UNIVERSITI TEKNOLOGI MALAYSIA
SEMESTER 1 - SESSION 2024/2025

ASSIGNMENT 1

SECTION 08

SECI1013 - DISCRETE STRUCTURE

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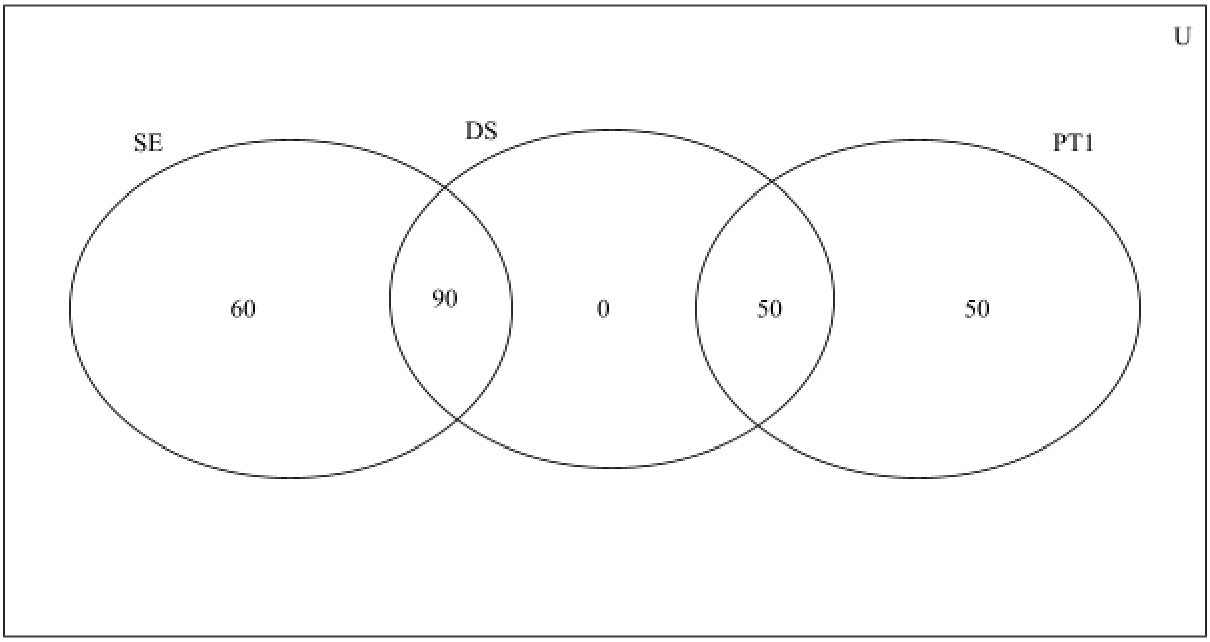
Question 1

a. There have 2 programs offered in Faculty of Computing, UTM. In this semester only 90 students out of total 150 from program 1 take discrete structure (DS) and software engineering (SE) subjects. While there have half of total 100 students from program 2 take discrete structure and programming technique 1 (PT1). All of the students from program 1 must take at least one of the subjects offered as stated either DS or SE while all of the students from program 2 must take at least one of the subjects offered as stated either DS or PT1.

Answer

i) Draw a Venn diagram to represent the above use case. (2 marks)



ii) How many students in total that take DS? (2 marks)

140 students.

iii) How many students have exactly took two subjects? (2 marks)

140 students.

iv) How many students took less than 2 subjects? (2 marks)

110 students.

b. Suppose, $A=\{n\in N|n$ odd$,1<n<20\}$, where $N=\{natural$ number$\}$ $B=\{n\in N|n$ is prime$,1<n<20\}$, $C=\{n\in N|n$ $d$ivisible by $5,1<x<10\}$

Answer

i) Find $|A|$, $|B|$and $|C|$, (3 marks)

$A = \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$
$B = \{2, 3, 5, 7, 11, 13, 17, 19\}$
$C = \{5\}$

$|A| = 9$, $|B| = 8$, $|C| = 1$

ii) Find the number of proper subsets of A. (3 marks)

Since $|A| = 9$,
The number of subset,
$|P(A)| = 2^9$
$= 512$
The number of proper subset of A
512-1 = 511

iii) Find $C\times B$ (2 marks)

$C\times B = \{(5,2), (5,3), (5,5), (5,7), (5,11), (5,13), (5,17), (5,19)\}$

Question 2

a. Formulate the symbolic expression in words using

m : You play table tennis.

n : You miss the midterm test.

o : You pass the subject.

Answer

i) $m \wedge n$ (2 marks)

You play table tennis and you miss the midterm test.

ii) $\neg(m \vee n) \vee o$ (2 marks)

You do not play table tennis and you do not miss the midterm test or you pass the subject

b. Show that $(a \rightarrow b) \equiv (\neg a \vee b)$. (6 marks)

Answer

$(a \rightarrow b)$ = If a, then b

a	b	$a \rightarrow b$	$\neg a$	$\neg a \vee b$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$(\neg a \vee b)$ = is know as Equivalent

c. Represent the given proposition symbolically by letting

x : You run 30 laps weekly .

y : You are healthy.

z : You take vegetables.

i) If you run 30 laps weekly, then you will be healthy. (2 marks)

$x \rightarrow y$

ii) If you do not run 30 laps weekly or do not take vegetables, then you will not be healthy. (2 marks)

$(\neg x \vee \neg z) \rightarrow \neg y$

iii) You will be healthy if and only if you run 30 laps weekly and take vegetable (2 marks)

$y \leftrightarrow (x \wedge z)$

Question 3

a. Let P(n) be the propositional function "n divides 15." Write each proposition in words and tell whether it is true or false. The domain of discourse is Z+.

Answer

i) P(5) (2 marks)
True.Because 15 can be divisible by 5

ii) $\forall nP(n)$ (2 marks)
False. Because not all positive integers can divide 15

iii) $\exists n\neg P(n)$ (2 marks)
True. Because at least one positive integer does not divide 15.

b. Let P(m, n) be the propositional function $m \geq n$. The domain of discourse is $Z+ \times Z+$.

i) Tell whether $\exists m \exists nP(m, n)$ is true or false. (2 marks)
True. Because for any $m = n$, $m \geq n$ holds.

ii) Write the negation of the proposition above. (2 marks)
$\neg(\exists m \exists nP(m,n)) \equiv \forall m \forall n \neg P(m,n)$

Question 4

Prove that if x and y are real numbers with x < y, there exists a rational number satisfying x < a < y. (6 marks)

$x < y$ = the difference y - x is a positive real number. $y - x > 0$. Real number can exists a rational number such as $= x < a < y$

For example $x = 2.1$ and $y = 2.9$. To find rational number $a = \frac{q}{p}$

$$2.1 < a < 2.9$$

For simplicity choose q = 10. After that multiply x and y by $q = 10$:

$$10x = 21 \text{ and } 10y = 29.$$

Integers between $10x$ and $10y$ are 22,23,24… 28. Choose one integer for example $p = 23$.

Divide $p = 23$, $q = 10$ to obtain the rational number : $a = \frac{q}{p} = \frac{23}{10} = 2.3$

To check if rational number is satisfied : $a = 2.3$ satisfied $x < a < y$

$$2.1 < 2.3 < 2.9$$

So a = $\frac{23}{10}$ is a rational number that satisfies $x < a < y$.

Question 5

a. List the elements of relation R on the set {1,2,3,4,5} defined by the rule (m,n) $\in$ R if 2 divides m − n. Afterwards, list the elements of R^{-1}. (4 marks)

R = {(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5)}
R^{-1} = {(1,1), (3,1), (5,1), (2,2), (4,2), (3,1), (3,3), (5,3), (2,4), (4,4), (1,5), (3,5), (5,5)}

b. Repeat question **a** for relation R on the set {1,2,3,4,5} defined by the rule (m,n) $\in$ R if m+n<=4. (4 marks)

R = {(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)}
R^{-1} = {(1,1), (2,1), (3,1), (1,2), (2,2), (1,3)}

c. Determine either relation of question a and b are symmetric. (2 marks)

When R = R^{-1}, the relation is symmetric. However, both question a and b are not symmetric because R $\neq$ R^{-1}, there are differences in the order of the pairs.

d. Determine whether each relation below defined on the set of positive integers is reflexive, symmetric, antisymmetric, transitive, and/or a partial order. (6 marks)

i) (x, y) $\in$ R if xy = 1.
Symmetric, antisymmetric and transitive only.

ii)(x, y) $\in$ R if x = y2.
Antisymmetric only.

iii) (x, y) $\in$ R if x = y.
Reflexive, symmetric, antisymmetric, transitive and it is a partial order since it is reflexive, symmetric and transitive.

Question 6

Determine whether each function below is one-to-one, onto, or both. Prove your answers. The domain of each function is the set of all integers. The codomain of each function is also the set of all integers.

Answer

i) f (n) = n + 1 (3 marks)

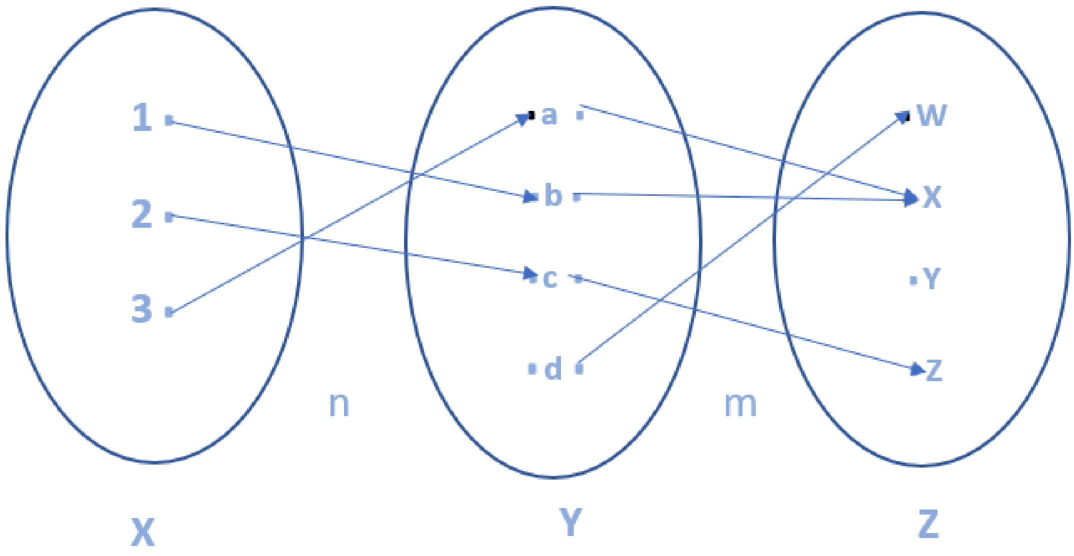
f(n1) = f(n2)
n1 + 1 = n2 + 1
n1 = n2
This shows that f is one-to-one

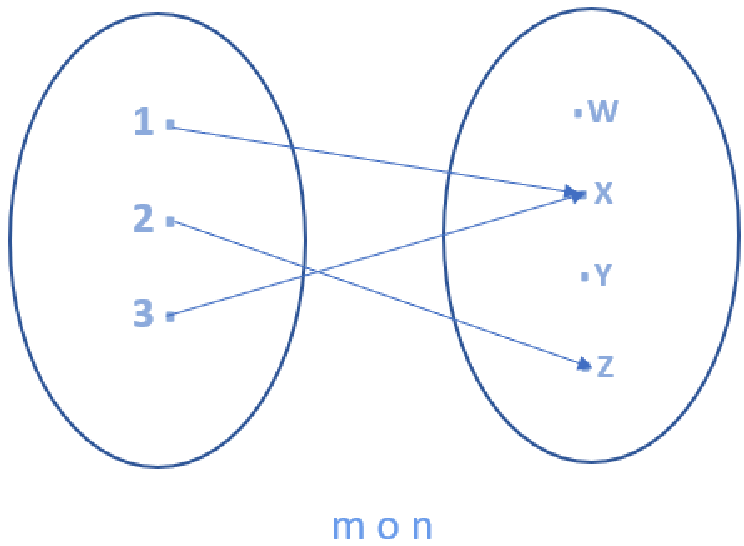
ii) f (n) = |n| (3 marks)

|n|= {n if x ≥ 0
-n if x <0}
f|n1| = f|n2|
n1 = -n2
f(3)=f(-3)
3≠-3
This shows that f is not one-to-one or onto

b. Given
n = {(1, b), (2, c), (3, a)}, a function from X = {1, 2, 3} to Y = {a, b, c, d}, and m= {(a, x), (b, x), (c, z), (d,w)}, a function from Y to Z = {w, x, y, z}, write m ∘ n as a set of ordered pairs and draw the arrow diagram of m ∘ n. (5 marks)

m o n = { (1,x),(2,z),(3,x) }





c. Let g = {(1, a), (2, c), (3, c)} be a function from X = {1, 2, 3} to Y = {a, b, c, d}. Let S = {1}, T = {1, 3}, U = {a}, and V = {a, c}. Find g(S), g(T), g−1(U), and g−1(V). (4 marks)

i) g(S) = g(1) =a
ii) g(T) = { g(1), g(3) } = { a,c}
iii) g−1(U) = 1
iv) g−1(V) = { 1,2,3 }