ASTR 660 Homework 3

Deadline: start of class on 10/31

There is one question, worth 50 marks, and an extra 50 bonus points.

Please combine answers into a single file and upload to your GitHub repository as `homework3.pdf`.

All code should also be committed to your GitHub repository and referenced in your answers.

Refer to presentation guidelines in homework 1. In addition, ANY USE OF CHATGPT OR SIMILAR AI TOOLS MUST BE CLEARLY DECLARED.

Question 1

1. Write a Python code to compute the second derivative of $y(x) = \cos(x)$ over $0 < x < 2\pi$ using the central difference method given in class, on a regularly-spaced grid with step size h.

   Compare your answer to the analytic solution by making a plot for a relatively large value of h, so the difference is visually obvious. Check, visually, that you code converges towards the analytic solution when you make h small enough.

   Define the relative error of the finite difference approximation in the previous part to be:

   $$\epsilon = \left| \frac{y_{\text{approx}} - y_{\text{true}}}{y_{\text{true}}} \right|.$$

   Plot $\log_{10} \epsilon$ against $\log_{10} h$ to find the approximate (i.e. within an order of magnitude) value of h that minimises the relative error over $0 < x < 2\pi$.

   Why is there a minimum value? In other words, why can't you make $\epsilon \to 0$ by making h arbitrarily small?

Bonus question

Repeat the above exercise in Fortran and comment on any differences you find. Marks will be awarded here for demonstration of the Fortran topics we have covered so far (e.g. data types, arrays, functions/subroutines, writing output to the terminal).