



INSTRUCTIONS:

- This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
- Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
- This assignment has 4 questions (80 marks), contributing 5% of overall course marks.

STRUCTURES:

- Chapter 2 Part 3: Recurrence Relation [20 Marks]
- Chapter 3 Part 1: Basic Principles [20 Marks]
- Chapter 3 Part 2 & 3: Permutations & Combinations [20 Marks]
- Chapter 3 Part 4: Pigeonhole Principle (First, Second, third Form) [20 Marks]

Q1. Recurrence Relation

- Solve the recurrence relation given;
 - $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1$ and $a_1 = 6$
 - $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;
initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$
 - $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1, a_1 = -2$ and $a_2 = -1$(12 marks)

- A sequence $a_1, a_2, a_3, a_4, \dots$ is given by

$$a_{n+1} = 5a_n - 3 ; a_1 = k$$

where k is a non-zero constant.

- Find the value of a_4 in terms of k .

(ii) Given that $a_4 = 7$, determine the value of k .

(8 marks)

Q2. Basic Principles

1. Refer to a set of five distinct computer science books, three distinct mathematics books, and two distinct art books.
 - a. In how many ways can these books be arranged on a shelf? (2 marks)
 - b. In how many ways can these books be arranged on a shelf if all books of the same discipline are grouped together? (3 marks)
 - c. There are 10 copies of one book and one copy each of 10 other books. In how many ways can we select 10 books? (2 marks)
2. Refer to the integers from 5 to 200, inclusive.
 - a. How many numbers are there? (2 marks)
 - b. How many are divisible by 5? (2 marks)
 - c. How many contain the digit 7? (5 marks)
 - d. How many have the digits in strictly increasing order? (Examples are 13, 147, 8.) (4 marks)

Q3. Permutations & Combinations

1. Determine how many strings can be formed by ordering the letters ABCDE subject to the conditions given.
 - a. Contains the substring ACE (2 marks)
 - b. Contains either the substring AE or the substring EA or both (3 marks)
 - c. In how many ways can five distinct Martians and eight distinct Jovians wait in line if no two Martians stand together? (3 marks)
 - d. In how many ways can five distinct Martians and five distinct Jovians wait in line? (2 marks)
2. In how many ways can we select a committee of three from a group of 11 persons? (3 marks)
3. Suppose that a pizza parlor features four specialty pizzas and pizzas with three or fewer unique toppings (no choosing anchovies twice!) chosen from 17 available toppings. How many different pizzas are there? (5 marks)
4. Let $X = \{a, b, c, d\}$.

Compute the number of 3-combinations of X.

(2 marks)

Q4. Pigeonhole Principle (First, Second, third Form)

1. How many students in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on a scale from 0 to 100 points. (5 marks)
2. what is the minimum number of students required in a Discrete Structure class so that at least six students will receive the same letter grade (A,B,C,D, or F) (5 marks)
3. Prove that among 35 students in a class, at least two have first names that start with the same letter. (5 marks)
4. Thirteen persons have first names Dennis, Evita, and Ferdinand and last names Oh, Pietro, Quine, and Rostenkowski. Show that at least two persons have the same first and last names. (5 marks)