168.17045, +0.00000) & (+14880.07985, +0.00000) & (+4311.77579, +0.00000) & (+1924.25027, +0.00000) & (+6$$

$$log(exp(SPD)eigs) = \begin{pmatrix} (+10.93611, +0.00000) & (+9.60778, +0.00000) & (+8.36911, +0.00000) & (+7.56229, +0.00000) & (+4.23666, +0$$

$$exp(Id) = \begin{pmatrix} +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +2.718 \end{pmatrix}$$

$$exp(Id)eigs = \begin{pmatrix} (+2.71828, +0.00000) & (+2.71828, +0.00000) & (+2.71828, +0.00000) & (+2.71828, +0.00000) & (+2.71828, +0.00000)$$

$$log(exp(Id)eigs) = \begin{pmatrix} (+1.00000, +0.00000) & (+1.00000, +0.00000) & (+1.00000, +0.00000) & (+1.00000, +0.00000) & (+1.00000, +0.000$$

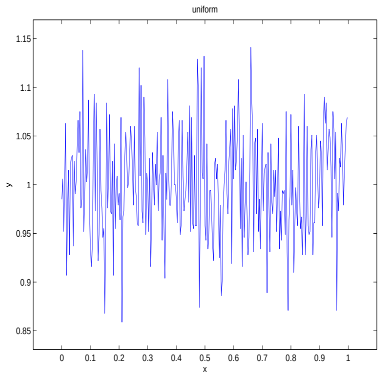
For $n \in \mathbb{Z}[16, 128)$ we calculate $|(SPD(n)Eigs - log(exp(SPD(n))eigs)|_{l^2}$
$|(SPD(n)Eigs - log(exp(SPD(n))eigs)|_{l^2} = \begin{pmatrix} (+5.36543, +0.00000) & (+5.36543, +0.00000) & (+5.36543, +0.00000) & (+5.36543, +0.0$

QueryPerformanceCounter = +0.03535

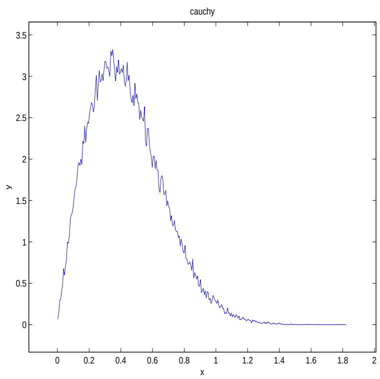
0.0.13 Random Number Generator

The sample size generated for this run is 100000.

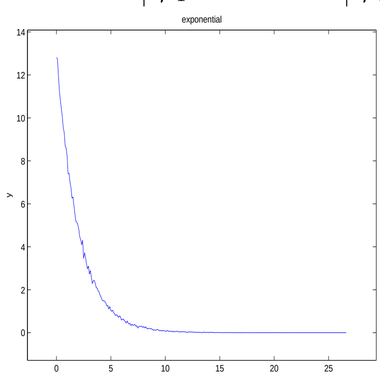
uniform	mean	variance	skewness	kurtosis
	$\mu_1 = +0.50030$	$\mu_2 = +0.08353$	$\mu_3 = +0.00339$	$\mu_4 = +1.80113$



cauchy	mean	variance	skewness	kurtosis
	$\mu_1 = +0.44288$	$\mu_2 = +0.05341$	$\mu_3 = +0.63935$	$\mu_4 = +3.28094$

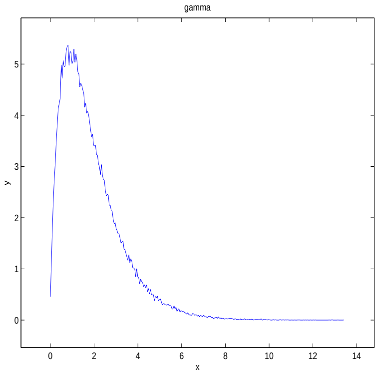


exponential	mean	variance	skewness	kurtosis
	$\mu_1 = +1.99647$	$\mu_2 = +3.99339$	$\mu_3 = +2.03097$	$\mu_4 = +9.30842$



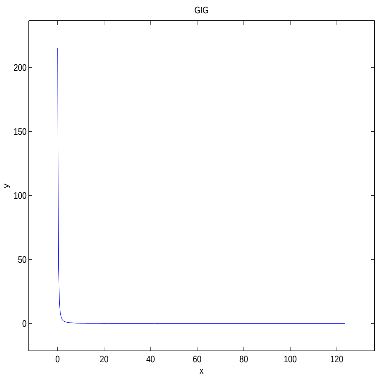
gamma

mean	variance	skewness	kurtosis
$\mu_1 = +1.89052$	$\mu_2 = +1.89855$	$\mu_3 = +1.46712$	$\mu_4 = +6.21783$



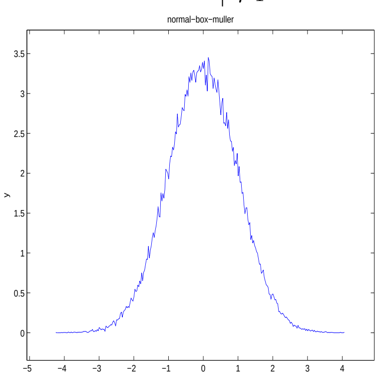
GIG

mean	variance	skewness	kurtosis
$\mu_1 = +0.80315$	$\mu_2 = +11.22582$	$\mu_3 = +15.31482$	$\mu_4 = +317.47280$



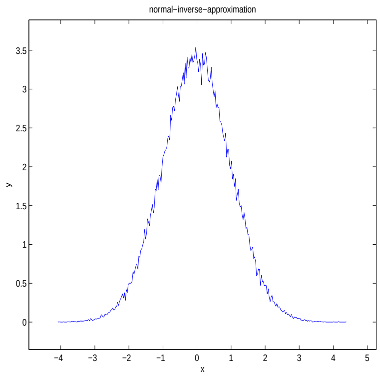
normal-box-muller

mean	variance	skewness	kurtosis
$\mu_1 = -0.00146$	$\mu_2 = +0.99607$	$\mu_3 = -0.00643$	$\mu_4 = +2.98875$



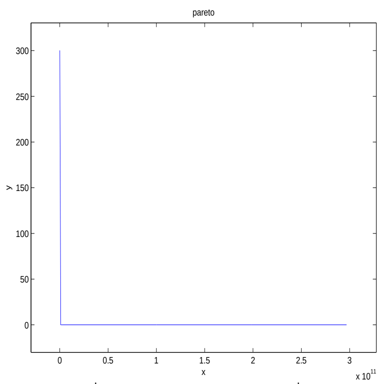
normal-inverse-approximation

mean	variance	skewness	kurtosis
$\mu_1 = +0.00230$	$\mu_2 = +1.00486$	$\mu_3 = +0.01163$	$\mu_4 = +2.99254$



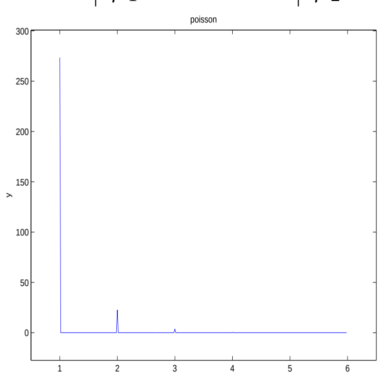
pareto

mean	variance	skewness	kurtosis
$\mu_1 = +3184578.26493$	$\mu_2 = +888468246174112900.00000$	$\mu_3 = +315.36997$	$\mu_4 = +99629.09819$

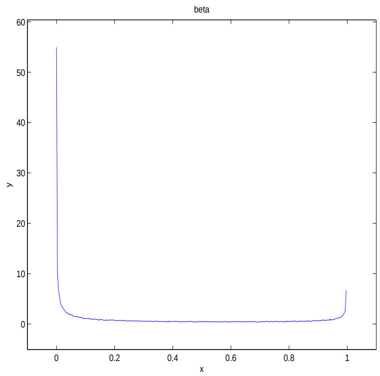


poisson

mean	variance	skewness	kurtosis
$\mu_1 = +1.10545$	$\mu_2 = +0.13057$	$\mu_3 = +3.94167$	$\mu_4 = +21.36551$



beta	mean	variance	skewness	kurtosis
	$\mu_1 = +0.33501$	$\mu_2 = +0.12730$	$\mu_3 = +0.67247$	$\mu_4 = +1.89726$



QueryPerformanceCounter = +24.11109

0.0.14 Multiclass Support Vector Machine

- Number or training points = 1024
- Feature dimension = 3
- Number or classes = 3

The mean vectors of the 3 classes

$\mu_1 = \begin{pmatrix} +1.90000 & +0.10000 & +0.10000 \end{pmatrix}$

$\mu_2 = \begin{pmatrix} +0.10000 & +1.90000 & +0.10000 \end{pmatrix}$

$\mu_3 = \begin{pmatrix} +0.00000 & +0.00000 & +1.90000 \end{pmatrix}$

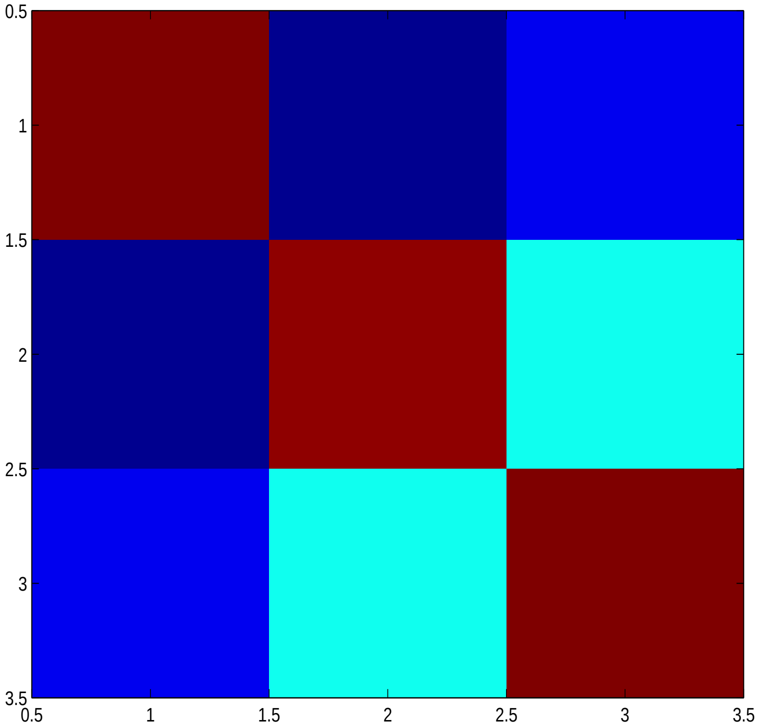
A random SPD covairance matrix is generated for each of the classes.

$$\rho_1 = \begin{pmatrix} +1.651 & -0.339 & -0.142 \\ -0.339 & +1.604 & +0.426 \\ -0.142 & +0.426 & +1.633 \end{pmatrix}$$
$$\rho_2 = \begin{pmatrix} +4.202 & +0.126 & +0.177 \\ +0.126 & +3.034 & +0.420 \\ +0.177 & +0.420 & +2.012 \end{pmatrix}$$
$$\rho_3 = \begin{pmatrix} +3.720 & +0.408 & -0.478 \\ +0.408 & +2.946 & -0.254 \\ -0.478 & -0.254 & +3.195 \end{pmatrix}$$

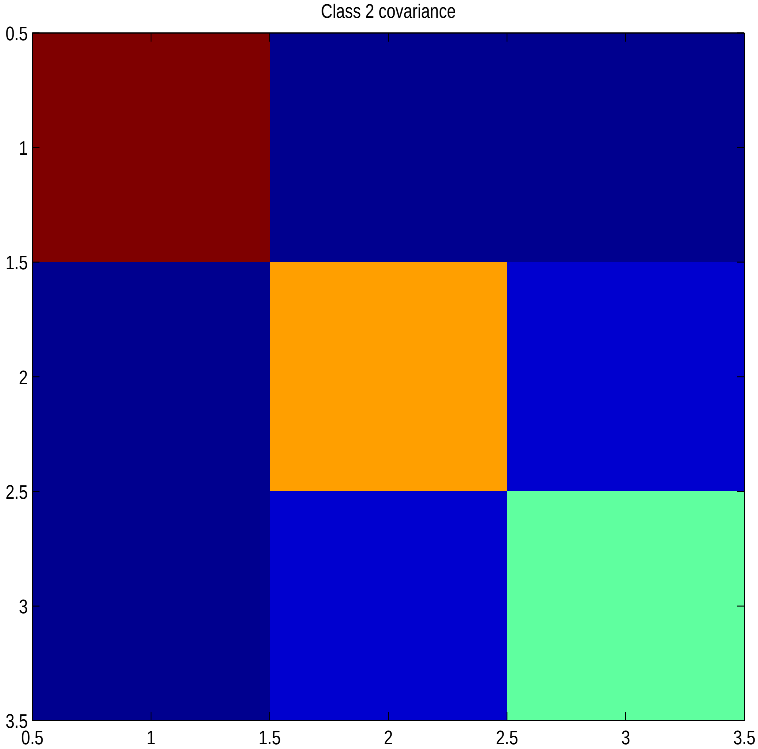
Verify L_1 condition number of covariance. The diagonal entries of the matrix have the form $(0.5 + U(0,1)) * dim(Dom(Cov))$ The lower-diagonal entries take the form $U(0,1) - 0.5$. The L_1 condition numbers are :

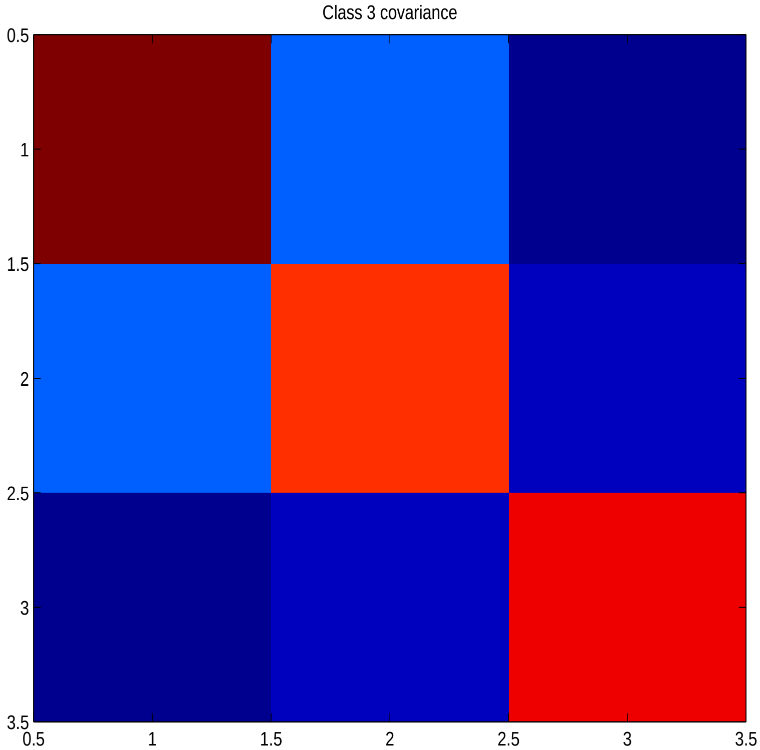
- +1.856
- +2.716
- +1.757

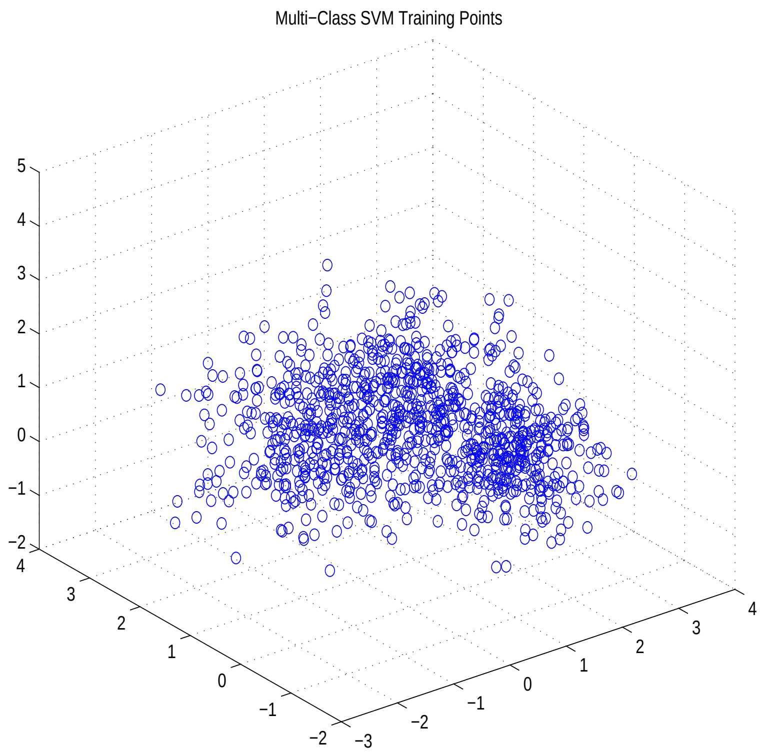
Class 1 covariance



Class 2 covariance

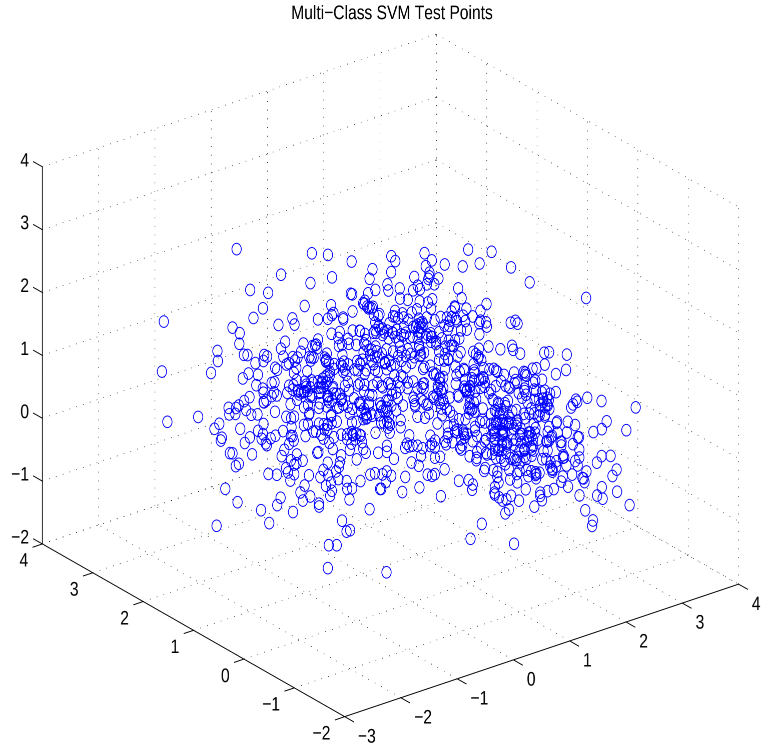






These are the SVM parameters - the RBF kernel is used

- allOutlierFraction=0.05
- mixingCoeff=0.3
- smoThresh=1.0/10000.0
- sigma=1

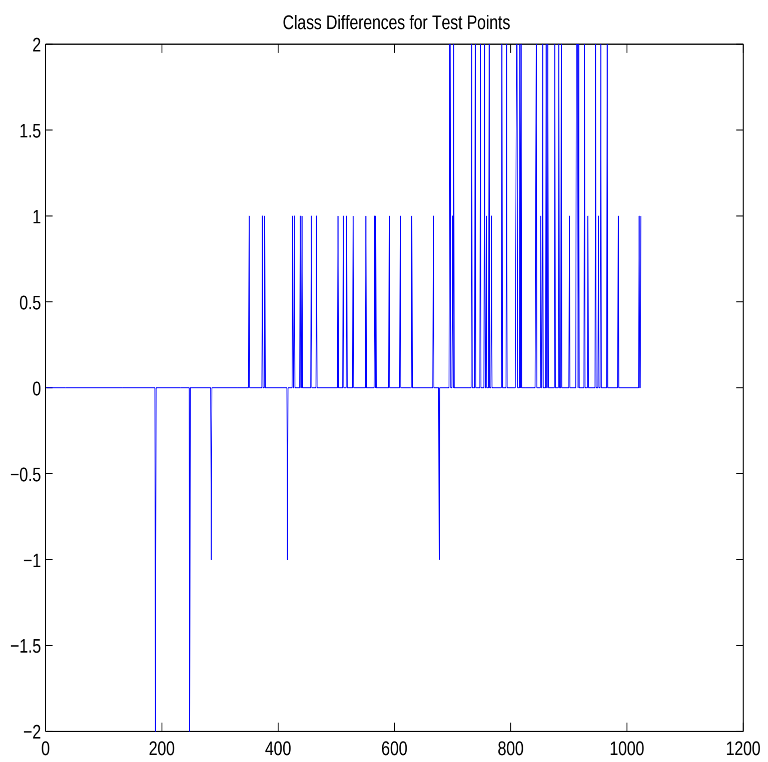


The marginal sample moments (mean var skew kurtosis) for training points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.666	+1.410	-0.091	+2.208
1	+0.700	+1.210	+0.476	+2.491
2	+0.708	+1.177	+0.607	+2.620

The marginal sample moments (mean var skew kurtosis) for test points.

Feature	μ_1	μ_2	μ_3	μ_4
0	+0.676	+1.283	+0.047	+2.169
1	+0.671	+1.200	+0.509	+2.465
2	+0.717	+1.134	+0.568	+2.559



The error rate for this run is +0.064
QueryPerformanceCounter = +9.336

0.0.15 Semidefinite Programming SDPA

QueryPerformanceCounter = +0.068

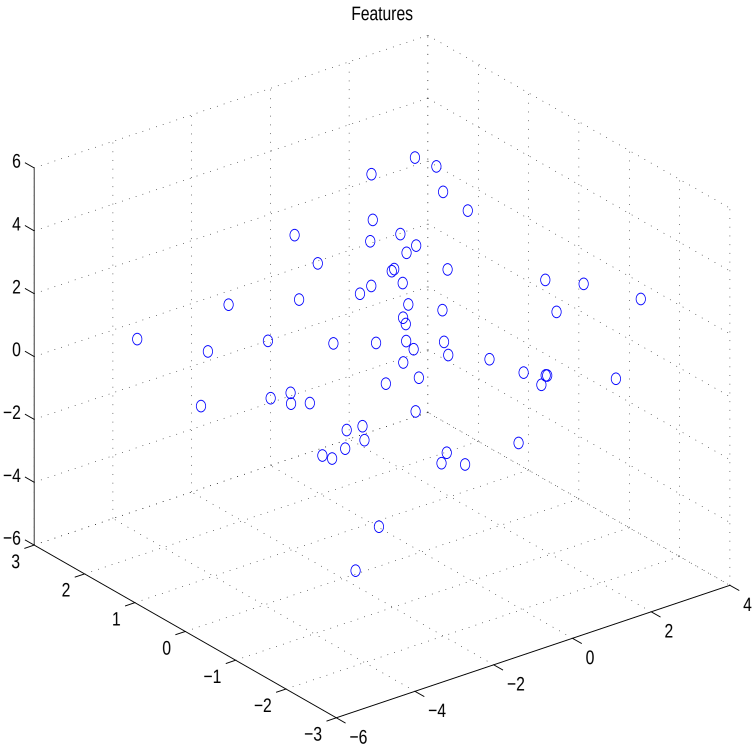
0.0.16 Linear Regression 3x1

0.0.17 3 x 1 Linear Regression

Sample size = 64

Number of features = 3

$$\sigma = \begin{pmatrix} +3.952 & -0.499 & -0.010 \\ -0.499 & +1.895 & +0.465 \\ -0.010 & +0.465 & +4.477 \end{pmatrix}$$



Beta +0.817, +0.999, +0.510

Response -0.857 +1.517 +3.649 +0.487 +0.061 +3.356 -3.453 +1.674 -3.623 +0.850 -1.287 +2.262 -4.347 +3.241 +1.591 +4.218 +0.724 +3.539 -0.392 +3.022 +1.310 +0.448 +0.148 -0.138 -0.852 +2.014 +2.813 -1.806 -2.514 +3.242 -1.518 +4.440 -0.415 +0.663 -6.367 -0.085 -0.088 +2.535 -0.815 +1.259 -0.826 -2.684 +0.991 +2.174 +3.978 +3.591 -2.041 -1.588 -0.583 -2.555 -3.100 -2.562 +0.366 +0.042 +2.006 -0.036 -1.843 +5.234 -0.297 +4.922 +1.822 -1.264 -2.779 -0.047 Estimate for Beta -62774385622041922000000000000000000000000000 -62774385622041922000000000000000000000000000000000000000000000000.000 -627743856220419220000000000000000000000000000000
Error: +0.000, -0.000, +0.000

QueryPerformanceCounter = +1.382

0.0.18 Matrix Norms

0.0.19 Haar Distributed Random Orthogonal Matrix $A \in O(n)$

Testing Operator Norm Number of Dimensions: 12

$$A = \begin{pmatrix} +0.238 & +0.244 & +0.342 & -0.223 & -0.202 & +0.303 & +0.210 & -0.034 & +0.219 & +0.484 & +0.351 & -0.365 \\ -0.161 & +0.022 & +0.156 & +0.053 & -0.097 & +0.258 & -0.459 & -0.115 & +0.417 & +0.197 & +0.104 & +0.650 \\ -0.010 & +0.490 & -0.118 & +0.270 & +0.549 & -0.103 & -0.169 & -0.478 & -0.173 & +0.220 & +0.140 & -0.072 \\ +0.033 & +0.235 & -0.118 & +0.046 & +0.142 & +0.162 & +0.751 & -0.148 & +0.337 & -0.214 & -0.112 & +0.353 \\ +0.362 & +0.288 & -0.076 & +0.095 & +0.344 & +0.198 & -0.261 & +0.584 & +0.315 & -0.108 & -0.269 & -0.144 \\ +0.157 & +0.109 & +0.134 & +0.637 & -0.356 & -0.487 & +0.124 & +0.132 & +0.101 & +0.312 & -0.174 & +0.069 \\ -0.243 & +0.020 & -0.690 & -0.129 & +0.005 & -0.102 & +0.107 & +0.374 & +0.026 & +0.445 & +0.278 & +0.103 \\ -0.113 & +0.036 & +0.003 & -0.365 & +0.065 & -0.612 & -0.108 & -0.169 & +0.612 & -0.126 & +0.006 & -0.209 \\ -0.564 & +0.491 & -0.109 & +0.162 & -0.420 & +0.213 & -0.095 & +0.015 & +0.053 & -0.280 & -0.068 & -0.293 \\ +0.600 & +0.129 & -0.443 & -0.007 & -0.406 & -0.005 & -0.182 & -0.245 & -0.002 & -0.300 & +0.273 & +0.068 \\ -0.009 & -0.480 & -0.331 & +0.297 & +0.017 & +0.314 & -0.027 & -0.341 & +0.337 & +0.190 & -0.275 & -0.361 \\ +0.104 & +0.245 & -0.119 & -0.439 & -0.192 & -0.003 & -0.056 & -0.178 & -0.184 & +0.324 & -0.706 & +0.116 \end{pmatrix}$$

$Det(A) : A \in O(n) = (+1.000,+0.000)$

$$L = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.939 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.405 & +0.118 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.261 & +0.123 & -0.390 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.017 & +0.805 & -0.368 & +0.159 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.015 & -0.781 & +0.926 & +0.993 & -0.318 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.055 & +0.372 & -0.126 & -0.053 & +0.373 & +0.115 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.604 & +0.343 & -0.460 & -0.041 & +0.694 & +0.191 & +0.062 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ -0.188 & +0.097 & +0.035 & -0.670 & -0.041 & -0.939 & -0.483 & -0.747 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.396 & +0.316 & -0.846 & -0.706 & +0.027 & -0.240 & +0.495 & +0.167 & +0.063 & +1.000 & +0.000 & +0.000 \\ +0.173 & +0.364 & -0.184 & -0.929 & -0.005 & -0.578 & -0.001 & -0.286 & +0.090 & +0.565 & +1.000 & +0.000 \\ -0.268 & +0.093 & -0.107 & +0.037 & -0.111 & +0.203 & -0.406 & +0.127 & +0.340 & +0.460 & -0.178 & +1.000 \end{pmatrix}$$

$$U = \begin{pmatrix} +0.600 & +0.129 & -0.443 & -0.007 & -0.406 & -0.005 & -0.182 & -0.245 & -0.002 & -0.300 & +0.273 & +0.068 \\ +0.000 & +0.612 & -0.524 & +0.156 & -0.801 & +0.208 & -0.266 & -0.215 & +0.051 & -0.562 & +0.188 & -0.229 \\ +0.000 & +0.000 & -0.807 & -0.150 & -0.065 & -0.128 & +0.065 & +0.300 & +0.019 & +0.390 & +0.367 & +0.157 \\ +0.000 & +0.000 & +0.000 & +0.562 & -0.177 & -0.561 & +0.230 & +0.339 & +0.103 & +0.611 & -0.126 & +0.141 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.191 & -0.228 & +0.030 & -0.253 & -0.223 & +0.714 & +0.148 & +0.148 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.079 & -0.516 & -1.207 & +0.187 & -0.994 & -0.291 & -0.777 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.928 & +0.235 & +0.387 & -0.059 & -0.180 & +0.495 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.349 & +0.407 & +0.168 & -0.371 & -0.014 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.332 & -0.539 & -0.689 & -0.580 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.319 & +0.525 & -0.484 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -1.380 & +0.221 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.538 \end{pmatrix}$$

$$L*U = \begin{pmatrix} +0.600 & +0.129 & -0.443 & -0.007 & -0.406 & -0.005 & -0.182 & -0.245 & -0.002 & -0.300 & +0.273 & +0.068 \\ -0.564 & +0.491 & -0.109 & +0.162 & -0.420 & +0.213 & -0.095 & +0.015 & +0.053 & -0.280 & -0.068 & -0.293 \\ -0.243 & +0.020 & -0.690 & -0.129 & +0.005 & -0.102 & +0.107 & +0.374 & +0.026 & +0.445 & +0.278 & +0.103 \\ +0.157 & +0.109 & +0.134 & +0.637 & -0.356 & -0.487 & +0.124 & +0.132 & +0.101 & +0.312 & -0.174 & +0.069 \\ -0.010 & +0.490 & -0.118 & +0.270 & +0.549 & -0.103 & -0.169 & -0.478 & -0.173 & +0.220 & +0.140 & -0.072 \\ -0.009 & -0.480 & -0.331 & +0.297 & +0.017 & +0.314 & -0.027 & -0.341 & +0.337 & +0.190 & -0.275 & -0.361 \\ +0.033 & +0.235 & -0.118 & +0.046 & +0.142 & +0.162 & +0.751 & -0.148 & +0.337 & -0.214 & -0.112 & +0.353 \\ +0.362 & +0.288 & -0.076 & +0.095 & +0.344 & +0.198 & -0.261 & +0.584 & +0.315 & -0.108 & -0.269 & -0.144 \\ -0.113 & +0.036 & +0.003 & -0.365 & +0.065 & -0.612 & -0.108 & -0.169 & +0.612 & -0.126 & +0.006 & -0.209 \\ +0.238 & +0.244 & +0.342 & -0.223 & -0.202 & +0.303 & +0.210 & -0.034 & +0.219 & +0.484 & +0.351 & -0.365 \\ +0.104 & +0.245 & -0.119 & -0.439 & -0.192 & -0.003 & -0.056 & -0.178 & -0.184 & +0.324 & -0.706 & +0.116 \\ -0.161 & +0.022 & +0.156 & +0.053 & -0.097 & +0.258 & -0.459 & -0.115 & +0.417 & +0.197 & +0.104 & +0.650 \end{pmatrix}$$

$Det(L) := (+1.000, +0.000) Det(U) := (+1.000, +0.000) Det(LU) := (+1.000, +0.000)$

$||A||_{L_1} = +3.200$

$||A||_{L_\infty} = +3.215$

$||A^{-1}||_{L_1} = +3.215$

$||A^{-1}||_{L_\infty} = +3.200$

$||A||_{L_\infty} * ||A^{-1}||_{L_\infty} = +10.290$

$||A||_{L_1} * ||A^{-1}||_{L_1} = +10.290$

Frobenious Norm $||A||_F$ via $\sum_{i,j=0}^{n} \|A_{i,j}|$ of $A \in O(n)$ +3.464

L_1 condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +9.534

$$A = \begin{pmatrix} +0.238 & +0.244 & +0.342 & -0.223 & -0.202 & +0.303 & +0.210 & -0.034 & +0.219 & +0.484 & +0.351 & -0.365 \\ -0.161 & +0.022 & +0.156 & +0.053 & -0.097 & +0.258 & -0.459 & -0.115 & +0.417 & +0.197 & +0.104 & +0.650 \\ -0.010 & +0.490 & -0.118 & +0.270 & +0.549 & -0.103 & -0.169 & -0.478 & -0.173 & +0.220 & +0.140 & -0.072 \\ +0.033 & +0.235 & -0.118 & +0.046 & +0.142 & +0.162 & +0.751 & -0.148 & +0.337 & -0.214 & -0.112 & +0.353 \\ +0.362 & +0.288 & -0.076 & +0.095 & +0.344 & +0.198 & -0.261 & +0.584 & +0.315 & -0.108 & -0.269 & -0.144 \\ +0.157 & +0.109 & +0.134 & +0.637 & -0.356 & -0.487 & +0.124 & +0.132 & +0.101 & +0.312 & -0.174 & +0.069 \\ -0.243 & +0.020 & -0.690 & -0.129 & +0.005 & -0.102 & +0.107 & +0.374 & +0.026 & +0.445 & +0.278 & +0.103 \\ -0.113 & +0.036 & +0.003 & -0.365 & +0.065 & -0.612 & -0.108 & -0.169 & +0.612 & -0.126 & +0.006 & -0.209 \\ -0.564 & +0.491 & -0.109 & +0.162 & -0.420 & +0.213 & -0.095 & +0.015 & +0.053 & -0.280 & -0.068 & -0.293 \\ +0.600 & +0.129 & -0.443 & -0.007 & -0.406 & -0.005 & -0.182 & -0.245 & -0.002 & -0.300 & +0.273 & +0.068 \\ -0.009 & -0.480 & -0.331 & +0.297 & +0.017 & +0.314 & -0.027 & -0.341 & +0.337 & +0.190 & -0.275 & -0.361 \\ +0.104 & +0.245 & -0.119 & -0.439 & -0.192 & -0.003 & -0.056 & -0.178 & -0.184 & +0.324 & -0.706 & +0.116 \end{pmatrix}$$

L_∞ condition number of Haar Distributed Random Orthogonal Matrix $A \in O(n)$ +10.290

Eigenvalues of $A \in O(n)$

(+0.283,+0.959), (+0.283,-0.959), (-0.472,+0.881), (-0.472,-0.881), (-0.823,+0.568), (-0.823,-0.568), (-0.920,+0.392), (-0.920,-0.392), (+0.971,+0.237), (+0.971,-0.237), (+0.751,+0.661), (+0.751,-0.661)

$|\lambda| : \lambda \in \sigma(A), A \in O(n)$
+1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000, +1.000
Calculating $A^\dagger A$, we expect $A^\dagger A \approx I$

$$A^\dagger A = \begin{pmatrix} +1.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\ -0.000 & +1.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\ -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 \\ +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 & -0.000 \\ +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 & -0.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 \\ +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 \\ -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating $A^{-1}, A \in O(n)$.

$$A^{-1} = \begin{pmatrix} +0.238 & -0.161 & -0.010 & +0.033 & +0.362 & +0.157 & -0.243 & -0.113 & -0.564 & +0.600 & -0.009 & +0.104 \\ +0.244 & +0.022 & +0.490 & +0.235 & +0.288 & +0.109 & +0.020 & +0.036 & +0.491 & +0.129 & -0.480 & +0.245 \\ +0.342 & +0.156 & -0.118 & -0.118 & -0.076 & +0.134 & -0.690 & +0.003 & -0.109 & -0.443 & -0.331 & -0.119 \\ -0.223 & +0.053 & +0.270 & +0.046 & +0.095 & +0.637 & -0.129 & -0.365 & +0.162 & -0.007 & +0.297 & -0.439 \\ -0.202 & -0.097 & +0.549 & +0.142 & +0.344 & -0.356 & +0.005 & +0.065 & -0.420 & -0.406 & +0.017 & -0.192 \\ +0.303 & +0.258 & -0.103 & +0.162 & +0.198 & -0.487 & -0.102 & -0.612 & +0.213 & -0.005 & +0.314 & -0.003 \\ +0.210 & -0.459 & -0.169 & +0.751 & -0.261 & +0.124 & +0.107 & -0.108 & -0.095 & -0.182 & -0.027 & -0.056 \\ -0.034 & -0.115 & -0.478 & -0.148 & +0.584 & +0.132 & +0.374 & -0.169 & +0.015 & -0.245 & -0.341 & -0.178 \\ +0.219 & +0.417 & -0.173 & +0.337 & +0.315 & +0.101 & +0.026 & +0.612 & +0.053 & -0.002 & +0.337 & -0.184 \\ +0.484 & +0.197 & +0.220 & -0.214 & -0.108 & +0.312 & +0.445 & -0.126 & -0.280 & -0.300 & +0.190 & +0.324 \\ +0.351 & +0.104 & +0.140 & -0.112 & -0.269 & -0.174 & +0.278 & +0.006 & -0.068 & +0.273 & -0.275 & -0.706 \\ -0.365 & +0.650 & -0.072 & +0.353 & -0.144 & +0.069 & +0.103 & -0.209 & -0.293 & +0.068 & -0.361 & +0.116 \end{pmatrix}$$

Calculating $A^{-1} * A, A \in O(n)$. We expect $A^{-1} * A \approx I$.

$$A^{-1} * A = \begin{pmatrix} +1.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 \\ +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & +0.000 & +0.000 & +1.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 \\ -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & -0.000 & +0.000 & +0.000 & -0.000 & -0.000 & +0.000 \\ -0.000 & -0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & -0.000 & -0.000 & -0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +1.000 & +0.000 & +0.000 & -0.000 & +0.000 \\ +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & +1.000 & -0.000 & -0.000 & -0.000 \\ +0.000 & +0.000 & -0.000 & +0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +1.000 & +0.000 & -0.000 \\ -0.000 & -0.000 & -0.000 & +0.000 & +0.000 & +0.000 & +0.000 & -0.000 & -0.000 & -0.000 & +1.000 & +0.000 \\ -0.000 & -0.000 & -0.000 & -0.000 & -0.000 & +0.000 & -0.000 & +0.000 & -0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

Calculating SVD of $A \in O(n)$

$$U = \begin{pmatrix} -0.091 & +0.066 & -0.281 & -0.694 & +0.052 & -0.168 & +0.012 & -0.345 & +0.178 & +0.025 & -0.425 & -0.252 \\ +0.214 & -0.274 & -0.325 & +0.179 & +0.041 & -0.279 & +0.657 & -0.146 & -0.348 & +0.160 & +0.088 & -0.233 \\ +0.030 & -0.311 & +0.422 & -0.335 & -0.443 & -0.009 & +0.105 & -0.315 & -0.342 & -0.100 & +0.022 & +0.421 \\ -0.238 & -0.244 & -0.342 & +0.223 & +0.202 & -0.303 & -0.210 & +0.034 & -0.219 & -0.484 & -0.351 & +0.365 \\ +0.820 & -0.259 & -0.063 & -0.011 & +0.100 & +0.298 & -0.069 & -0.006 & +0.207 & -0.246 & -0.216 & +0.051 \\ +0.028 & +0.502 & +0.206 & +0.250 & -0.249 & +0.172 & +0.197 & -0.059 & -0.293 & -0.320 & -0.492 & -0.280 \\ +0.157 & +0.083 & +0.412 & +0.064 & +0.032 & -0.696 & +0.095 & -0.042 & +0.394 & -0.336 & +0.132 & -0.115 \\ -0.308 & -0.254 & +0.132 & +0.057 & +0.330 & +0.396 & +0.104 & -0.433 & +0.084 & -0.432 & +0.271 & -0.297 \\ -0.185 & -0.473 & +0.332 & +0.325 & -0.014 & -0.007 & -0.026 & -0.063 & +0.246 & +0.411 & -0.522 & -0.140 \\ +0.161 & +0.089 & +0.391 & -0.163 & +0.679 & -0.104 & -0.174 & +0.017 & -0.491 & +0.187 & -0.072 & -0.049 \\ +0.089 & -0.264 & -0.064 & +0.001 & -0.338 & -0.151 & -0.566 & +0.056 & -0.298 & -0.078 & +0.154 & -0.582 \\ -0.164 & -0.270 & +0.140 & -0.350 & +0.019 & +0.097 & +0.312 & +0.747 & -0.016 & -0.239 & -0.084 & -0.168 \end{pmatrix}$$

$$S = \begin{pmatrix} +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 & +0.000 \\ +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +0.000 & +1.000 \end{pmatrix}$$

$$V = \begin{pmatrix} +0.000 & -0.000 & -0.000 & -1.000 & +0.000 & -0.000 & +0.000 & -0.000 & -0.000 & +0.000 & +0.000 & -0.000 \\ -0.208 & -0.699 & +0.179 & -0.000 & +0.000 & -0.381 & -0.140 & -0.075 & +0.145 & -0.194 & -0.285 & -0.354 \\ +0.022 & +0.085 & -0.393 & -0.000 & -0.216 & +0.061 & -0.014 & +0.158 & -0.169 & +0.477 & -0.126 & -0.701 \\ +0.067 & +0.222 & -0.024 & +0.000 & +0.149 & +0.182 & +0.097 & +0.156 & -0.150 & -0.295 & -0.864 & +0.037 \\ -0.074 & -0.374 & -0.672 & +0.000 & +0.477 & +0.139 & +0.021 & -0.235 & -0.040 & +0.159 & -0.047 & +0.267 \\ -0.385 & +0.241 & -0.094 & +0.000 & -0.119 & +0.205 & +0.367 & -0.571 & +0.402 & -0.210 & -0.014 & -0.249 \\ +0.067 & +0.251 & -0.373 & +0.000 & -0.358 & -0.506 & -0.378 & -0.363 & -0.231 & -0.238 & -0.068 & +0.139 \\ +0.584 & -0.157 & -0.156 & +0.000 & -0.119 & -0.292 & +0.696 & +0.016 & +0.028 & -0.145 & +0.052 & -0.026 \\ +0.043 & -0.333 & -0.215 & +0.000 & -0.477 & +0.560 & -0.113 & +0.167 & -0.131 & -0.466 & +0.159 & +0.004 \\ +0.004 & +0.098 & +0.097 & +0.000 & +0.477 & +0.011 & +0.034 & -0.142 & -0.572 & -0.390 & +0.301 & -0.403 \\ +0.195 & +0.208 & -0.289 & +0.000 & +0.298 & -0.094 & -0.290 & +0.352 & +0.589 & -0.337 & +0.143 & -0.218 \\ +0.642 & -0.076 & +0.223 & +0.000 & +0.060 & +0.304 & -0.330 & -0.509 & +0.138 & +0.137 & -0.107 & -0.135 \end{pmatrix}$$

$$USV = \begin{pmatrix} -0.444 & -0.353 & +0.200 & +0.091 & -0.176 & -0.037 & -0.191 & -0.079 & -0.244 & +0.165 & +0.599 & +0.332 \\ -0.021 & +0.476 & -0.137 & -0.214 & +0.186 & -0.497 & -0.295 & -0.026 & -0.308 & -0.187 & -0.057 & +0.454 \\ +0.171 & +0.495 & +0.247 & -0.030 & -0.204 & -0.008 & -0.377 & -0.145 & +0.088 & +0.561 & +0.196 & -0.313 \\ +0.320 & -0.090 & +0.234 & +0.238 & +0.101 & +0.192 & +0.046 & -0.076 & +0.057 & +0.419 & -0.339 & +0.655 \\ -0.080 & +0.050 & -0.084 & -0.820 & -0.228 & +0.354 & +0.184 & -0.193 & +0.099 & +0.073 & +0.009 & +0.219 \\ -0.442 & -0.099 & +0.202 & -0.028 & -0.386 & -0.422 & +0.150 & -0.112 & -0.075 & +0.202 & -0.575 & -0.120 \\ +0.211 & -0.286 & -0.315 & -0.157 & -0.326 & +0.001 & -0.387 & +0.642 & -0.168 & +0.183 & -0.139 & +0.004 \\ -0.500 & +0.277 & -0.533 & +0.308 & -0.078 & +0.243 & -0.154 & +0.023 & +0.397 & +0.100 & -0.090 & +0.170 \\ -0.086 & +0.297 & -0.087 & +0.185 & -0.098 & +0.436 & +0.240 & +0.025 & -0.766 & +0.051 & -0.085 & -0.089 \\ -0.099 & -0.220 & -0.384 & -0.161 & +0.587 & -0.165 & +0.119 & -0.117 & -0.149 & +0.559 & +0.022 & -0.184 \\ -0.226 & +0.286 & +0.304 & -0.089 & +0.183 & -0.070 & +0.408 & +0.694 & +0.151 & +0.163 & +0.156 & +0.081 \\ +0.328 & +0.077 & -0.382 & +0.164 & -0.427 & -0.361 & +0.514 & -0.086 & +0.043 & +0.128 & +0.308 & +0.127 \end{pmatrix}$$

0.0.20 Wishart Matrix $A \in W(n)$

L_1 condition number of Wishart Matrix +56267.800 L_infty condition number of Wishart Matrix +56267.800

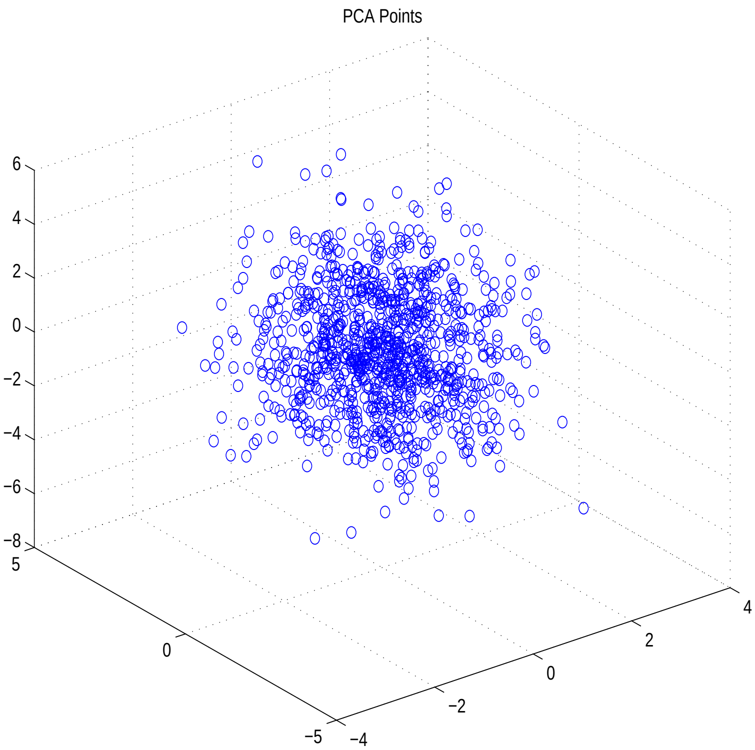
0.0.21 Gaussian Orthogonal Ensemble $A \in GOE(n)$

L_1 condition number of GOE Matrix +470.231 L_∞ condition number of GOE Matrix +470.231

0.0.22 The Identity Matrix $I \in M(n)$

L_1 condition number of $I = +1.000$ L_∞ condition number of $I = +1.000$ QueryPerformanceCounter = +1.815

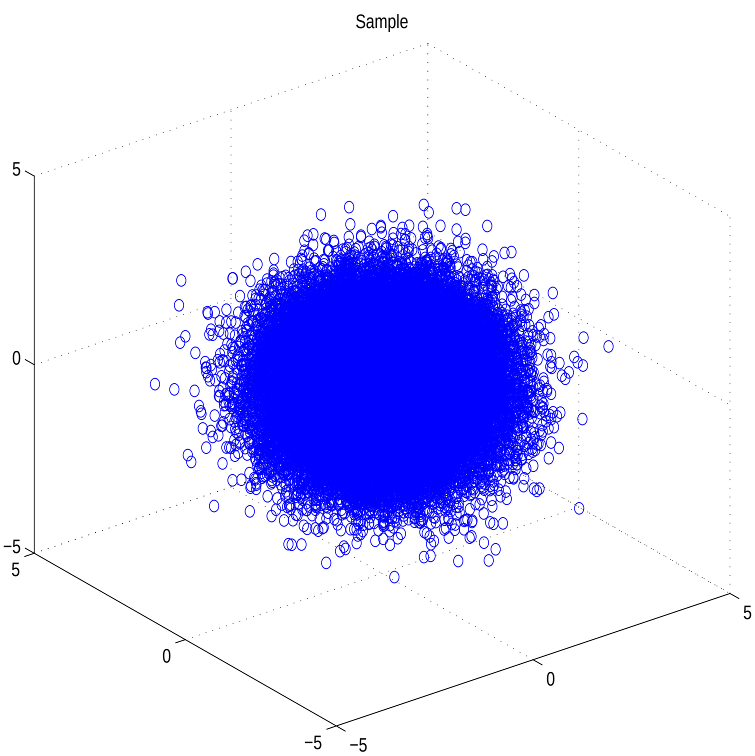
0.0.23 Principal Components Matlab

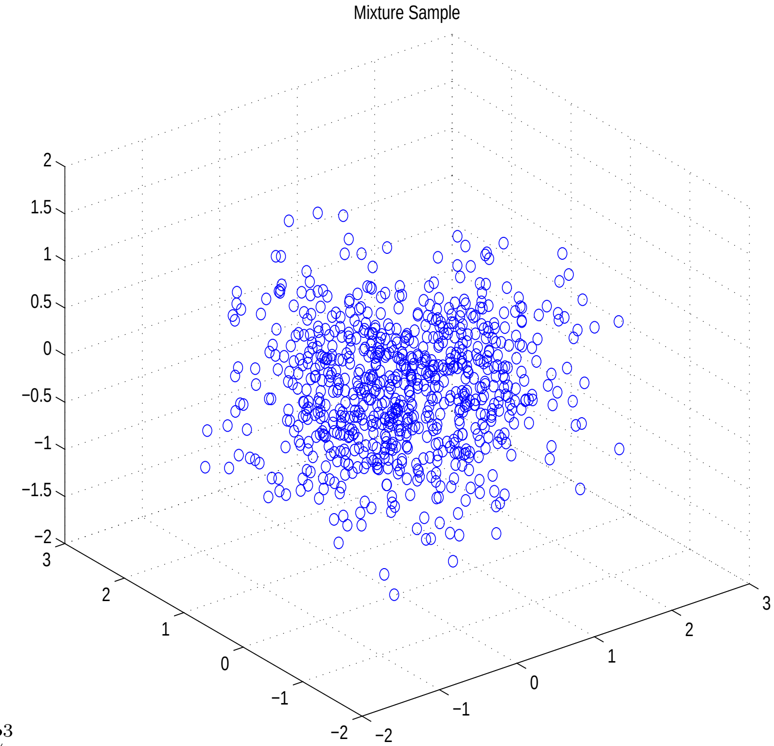


The eigenvectors: +0.065, +0.220, +0.973 +0.087, +0.970, -0.225 -0.994, +0.099, +0.044
All of the eigenvalues of the covariance matrix: (+0.958,+0.000), (+2.025,+0.000), (+3.017,+0.000)
QueryPerformanceCounter = +1.423

0.0.24 Multi Variate Random Number Generator

Sample from $N(\mu, \Sigma)$ mean= -0.002, variance=+1.004, skewness=+0.006, kurtosis=+3.003 mean= -0.001, variance=+1.017, skewness=-0.005, kurtosis=+2.988 mean= -0.002, variance=+1.006, skewness=-0.016, kurtosis=+3.014 Covariance Matrix +1.004, +0.009, +0.003 +0.009, +1.017, -0.003 +0.003, -0.003, +1.006





Generate a sample from a unifom mixture of three Gaussians in R^3
QueryPerformanceCounter = +33.721

0.0.25 Matrix Multiply

Comparing naive matrix multiply verus Intel MKL dgemm for matrix of size 2048. This is for type double (hence the d in dgemm). Naive type double matrix multiply tic toc = +3.607 dgemm plus row to column major transpose operation tic toc = +2.536 Comparing naive matrix multiply verus Intel MKL sgemm for matrix of size 2048. This is for type float (hence the s in dgemm). Naive type float matrix multiply tic toc = +3.601 sgemm plus row to column major transpose operation tic toc = +2.539 QueryPerformanceCounter = +13.613

0.0.26 Descriptive Statistics

Mean N(0,1): +0.003 Variance N(0,1): +1.006 Mean N(0,1) [recurrence relation method] :+0.003 Variance [recurrence relation method] :+1.006 Skewness : +0.007 Kurtosis : +2.997 QueryPerformanceCounter = +0.053

0.0.27 Time Series

+0.093 +0.726 +0.011 +2.178 QueryPerformanceCounter = +0.203

0.0.28 Matrix

QueryPerformanceCounter = +1.342 QueryPerformanceCounter = +1.308