Interpreting a trained CNN

-- What did my convolutional network learn?

Columbus Machine Learners Meetup

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Outline

Interpreting a trained CNN

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   2. Convolution
   3. Convolutional network
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   2. My CNN encoder and interpreters – some hybrid models
   3. Did my CNN learn translational invariance?
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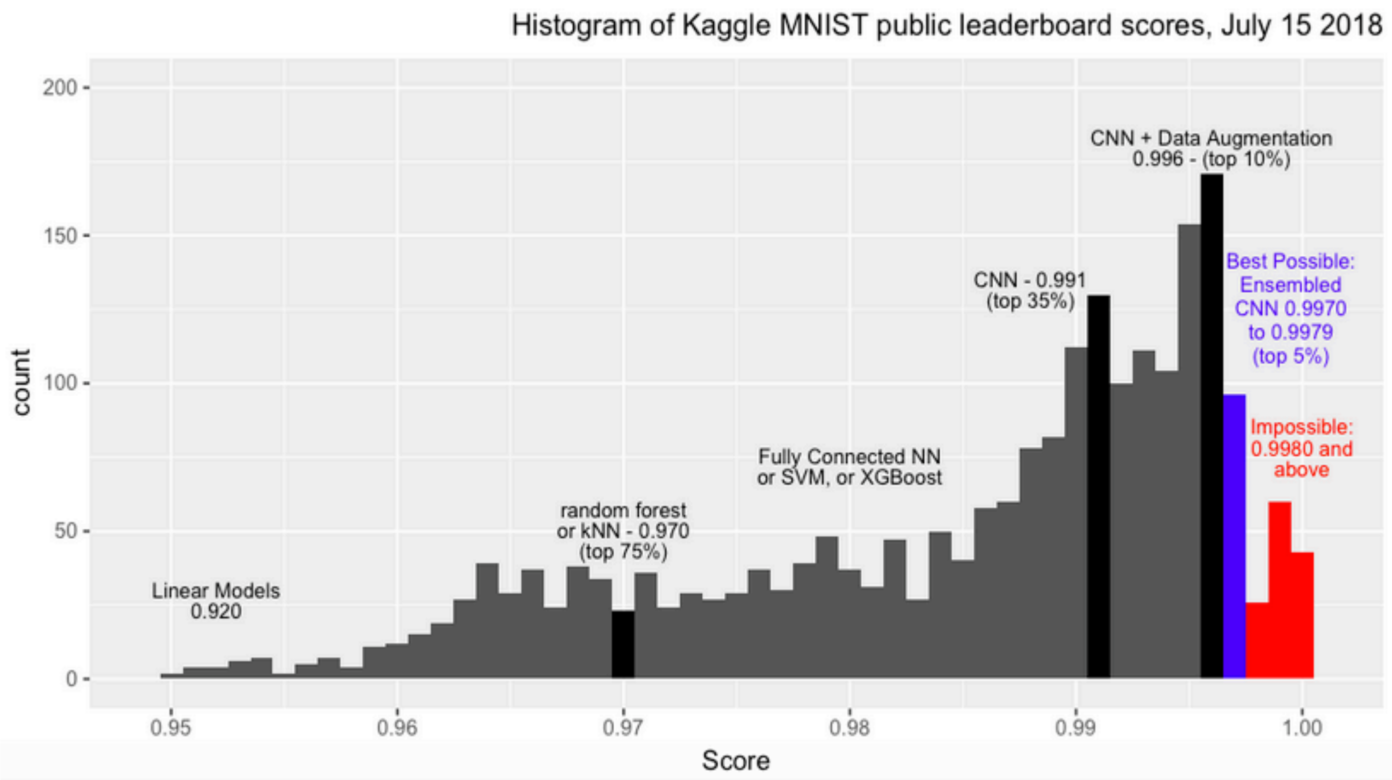
I. Motivation and Introduction to CNN

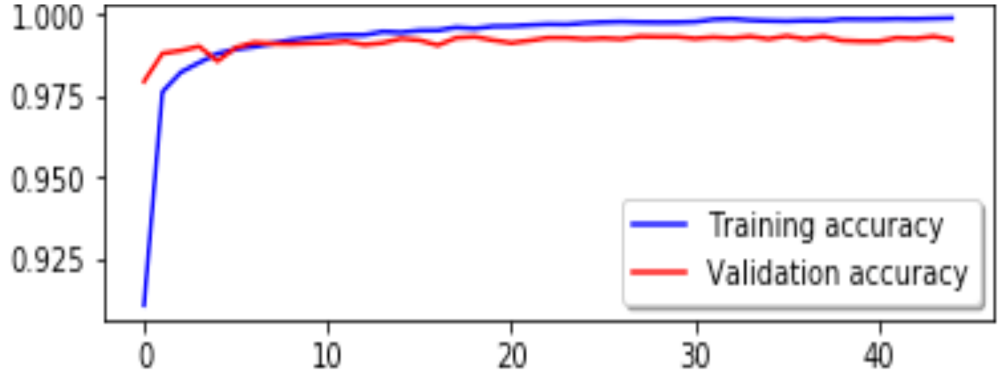
1. Motivation
2. Neural network
3. Convolution
4. Convolutional network

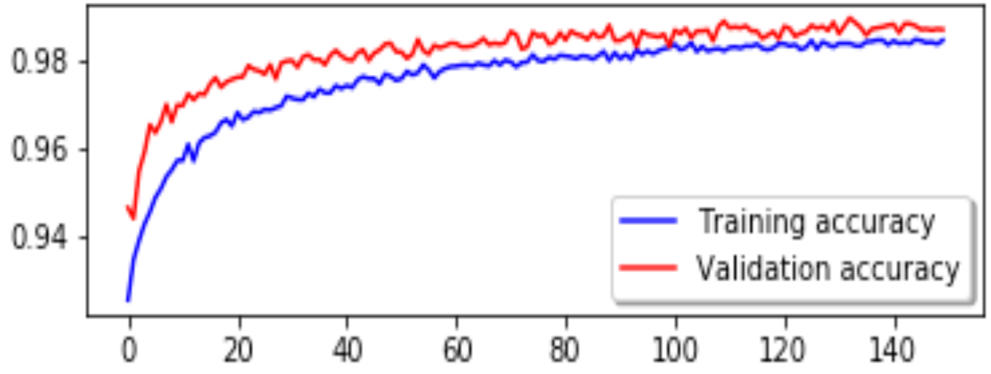
Motivation

1. My CNN classifier achieved 99.3% accuracy on MNIST hand-written digit dataset, but I cannot interpret it!
2. Knowing the information it did/did not learn can be the key to improvements.
3. *Validation accuracy exceeds training accuracy after data augmentation!*



https://www.kaggle.com/cdeotte/25-million-images-0-99757-mnist

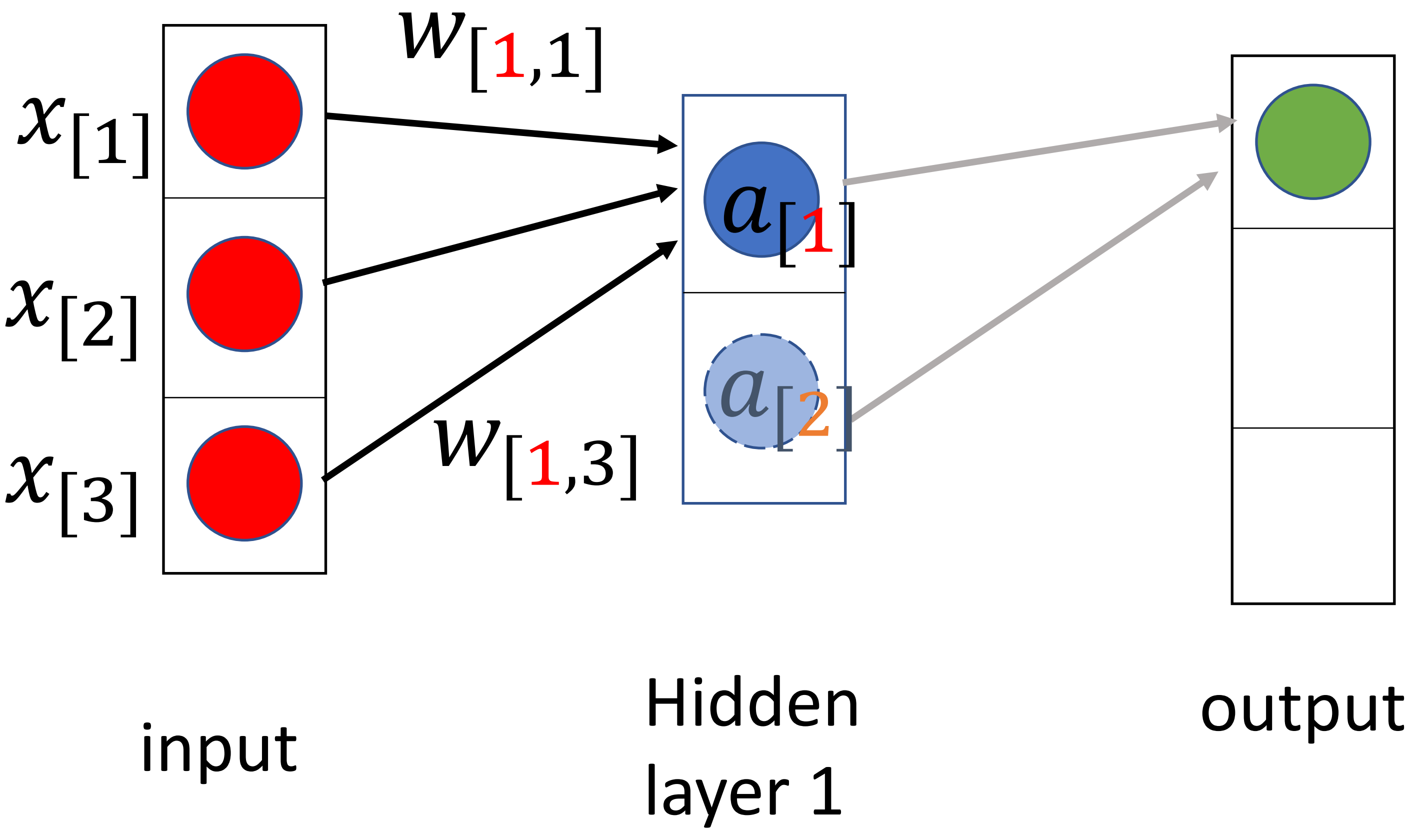




Neural network

$$z_{[1]} = w_{[1,1]}x_{[1]} + w_{[1,2]}x_{[2]} + w_{[1,3]}x_{[3]} + b_{[1]} \cdot 1$$

$$a_{[1]} = \sigma\left(z_{[1]}\right) \qquad \sigma: nonlinear\ activation\ function$$



TL;DR

1. Weigh
2. Bias
3. Activate

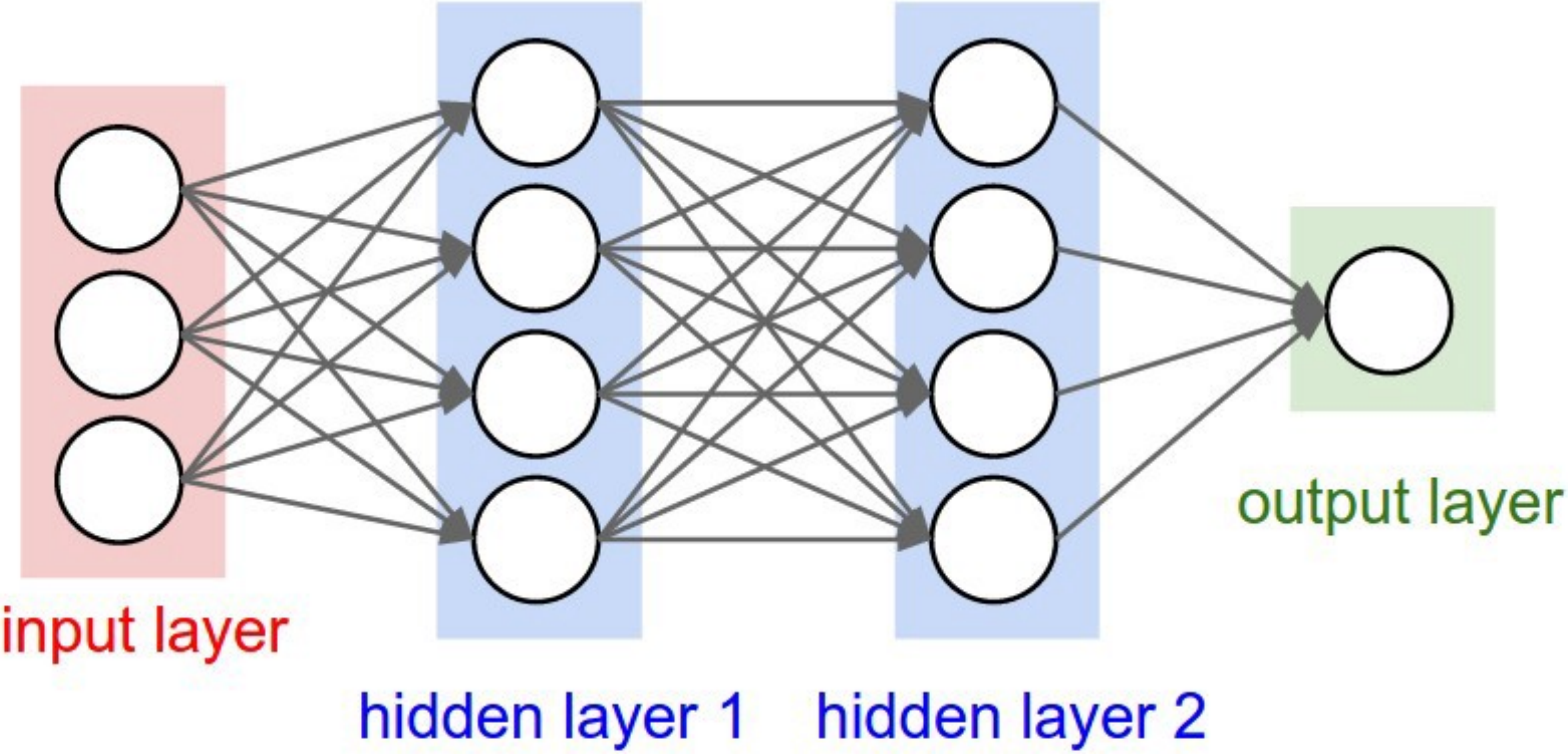
$$\vec{a} = \sigma\left(\overleftrightarrow{w} \cdot \vec{x} + \vec{b}\right)$$

$$\vec{a}^{(1)} = \sigma\left(\overleftrightarrow{w}^{(1)} \cdot \vec{x} + \vec{b}^{(1)}\right)$$

$$\vec{a}^{(2)} = \sigma\left(\overleftrightarrow{w}^{(2)} \cdot \vec{a}^{(1)} + \vec{b}^{(2)}\right)$$

$$\vec{a}^{(h)} = \sigma\left(\overleftrightarrow{w}^{(h)} \cdot \vec{a}^{(h-1)} + \vec{b}^{(h)}\right)$$

Neural Network – fully connected



Pixels

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	87	144	62	19	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	139	204	253	253	253	172	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	210	253	253	157	44	108	253	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	45	228	253	215	83	1	0	4	107	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	119	227	253	190	31	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	66	209	255	253	136	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	99	248	253	185	55	4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	99	253	253	166	44	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	100	248	253	162	9	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	10	183	247	253	166	9	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	57	250	254	243	131	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	47	188	253	239	74	122	207	177	177	163	67	41	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	14	207	253	245	73	0	79	253	253	253	254	253	236	9	0	0	0	0	0	0	0	0
0	0	0	0	0	0	197	253	201	62	0	0	1	10	10	59	122	149	250	234	61	5	0	0	0	0	0	0
0	0	0	0	0	0	254	253	129	0	0	0	0	0	0	0	0	0	138	253	253	79	0	0	0	0	0	0
0	0	0	0	0	0	254	253	82	0	0	0	0	0	0	0	0	24	153	253	253	127	0	0	0	0	0	0
0	0	0	0	0	0	196	253	202	64	12	12	12	12	45	122	233	238	253	243	145	5	0	0	0	0	0	0
0	0	0	0	0	0	13	207	253	253	253	253	253	253	253	253	229	180	99	4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	46	187	253	253	253	216	176	142	66	29	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	13	33	33	33	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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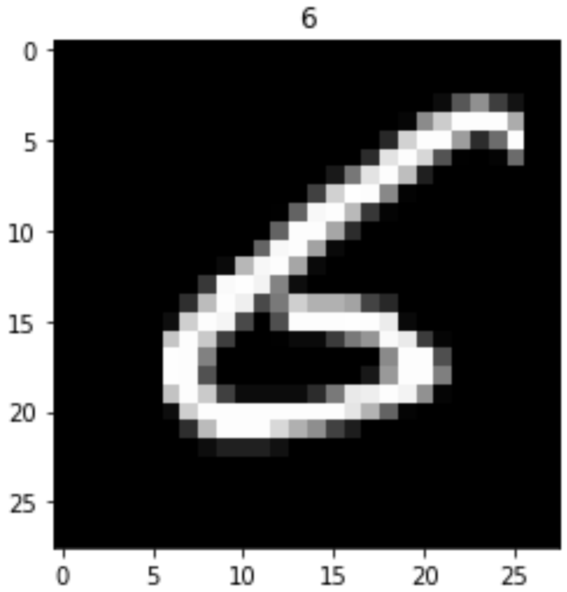
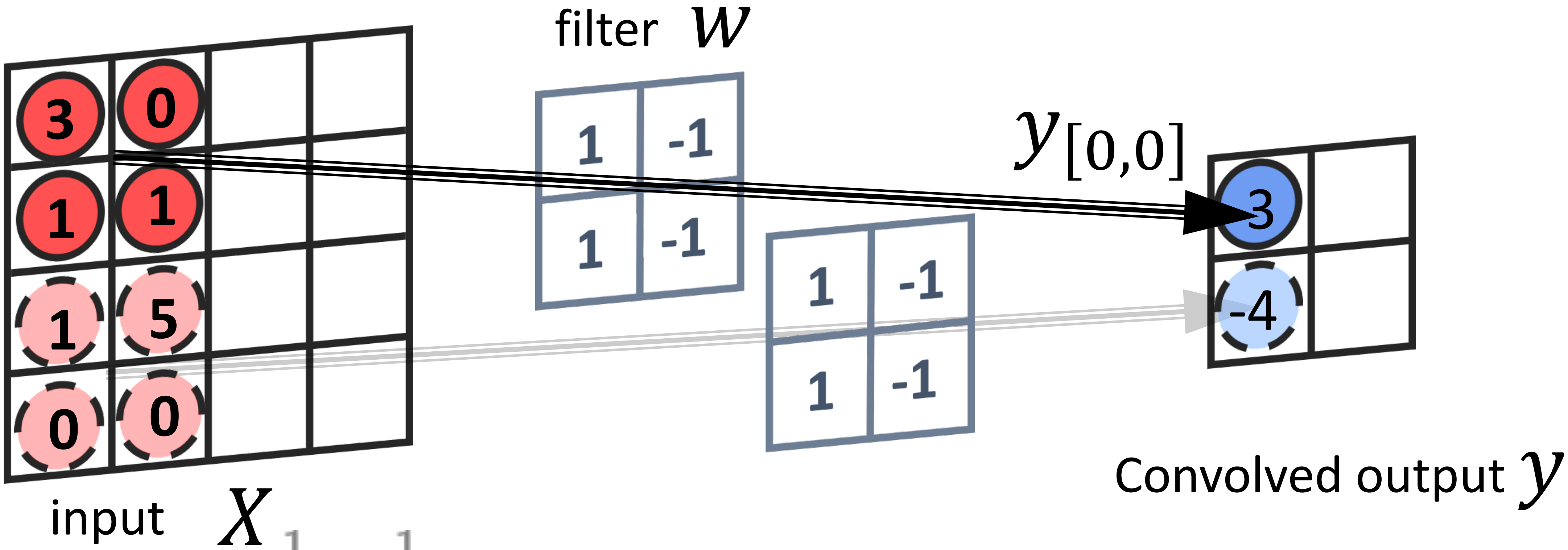
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



Convolution

$$y_{[0,0]} = \sum_{t=0}^{1}\sum_{s=0}^{1} w_{[s,t]} X_{[0+s,0+t]}$$



$$y_{[i,j]} = \sum_{t=0}^{1}\sum_{s=0}^{1} w_{[s,t]} X_{[2i+s,2j+t]}$$

$$y = w * X$$

Convolution

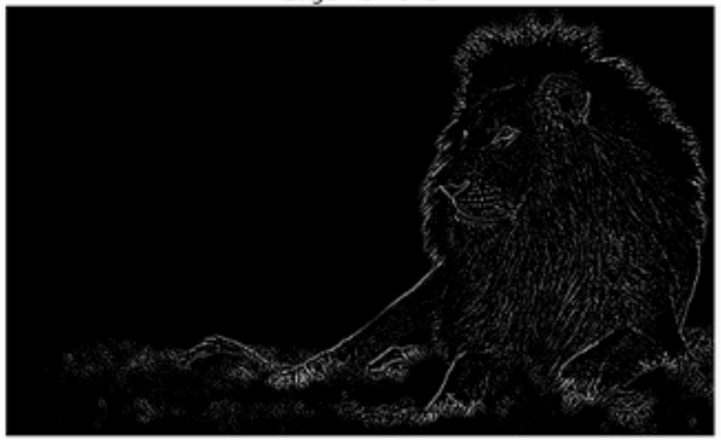
- Convolution can do edge detection, blurring, sharpening and etc.

```
edge_kernel_1 = np.array([[1, 0, -1],
                          [0, 0, 0],
                          [-1, 0, 1]])

edge_kernel_2 = np.array([[0, 1, 0],
                          [1, -4, 1],
                          [0, 1, 0]])

edge_kernel_3 = np.array([[-1, -1, -1],
                          [-1, 8, -1],
                          [-1, -1, -1]])
```

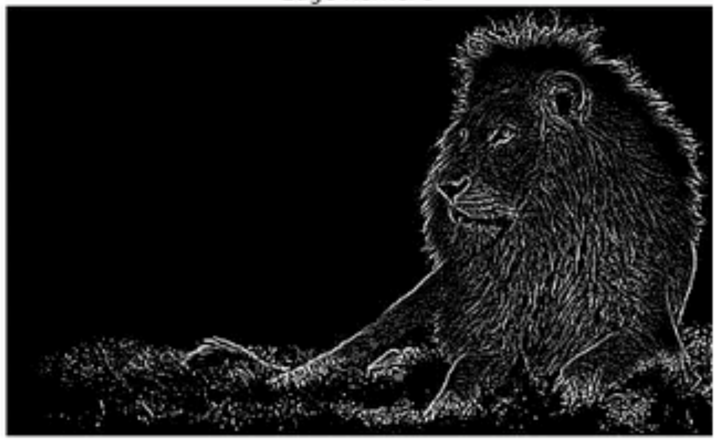
Original Image

Edge Kernel 1

Edge Kernel 2

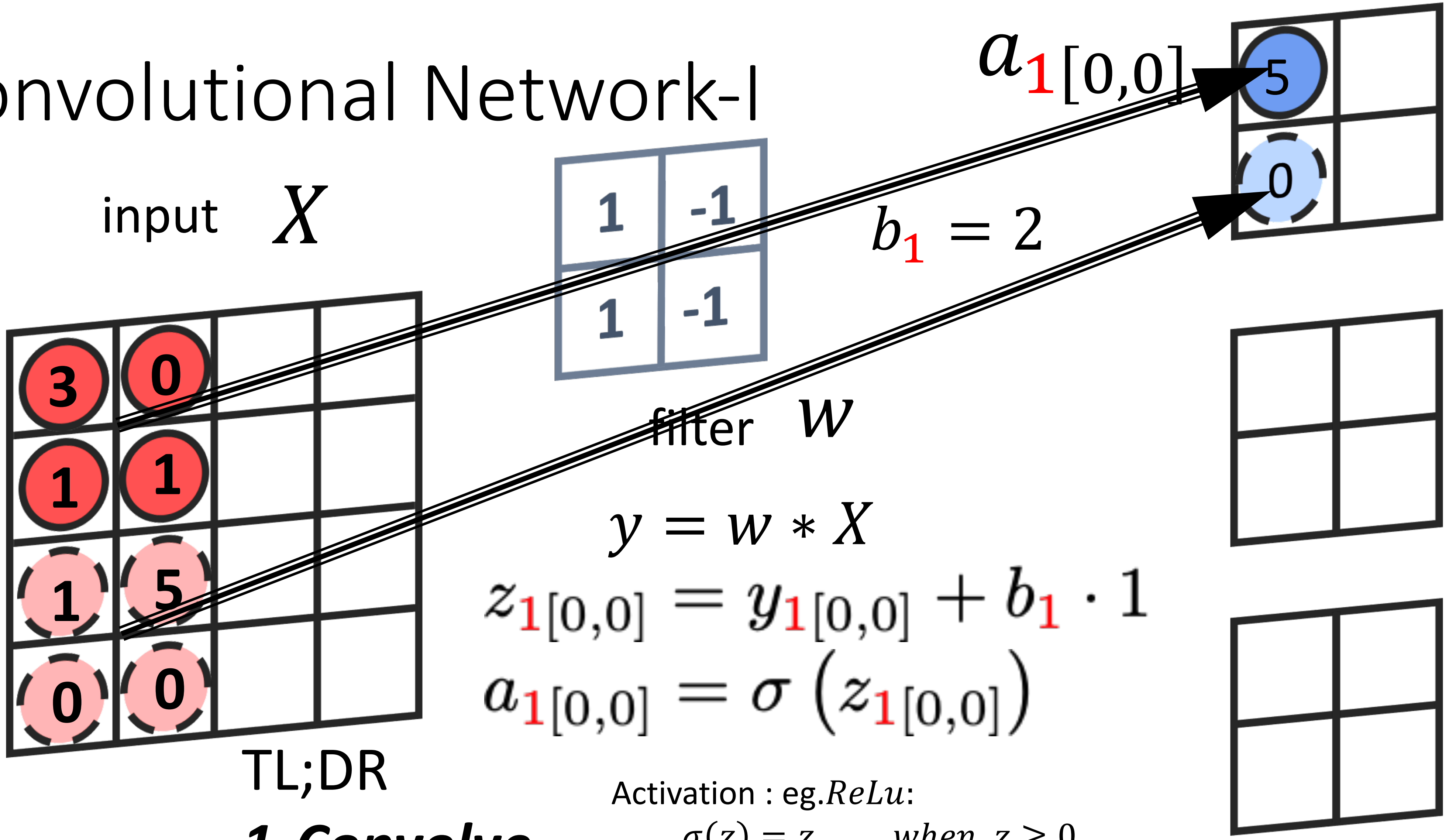


Edge Kernel 3



https://beckernick.github.io/convolutions/

Convolutional Network-I



$$y = w * X$$

$$z_{1[0,0]} = y_{1[0,0]} + b_1 \cdot 1$$

$$a_{1[0,0]} = \sigma\left(z_{1[0,0]}\right)$$

Activation : eg.$ReLu$:

$$\sigma(z) = z, \quad when\ \ z \geq 0$$
$$= 0, \quad when\ \ z < 0$$

TL;DR

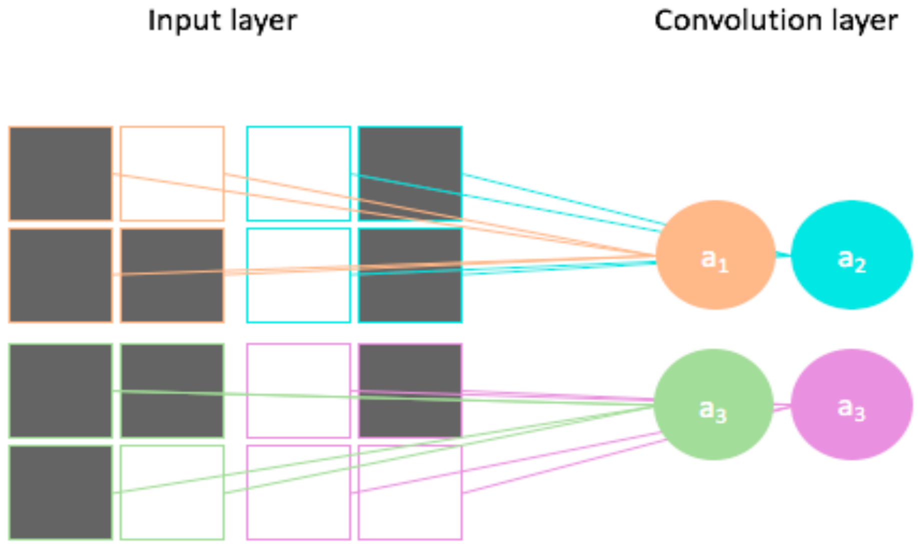
1. Convolve

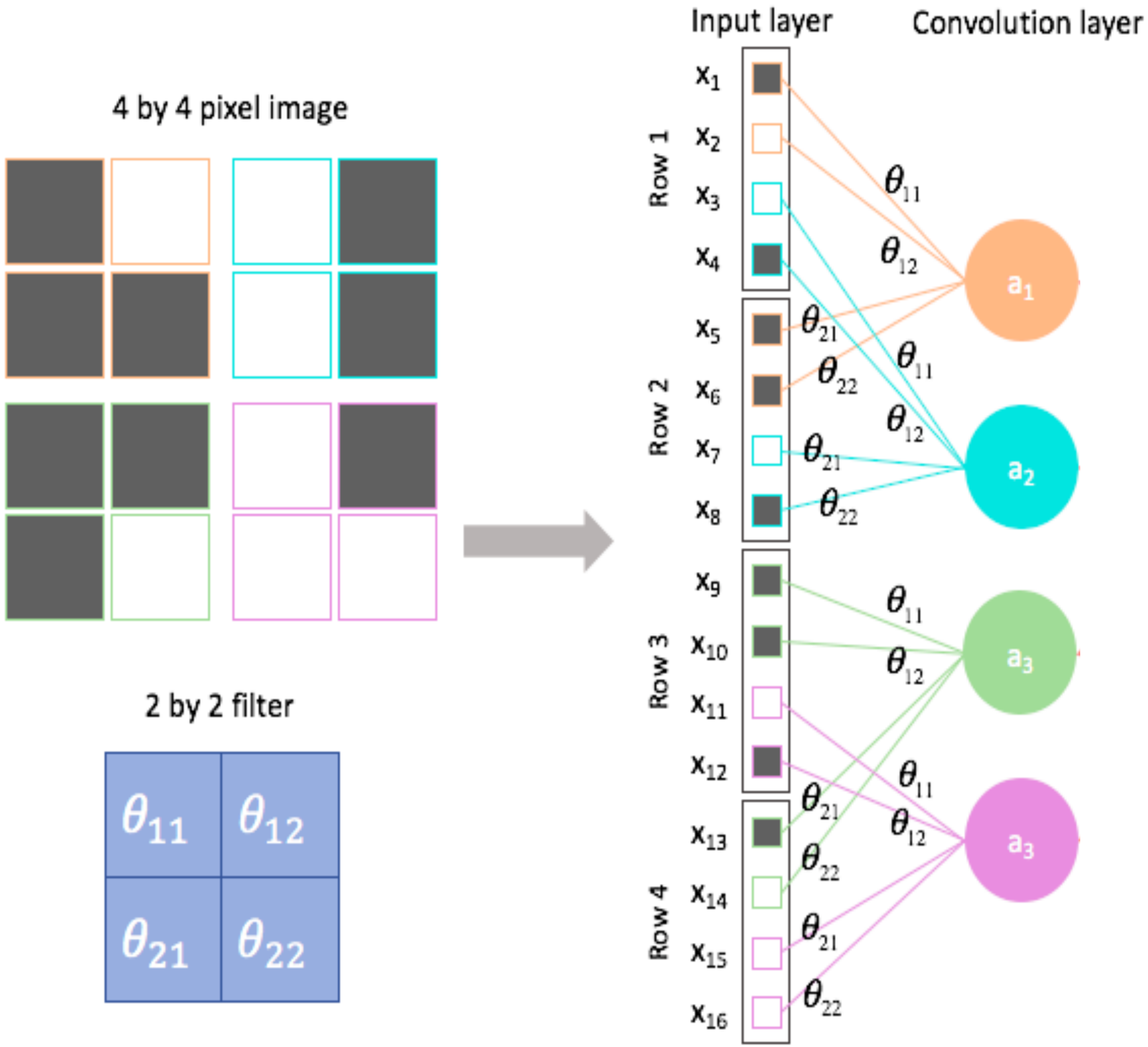
2. Bias

3. Activate

Conv Net-II

1. Sparsely connected network
2. Locality is preserved with convolution





https://www.jeremyjordan.me/convolutional-neural-networks/

The approaches

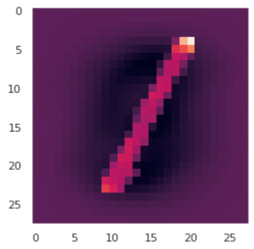
1. Visualize the intermediate activations
2. CNN encoder and interpreters
3. Did my CNN learn translational invariance?

Visualize the intermediate activations-I

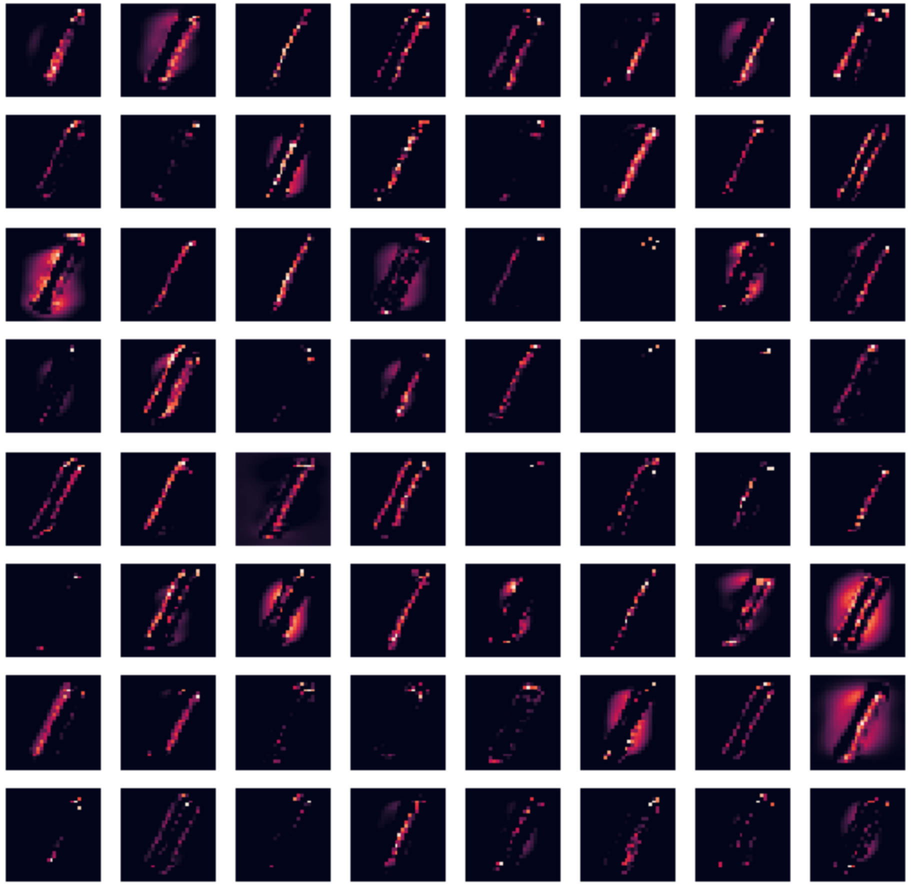
h:hiddne layer index

conv2d_1 $h = 1$

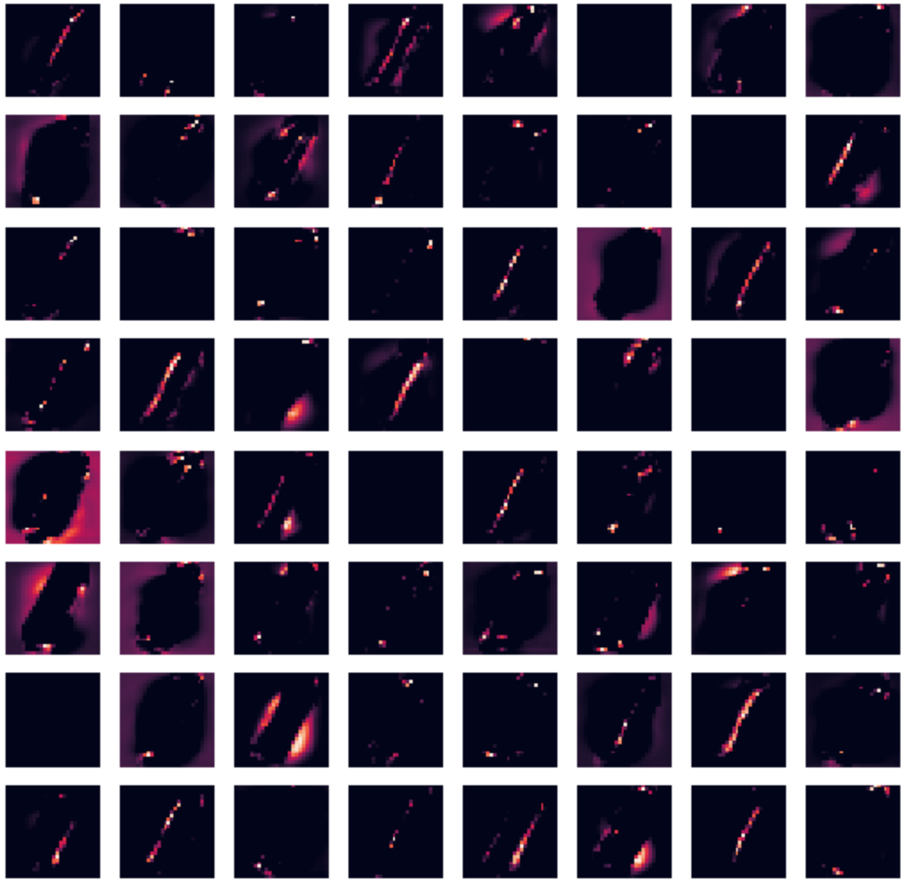
conv2d_2 $h = 2$



28*28=784



(28*28)*64=50,176



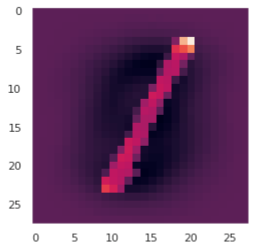
(28*28)*64=50,176

Visualize the intermediate activations-II

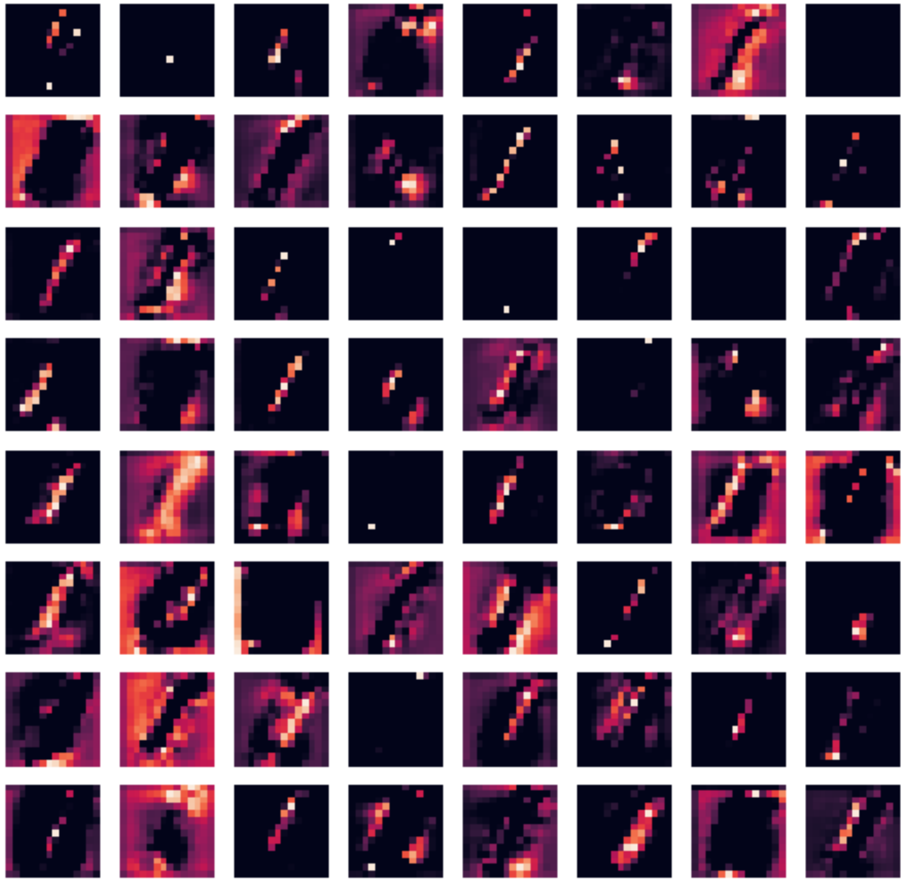
h:hiddne layer index

conv2d_3 $h = 5$

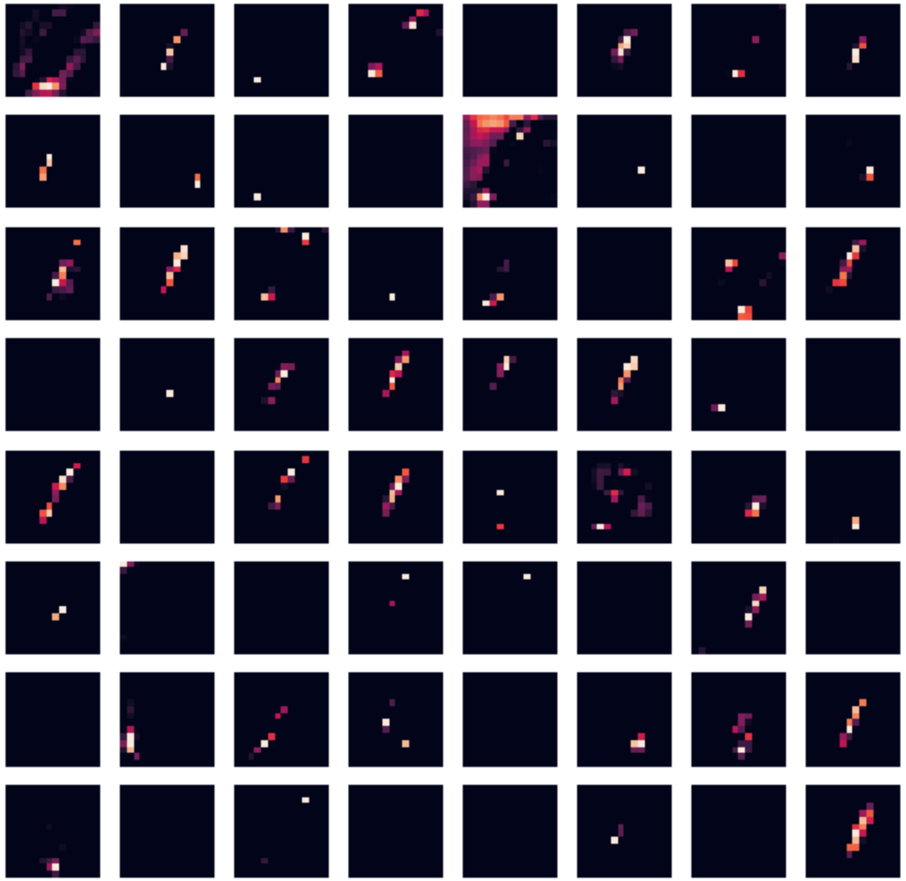
conv2d_4 $h = 6$



28*28=784



(14*14)*64=12,544

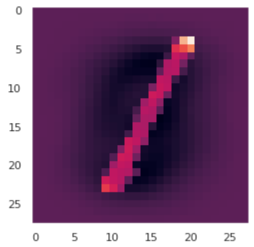


(14*14)*64=12,544

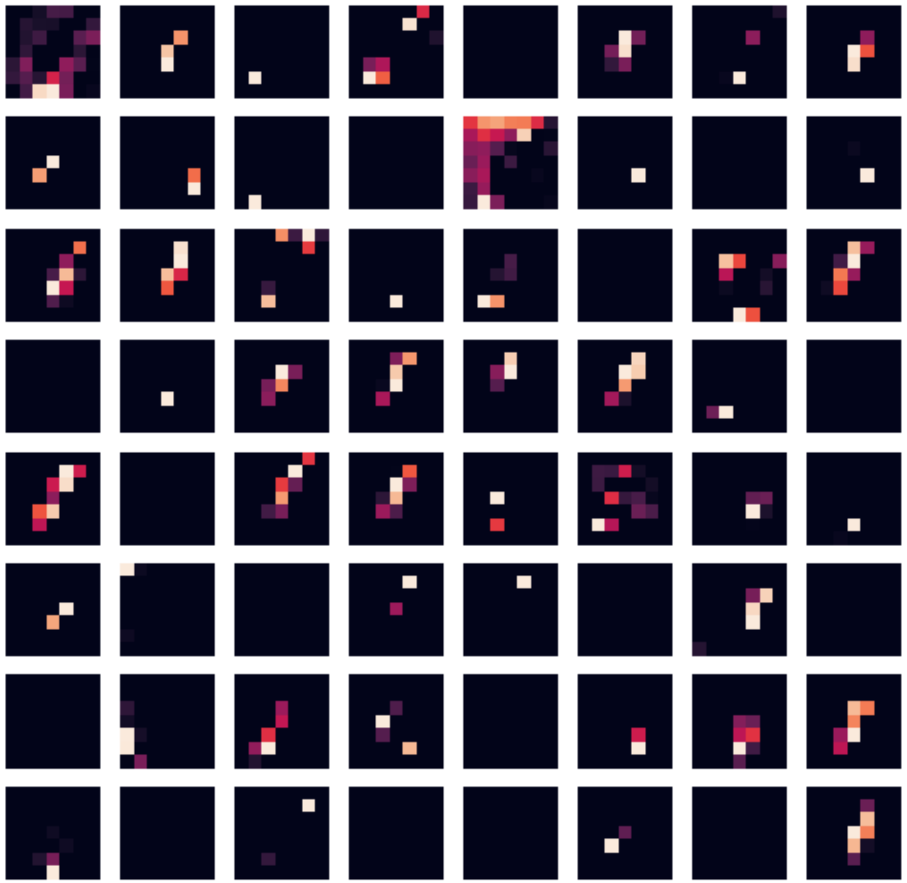
Visualize the intermediate activations-III

h:hiddne layer index

dropout_2 $h = 8$

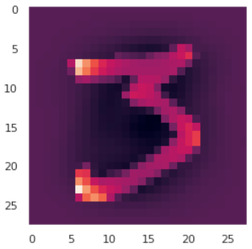


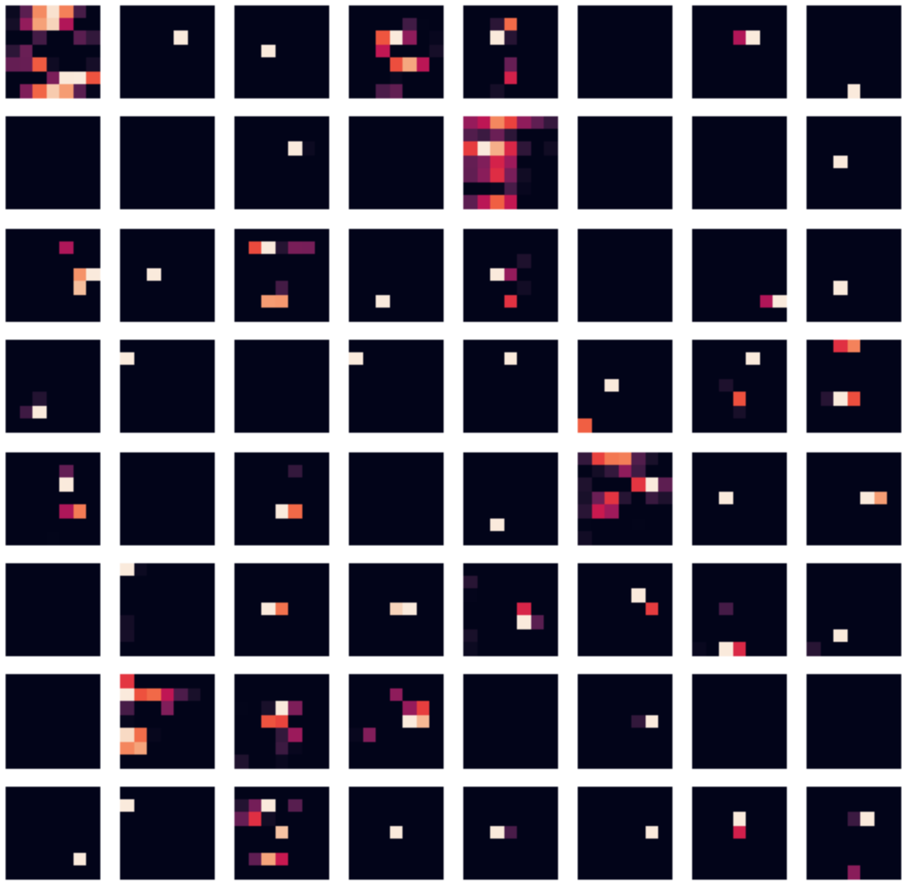
28*28=784



(7*7)*64=3,136

dropout_2 $h = 8$





(7*7)*64=3,136

CNN encoder and interpreters

the architecture

```
#The model structure
my_CNN_clf=Sequential([
    ############################## CNN-encoder
    Conv2D(filters = 64, activation = 'relu',kernel_size=(5,5),padding='Same',input_shape = (28,28,1)),
    Conv2D(filters = 64, activation = 'relu',kernel_size=(5,5),padding='Same'),
    MaxPool2D(pool_size=(2,2),strides=(2,2)),
    Dropout(0.25),
    Conv2D(filters = 64, activation = 'relu',kernel_size=(3,3),padding='Same'),
    Conv2D(filters = 64, activation = 'relu',kernel_size=(3,3),padding='Same'),
    MaxPool2D(pool_size=(2,2),strides=(2,2)),
    Dropout(0.25),
    Flatten(), # d=64*7*7=3136
    ############################## CNN-encoder

    ##################            compressor
    Dense(256, activation='relu'),
    Dropout(0.5), #d = 256
    ##################            compressor

    ############################## interpretor
    Dense(10, activation='softmax'), #a normalized exponential functions for probability
    ############################## interpretor
    ])
```

$h = 1$

$h = 2$

$h = 5$

$h = 6$

$h = 8$

Encoder: input→ 3,136-dimensional vector

$h = 10$

Compressor: 3,136 → 256 dimensional

<u>The code vector</u>

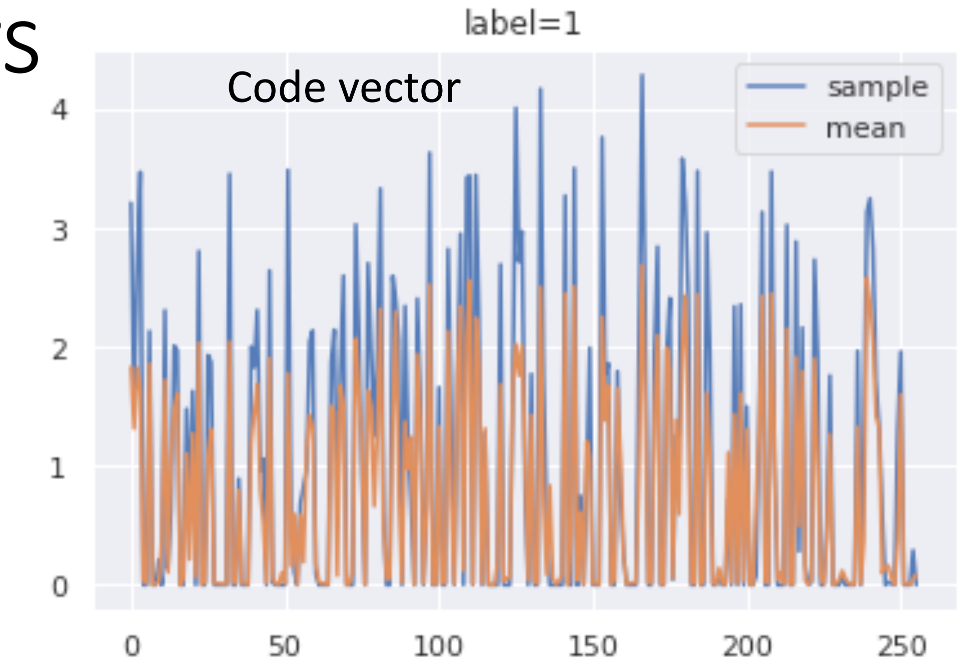
Interpreter: 256 → 10 dim. probability vector for predictions

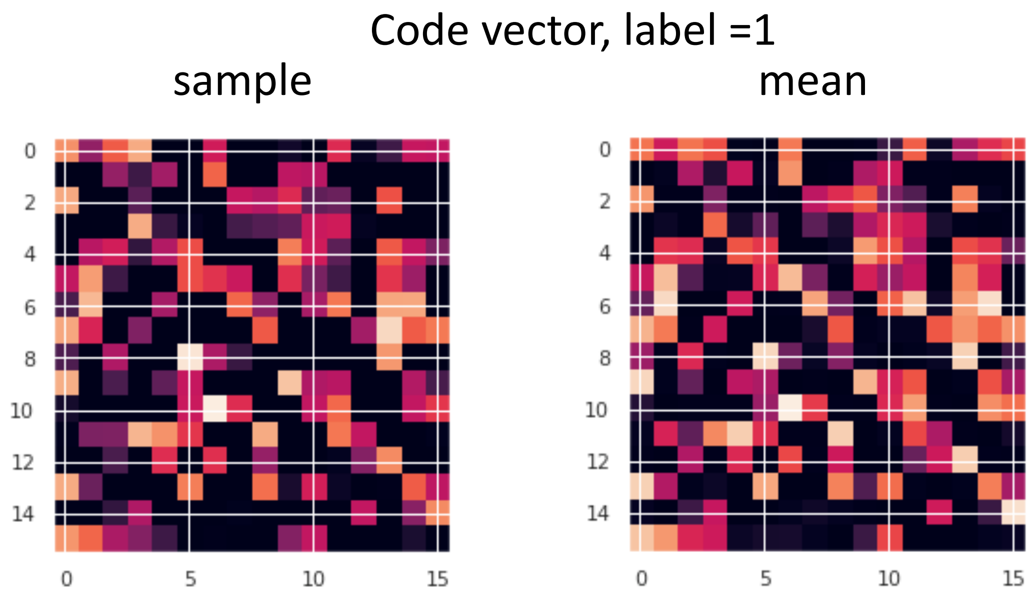
CNN encoder and interpreters

the code vectors

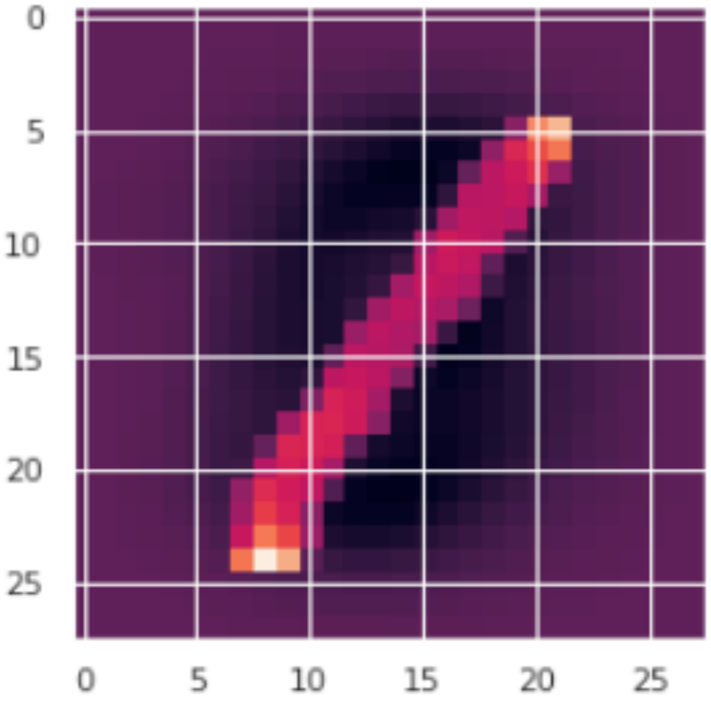
1. Consistent and self-similarity within a class
2. Different interpreters show similar accuracy
   => Conv. encoder is the key

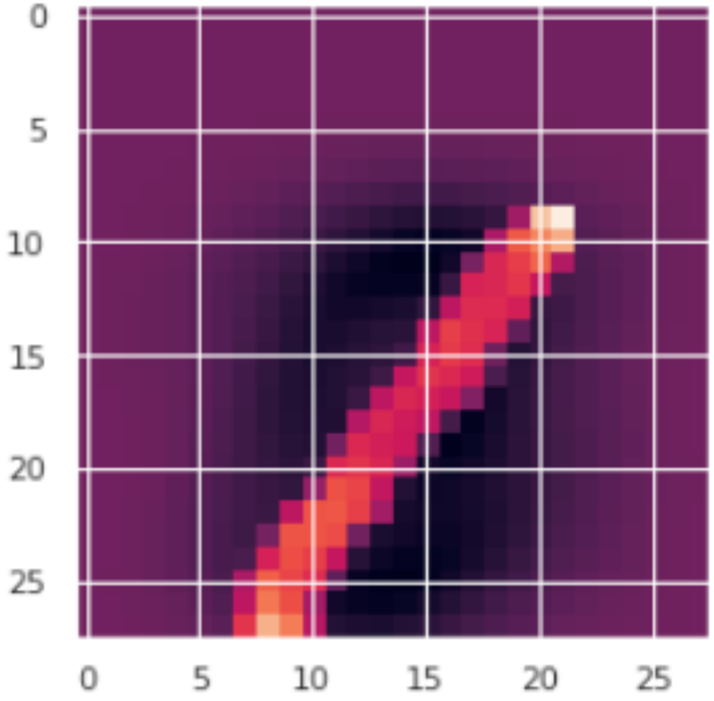
Interpreters	accuracy
1 Dense layer NN	99.44%
SVM	99.35%
Random Forests (100 trees)	99.32%
Gradient Boost (100 trees)	99.24%
Decision Tree	98.46%





Translational Invariance

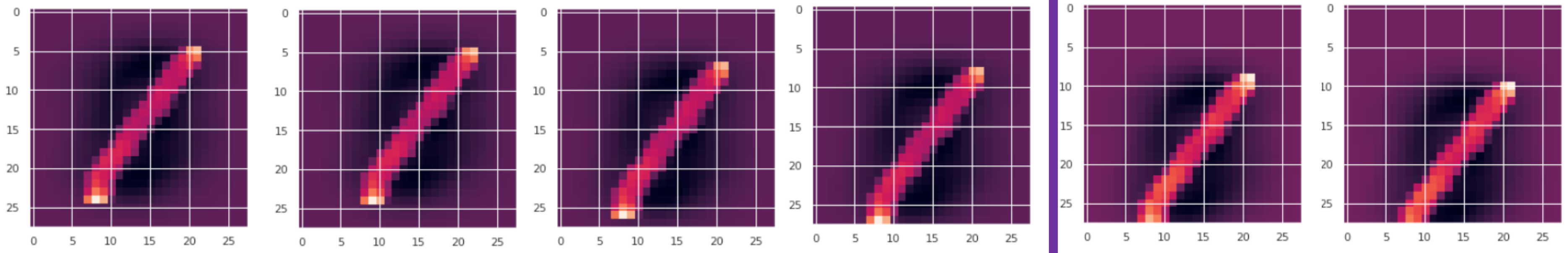




Did my CNN learn translational invariance?

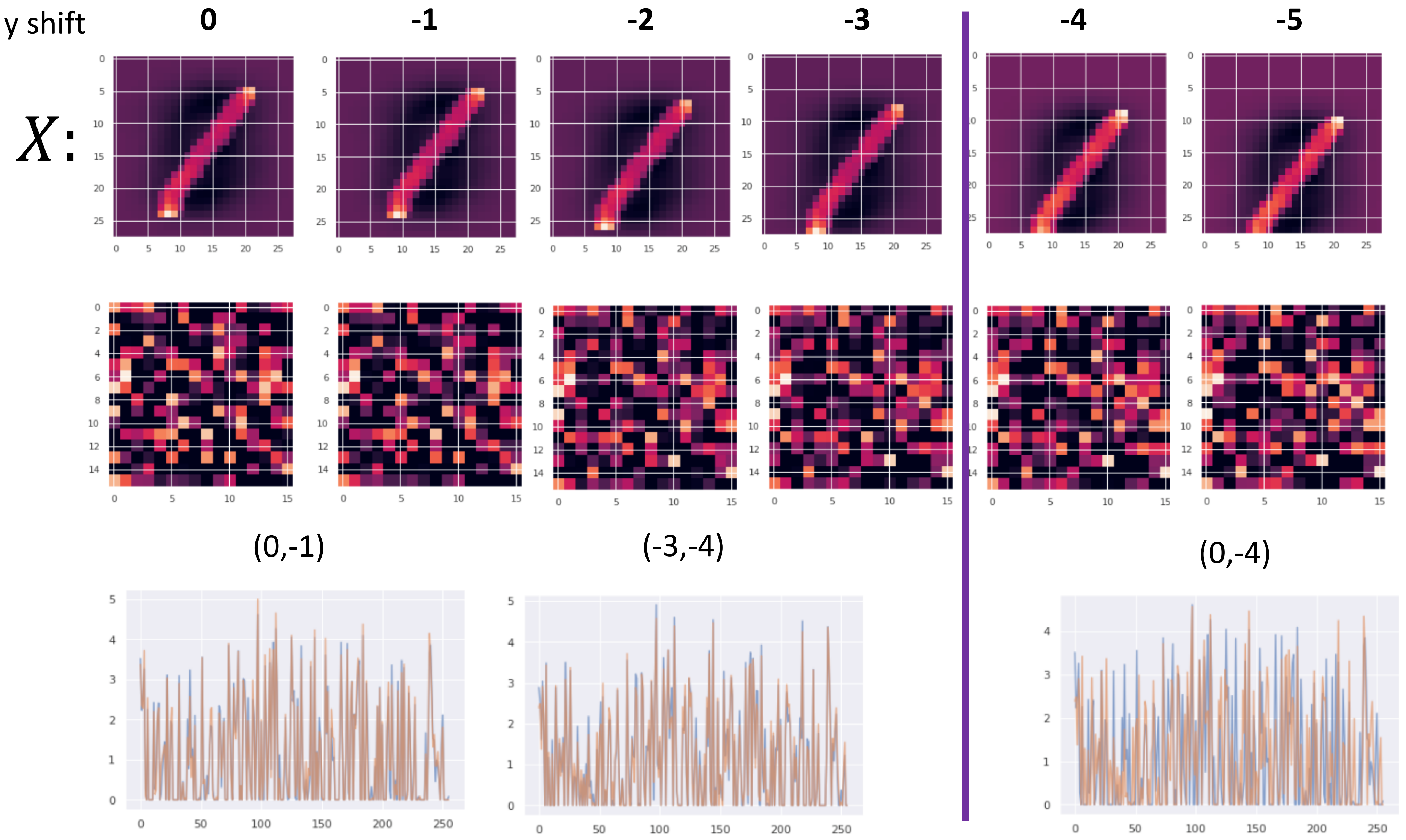
$Prob(y = 1|X)$

X:



y shift (pixels)	0	-1	-2	-3	-4	-5
CNN (CNN+NN) (acc = 99.3%)	100%	100%	99.99%	99.4%	0.71% (7: 99.29%)	1.4E-4% (7: 99.99%)
CNN+RF, T=1000 (acc = 99.3%)	100%	96.6%	67.7% (7: 18.4)	52.8% (7: 28.3%)	44.4% (7: 35.5%)	34.8% (7: 47.6 %)
CNN+SVM (acc = 99.3%)	99.9%	99.8%	97.8%	79.9% (9: 18.8%)	21% (9: 77.76%)	1% (9: 98.9%)
Random Forest (RF) (acc = 96.6%)	99.5%	84.9% (7: 9.8%)	34.1% (7: 39.6%)	9.1% (7: 56.3%)	5.4% (7: 70.1%)	4.8% (7: 70.2 %)
SVM (acc = 96.1%)	99.6%	42.8% (7: 50.8%)	0.85% (7: 94.5%)	0.42% (7: 81.3 %)	0.43% (7: 79.5%)	0.42% (80.8%)

Did my CNN learn translational symmetry?

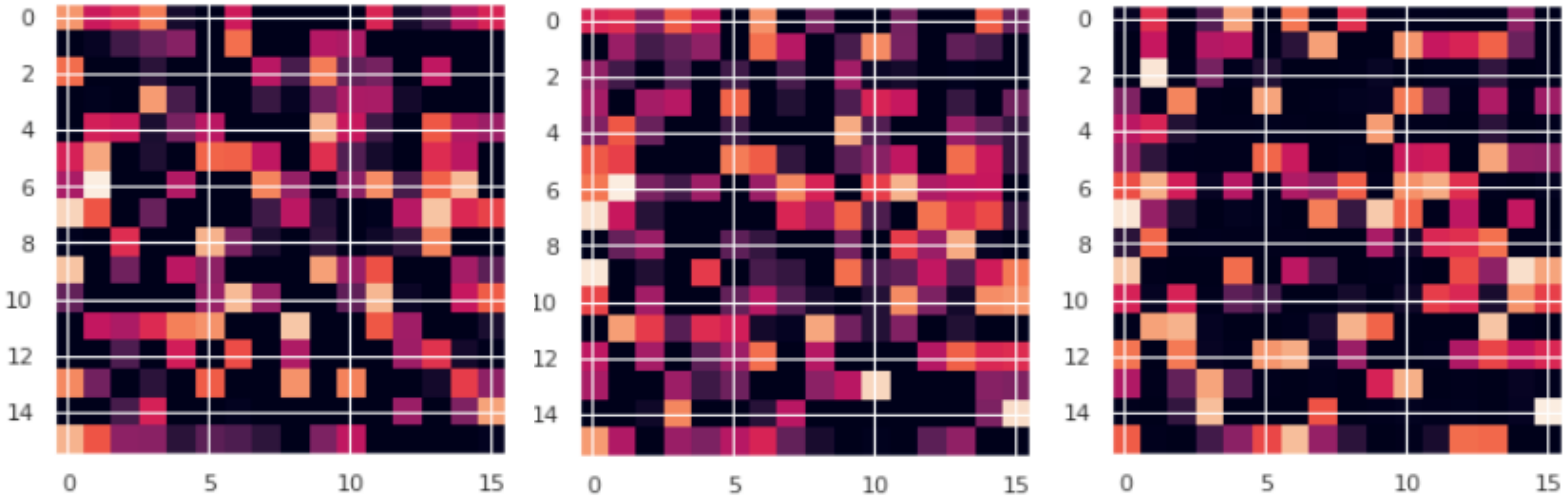


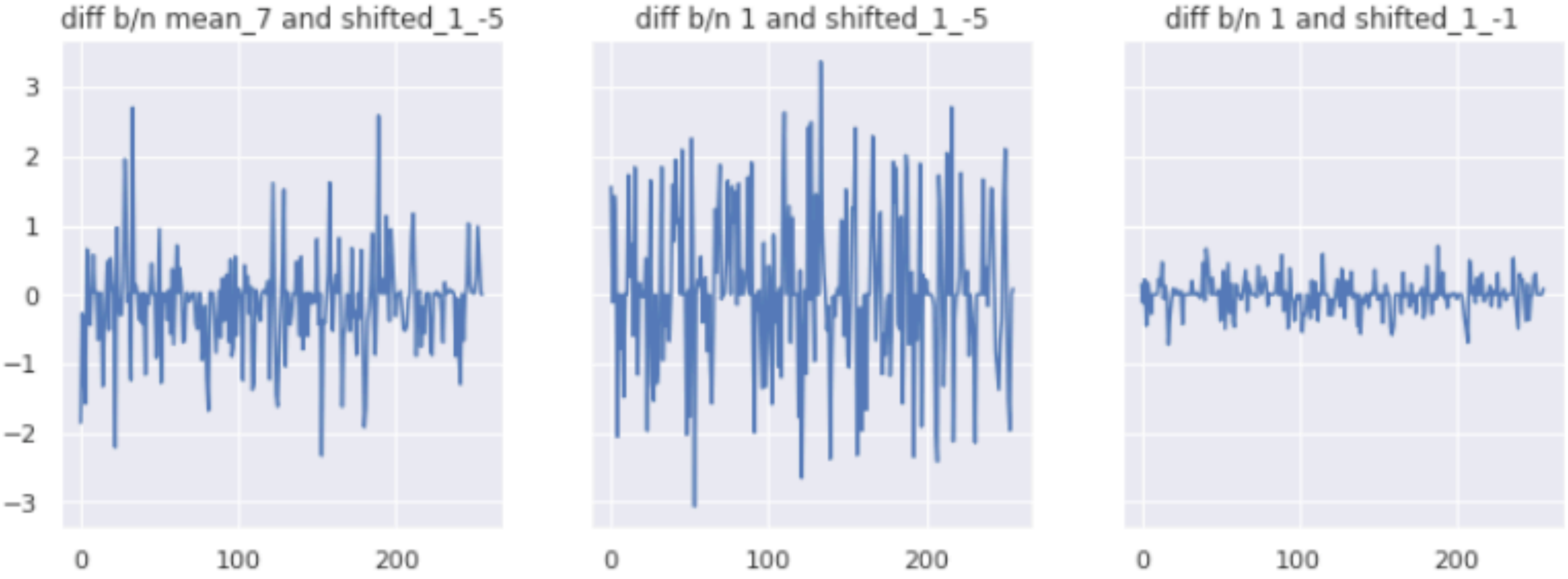
Conclusion

- Convolution reduces the neural network complexity and comprehend locality (relative positions) of the image (image pixels).
- The deep convolutional encoder is the key for the performance
- CNN encoding models learn translational invariance better than others, though not perfect.

Conclusion

- Code vectors in hidden layers of CNN reduce the dimensionality of images.
- Information learned in CNN is **position sensitive**. And convolution is the key for high accuracy performance.
- The predicted probabilities of CNN are not continuous with respect to translations.
- Translations deform CNN code vectors more softly.


0
2
4
6
8
10
12
14
0
5
10
15
0
2
4
6
8
10
12
14
0
5
10
15
0
2
4
6
8
10
12
14
0
5
10
15


diff b/n mean_7 and shifted_1_-5
diff b/n 1 and shifted_1_-5
diff b/n 1 and shifted_1_-1
3
2
1
0
−1
−2
−3
0
100
200