MTH 600: Computational Methods in Mathematics
Lab 5 Exercise

Question 1.

Assume the stock price follows a geometric Brownian motion

$$dS_t = rS_t dt + \sigma(S_t, t)S_t dZ_t,$$

where $S_0 = 20$, annual risk free interest rate $r = 5\%$, and

$$\sigma(S_t, t) = 0.281 + 0.002538(t + t^2) + (0.207 + 0.033t + 0.218t^2) \times \tanh\left(\frac{-27.42 - 4.71t}{1 + 28.27t} \ln\left(\frac{S_t}{S_0}\right) + \frac{0.025 + 0.29t}{1 + 1.85t}\right).$$

Price an Asian option matures in 1 year with strike price K = 20 by the Euler-Marayama method. The payoff function of the Asian option is

$$\text{payoff} = \max\{(A_N - K)^+\}, \text{ where } A_N = \frac{1}{N}\sum_{i=1}^{N} S_i,$$

where $N = 1/\Delta t$. In the Euler-Marayama method, we set the time step $\Delta t = 0.1, 0.01, 0.005, 0.001$, respectively, and the number of paths can be determined as $M = 1/(\Delta t)^2$. Plot a graph of the computed Asian option prices with respect to Δt.