Assignment 1 – Chapter 1.1 – Chapter 2.2

Q1 (Chapter 1.1)

a. There have 2 programs offered in Faculty of Computing, UTM. In this semester only 90 students out of total 150 from program 1 take discrete structure (DS) and software engineering (SE) subjects. While there have half of total 100 students from program 2 take discrete structure and programming technique 1 (PT1). All of the students from program 1 must take at least one of the subjects offered as stated either DS or SE while all of the students from program 2 must take at least one of the subjects offered as stated either DS or PT1.

   i) Draw a Venn diagram to represent to above use case. (2 marks)
   ii) How many students in total that take DS? (2 marks)
   iii) How many students have exactly took two subjects? (2 marks)
   iv) How many students took less than 2 subjects? (2 marks)

b. Suppose, A={$n \in N | n$ odd,1<n<20}, where N={$n$atural number}
   B={$n \in N | n$ is prime,1<n<20}, C={$n \in N | n$ $d$ivisible by 5,1<x<10}
   i) Find $|A|$, $|B|$and $|C|$, (3 marks)
   ii) Find the number of proper subsets of A. (3 marks)
   iii) Find $C \times B$ (2 marks)

Q2 (Chapter 1.2)

a. Formulate the symbolic expression in words using
   m : You play table tennis.
   n : You miss the midterm test.
   o : You pass the subject.

   i) m $\wedge$ n (2 marks)
   ii) $\neg$(m $\vee$ n) $\vee$ o (2 marks)

b. Show that (a $\rightarrow$ b) $\equiv$ ($\neg$a $\vee$ b). (6 marks)

c. Represent the given proposition symbolically by letting
   x : You run 30 laps weekly .
   y : You are healthy.
   z : You take vegetable.

   i) If you run 30 laps weekly, then you will be healthy. (2 marks)
   ii) If you do not run 30 laps weekly or do not take vegetable, then you will not be healthy. (2 marks)
   iii) You will be health if and only if you run 30 laps weekly and take vegetable (2 marks)

Q3 (Chapter 1.3)

a. Let P(n) be the propositional function “n divides 15.” Write each proposition in words and tell whether it is true or false. The domain of discourse is Z+.

i) P(5) (2 marks)
ii) ∀nP(n) (2 marks)
iii) ∃n¬P(n) (2 marks)

b. Let P(m, n) be the propositional function m ⩾ n. The domain of discourse is Z+ × Z+. Tell whether ∃m∃nP(m, n) is true or false. (2 marks)
Write the negation of the proposition above. (2 marks)

Q4 (Chapter 1.4)

Prove that if x and y are real numbers with x < y, there exists a rational number a satisfying x < a < y. (6 marks)

Q5 (Chapter 2.1)

a. List the elements of relation R on the set {1,2,3,4,5} defined by the rule (m,n) ∈ R if 2 divides m − n. Afterwards, list the elements of R^{-1}. (4 marks)
b. Repeat question **a** for relation R on the set {1,2,3,4,5} defined by the rule (m,n) ∈ R if m+n<=4. (4 marks)
c. Determine either relation of question a and b are symmetric. (2 marks)
d. Determine whether each relation below defined on the set of positive integers is reflexive, symmetric, antisymmetric, transitive, and/or a partial order. (6 marks)
   i) (x, y) ∈ R if xy = 1.
   ii) (x, y) ∈ R if $x = y^2$.
   iii) (x, y) ∈ R if x = y.

Q6 Chapter 2.2)

a. Determine whether each function below is one-to-one, onto, or both. Prove your answers. The domain of each function is the set of all integers. The codomain of each function is also the set of all integers.
   i) f (n) = n + 1 (3 marks)
   ii) f (n) = |n| (3 marks)

b. Given
   n = {(1, *b*), (2, *c*), (3, *a*)}, a function from *X* = {1, 2, 3} to *Y* = {*a, b, c, d*}, and
   m= {(*a, x*), (*b, x*), (*c, z*), (*d,w*)}, a function from *Y* to *Z* = {*w, x, y, z*}, write *m* ◦ *n* as a set of ordered pairs and draw the arrow diagram of *m* ◦ *n*. (5 marks)

c. Let g = {(1, a), (2, c), (3, c)} be a function from X = {1, 2, 3} to Y = {a, b, c, d}. Let S = {1}, T = {1, 3}, U = {a}, and V = {a, c}. Find g(S), g(T), g−1(U), and g−1(V). (4 marks)