Wayne Tzu-Wen Wu

PC3236: Assignment 3 Report

Problem 1

In Problem 1, a system of coupled ordinary differential equations are given to be solved numerically. To begin with, it is noted that the first equation is in fact a 2nd order differential equation. In order to apply initial value problem's numerical schemes such as Euler's or Runge-Kutta's methods, the equation must first be broken down into two 1st order differential equations. With simple changes of variables, the system of equations can be rewritten as:

$$x_1' = x_2 \,, x_2' = \frac{3x_1}{y} \,, y' = \frac{{x_1}^2}{500} + y\cos t$$

with the initial conditions of $x_1(0) = 10, x_2(0) = 0, y(0) = 10$.

To solve the differential equations, the 4th order Runge-Kutta's (RK4) method is used. The RK4 equations as given in Equation 5.32 and 5.33 from the textbook can be rewritten as below in vector form with t as the independent variable instead of x:

$$\boldsymbol{f_0} = \boldsymbol{F}(t, \boldsymbol{Y})$$

$$\boldsymbol{f_1} = \boldsymbol{F}\left(t + \frac{h}{2}, \boldsymbol{Y} + \frac{h}{2}\boldsymbol{f_0}\right)$$

$$\boldsymbol{f_2} = \boldsymbol{F}\left(t + \frac{h}{2}, \boldsymbol{Y} + \frac{h}{2}\boldsymbol{f_1}\right)$$

$$\boldsymbol{f_3} = \boldsymbol{F}(t + h, \boldsymbol{Y} + h\boldsymbol{f_2})$$

$$\boldsymbol{Y}(t_0 + h) = \boldsymbol{Y}(t_0) + \frac{h}{6}(\boldsymbol{f_0} + \boldsymbol{2f_1} + \boldsymbol{2f_2} + \boldsymbol{f_3})$$

The vectors **Y** and **F** are constructed from the system of differential equations given above:

$$\boldsymbol{Y} = \begin{bmatrix} x_1 \\ x_2 \\ y \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} x_2 \\ \dfrac{3x_1}{y} \\ \dfrac{{x_1}^2}{500} + y\cos t \end{bmatrix}$$

(Note: Y is simply a vector that encompasses all three variables. It has nothing to do with the variable y.)

In MATLAB, a grid is created which stores the values of the vector **Y** across all time steps. The essence of the numerical scheme is then evaluating the current **Y** based on the previous **Y** value as related through the RK4 equations. From the initial conditions at t = 0 up to t = 10, the values of x(t) and y(t) can be obtained which are plotted in Figure 1. To verify that the step size is indeed small enough, both h and h/2 were used which yielded almost identical plots, thus suggesting convergence.

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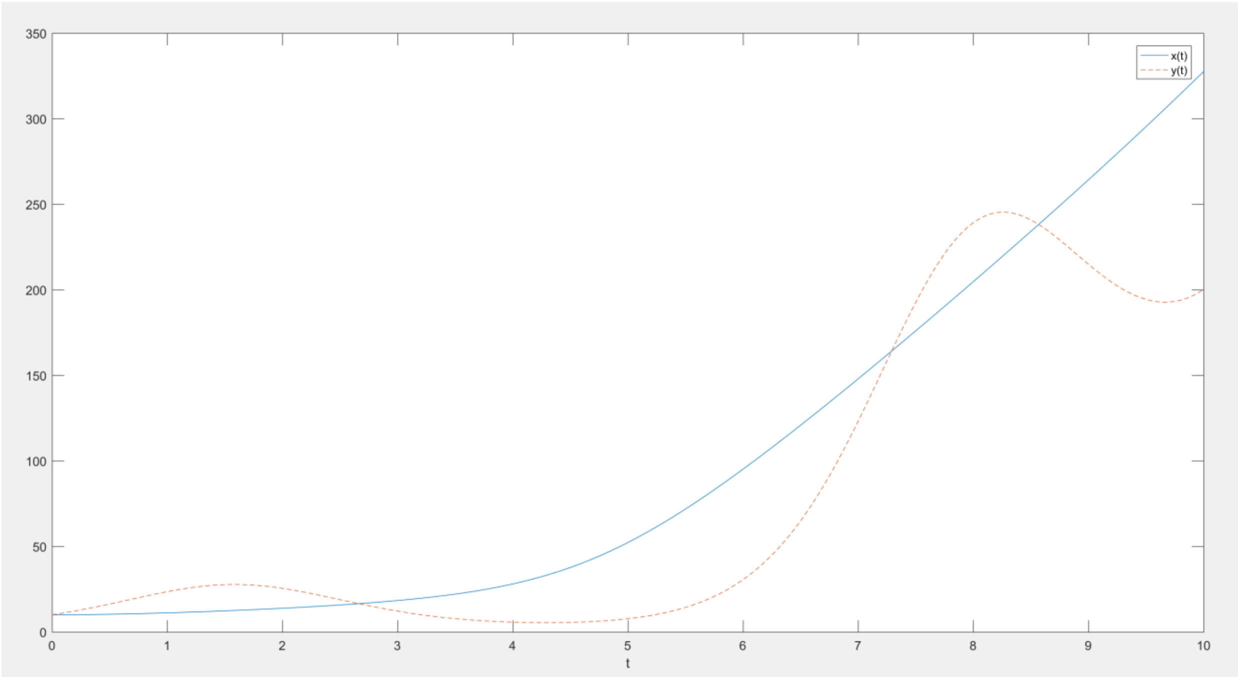

Figure 1