Parameterized Algorithms 22/23 — homework 1

Basic techniques

Deadline: November 22nd, 2024, 20:00 CET

Problem 1. The *cutwidth* of a graph G is the least k such that the vertices of G can be ordered as $u_1, \ldots, u_n$ so that for every $i \in \{1, \ldots, n-1\}$, there are at most k edges with one endpoint in $\{u_1, \ldots, u_i\}$ and the other in $\{u_{i+1}, \ldots, u_n\}$. Prove that the cutwidth of an n-vertex graph can be computed in time $2^n \cdot n^{\mathcal{O}(1)}$.

Problem 2. In the Convex Deletion problem we are given a set of n points in the plane, no three of them collinear, and an integer k. The task is to decide whether one can remove at most k of those points so that the remaining ones form the vertices of a convex polygon. Prove that this problem is fixed-parameter tractable when parameterized by k.

Problem 3. Consider the variant of the Hitting Set problem where the input family $\mathcal{F}$ satisfies the following property: for any two distinct $A, B \in \mathcal{F}$, we have $|A \cap B| \leqslant 10$. Prove that this problem has a polynomial kernel when parameterized by k (the size of the sought hitting set for $\mathcal{F}$).

Note: To give a polynomial kernel, one must reduce both the size of $\mathcal{F}$ and the total number of elements in $\bigcup \mathcal{F}$ to polynomial in k.

Problem 4. A graph G is a *split graph* if there is a partition (C, I) of the vertex set of G so that C is a clique in G and I is an independent set. In the Split Vertex Deletion problem we are given a graph G and an integer k, and the task is to decide whether there is a subset of vertices X of size at most k such that $G - X$ is a split graph. Prove that this problem can be solved in time $3^k \cdot n^{\mathcal{O}(1)}$.