MTH 600: Computational Methods in Mathematics
Lab 8 Exercise

Question 1.

Assume the stock price follows a geometric Brownian motion

$$dS = rSdt + \sigma SdZ_t.$$

The European Call option price, $V(S, \tau)$, satisfies the Black-Scholes equation, i.e,

$$V_\tau - \frac{\sigma^2}{2}S^2 V_{SS} - rSV_S + rV = 0, \qquad V(S, 0) = (S - K)^+,$$

where $\tau \equiv T - t$ is the time to maturity.

a). Implement the **Crank-Nicolson** method with central differencing and upstream weighting to solve the B-S equation on a general grid $[0,\ S_{max}] \times [0,\ T]$ with $\Delta S = S_{max}/M$ and $\Delta\tau = T/N$.

b). Compute the European call option price through your implementation in (a) with the current value of S as 1, $\sigma = 0.3$, $r = 0.05$, $K = 1$, $T = 0.25$, $S_{max} = 3$,$M = 100$ and $N = 200$.