UNIVERSITI TEKNOLOGI MALAYSIA
SEMESTER 1 - SESSION 2024/2025

ASSIGNMENT 2

SECTION 08

SECI1013 - DISCRETE STRUCTURE

LECTURER'S NAME: Ts. Dr. Goh Eg Su

NAME	MATRIC NO
ADRIANA MAISARAH BINTI AZNAN	B21EC0004
NURUL ATHIRAH BINTI ZAKARIA	A24CS0175
SITI ATHIRAH BINTI ABD MALIK	A24CS0189
WARDINA SAFIAH BINTI HARUN	A24CS0209

Q1. Recurrence Relation

(i) $a_n=6a_{n-1}-9a_{n-2}$; initial conditions $a_0=1$ *and* $a_1=6$

i) $a_n = 6a_{n-1} - 9a_{n-2}$: $a_0 = 1$, $a_1 = 6$ $n \geq 1$

$a_2 = 6(6) - 9(1) = 27$

$a_3 = 6(27) - 9(6) = 108$

$a_4 = 6(108) - 9(27) = 405$

$a_5 = 6(405) - 9(108) = 1458$

1, 6, 27, 108, 405, 1458, 5103, 17496, ...

(ii) $a_n=6a_{n-1}-11a_{n-2}+6a_{n-3}$;
initial conditions $a_0=2$,$a_1=5$ *and* $a_2=15$

ii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$: $a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$n \geq 2$

$a_3 = 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6(47) - 11(15) + 6(5) = 147$

$a_5 = 6(147) - 11(47) + 6(15) = 455$

$a_6 = 6(455) - 11(147) + 6(47) = 1395$

2, 5, 15, 47, 147, 455, 1395, 4247, 126867, ...

(iii) $a_n=-3a_{n-1}-3a_{n-2}+a_{n-3}$ initial conditions $a_0=1$,$a_1=-2$ *and* $a_2=-1$

iii) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$: $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$n \geq 2$

$a_3 = -3(-1) - 3(-2) + 1 = 10$

$a_4 = -3(10) - 3(-1) + (-2) = -29$

$a_5 = -3(-29) - 3(10) + (-1) = 56$

$a_6 = -3(56) - 3(-29) + 10 = -71$

1, 2, -1, 10, -29, 56, -71, 16, 221, -782, ...

2. A sequence $a1, a2$, $a3$, $a4$, … is given by $an+1 = 5an - 3$; $a1 = k$ where k is a non-zero constant.

(i) Find the value of $a4$ in terms of k .

Soalan 2

$a_{n+1} = 5a_n - 3$; $a_1 = k$

i) $a_2 = 5a_1 - 3 = 5k - 3$

$a_3 = 5a_2 - 3 = 5(5k-3) - 3$

$= 25k - 15 - 3$

$= 25k - 18$

$a_4 = 5a_3 - 3$

$= 5(25k - 18) - 3$

$= 125k - 90 - 3$

$= 125k - 93$

(ii) Given that $a4= 7$, determine the value of k .

ii) $a_4 = 7$

$7 = 125k - 93$

$100 = 125k$

$k = \frac{4}{5}$

Q2. Basic Principles

1. Refer to a set of five distinct computer science books, three distinct mathematics books, and two distinct art books.

   a. In how many ways can these books be arranged on a shelf? (2 marks)
      Total number of books = 5+3+2
      = 10 books
      Ways it can be arranged = 10!
      = 3628800 ways

   b. In how many ways can these books be arranged on a shelf if all books of the same discipline are grouped together? (3 marks)
      All of the books can be arranged in = 3!

      Each of the discipline can be arranged in;
      Computer science = 5!
      Mathematics = 3!
      Art = 2!

      The total number of arrangements = 3!×5!×3!×2!
      = 8640 ways

   c. There are 10 copies of one book and one copy each of 10 other books. In how many ways can we select 10 books? (2 marks)
      Case 1: If you pick 0 book A, you must pick all 10 books from the other 10 books
      = C(10,0)
      = 1 way
      Case 2: If you pick 1 book A, you must pick 9 books from the other 10 books
      = C(10,1)
      = 10 ways
      The pattern continue until you pick all 10 books from A

      Ways we can select 10 books
      = 1+10+45+120+210+252+210+120+45+10+1
      = 1024 ways

2. Refer to the integers from 5 to 200, inclusive.

   a. How many numbers are there? (2 marks)
      $(5 \leq x \leq 200)$
      = 200-5+1
      = 196 numbers

   b. How many are divisible by 5? (2 marks)
      Numbers that are divisible by 5
      = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195, 200
      = 40 numbers

   c. How many contain the digit 7? (5 marks)
      Numbers with 7 in the tens place:
      - From 70 to 79 = 10 numbers
      - From 170 to 179 = 10 numbers

      Numbers with 7 in the units place:
      - Numbers ending in 7 = 20 numbers

      Double-counted:
      - 77 and 177 = 2 numbers

      Numbers that contain the digit 7 = 10+10+20-2 = 38 numbers

   d. How many have the digits in strictly increasing order? (Examples are 13, 147, 8.) (4 marks)
      Single-digit numbers
      - 5,6,7,8,9 = 5 numbers

      Double-digit numbers
      - 12,13,14,15,16,17,18,19,23,24,25,26,27,28,29,34,35,36,37,38,39,45,46,47,48,49, 56,57,58,59,67,68,69,78,79,89
        = 36 numbers

      Three-digit numbers
      - 123,124,125,126,127,128,129,134,135,136,137,138,139,145,146,147,148,149,156 ,157,158,159,167,168,169,178,179,189
        = 28 numbers

      The total numbers of numbers with digits in strictly increasing order between 5 and 200
      = 5+36+28 = 69 numbers

Q3. Permutations & Combinations

1. Determine how many strings can be formed by ordering the letters ABCDE subject to the conditions given.

a. Contains the substring ACE (2 marks)

ordering the Letters ABCDE. substring ACE = one block.

ACE, B, D = 3 Blocks

3 blocks = 3! =6 ways

Number of strings containing in "ACE" is 6.

b. Contains either the substring AE or the substring EA or both (3 marks)

Ordering letters ABCDE. Substring AE = one block.

AE, B,C,D = 4 blocks

4! =24 ways.

Count string EA = 1 block

EA, B,C, D

4! =24 ways.

String with both AE and EA

24 + 24 =48 ways.

c. In how many ways can five distinct Martians and eight distinct Jovians wait in line if no two Martians stand together? (3 marks)

Jovians = 8! Ways.

9 slots between the jovians. Place 5 martians in 5 slots.

$$\binom{9}{5}$$

5 martians = 5! Ways.

$$8! \cdot \binom{9}{5} \cdot 5! \quad = 40320$$

$$40320 \cdot 126 \cdot 120 = 609,638,480.$$

d. In how many ways can five distinct Martians and five distinct Jovians wait in line? (2 marks)

If no restriction arrange all 10 individuals.

Total number arrangement: 10! =3,628,800

2. In how many ways can we select a committee of three from a group of 11 persons? (3 marks)

Committee of 3 from group of 11

Formula of combinations : $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$\binom{11}{3} = \frac{11!}{3! \cdot 8!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165.$$

There are 165 ways to select the committee

3. Suppose that a pizza parlor features four specialty pizzas and pizzas with three or fewer unique toppings (no choosing anchovies twice!) chosen from 17 available toppings. How many different pizzas are there? (5 marks)

There are 4 speciality pizzas.

Pizzas with 1,2, or 3 toppings: choose 17 toppings.

Pizzas with 1 toppings : $\binom{17}{1} = 17$

Pizzas with 2 toppings : $\binom{17}{2} = \frac{17 \cdot 16}{2} = 136$

Pizzas with 3 toppings : $\binom{17}{3} = \frac{17 \cdot 16 \cdot 15}{3 \cdot 2 \cdot 1} = 680.$

Total pizzas : 4 + 17 + 136 + 680 =837

There are 837 different pizzas

4. Let X = {a, b, c, d.} Compute the number of 3-combinations of X.

Let x = { a,b,c,d}

Formula combinations: $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

The size of set n = 4

The combination r = 3

$$\binom{4}{3} = \frac{4!}{3!\,(4-3)!} = \frac{4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(1)} = \frac{24}{6} = 4$$

The number of combinations x is 4

Combinations = {a,b,c} ,{ a,b,d} , {a,c,d} , {b,c,d}

Q4. Pigeonhole Principle (First, Second, third Form)

1. How many students in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on a scale from 0 to 100 points. (5 marks)

Pigeons , n : Students = ?

Pigeonholes , m : Student's possible exam score , $|\{0, 1, 2, \cdots, 100\}| = 101$

- If 1 pigeonhole contain more than 1 pigeon , then it must be $n > m$.
- If there is 101 students , it might not be possible to have at least 2 students received the same score because it is possible that all 101 students have different scores between them.
- There must be 102 students in a class to guarantee that at least 2 students received the same score on the final exam.

2. What is the minimum number of students required in a Discrete Structure class so that at least six student will receive the same letter grade (A, B, C, D, or F) (5 marks)

· Pigeons, n : Students = ?

Pigeonholes, m : grades , $|\{A, B, C, D, F\}| = 5$

one grade at least 6 students , $\left\lceil \frac{n}{5} \right\rceil \geq 6$

If $\frac{n}{5} = 6$, then $n = 30$. Therefore, $\left\lceil \frac{30}{5} \right\rceil = 6$ satisfies the condition.

$\therefore$ The minimum number of students required in the class is 30.

3. Prove that among 35 students in a class, at least two have first names that start with the same letter. (5 marks)

Pigeons, n : Students = 35

Pigeonholes, m : possible starting letters $|\{A, B, C, \ldots, Z\}| = 26$

Thus, $k = \left\lceil \frac{n}{m} \right\rceil = \left\lceil \frac{35}{26} \right\rceil = 2$ $\therefore$ Proven

4. Thirteen persons have first names Dennis, Evita, and Ferdinand and last names Oh, Pietro, Quine, and Rostenkowski. Show that at least two persons have the same first and last names. (5 marks)

First name choice = 3

Last name choice = 4

Total possible combinations : $3 \times 4 = 12$

Pigeons, n : Students = 13

Pigeonholes, m : possible combinations = 12

In the first form of Pigeonhole Principle, if $n > m$, some pigeonhole contains at least 2 pigeons. Thus, at least 2 persons have the same first and last names.