Parameterized Algorithms 24/25 — homework 2

Treewidth, color coding, and algebraic techniques

Deadline: December 23rd, 2024, 20:00 CET

Note: In all problems except Problem 4, the algorithms should be deterministic, but correct randomized algorithms will be awarded 5 points. In Problem 4, we allow randomized algorithms.

Problem 1. In the Tree Deletion Set problem we are given a graph G and an integer $k \in \mathbb{N}$, and the question is to decide whether there exist a subset of vertices X such that $|X| \leqslant k$ and $G - X$ is a tree (that is, a *connected* graph without cycles). Prove that this problem parameterized by k is fixed-parameter tractable.

Problem 2. In the Short Directed Disjoint Paths problem we are given a directed graph D, k pairs of vertices $(s_1, t_1), \ldots, (s_k, t_k)$, all pairwise different, and an integer $\ell \in \mathbb{N}$. The question is whether there are pairwise vertex-disjoint directed paths $P_1, \ldots, P_k$ such that each P_i leads from s_i to t_i and has length at most ℓ. Prove that this problem parameterized by $k + \ell$ is fixed-parameter tractable.

Problem 3. In the Directed Feedback Arc Set problem we are given a directed graph D and an integer $k \in \mathbb{N}$, and the question is whether one can remove k arcs from D in order to make it acyclic (i.e., into a DAG). Prove that given a tree decomposition of (the underlying undirected graph of) D of width t, this problem can be solved in time $2^{\mathcal{O}(t \log t)} \cdot n^{\mathcal{O}(1)}$.

Note: Solutions proving fixed-parameter tractability for the parameterization by t, but achieving a worse running time, will be awarded points according to the obtained complexity.

Problem 4. In the Matched Perfect Matching problem we are given a bipartite graph $G = (A, B, E)$, where A and B are the sides, each of size n, and k subsets of edges $E_1, \ldots, E_k \subseteq E$. The question is whether G contains a perfect matching M that contains k distinct edges $e_1, \ldots, e_k$ such that $e_i \in E_i$, for each $i \in \{1, \ldots, k\}$. Prove that this problem can be solved in fixed-parameter time when parameterized by k.