# Artificial Intelligence

Instructor: Jie Shen

Dept. of Computer Science

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# AI applications...

- ChatGPT
- content generation
- auto driving, robotics
- ...

# But...

Theoretical Foundation of AI

# Syllabus

- Review of calculus, probability, linear algebra
- random projection
- singular value decomposition, principal component analysis
- dictionary learning and sparse coding
- low-rank matrix estimation, with applications to recommender systems
- Large language models: the Transformer
- GPT, Bert, DistillBert
- Scaling laws, chain of thought
- Contrastive learning
- Deep learning in bioinformatics

# Course Staff

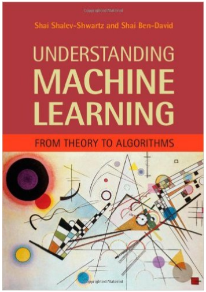
Instructor: Jie Shen (jie.shen@stevens.edu)

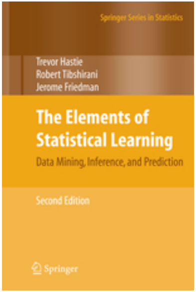
TA - Section A: Ziruo (Rosie) Zhao (zzhao83@stevens.edu)

TA - Section B: Krishna Deb (kdeb@stevens.edu)

Office Hours: 1:00 - 2:00 Friday at GS 351

- No Required Textbook
- Recommended (available online)




- Research Papers in NeurIPS, ICML, COLT

# Grading

- Midterm Exam (60%)
  - closed-book
  - **Section A: Oct 22 - mark it in your calendar!**
  - **Section B: Oct 25 - mark it in your calendar!**
- Final Paper Presentation (40%)
  - **Section A: Dec 10, or Dec 3 + Dec 10**
  - **Section B: Dec 13, or Dec 6 + Dec 13**
  - more details will be announce in October
- Final Grade

| 90 - 100 | 85 - 89 | 80 - 84 | 75 - 79 | 70 - 74 | <70 |
|---|---|---|---|---|---|
| A | A- | B+ | B | B- | Fail |

# About the Course

Tough

- Not for introductory purpose
- Research oriented
  - Emphasize on both theoretical and application aspects
  - Analyze computational cost
  - Understand statistical accuracy
- Strong background in calculus, linear algebra, and probability
  - If cannot do Quiz 0, consider dropping the course

# About the Course

Overarching goal: Students can do independent research

- read research articles
- implement algorithms
- push the frontier of AI

General paradigm

- paper reading is assigned every week
- Not graded, but you are welcome to discuss during office hours

# Quiz 0 (20 min)

1. Let $\boldsymbol{x} = (1\ 2\ 3)$, $\boldsymbol{y} = (1\ 1\ 1)$. Calculate $\boldsymbol{x}\boldsymbol{y}^\top$ and $\boldsymbol{x}^\top\boldsymbol{y}$.
2. Show that for all $x > 0$, $\log(1+x) \leq x - \frac{x^2}{2} + \frac{x^3}{3}$.
3. Show that $\frac{1}{2}(e^x + e^{-x}) \leq e^{x^2/2}$ for all $x \in \mathbb{R}$, where $e$ is the base of the natural logarithm.

# Linear Algebra Overview

A $d$-dimensional column vector $\boldsymbol{x}$ is a set of $d$ numbers

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix}$$

- Bold lowercase letters for vectors
- Almost all the data is vector

"All the News That's Fit to Print"

The New York Times

LATE CITY EDITION

10 CENTS

MEN WALK ON MOON

ASTRONAUTS LAND ON PLAIN; COLLECT ROCKS, PLANT FLAG

Voice From Moon: 'Eagle Has Landed'

A Powdery Surface Is Closely Explored

# Vector Operations

Suppose $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^d$ are column vectors, $a, b \in \mathbb{R}$

- $\boldsymbol{x}^\top \stackrel{\text{def}}{=} (x_1 \quad x_2 \quad \dots \quad x_d)$
- $a\boldsymbol{x} \stackrel{\text{def}}{=} (ax_1 \quad ax_2 \quad \dots \quad ax_d)^\top$
- $\boldsymbol{x} + \boldsymbol{y} \stackrel{\text{def}}{=} (x_1 + y_1 \quad x_2 + y_2 \quad \dots \quad x_d + y_d)$
- $a\boldsymbol{x} + b\boldsymbol{y}$
- $\langle \boldsymbol{x}, \boldsymbol{y} \rangle \stackrel{\text{def}}{=} \sum_{i=1}^d x_i y_i \in \mathbb{R}$
    - Sometimes use $\boldsymbol{x}^\top \boldsymbol{y}$, $\boldsymbol{x} \cdot \boldsymbol{y}$

# Vector Norms

- $\|\boldsymbol{x}\|_2 \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^{d} x_i^2}$
    - Broadly used
    - $\|\boldsymbol{x} - \boldsymbol{y}\|_2$
- $\|\boldsymbol{x}\|_1 \stackrel{\text{def}}{=} \sum_{i=1}^{d} |x_i|$
- $\|\boldsymbol{x}\|_\infty \stackrel{\text{def}}{=} \max_{1 \leq i \leq d} |x_i|$

# Matrix

Vector: a set of numbers
Matrix: a set of vectors

- Bold capital letters $\boldsymbol{X} \in \mathbb{R}^{d \times n}$
- $\boldsymbol{X} = (x_{ij})_{1 \leq i \leq d, 1 \leq j \leq n} = (\boldsymbol{x}_1 \ \boldsymbol{x}_2 \ \ldots \ \boldsymbol{x}_n)$
- $a\boldsymbol{X}$ for $a \in \mathbb{R}$
- $a\boldsymbol{X} + b\boldsymbol{Y}$ when $\boldsymbol{X}, \boldsymbol{Y}$ have the same size
- Multiplication: $\boldsymbol{X} \in \mathbb{R}^{d \times n}$, $\boldsymbol{Y} \in \mathbb{R}^{p \times m}$
  - Can do $\boldsymbol{X}\boldsymbol{Y}$ only when $n = p$
  - $\boldsymbol{X}\boldsymbol{Y} \in \mathbb{R}^{d \times m}$
  - For $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^d$, $\boldsymbol{x}^\top \boldsymbol{y} \in \mathbb{R}$, $\boldsymbol{x}\boldsymbol{y}^\top \in \mathbb{R}^{d \times d}$
- Transpose
- Symmetric matrix, diagonal
- Inverse of a square matrix

# Probability Overview

Probability: measure of likelihood that an event will occur.

- From 0 to 1
- Coin tossing (heads or tails)

- Random variable $X$
- Events $= \{0, 1\}$
- $X$ has distribution $\mathcal{D}$

# Probability Overview

- If $X$ is discrete, probability mass function $p(x) = P(X = x)$
  - Takes value from a countable set
  - $\{0, 1\}$
  - $\{1, 2, 3, \dots\}$
- If $X$ is continuous, probability density function (PDF) $p(x)$

$$P(X \leq x) = \int_{-\infty}^{x} p(z)dz$$

  - Uniform distribution
  - Normal distribution
- $P(X \leq x)$: cumulative density function

# Expected Value

Expected value

- Discrete: $\mathbb{E}[X] = \sum xp(x)$
- Continuous: $\mathbb{E}[X] = \int xp(x)dx$
- Practice: Play a game for money. Each time

$$\Pr(X = 1) = 0.6, \quad \Pr(X = -1) = 0.4.$$

  When can we win 100 dollars?

# Expectation

- Average of multiple outcomes
- Not quite useful in practice
    - gambling
    - weather forecasting (rainy, sunny, dry)
    - in expectation = I guess
- But, $\mathbb{E}[X]$ implies $P(X)$

# Markov's Inequality

**Theorem.** If $X > 0$, $P(X \geq t) \leq \frac{\mathbb{E}[X]}{t}$ for all $t > 0$.

- Proof of correctness
- Proof of tightness
- Negative random variables
    - moment-generating function

# Variance

- $\mathrm{Var}(X) \stackrel{\text{def}}{=} \mathbb{E}[X - \mathbb{E}X]^2 = \mathbb{E}X^2 - (\mathbb{E}X)^2$
- $X_1, X_2, \ldots X_n$ are independent, then $\mathrm{Var}(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} \mathrm{Var}(X_i)$

# Chebyshev's Inequality

# Hoeffding's Inequality

Symmetric Bernoulli distribution: $P(X = 1) = P(X = -1) = 1/2$

**Theorem.** Let $X_1, X_2, \ldots, X_n$ be independent symmetric Bernoulli random variables. Let $\boldsymbol{a} = (a_1, a_2, \ldots, a_n) \in \mathbb{R}^n$. Then, for any $t \geq 0$,

$$P\left(\sum_{i=1}^{n} a_i X_i \geq t\right) \leq \exp\left(-\frac{t^2}{2\left\|\boldsymbol{a}\right\|_2^2}\right)$$

- Proof of correctness
- Generalize to non-symmetric distribution

$$P(X = 1) = p \in [0, 1], \quad P(X = -1) = 1 - p$$