



### INSTRUCTIONS:

- This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
- Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
- This assignment has 5 questions (70 marks), contributing 5% of overall course marks.

### STRUCTURES:

- |                                                |            |
|------------------------------------------------|------------|
| 1. Chapter 3.5 : Discrete Probability Theory   | [15 Marks] |
| 2. Chapter 3.5 : Bayes' Theorem                | [15 Marks] |
| 3. Chapter 4.1 : Graph Definition and Notation | [15 Marks] |
| 4. Chapter 4.2 : Representation of Graphs      | [10 Marks] |
| 5. Chapter 4.3 : Isomorphism of Graph          | [15 Marks] |

### Q1. Discrete Probability Theory

1. [4 Marks]

A parent-teacher committee consisting of 4 people is to be formed from 20 parents and 5 teachers. Find the probability that the committee will consist of these people :

- All teachers
- 2 teacher and 2 parents

2. [3 Marks]

The probability that Kamal will live on campus and buy a new car is 0.37. If the probability that he will live on campus is 0.73, find the probability that he will buy a new car, given that he lives on campus.

3. [8 Marks]

Two dice are rolled:

- List the members of the event "the sum of the numbers on the dice is even."
- The probability that the sum of the numbers on the dice is 9?
- The probability that the sum of the numbers on the dice is 7 or 8?

## Q2. Bayes' Theorem

1. [7 Marks]

A company buys computers from three vendors and tracks the number of defective machines. The following table shows the results.

	Vendor		
	Acme	DotCom	Nuclear
Percent purchased	55	10	35
Percent defective	1	3	3

Let  $A$  denote the event "the computer was purchased from Acme," let  $D$  denote the event "the computer was purchased from DotCom," let  $N$  denote the event "the computer was purchased from Nuclear," and let  $B$  denote the event "the computer was defective."

- Find  $P(A)$ ,  $P(D)$ , and  $P(N)$
- Find  $P(B)$

2. [8 Marks]

Coronavirus disease 2019 (COVID-19) is a contagious disease caused by the coronavirus SARS-CoV-2. The RT-PCR test is used to detect detecting active infection of the Covid-19 virus. Approximately, 15 percent of the patients at one clinic have the Covid-19 virus. Furthermore, among those that have the Covid-19 active infection, approximately 95 percent test positive on the RT\_PCR test. Among those that do not have the Covid-19 active infection, approximately 2 percent test positive on the RT-PCR test.

- Find the probability that a patient has the active Covid-19 infection if the RT-PCR test is positive.
- How small would  $P(H)$  have to be so that the conclusion would be "no active infection of Covid-19" even if the result of the test is positive?

### Q3. Graph Definition and Notation

1. [10 Marks]

a) Draw the graph based on the following adjacency matrix:

	A	B	C	D	E
A	1	1	0	1	0
B	1	0	1	1	0
C	0	1	0	1	0
D	1	1	0	0	1
E	1	0	0	1	0

b) Based on your answer in Question 3 above, find the degree of each vertex A, B, C, D and E.

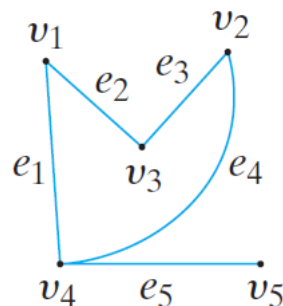
2. [5 Marks]

Give the definitions of the below terms:

- a) Vertice
- b) Edge
- c) Loop
- d) Parallel edges
- e) Degree of a vertex

### Q4. Representation of Graphs

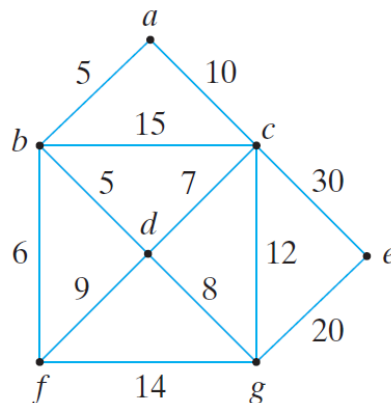
1. [6 Marks]



- a) Based on the graph above, specify the disjoint vertex sets.
- b) What type of the graph above?

2. [4 Marks]

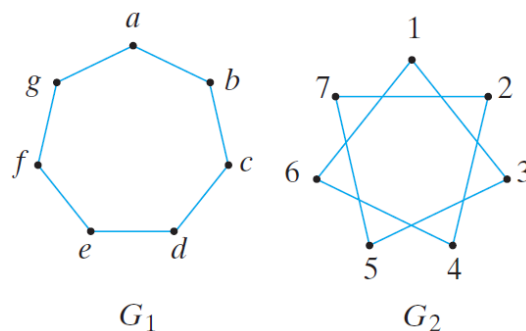
In the following graph the vertices represent cities and the numbers on the edges represent the costs of building the indicated roads. Find a least-expensive road system that connects all the cities.



### Q5. Isomorphism of Graph

1. [7 Marks]

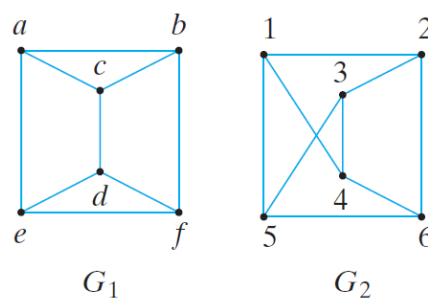
Given two graphs  $G_1$  and  $G_2$  as below, prove that  $G_1$  and  $G_2$  are isomorphic. You must prove through its properties. Afterwards, show the adjacency matrices of both  $G_1$  and  $G_2$  are equal.



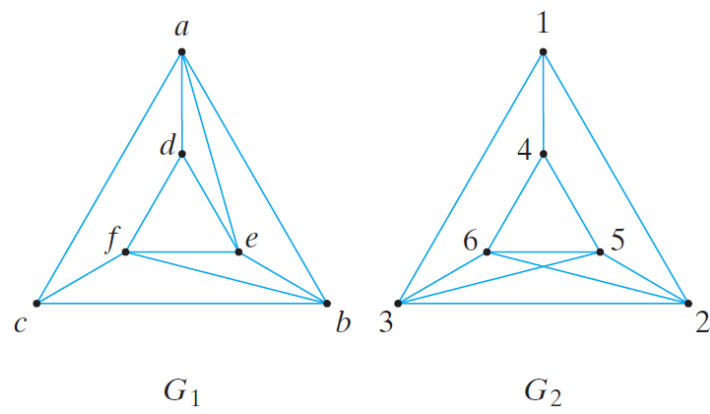
2. [4 Marks]

Given two graphs  $G_1$  and  $G_2$  as below, prove that either  $G_1$  and  $G_2$  are isomorphic. Give your reason.

a)



b)



3.

Draw all nonisomorphic simple graphs having three vertices. [4 Marks]