Computational complexity, homework problem 4, Fall 2024

Consider the following problem. On input, we are given an integer n and an undirected graph G with vertex set $V(G) = \{1, \ldots, n\} \times \{1, \ldots, n\}$ with the following properties:

- for every $1 \leq b \leq n$ there is no edge between vertices $\{(a, b) \mid 1 \leq a \leq n\}$;
- for every $1 \leq b, b' \leq n$, $b \neq b'$ and $1 \leq a \leq n$, there are at most $\log_2 n$ integers $1 \leq a' \leq n$ such that $(a, b)(a', b')$ is an edge of G.

We ask if G contains a clique of size n (i.e., a set of n pairwise adjacent vertices; note that they need to be of the form $(a_1, 1)$, $(a_2, 2)$, $\ldots$, (a_n, n) for some $1 \leq a_1, a_2, \ldots, a_n \leq n$).

Prove that if there exists a constant $c > 1$ and an algorithm solving the problem above in time $\mathcal{O}(c^n)$, then the Exponential Time Hypothesis fails.

Rules

1. This problem is worth 10 point.
2. You can use any statement proved or stated on lecture or tutorials.
3. You should work on your own. It is forbidden to search for solutions in the Internet or to post questions on web services such as stackexchange. Violating this rule may result in failing the course.
4. The deadline is at 8pm on 2025-01-24. The solution should be written in English and submitted via Moodle. Scans of handwritten solutions are accepted, but we prefer solutions in PDF typed in LaTeX.
5. Questions can be sent to Marcin Pilipczuk (`m.pilipczuk@uw.edu.pl`).