UTM
UNIVERSITI TEKNOLOGI MALAYSIA
Faculty of Computing

UNIVERSITI TEKNOLOGI MALAYSIA
SEMESTER 1 - SESSION 2024/2025

ASSIGNMENT 3

SECTION 08

SECI1013 - DISCRETE STRUCTURE

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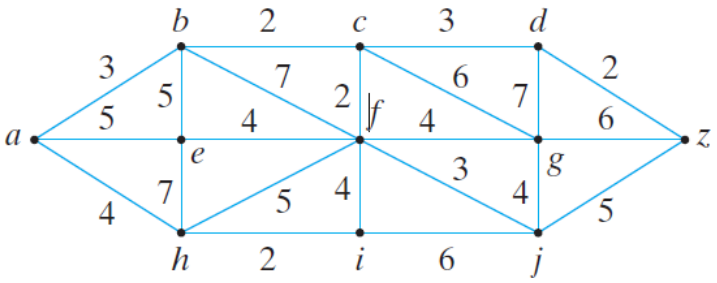
Q1. Dijkstra's Shortest Path Algorithm

1. [3 Marks]

Describe Dijkstra's shortest-path algorithm.

2. [9 Marks]



Based on the graph above, find the length of a shortest path and a shortest path between each pair of vertices in the weighted graph as stated below:

a) a,f

b) b,j

c) a,g

Answers Q1:-

1. - Initially, S :=∅
   - N:= V
   - For all vertices, u ∈ V, u ≠ a, L(u):= ∞
   - L(a):=0
   - While z ∉ S do,
     - Let v ∈ N be such that L(v)=min{L(u)|u ∈ N}
     - S:= S ∪ {v}
     - N:=N − {v}
     - For all w ∈ N such that there is an edge from v to w
     - If L(v)+W[v,w] < L(w) then L(w)=L(v)+W[v,w]

2.

I	S	N	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(h)	L(i)	L(j)	L(z)
0	{}	{a, b, c, d, e, f, g, h, i, j, z}	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
1	{a}	{b, c, d, e, f, g, h, i, j, z}	0	3	∞	∞	5	∞	∞	4	∞	∞	∞
2	{a, b}	{c, d, e, f, g, h, i, j, z}	0	3	5	∞	5	10	∞	4	∞	∞	∞
3	{a, b, h}	{c, d, e, f, g, i, j, z}	0	3	5	∞	5	9	∞	4	6	∞	∞
4	{a, b, c, h}	{d, e, f, g, i, j, z}	0	3	5	8	5	7	11	4	6	∞	∞
5	{a, b, c, e, h}	{d, f, g, i, j, z}	0	3	5	8	5	7	11	4	6	∞	∞
6	{a, b, c, e, h, i}	{d, f, g, j, z}	0	3	5	8	5	7	11	4	6	12	∞
7	{a, b, c, e, f, h, i}	{d, g, j, z}	0	3	5	8	5	7	11	4	6	10	∞
8	{a, b, c, d, e, f, h, i}	{g, j, z}	0	3	5	8	5	7	11	4	6	10	10
9	{a, b, c, d, e, f, h, i, j}	{g, z}	0	3	5	8	5	7	11	4	6	10	10
10	{a, b, c, d, e, f, h, i, j, z}	{g}	0	3	5	8	5	7	11	4	6	10	10

a) The shortest path from a to f is 7

   a → b → c → f

   3+2+2 = 7

b) The shortest path from b to j is (10-3 = 7)

   b → c → f → j

   2+2+3 = 7

c) The shortest path from a to g is 11

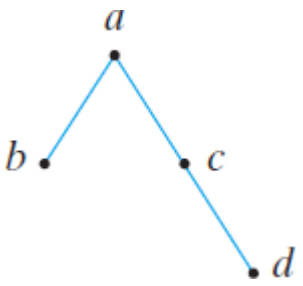
   a → b → c → g

   3+2+6 = 11

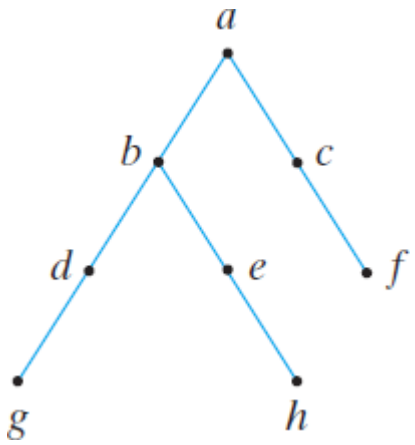
Q2. Trees

1. [4 Marks]

Determine either each of the following binary tree is balance.

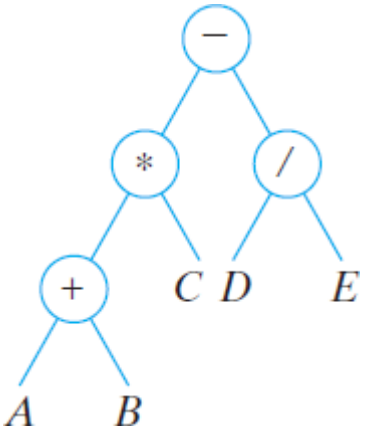


a



b

2. [3 Marks]



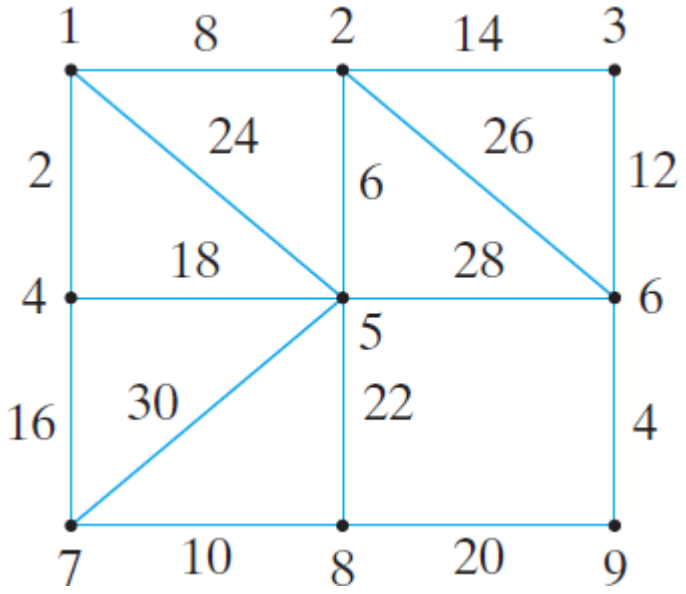
From the above tree, form the expression using inorder traversal.

3. [7 Marks]

Represent the expression below as a binary tree and write the prefix and postfix forms of the expression.

$$(((A + B) * C + D) * E) - ((A + B) * C - D)$$

4. [4 Marks]



Find a minimal spanning tree for the above graph.

Answer Q2:

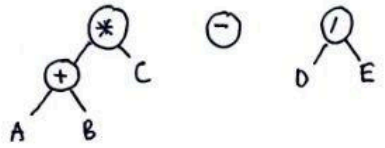
1. Tree a

   Height difference = 2 - 1 = 1 (balanced)

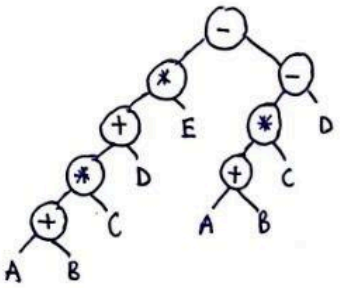
   Tree b

   Height difference = 3 - 2 = 1 (balanced)

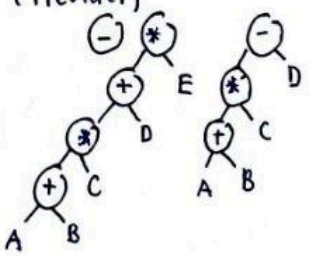
   ∴ Both trees are balanced since the height difference does not exceed 1.

2. 

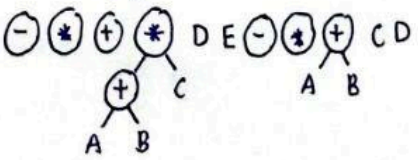
   A + B * C - D / E

3. 

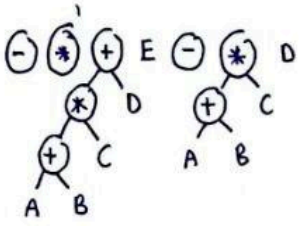
   Prefix (Preorder)

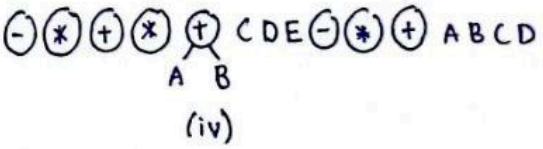
   (i)

   (iii)

   (ii)

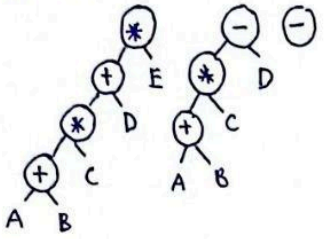
   (iv)

   ∴ Prefix form:

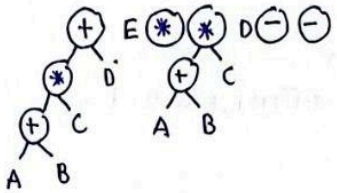
   - * + * + A B C D E - * + A B C D

Postfix (Postorder):

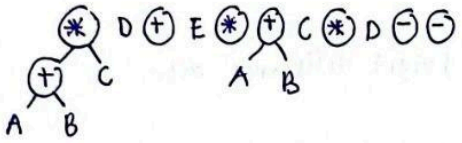
(i)



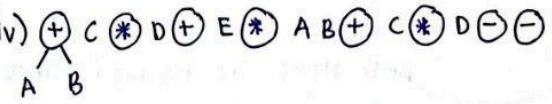
(ii)



(iii)



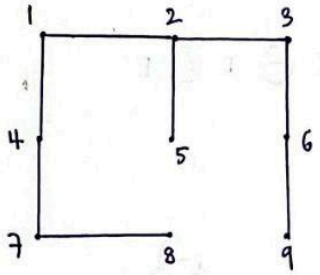
(iv)



∴ Postfix form : AB+C*D+E*AB+C*D--

4. Based on Krustal's Algorithm,

1,4	2	✓
6,9	4	✓
2,5	6	✓
1,2	8	✓
7,8	10	✓
3,6	12	✓
2,3	14	✓
4,7	16	✓
4,5	18	
8,9	20	
5,8	22	
1,5	24	
2,6	26	
5,6	28	
5,7	30	



2+4+6+8+10+12+14+16 = 72

Q3. Deterministic Finite Automata

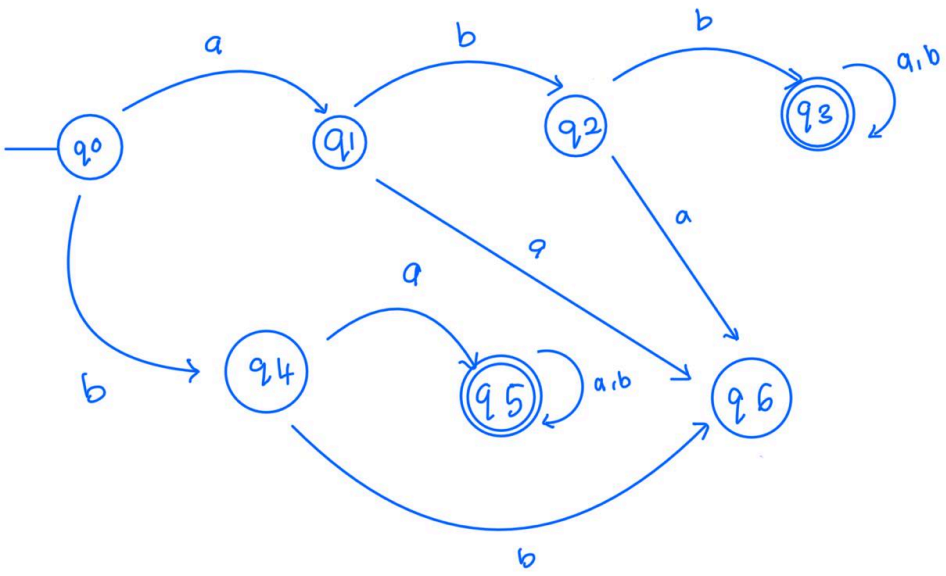
1. [8 Marks]

Design deterministic finite-state automata that accept the strings over {a, b} having the properties specified in:

a) Starting either abb or ba

Answer Q3:

1.
a)



Q4. Finite State Machines

1. [3 Marks]

Define *finite-state machine*.

2. [13 Marks]

Draw each of the transition diagram of the finite-state machine (I,O, S, f , g, σ_0) based on the following tables

a) $\mathcal{I} = \{a, b\}, \mathcal{O} = \{0, 1\}, \mathcal{S} = \{\sigma_0, \sigma_1\}$

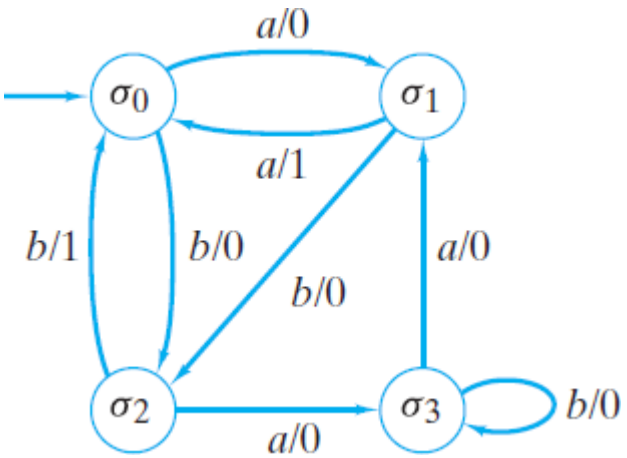
	f		g	
$\mathcal{S}$ \ $\mathcal{I}$	a	b	a	b
σ_0	σ_1	σ_1	1	1
σ_1	σ_0	σ_1	0	1

b) $\mathcal{I} = \{a, b, c\}, \mathcal{O} = \{0, 1\}, \mathcal{S} = \{\sigma_0, \sigma_1, \sigma_2\}$

	f			g		
$\mathcal{S}$ \ $\mathcal{I}$	a	b	c	a	b	c
σ_0	σ_0	σ_1	σ_2	0	1	0
σ_1	σ_1	σ_1	σ_0	1	1	1
σ_2	σ_2	σ_1	σ_0	1	0	0

3. [6 Marks]

Find the sets I, O, and S, the initial state, and the table defining the next-state and output functions for the finite-state machine below.



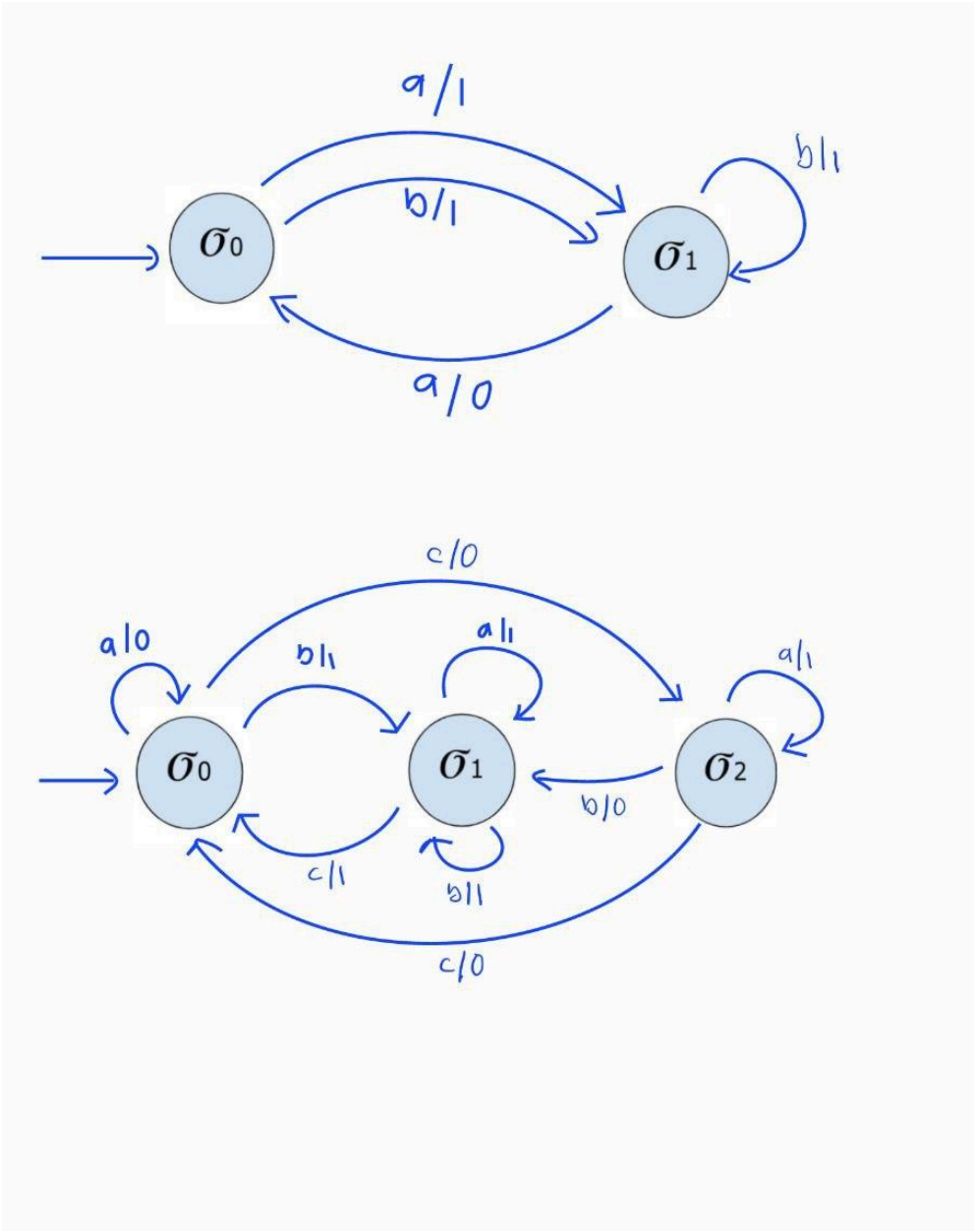
Answer Q4:-

1.

- automata with input as well as output
- every state has input and corresponding to the input the state also has an output
- A finite state machine is a sextuple
  $M=\{S, I, O, q_0, f_s, f_0\}$

  where,
  S is a finite nonempty of states
  I is the input of alphabet
  O is the output of alphabet
  q_0 is the initial state
  f_s is the state transition function
  f_0 is the output function

2. a) and b)



3.

I = {a, b}

O={0, 1}

S={σ_0, σ_1, σ_2, σ_3}

Initial state=σ_0

	fs		fg	
S	**a**	**b**	**a**	**b**
σ_0	σ_1	σ_2	0	0
σ_1	σ_0	σ_2	1	0
σ_2	σ_3	σ_0	0	1
σ_3	σ_1	σ_3	0	0