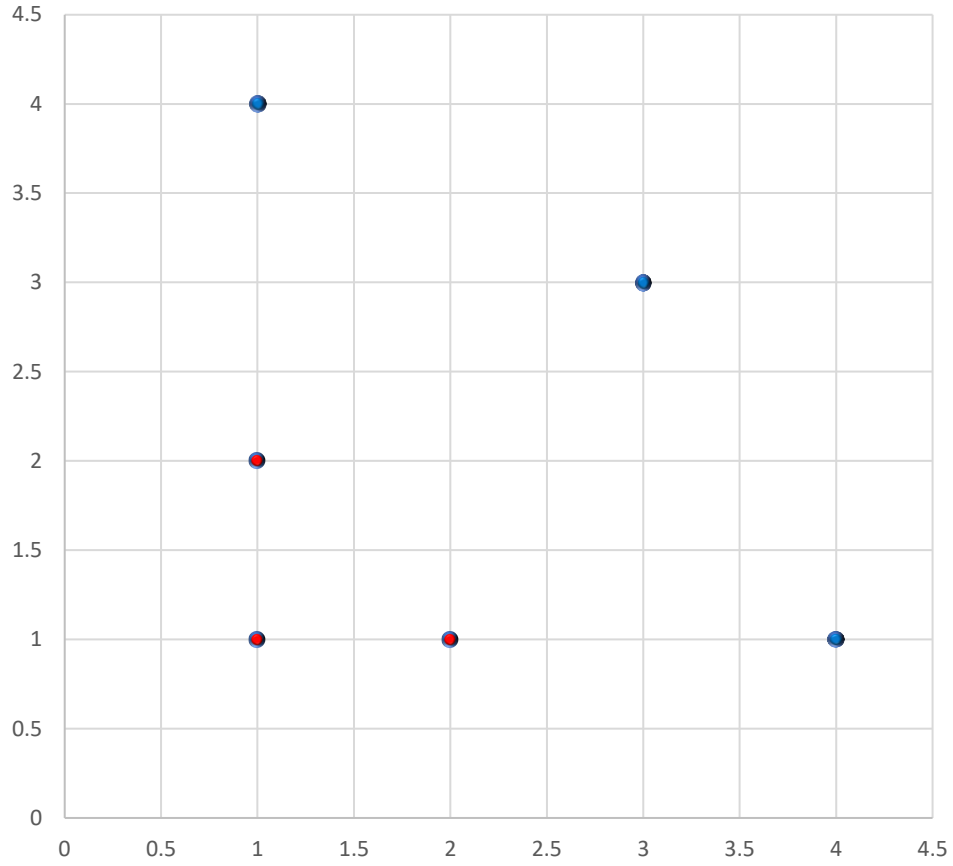
Support Vector Machine

Support Vector Machine



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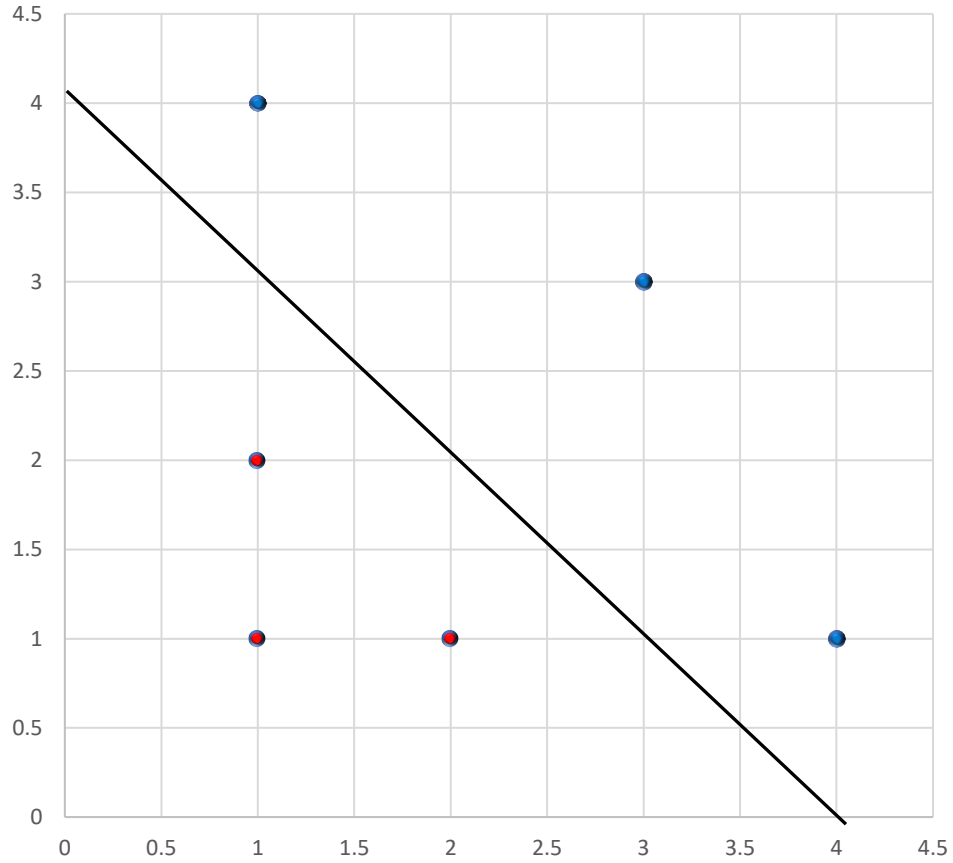

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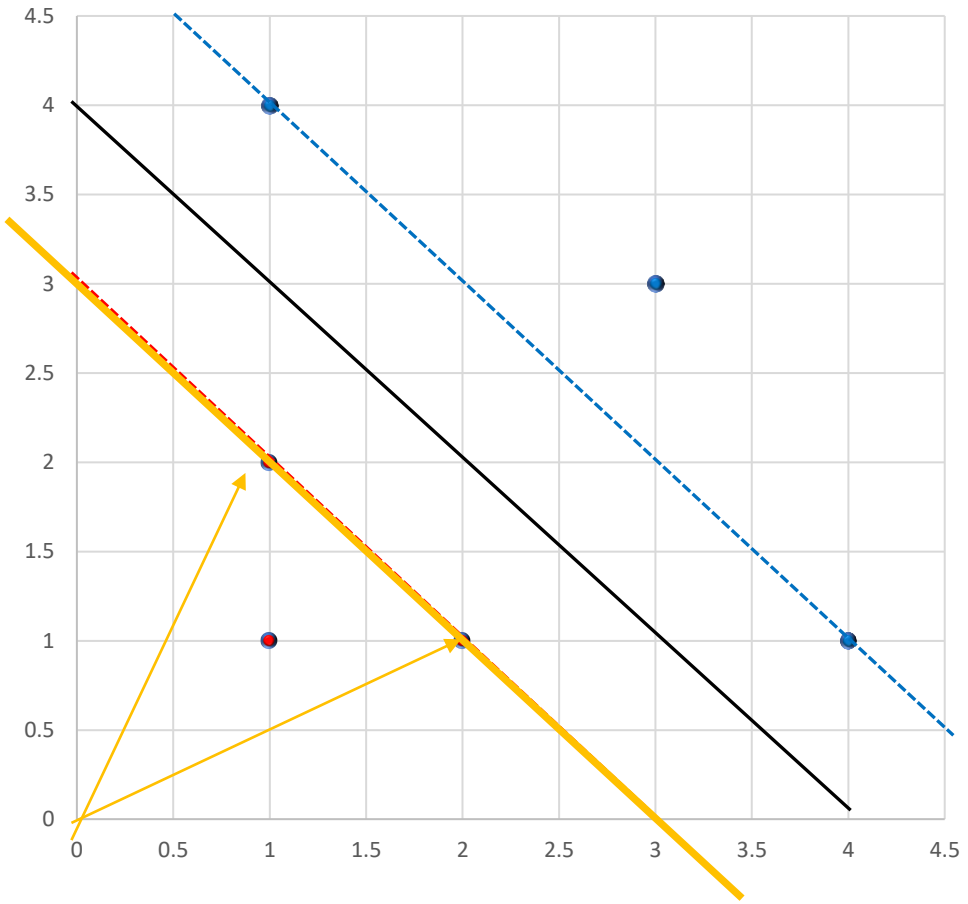

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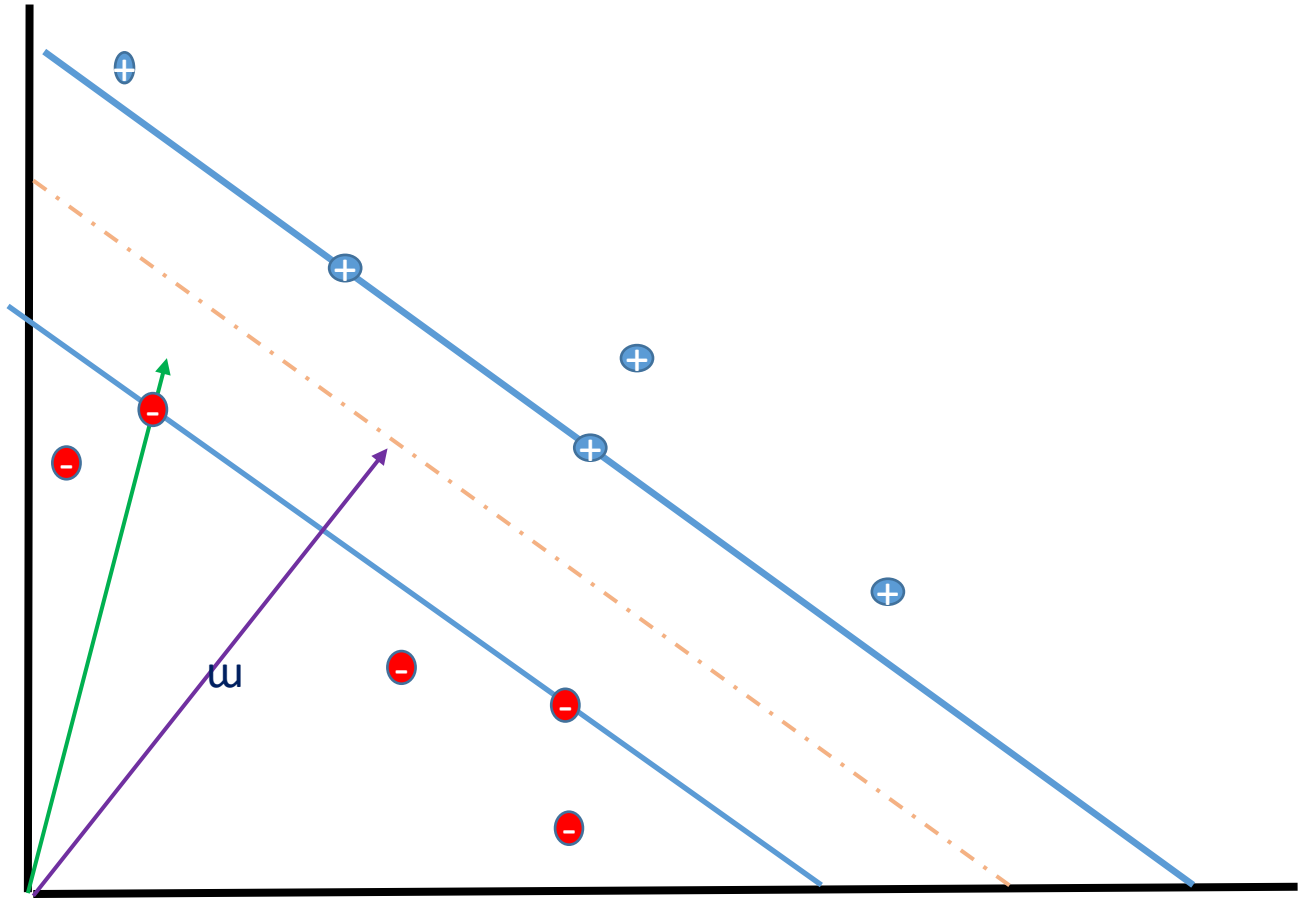

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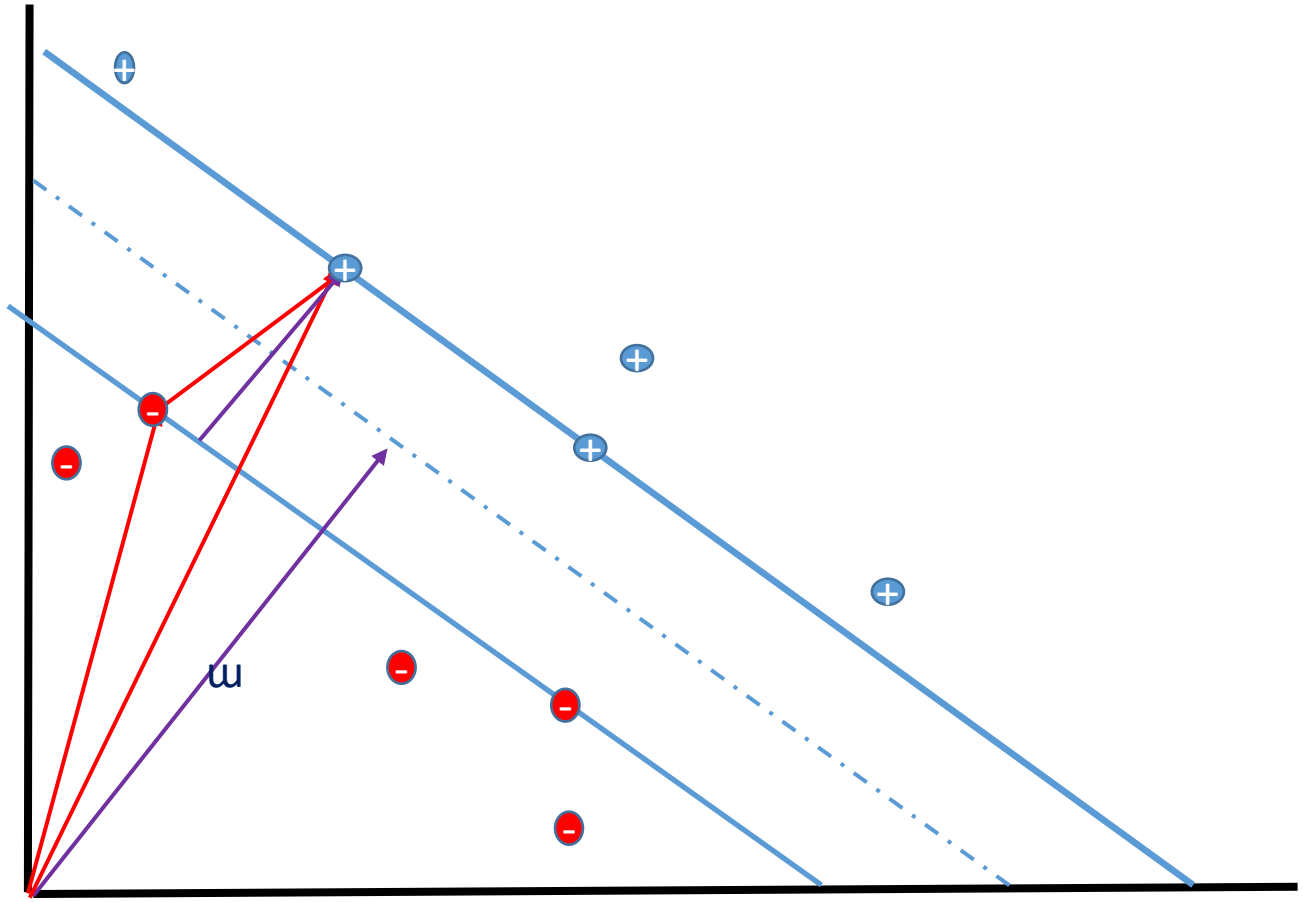


$\bar{w} \cdot \bar{u} \geq c \qquad b = -c$

$\bar{w} \cdot \bar{u} + b \geq 0$

$\bar{w} \cdot \bar{x}_i^+ + b \geq 1$

$\bar{w} \cdot \bar{x}_i^- + b \leq -1$

Define $y_i = 1$ for $\bar{x}_i^+$

$y_i = -1$ for $\bar{x}_i^-$

$$\begin{cases} y_i (\bar{w} \cdot \bar{x}_i^+ + b) \geq 1 \\ y_i (\bar{w} \cdot \bar{x}_i^- + b) \geq 1 \end{cases}$$

for x_i in the gutter

$$\begin{cases} y_i (\bar{w}\, \bar{x}_i^+ + b) - 1 = 0 \\ y_i (\bar{w}\, \bar{x}_i^- + b) - 1 = 0 \end{cases}$$

width of the street

$$(\bar{x}_i^+ - \bar{x}_i^-) \cdot \frac{\bar{w}}{\|w\|} = \frac{\bar{w} \cdot \bar{x}^+ - \bar{w}\,\bar{x}^-}{\|w\|}$$

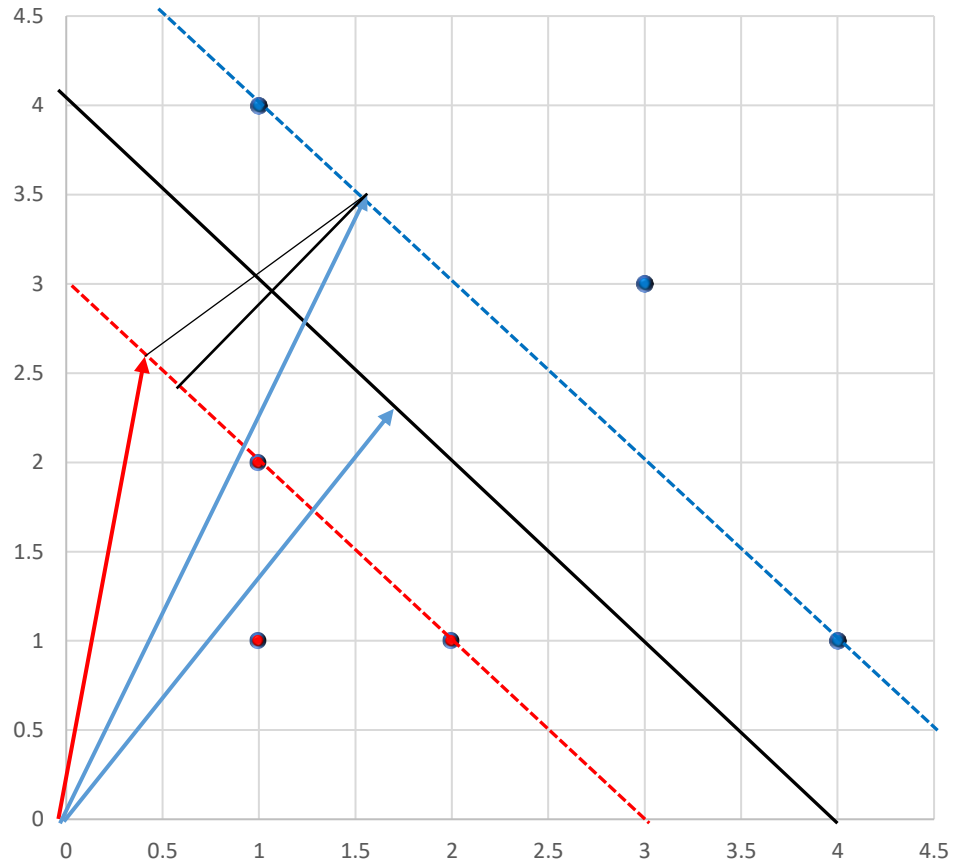
for x^+ $y_i = 1$ then $\bar{w}\, x^+ = 1 - b$

for $\bar{x}^-$ $y_i = -1 - b$

$$\frac{\bar{w}\, x^+ - \bar{w}\, x^-}{\|w\|} = \frac{(1-b) - (-1-b)}{\|w\|} = \frac{2}{\|w\|} = \text{width of the street}$$

$$\text{Max } \frac{2}{\|w\|} \quad \text{s.t.} \quad y_i (\bar{w}\, x_i + b) - 1 = 0$$

$$\Rightarrow \text{Min } \|w\| \quad \text{s.t.} \quad y_i (\bar{w}\, x_i + b) - 1 = 0$$



$\Rightarrow \min \frac{1}{2}\|w\|^2$

s.t. $= y_i(\bar{w}x_i + b) - 1 = 0$ for all i

$\Rightarrow L = \min \frac{1}{2}\|w\|^2 - \sum_i \alpha_i [y_i(\bar{w}x_i + b) - 1]$

$$\begin{cases} \frac{\partial L}{\partial w} = w - \sum_i \alpha_i y_i x_i = 0 \Rightarrow \boxed{w = \sum_i \alpha_i y_i x_i} \\ \frac{\partial L}{\partial b} = \boxed{\sum \alpha y_i = 0} \end{cases}$$

$\max L = \frac{1}{2} \sum_i \alpha_i y_i x_i \sum_j \alpha_j y_j x_j - \sum_i \alpha_i y_i x_i \sum_j \alpha_j y_j \alpha_j$

$- \sum_i \alpha_i y_i \overset{0}{b} + \sum_i \alpha_i$

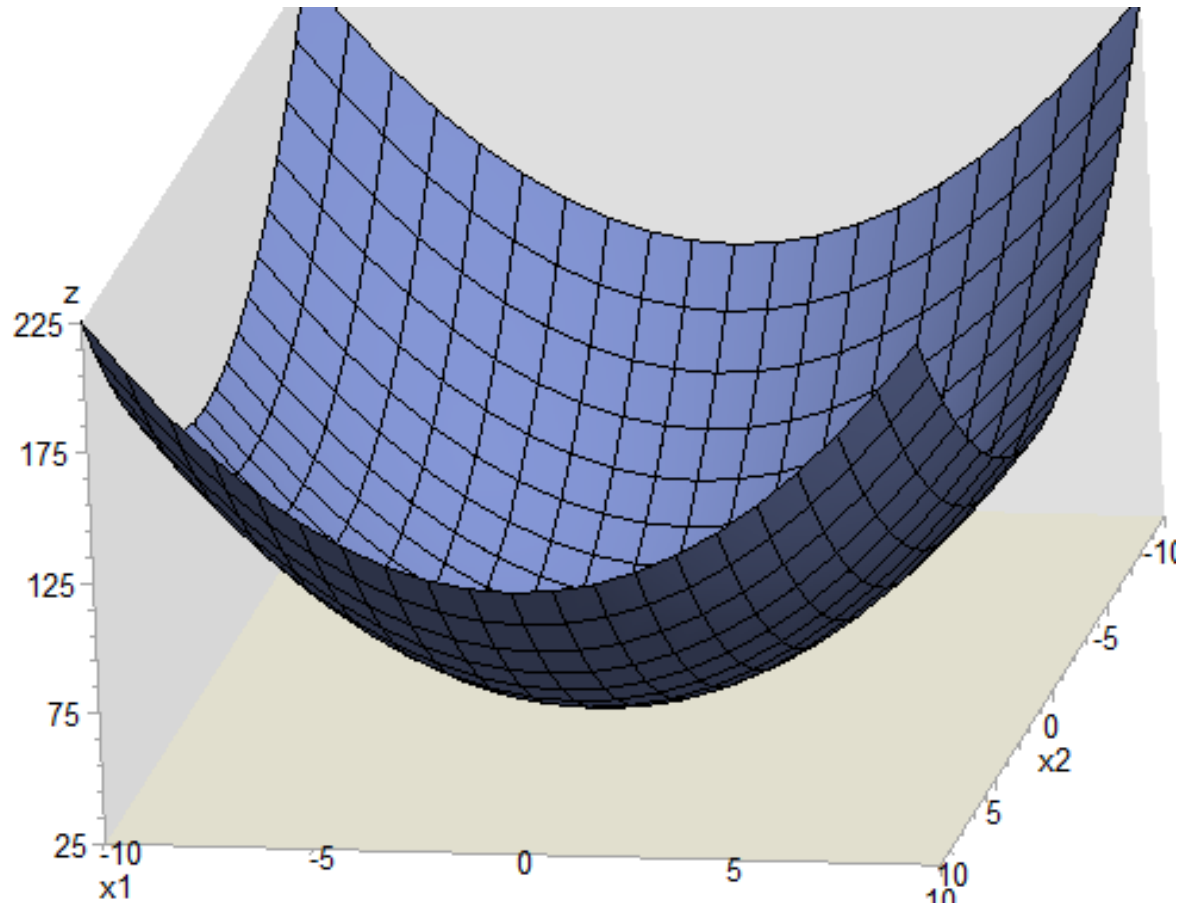
$\max L = \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j x_i x_j$

Define $H_{ij} = y_i x_i \cdot y_j x_j$

$\max$

$\Rightarrow L = \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i H_{ij} \alpha_j$

$\max_{\alpha} L = \sum \alpha_i - \frac{1}{2} \alpha^T H \alpha$


z
225
175
125
75
25
-10
x1
-5
0
5
10
-10
-5
0
x2
5
10