Parameterized Algorithms 24/25 — homework 3

Important separators, representative sets, and lower bounds

Deadline: January 31st, 2025, 20:00 CET

Problem 1. In the Cut with Forbidden Pairs problem we are given a directed graph D, two vertices s and t, an integer k, and a family $\mathcal{S}$ consisting of pairs of arcs. The question is to decide whether there exists a subset of arcs F of size at most k that intersects every directed path from s to t, but $|F \cap \{a, a'\}| \leqslant 1$ for each $\{a, a'\} \in \mathcal{S}$. Prove that

- this problem is W[1]-hard when parameterized by k, [5pts]
- it does not admit an $f(k) \cdot n^{o(k)}$-time algorithm for any computable f, assuming the Exponential Time Hypothesis. [1pt]

Problem 2. In the Double Cut problem we are given a directed graph D, two vertices s and t, and an integer k. The task is to decide whether there exists a subset of arcs F of size at most k such that every directed path from s to t contains at least two arcs from F. Prove that this problem is fpt when parameterized by k.

Problem 3. In the Short Cycle Packing problem we are given an n-vertex graph G and an integer k. The question is whether in G one can find k vertex-disjoint induced cycles of length at most 5. Show a deterministic FPT algorithm for Short Cycle Packing, running in time $c^k n^{O(1)}$ for a constant $c < (2e)^5$ (in other words: beat the standard color coding based algorithm, perhaps by using a different approach).

Problem 4. In the Max Cut problem we are given a graph G and the goal is to find a coloring of the vertices of G into two colors so that the number of edges with endpoints of different colors is maximized. Suppose the input graph G is given together with a tree decomposition of width t. Prove that then the Max Cut problem can be solved in time $2^t \cdot n^{\mathcal{O}(1)}$, but the existence of an algorithm with running time $(2-\varepsilon)^t \cdot n^{\mathcal{O}(1)}$, for any $\varepsilon > 0$, would contradict the Strong Exponential Time Hypothesis.