

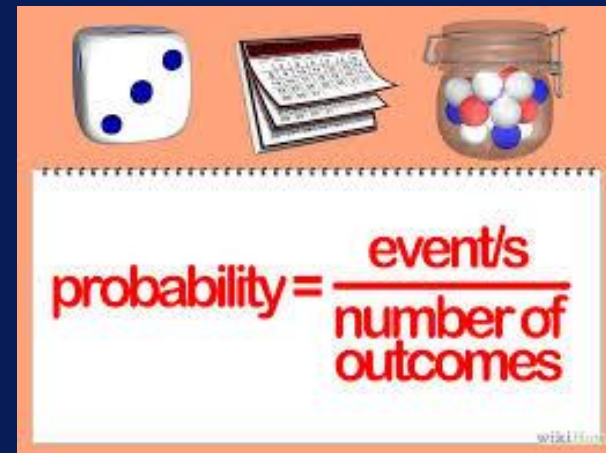
# Naïve Bayes

# After this video you will be able to..

- Discuss how a Naïve Bayes model works for classification
- Define the components of Bayes' Rule
- Explain what the 'naïve' means in Naïve Bayes

# Naïve Bayes Overview

- Probabilistic approach to classification
  - Relationships between input features and class expressed as probabilities
  - Label for sample is class with highest probability given input



# Naïve Bayes Classifier

**Classification  
Using  
Probability**



**Bayes  
Theorem**



**Feature  
Independence  
Assumption**

# Probability of Event

Probability is measure of how likely an event is

## Probability of Event 'A' Occurring

$$P(A) = \frac{\text{\# ways for A}}{\text{\# possible outcomes}}$$

# Probability of Event

What is the probability of rolling a die and getting 6?

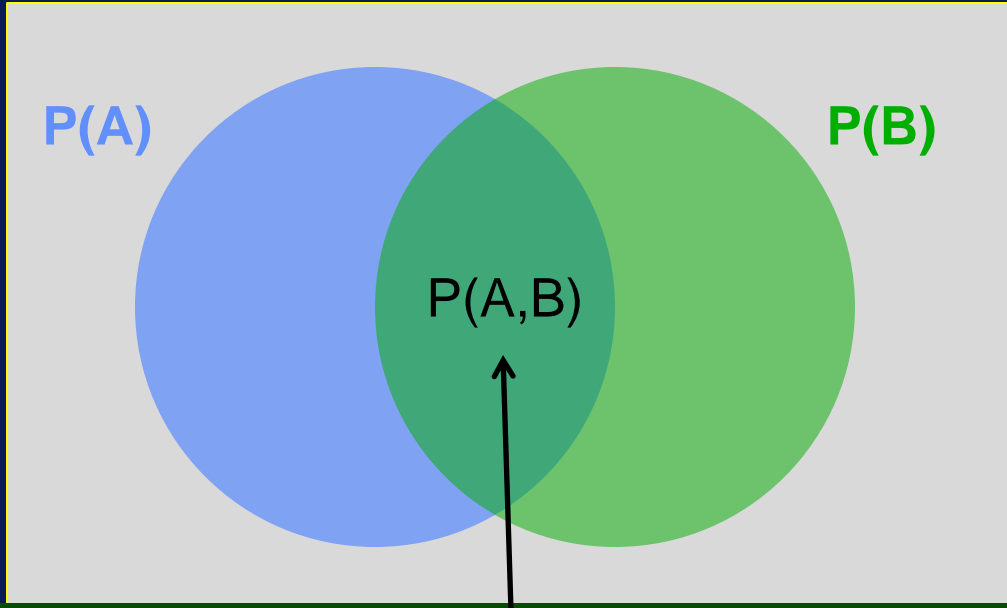


## Probability of Rolling 6 on a Die

$$P(6) = \frac{\text{\# ways for getting 6}}{\text{\# possible outcomes}} = \frac{1}{6}$$

# Joint Probability

Probability of events A and B occurring together



**Joint Probability of A and B**

# Joint Probability Example

What is the probability of two 6's when rolling two dice?



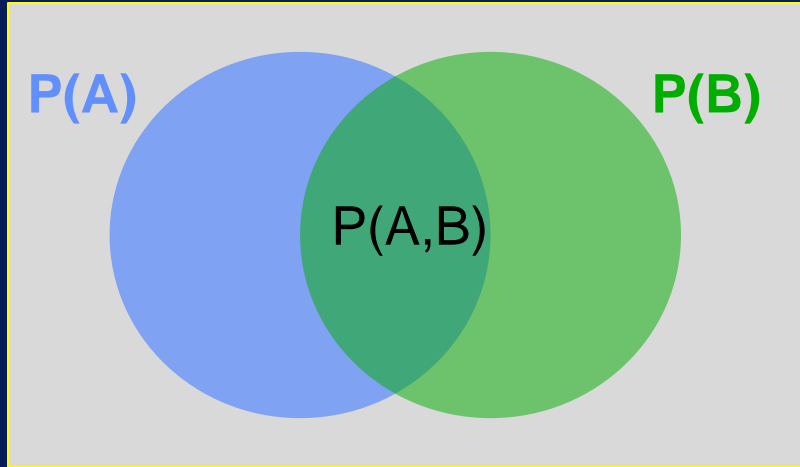
**Probability of Rolling Two 6's**

$$P(A,B) = P(A) * P(B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$$



# Conditional Probability

Probability of event A occurring, given that event B occurred



$$P(A | B) = \frac{P(A,B)}{P(B)}$$

**Conditional  
Probability**

# Bayes' Theorem

- Relationship between  $P(B | A)$  and  $P(A | B)$  can be expressed through Bayes' Theorem

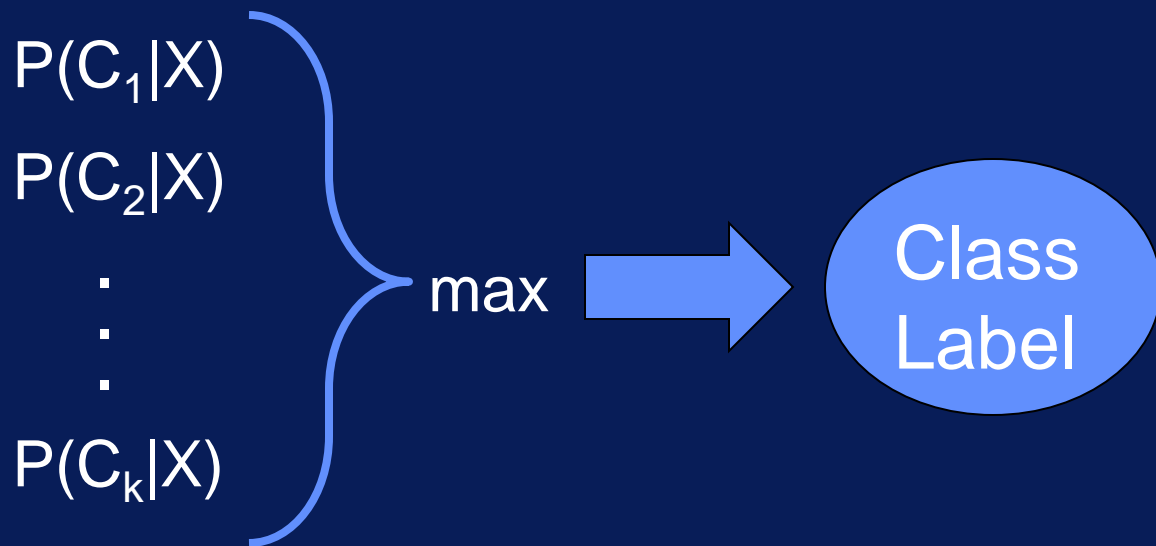
$$P(B | A) = \frac{P(A | B) * P(B)}{P(A)}$$

**Bayes' Theorem**

# Classification with Probabilities

Given features  $X=\{X_1, X_2, \dots, X_n\}$ , predict class  $C$

Do this by finding value of  $C$  that maximizes  $P(C | X)$



# Bayes' Theorem for Classification

- But estimating  $P(C|X)$  is difficult
- Bayes' Theorem to the rescue!
  - Simplifies problem



# Bayes' Theorem for Classification

Posterior Probability

Class-Conditional Probability

Prior Probability

$$P(C | X) = \frac{P(X | C) * P(C)}{P(X)}$$

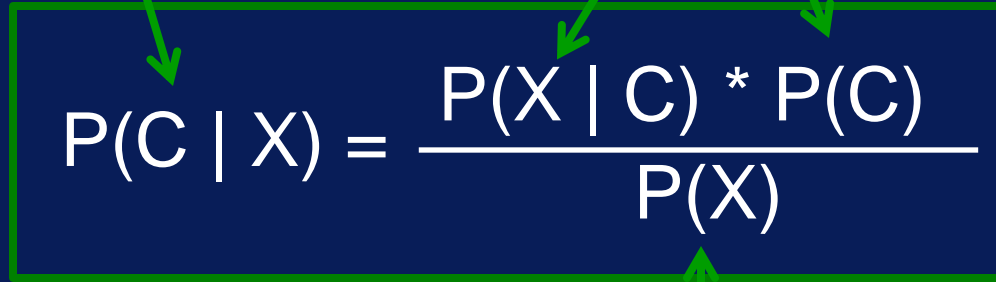
Probability of observing values for input features

The diagram illustrates Bayes' Theorem for Classification. The equation  $P(C | X) = \frac{P(X | C) * P(C)}{P(X)}$  is enclosed in a green rectangular box. Four orange arrows point from descriptive labels to parts of the equation: one from 'Posterior Probability' to  $P(C | X)$ , one from 'Class-Conditional Probability' to  $P(X | C)$ , one from 'Prior Probability' to  $P(C)$ , and one from 'Probability of observing values for input features' to  $P(X)$  in the denominator.

# Bayes' Theorem for Classification

Need to  
calculate this

Can be estimated from data!



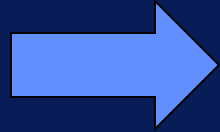
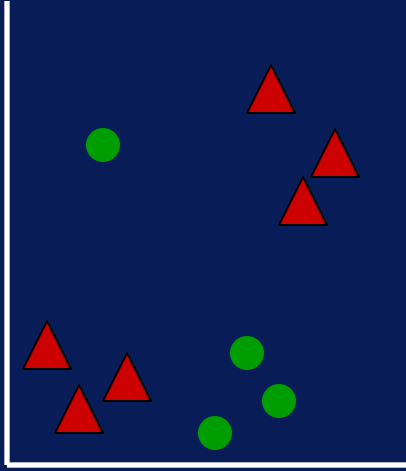
The diagram shows the Bayes' Theorem formula enclosed in a green rectangular box. Three green arrows point from external text to parts of the formula: one from 'Need to calculate this' to the left side  $P(C | X)$ , one from 'Can be estimated from data!' to the numerator  $P(X | C) * P(C)$ , and another from the same text to the denominator  $P(X)$ . A fourth green arrow points from the text 'Constant (can be ignored)' below to the denominator  $P(X)$ .

$$P(C | X) = \frac{P(X | C) * P(C)}{P(X)}$$

Constant (can be ignored)

To get  $P(C | X)$ , only need to find  $P(X | C)$  and  $P(C)$ , which can be estimated from the data!

# Estimating $P(C)$



$$P(\bullet) = 4/10 = 0.4$$

$$P(\blacktriangle) = 6/10 = 0.6$$

To estimate  $P(C)$ , calculate fraction of samples for class  $C$  in training data.

# Estimating $P(X | C)$

## Independence Assumption

- Features are independent of one another:

$$P(X_1, X_2, \dots, X_n | C) = P(X_1 | C) * P(X_2 | C) * \dots * P(X_n | C)$$

To estimate  $P(X | C)$ , only need to estimate  $P(X_i | C)$  individually → Much simpler!



# Estimating $P(X_i | C)$

| Home Owner | Marital Status | Loan Default |
|------------|----------------|--------------|
| Yes        | Single         | No           |
| No         | Married        | No           |
| No         | Single         | No           |
| Yes        | Married        | No           |
| No         | Divorced       | Yes          |
| No         | Married        | No           |
| Yes        | Divorced       | No           |
| No         | Single         | Yes          |
| No         | Married        | No           |
| No         | Single         | Yes          |

$P(\text{Home Owner} = \text{Yes} | \text{No}) = 3/7 = 0.43$



$P(\text{Marital Status} = \text{Single} | \text{Yes}) = 2/3 = 0.67$



# Naïve Bayes Classification

- **Fast and simple**
- **Scales well**
- **Independence assumption may not hold true**
  - In practice, still works quite well
- **Does not model interactions between features**

# Naïve Bayes Classifier

**Classification  
Using  
Probability**



**Bayes  
Theorem**



**Feature  
Independence  
Assumption**