

Computational complexity, homework problem 2, Fall 2024

In this problem, all graphs are undirected and given on an input tape as: first the number of vertices n , and then a list of edges, where each edge is a pair of vertices and the vertices are labelled by numbers $1, 2, \dots, n$ written in binary. A *tree* is a connected graph without cycles. A tree T has *radius at most 2* if there exists a vertex v in T such that every vertex of T is within distance at most 2 from v . A graph H is an *induced subgraph* of a graph G if $V(H) \subseteq V(G)$ and $E(H)$ consists of all edges of G with both endpoints in $V(H)$.

1. Consider a language consisting of pairs of trees (T_1, T_2) such that T_1 is of radius at most 2 and T_2 contains a subgraph isomorphic to T_1 . Prove that this language belongs to the class L.

Note: You do not need to argue that checking input syntax (i.e., checking if the input indeed consists of two graphs T_1 and T_2) can be done in L. However, observe that you need to reject inputs where T_1 and T_2 are correctly formatted graphs, but one of them is not a tree.

2. Consider now a language consisting of pairs (T, G) where T is a tree of radius at most 2, G is a graph, and T is isomorphic to an induced subgraph of G . Prove that this language is NP-complete.

Rules

1. This problem is worth 10 point (5 points for each part).
2. You can use any statement proved or stated on lecture or tutorials.
3. You should work on your own. It is forbidden to search for solutions in the Internet or to post questions on web services such as stackexchange. Violating this rule may result in failing the course.
4. The deadline is at 8pm on 2024-11-21. The solution should be written in English and submitted via Moodle. Scans of handwritten solutions are accepted, but we prefer solutions in PDF typed in LaTeX.
5. Questions can be sent to Jakub Gajarský, gajarsky@mimuw.edu.pl.