UNIVERSITI TEKNOLOGI MALAYSIA
SEMESTER 1 - SESSION 2024/2025

ASSIGNMENT 3

SECTION 08

SECI1013 - DISCRETE STRUCTURE

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Q1. Discrete Probability Theory

1. A parent-teacher committee consisting of 4 people is to be formed from 20 parents and 5 teachers. Find the probability that the committee will consist of these people :
a) All teachers
b) 2 teacher and 2 parents

2. The probability that Kamal will live on campus and buy a new car is 0.37. If the probability that he will live on campus is 0.73, find the probability that he will buy a new car, given that he lives on campus.

3. Two dice are rolled:
a) List the members of the event “the sum of the numbers on the dice is even.”
b) The probability that the sum of the numbers on the dice is 9?
c) The probability that the sum of the numbers on the dice is 7 or 8?

Answers Q1;

1. a) Total people = 20 + 5 = 25

To select 4 people from 25 people, there are $C(25,4)$ ways

To select all teachers, there are $C(5,4)$ ways

The probability that the committee will consist of all teachers:

$$\frac{C(5,4)}{C(25,4)} = \frac{5}{12650} = \frac{1}{2530}$$

b) To select 4 people from 25 people, there are $C(25,4)$ ways

To select 2 teacher, $C(5,2)$ ways

To select 2 parents, $C(20,2)$ ways

$$\frac{C(5,2) \times C(20,2)}{C(25,4)} = \frac{10 \times 190}{12650} = \frac{38}{253} = 0.1502$$

2. Let C denotes that Kamal live on campus and N denotes that Kamal buy a new car

$P(C) = 0.73$

$P(N \cap C) = 0.37$

$$P(N \mid C) = \frac{P(N \cap C)}{P(C)} = \frac{0.37}{0.73} = 0.5068$$

3. a) Let A denotes the event "the sum of the numbers on the dice is even"

$A = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5),$
$(4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

b) Let B denotes the event to obtain the sum is 9. There are 4 ways:

$(3,6), (4,5), (5,4), (6,3)$

$$P(B) = \frac{4}{36} = \frac{1}{9}$$

c) Let C denotes the event to obtain the sum is 7. There are 6 ways:

$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$

$$P(C) = \frac{6}{36} = \frac{1}{6}$$

Let D denotes the event to obtain the sum is 8. There are 5 ways:

$(2,6), (3,5), (4,4), (5,3), (6,2)$

$$P(D) = \frac{5}{36}$$

$C \cap D$ is the event of obtaining the sum is 7 and 8.

There is no way to obtain this event. Thus,

$P(C \cap D) = 0$

$$P(C \cup D) = P(C) + P(D) - P(C \cap D) = \frac{1}{6} + \frac{5}{36} - 0 = \frac{11}{36}$$

Q2. Bayes' Theorem

1. A company buys computers from three vendors and tracks the number of defective machines. The following table shows the results.

	Vendor		
	Acme	*DotCom*	*Nuclear*
Percent purchased	55	10	35
Percent defective	1	3	3

Let A denote the event "the computer was purchased from Acme," let D denote the event "the computer was purchased from DotCom," let N denote the event "the computer was purchased from Nuclear," and let B denote the event "the computer was defective."

a) Find P(A), P(D), and P(N)
b) Find P(B)

2. Coronavirus disease 2019 (COVID-19) is a contagious disease caused by the coronavirus SARS-CoV-2. The RT-PCR test is used to detect active infection of the Covid-19 virus. Approximately, 15 percent of the patients at one clinic have the Covid-19 virus. Furthermore, among those that have the Covid-19 active infection, approximately 95 percent test positive on the RT-PCR test. Among those that do not have the Covid-19 active infection, approximately 2 percent test positive on the RT-PCR test.

a) Find the probability that a patient has the active Covid-19 infection if the RT-PCR test is positive.
b) How small would P(H) have to be so that the conclusion would be "no active infection of Covid-19" even if the result of the test is positive?

Answers Q2;

A) Find P(A), P(D), and P(N)

P(A) = The computer was purchased from Acme
P(D) = The computer was purchased from DotCom
P(N) = The computer was purchased from Nuclear

P(A) = 55% ($\frac{55}{100}$ = 0.55)

P(D) = 10% ($\frac{10}{100}$ = 0.10)

P(N) = 35% ($\frac{35}{100}$ = 0.35)

B) Find P(B)

$$P(B) = P\left(\frac{B}{A_1}\right) * P(A_1) + P\left(\frac{B}{A_2}\right) * P(A_2) + \cdots + P\left(\frac{B}{A_K}\right) * P(A_K)$$

percentage defective.

$P\left(\frac{B}{A}\right)$ = Acme $\frac{1}{100} = 0.01$

$P\left(\frac{B}{D}\right)$ = Dot com $\frac{3}{100} = 0.03$

$P\left(\frac{B}{N}\right)$ = Nuclear $\frac{3}{100} = 0.03$

$P\left(\frac{B}{A}\right) = (0.55)(0.01) + (0.10)(0.03) + (0.35)(0.03)$

$P\left(\frac{B}{A}\right) = 0.0055 + 0.003 + 0.0105 = 0.019$

$P\left(\frac{B}{A}\right) = 0.019 = 1.9\%$

A) Find the probability that a patient has the active Covid-19 infection if the RT-PCR test is positive.

x = patient have positive result

y = Test is positive

using Bayes Theorem:

$$P(x) = P\left(\frac{y}{x}\right) P(x) + P\left(\frac{y}{x^2}\right) P(x^2)$$

$$P(x) = \frac{15}{100} = 0.15$$

$$P(x^2) = 1 - P(x) = 0.85$$

$$P\left(\frac{y}{x}\right) = \frac{95}{100} = 0.95$$

$$P\left(\frac{y}{x^2}\right) = \frac{2}{100} = 0.02$$

① $P(y) = P(y) = (0.15)(0.95) + (0.02)(0.85)$

$= P(y) = 0.1425 + 0.017 = 0.1595$

② $P\left(\frac{x}{y}\right) = P\left(\frac{x}{y}\right) = \frac{(0.95)(0.15)}{0.1595}$

$$P\left(\frac{x}{y}\right) = \frac{0.1425}{0.1595} = 0.893$$

B) How small would P(H) have to be so that the conclusion would be "no active infection of Covid-19" even if the result of the test is positive?

x = patient have positive result

y = Test is positive

$P\left(\frac{x}{y}\right) < 0.5$

$$P\left(\frac{x}{y}\right) = \frac{P\left(\frac{y}{x}\right) P(x)}{P(y)}$$

① $P\left(\frac{x}{y}\right) < 0.5$

$$\frac{P\left(\frac{y}{x}\right) P(x)}{P\left(\frac{y}{x}\right) P(x) + P\left(\frac{y}{x^2}\right) P(x^2)} < 0.5$$

② $P(x) = \frac{0.95}{0.95a + 0.02(1-a)} < 0.5$

$= 0.95a < 0.5\,[0.95a + 0.02 - 0.02a]$

$0.95a < 0.475a + 0.01$

$0.475a < 0.01$

$a < \frac{0.01}{0.475} \approx 0.021$

$P(x) = 2.1\%$

Q3. Graph Definition and Notation

1. a) Draw the graph based on the following adjacency matrix:

	A	B	C	D	E
A	1	1	0	1	0
B	1	0	1	1	0
C	0	1	0	1	0
D	1	1	0	0	1
E	1	0	0	1	0

b) Based on your answer in Question 3 above, find the degree of each vertex A, B, C, D and E.

2. Give the definitions of the below terms:

a) Vertice

b) Edge

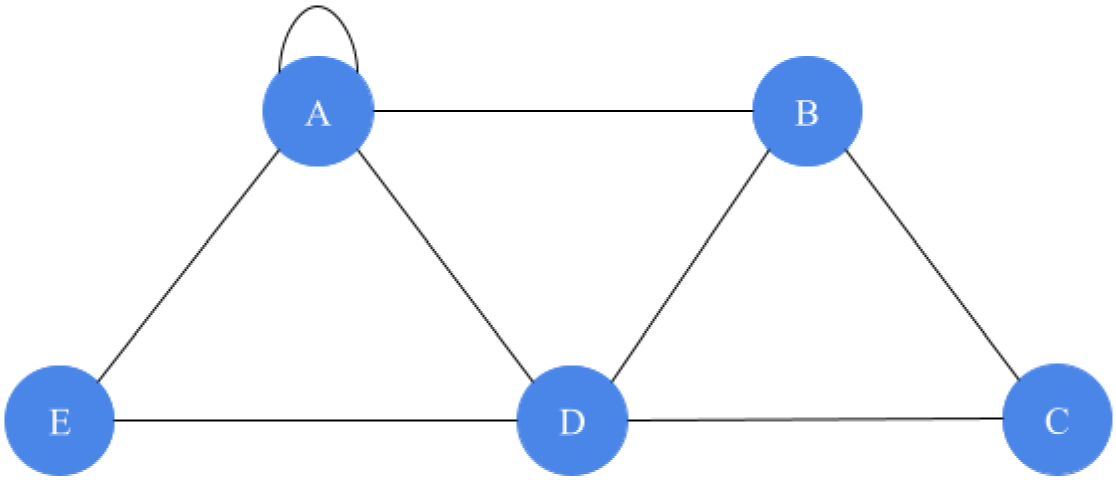
c) Loop

d) Parallel edges

e) Degree of a vertex

Answers Q3;

1. a)



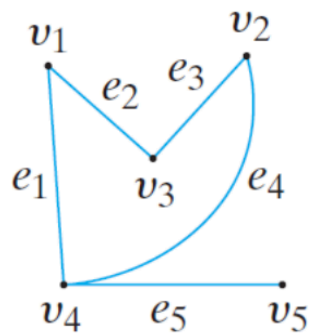
b) deg(A) = 5
deg(B) = 3
deg(C) = 2
deg(D) = 4
deg(E) = 2

2.

a) **Vertice** - The plural form of "vertex," which represents a point or a node in a graph where edges connect.
b) **Edge** - A line segment that joins two vertices in a graph, representing a connection or relationship between them.
c) **Loop** - Is an edge with just one endpoint, it starts and ends at the same vertex.
d) **Parallel edges** - Two or more distinct edges with the same set of endpoints
e) **Degree of a vertex** - The number of edges incident to a vertex. A loop is counted twice when determining the degree.

Q4. Representation of Graphs

1.



a) Based on the graph above, specify the disjoint vertex sets.

b) What type of the graph above?

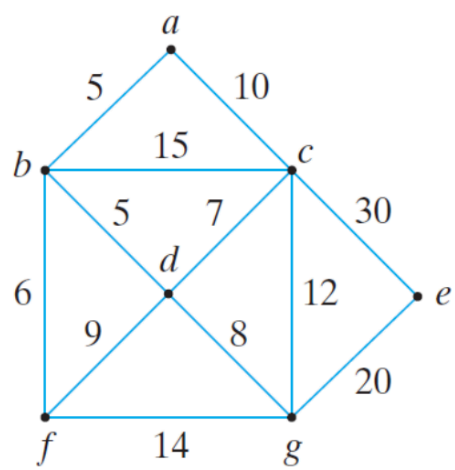
$$A_g = \begin{array}{c c} & \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

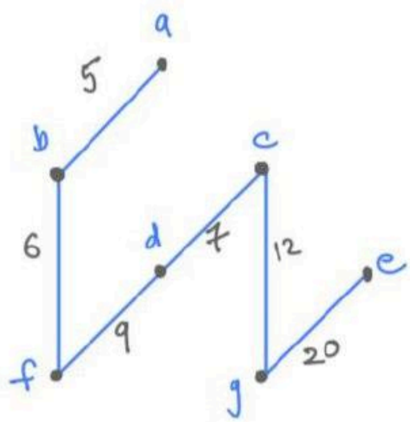
a) disjoint vertex sets :

$\{V_1, V_1\}$, $\{V_1, V_2\}$, $\{V_1, V_5\}$,
$\{V_2, V_1\}$, $\{V_2, V_2\}$, $\{V_2, V_5\}$,
$\{V_3, V_3\}$, $\{V_3, V_4\}$, $\{V_3, V_5\}$,
$\{V_4, V_3\}$, $\{V_4, V_4\}$,
$\{V_5, V_2\}$, $\{V_5, V_3\}$, $\{V_5, V_4\}$, $\{V_5, V_5\}$

b) simple and connected graph because no Loop, or any parallel edges and all vertices are connected

2. In the following graph the vertices represent cities and the numbers on the edges represent the costs of building the indicated roads. Find a least-expensive road system that connects all the cities.





least-expensive roads:

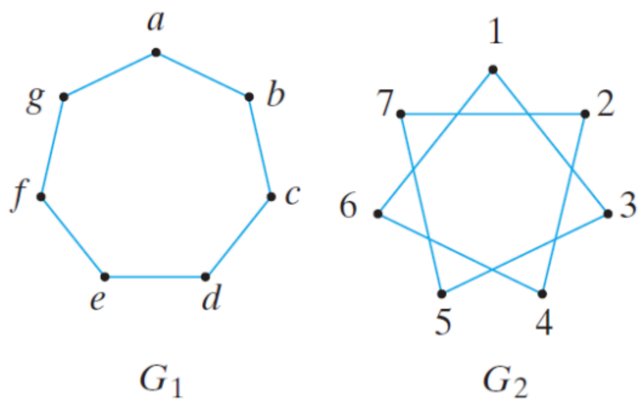
= (a, b), (b, f), (f, d), (d, c), (c, g), (g, e)

= 5 + 6 + 9 + 7 + 12 + 20

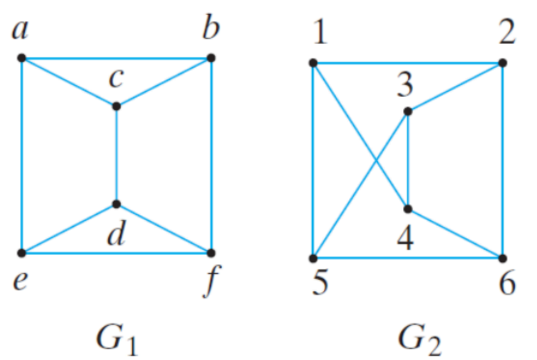
= 59

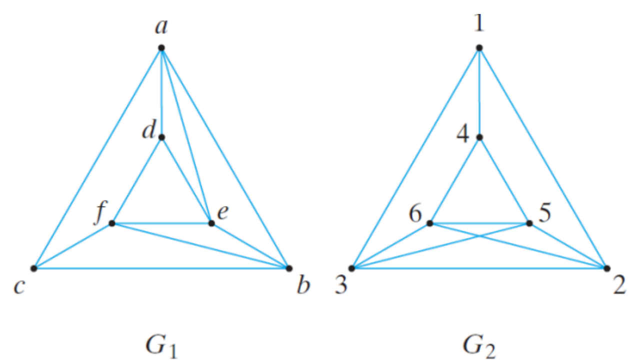
Q5. Isomorphism of Graph

1. Given two graphs G1 and G2 as below, prove that G1 and G2 are isomorphic. You must prove through its properties. Afterwards, show the adjacency matrices of both G1 and G2 are equal.



2. Given two graphs G1 and G2 as below, prove that either G1 and G2 are isomorphic. Give your reason.

a)


b)


3. Draw all nonisomorphic simple graphs having three vertices.

Answers Q5;

1. - Both have 7 vertices and 7 edges
   - Both are connected and simple graph
   - Both have 7 vertices with 2 degree
   - f: G1→ G2,

     where G1={a, b, c, d, e, f, g} and G2={1, 2, 3, 4, 5, 6, 7}

     f(a) = 1, f(b) = 3, f(c) = 5, f(d) = 7, f(e) = 2, f(f) = 4, f(g) = 6

$$\text{Adjacency Matrix, } A_{G1} = \begin{array}{cc} & \begin{matrix} a & b & c & d & e & f & g \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\text{Adjacency Matrix, } A_{G2} = \begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

∴ G1 and G2 are isomorphic

2.

a)

$$A_{G_1} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_{G_2} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 4 & 3 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

∴ These graphs are not isomorphic because the adjacency matrices for both graphs dont match.

b)

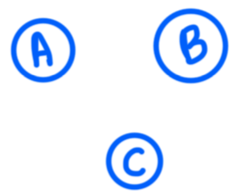
$$A_{G_1} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_{G_2} = \begin{array}{c} \\ 2 \\ 3 \\ 1 \\ 5 \\ 6 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 2 & 3 & 1 & 5 & 6 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

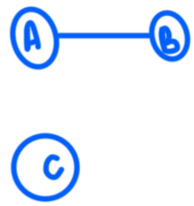
∴ These graphs are not isomorphic because the adjacency matrix for both graphs dont match each other.

3. Draw all nonisomorphic simple graphs having three vertices.

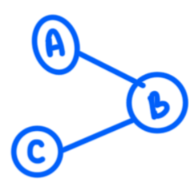
Graph 1:



Graph 2:



Graph 3:



Graph 4: