CSC343 A3
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1. a)

```
-- age, sName, rating could be redundant
/*
Reservation
sID age length  sName   day                 cName  rating  cID
---------------------------------------------------------------
s1  45  17      sName1  2021-04-15 8:00:00  cName1  3       c1
s1  45  15      sName1  2021-04-15 9:00:00  cName2  3       c2
```

2.a)

- KOQ^+ = KOQPSR, so KOQ not a superkey and KOQ → PS violates BCNF
- L^+ = LKN , so L not a superkey and L → KN violates BCNF
- KQ^+ = KQRS, so KQ not a superkey and KQ → RS violates BCNF

b)

- Decompose F using FD KOQ → PS
  KOQ^+ = KOQPSR
  R1 = KOPQRS   R2 = KOLMNQ

- Project FDs onto R1

	Closure	FD
K	$K^+ = K$	nothing
O	$O^+ = O$	nothing
P	$P^+ = P$	nothing
Q	$Q^+ = Q$	nothing
R	$R^+ = R$	nothing
S	$S^+ = S$	nothing

KO	$KO^+ = KO$	nothing
KP	$KP^+ = KP$	nothing
KQ	$KQ^+ = KQRS$	$KQ \rightarrow RS$: violates BCNF, abort projection

- Decompose R1 further using FD $KQ \rightarrow RS$ into
  R3 = KQRS R4 = KOPQ

- Project FDs onto R3 = KQRS

	Closure	FD
K	$K^+ = K$	nothing
Q	$Q^+ = Q$	nothing
R	$R^+ = R$	nothing
S	$S^+ = S$	nothing
KQ	$KQ^+ = KQRS$	$KQ \rightarrow RS$
KR	$KR^+ = KR$	nothing
KS	$KS^+ = KS$	nothing
QR	$QR^+ = QR$	nothing
QS	$QS^+ = QS$	nothing
RS	$RS^+ = RS$	nothing
KRS	$KRS^+ = KRS$	nothing
QRS	$QRS^+ = QRS$	nothing

Therefore R3 satisfies BCNF

- Project FDs onto R4 = KOPQ

	Closure	FD
K	$K^+ = K$	nothing

O	$O^+ = O$	nothing
P	$P^+ = P$	nothing
Q	$Q^+ = Q$	nothing
KO	$KO^+ = KO$	nothing
KP	$KP^+ = KP$	nothing
KQ	$KQ^+ = KQRS$	nothing
OP	$OP^+ = OP$	nothing
OQ	$OQ^+ = OQ$	nothing
PQ	$PQ^+ = PQ$	nothing
KOP	$KOP^+ = KOP$	nothing
KOQ	$KOQ^+ = KOQPSR$	KOQ → P
KPQ	$KPQ^+ = KPQRS$	nothing
OPQ	$OPQ^+ = OPQ$	nothing

Therefore R4 satisfies BCNF

- Now return to R2 = KOLMNQ and project FDs onto it

	Closure	FD
K	$K^+ = K$	nothing
O	$O^+ = O$	nothing
L	$L^+ = LKN$	L → KN : violates BCNF, abort projection

- Need to further decompose R2 using FD L → KN
  R5 = KLN R6 = LMOQ

- Project FDs onto R5 = KLN

	Closure	FD
K	$K^+ = K$	nothing
L	$L^+ = LKN$	$L \rightarrow KN$
N	$N^+ = N$	nothing
Supersets of L	irrelevant	can only generate weaker FDs than what we already have
KN	$KN^+ = KN$	nothing

Therefore R5 satisfies BCNF

- Project FDs onto R6 = LMOQ

	Closure	FD
L	$L^+ = LKN$	nothing
M	$M^+ = M$	nothing
O	$O^+ = O$	nothing
Q	$Q^+ = Q$	nothing
LM	$LM^+ = LMKN$	nothing
LO	$LO^+ = LOKN$	nothing
LQ	$LQ^+ = LQKNRS$	nothing
MO	$MO^+ = MO$	nothing
MQ	$MQ^+ = MQ$	nothing
OQ	$OQ^+ = OQ$	nothing
LMO	$LMO^+ = LMOKN$	nothing
LMQ	$LMQ^+ = LMQKNRS$	nothing
LOQ	$LOQ^+ = LOQKNRSP$	nothing
MOQ	$MOQ^+ = MOQ$	nothing

Therefore relation R6 satisfies BCNF

FINAL DECOMPOSITION:

a) R3 = KQRS with FD KQ → RS
b) R4 = KOPQ with FD KOQ → P
c) R5 = KLN with FD L → KN
d) R6 = LMOQ with no FDs

c) The original FDs from set G seem to be preserved except for KOQ → PS. However, if we split the FD into KOQ → P and KOQ → S we notice that KOQ → P is preserved in the final schema and so is KOQ → S since we have a stronger FD KQ → S that holds in our final schema (got by splitting KQ → RS). Therefore, the final schema preserves dependencies.

d) Chase Test:

let t = (k, l, m, n, o, p, q, r, s)

K	L	M	N	O	P	Q	R	S
k	l_1	m_1	n_1	o_1	p_1	q	r	s
k	l_2	m_2	n_2	o	p	q	r_2	s_2
k	l	m_3	n	o_3	p_3	q_3	r_3	s_3
k4	l	m	n_4	o	p_4	q	r_4	s_4

↓

K	L	M	N	O	P	Q	R	S
k	l_1	m_1	n_1	o_1	p_1	q	r	s
k	l_2	m_2	n_2	o	p	q	r_2	s_2
k	l	m_3	n	o_3	p_3	q_3	r_3	s_3
k	l	m	n	o	p_4	q	r_4	s_4

↓

K	L	M	N	O	P	Q	R	S
k	l_1	m_1	n_1	o_1	p_1	q	r	s
k	l_2	m_2	n_2	o	p	q	r	s
k	l	m_3	n	o_3	p_3	q_3	r_3	s_3
k	l	m	n	o	p_4	q	r	s

↓

K	L	M	N	O	P	Q	R	S
k	l_1	m_1	n_1	o_1	p_1	q	r	s
k	l_2	m_2	n	o	p	q	r	s
k	l	m_3	n	o_3	p_3	q_3	r_3	s_3
k	l	m	n	o	p	q	r	s

Thus $T \in F$ and the decomposition is lossless.

3. a)

Original FDs

1. ACDE→B
2. B→C
3. B→F
4. CD→A
5. CD→F
6. BCF→A
7. BCF→D
8. ABF→H

Reducing LHS

1. CDE→B (CDE$^+$=CDEAFBH)
2. B→C
3. B→F
4. CD→A
5. CD→F
6. B→A (B$^+$=BCFADH)
7. B→D (Same reason as 6)
8. B→H (Same reason as 6)

Removing redundant FDs

FD	Exclude	Closure	Decision
1	1	CDE$^+$=CDEAF	Keep
2	2	B$^+$=BFADH	Keep
3	3	B$^+$=BCADHF	Discard
4	3,4	CD$^+$=CDF	Keep
5	3,5	CD$^+$=CDA	Keep
6	3,6	B$^+$=BCDHAF	Discard
7	3,6,7	B$^+$=BCH	Keep
8	3,6,8	B$^+$=BCDAF	Keep

Minimal basis is:

CDE→B
B→C
CD→A
CD→F
B→D

$B \rightarrow H$

b)
G is in every key (it doesn't appear in any of the FDs)
E is in every key (it only appears on the LHS of the FDs)
H isn't in any key (it only appears on the RHS of the FDs)
A,B,C,D,F must be checked

$EGA^+=EGA$
$EGB^+=EGBCDHAF$ so EGB is a key
$EGC^+=EGC$
$EGD^+=EGD$
$EGF^+=EGF$
$EGAC^+=EGAC$
$EGAD^+=EGAD$
$EGAF^+=EGAF$
$EGCD^+=EGCDAFBH$ so EGCD is a key
$EGCF^+=EGCF$
$EHDF^+=EGDF$
$EGACF^+=EGACF$
$EGADF^+=EGADF$

All other possibilities contain either EGB or EGCD, so there are two keys. EGB and EGCD.

c)
Merge the RHS of the FDs
$CDE \rightarrow B$
$B \rightarrow CDH$
$CD \rightarrow AF$

R1(C,D,E,B), R2(B,C,D,H), R3(C,D,A,F)

No relation is a superkey. Add R4(E,G,C,D), a relation whose schema is the key .

d) R1, R2, and R3, are relations that are formed by $CDE \rightarrow B$, $B \rightarrow CDH$, and $CD \rightarrow AF$ respectively, and so the LHS of these FDs are superkeys for their respective relations. This is not necessarily the case for other relations. Consider $B \rightarrow CDH$. From this FD we can get $B \rightarrow CD$ which can project onto relation R1. The closure of B is $B^+=BCDHAF$, and the FD generated is $B \rightarrow CD$. So B is not a super key of relation R1(C,D,E,B) and $B \rightarrow CDH$ violates BCNF. Therefore this schema allows redundancies.