Introduction to Cosmology

ASTR 434

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Lecture 5 / Dark Matter and Dark Energy / October 15th 2024

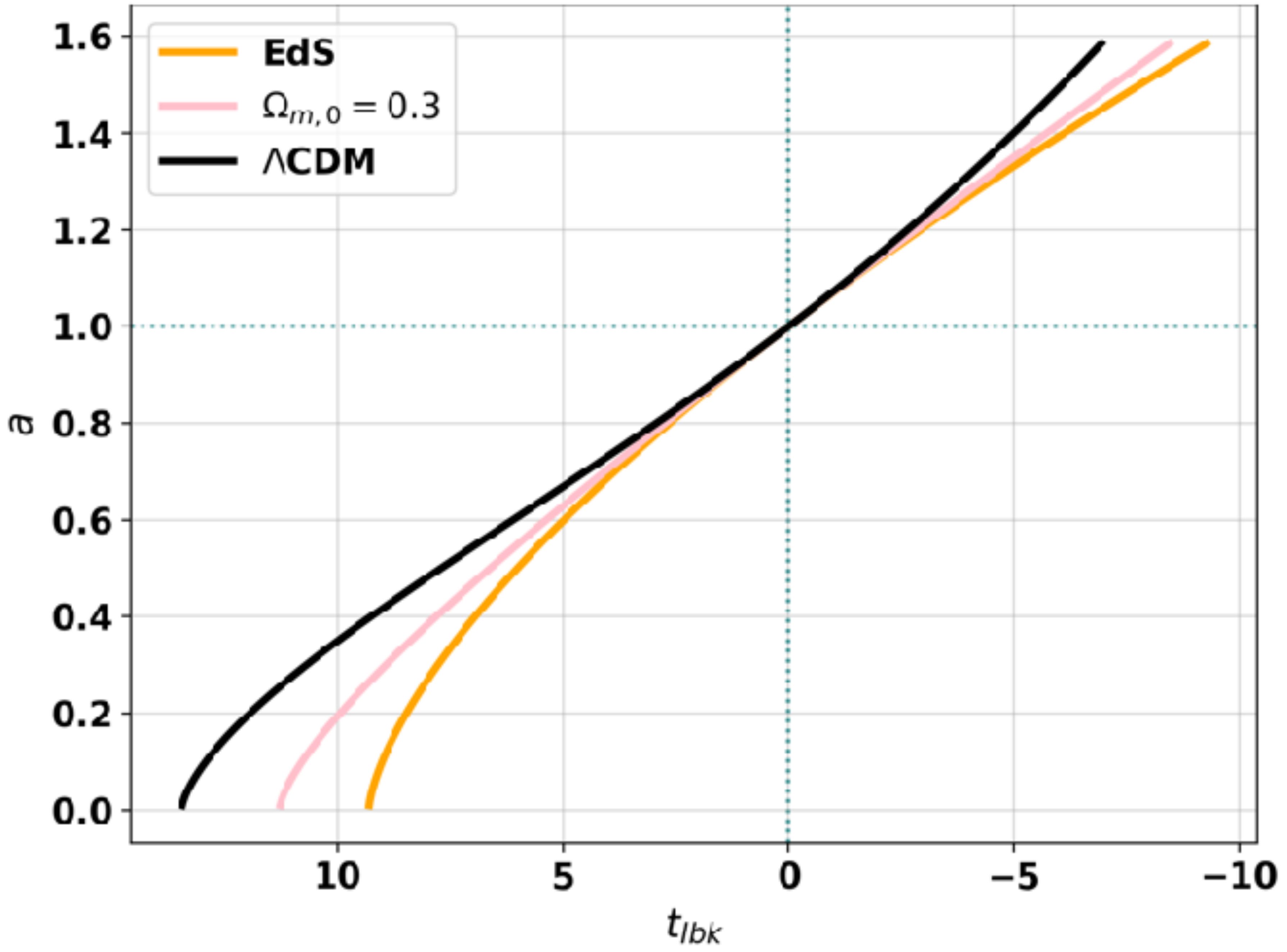
Recap

Distances: $D_L(z) = S_\kappa(r)(1+z)$ and $D_A(z) = \dfrac{S_\kappa(r)}{1+z}$, hence $\dfrac{D_L}{D_A} = (1+z)^2$

Solving for these requires $r(z)$ from FLRW, given $a(t)$ from Friedmann.

Friedmann: $H^2(t) = H_0^2\left(\Omega_{m,0}\,a^{-3} + \Omega_{r,0}\,a^{-4} + \Omega_{\Lambda,0} + (1-\Omega_0)\,a^{-2}\right)$

A Friedmann model dominated by constant energy density expands **exponentially:** $a(t) = e^{H_0(t-t_0)}$**.**



This lecture: ΛCDM

- "The" ΛCDM model
- Evidence for accelerating expansion
- Dark energy
- Evidence for dark matter
- Neutrinos

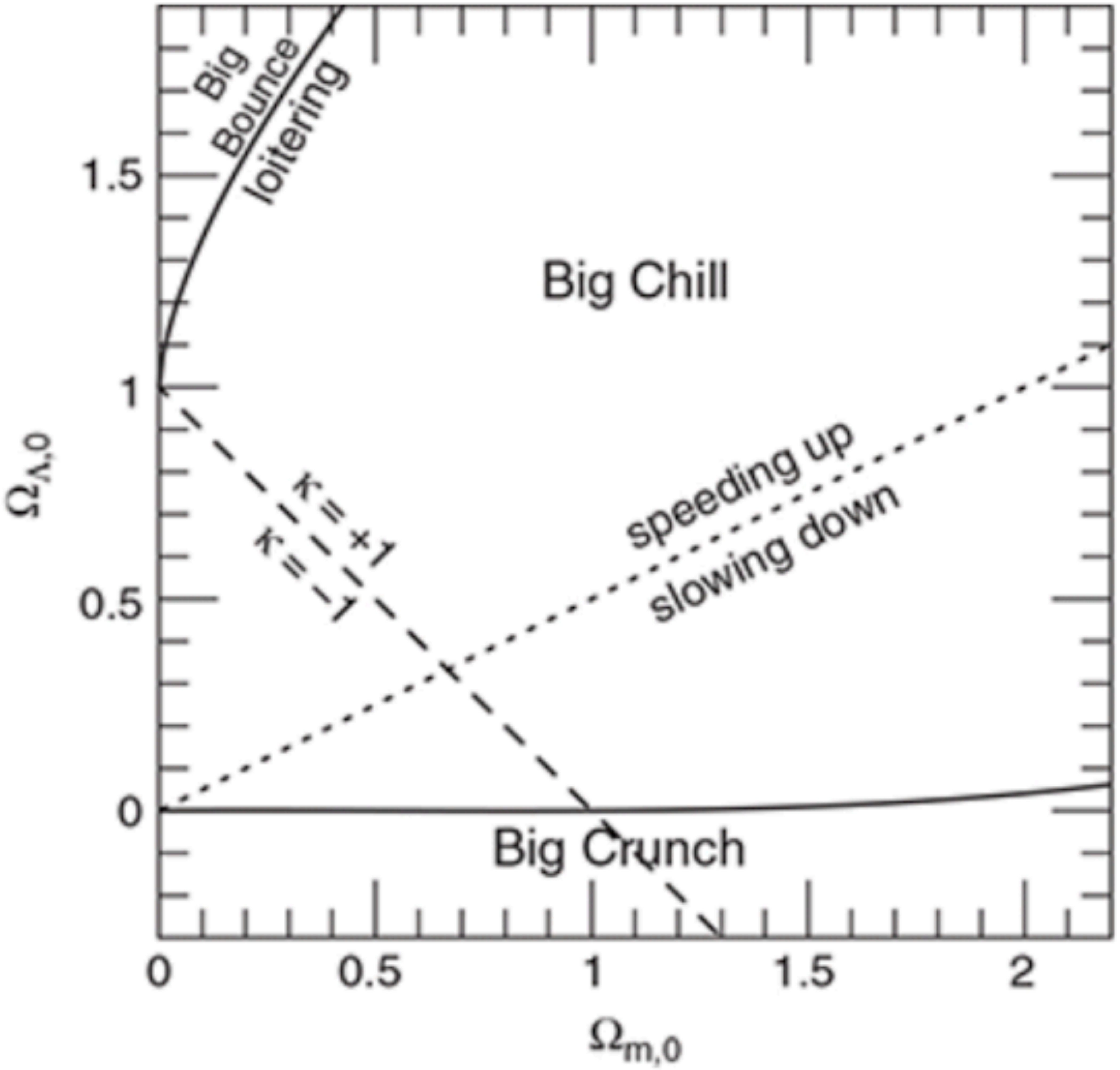
The ΛCDM Model

Matter + Cosmological Constant Models

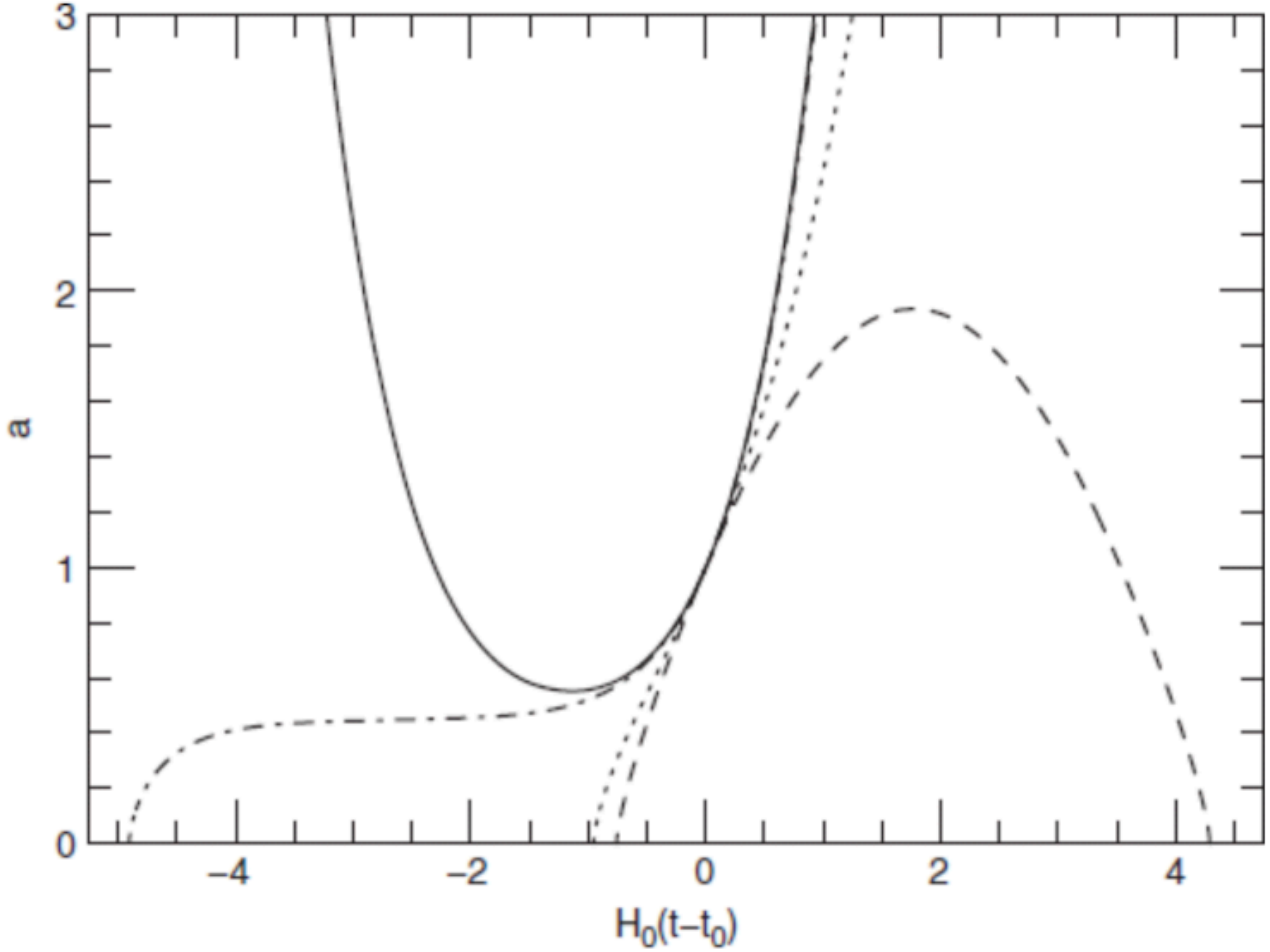
This diagram shows the different possible fates of a universe governed by the Friedman equation with these components:

$$H^2 = H_0^2 \left[\Omega_\mathrm{m} + \Omega_\Lambda + \Omega_\mathrm{k}\right]$$

We might call all these "ΛCDM models" (although we have not yet talked about the matter being Cold Dark Matter).



Bouncing, loitering, collapsing (all ruled out)

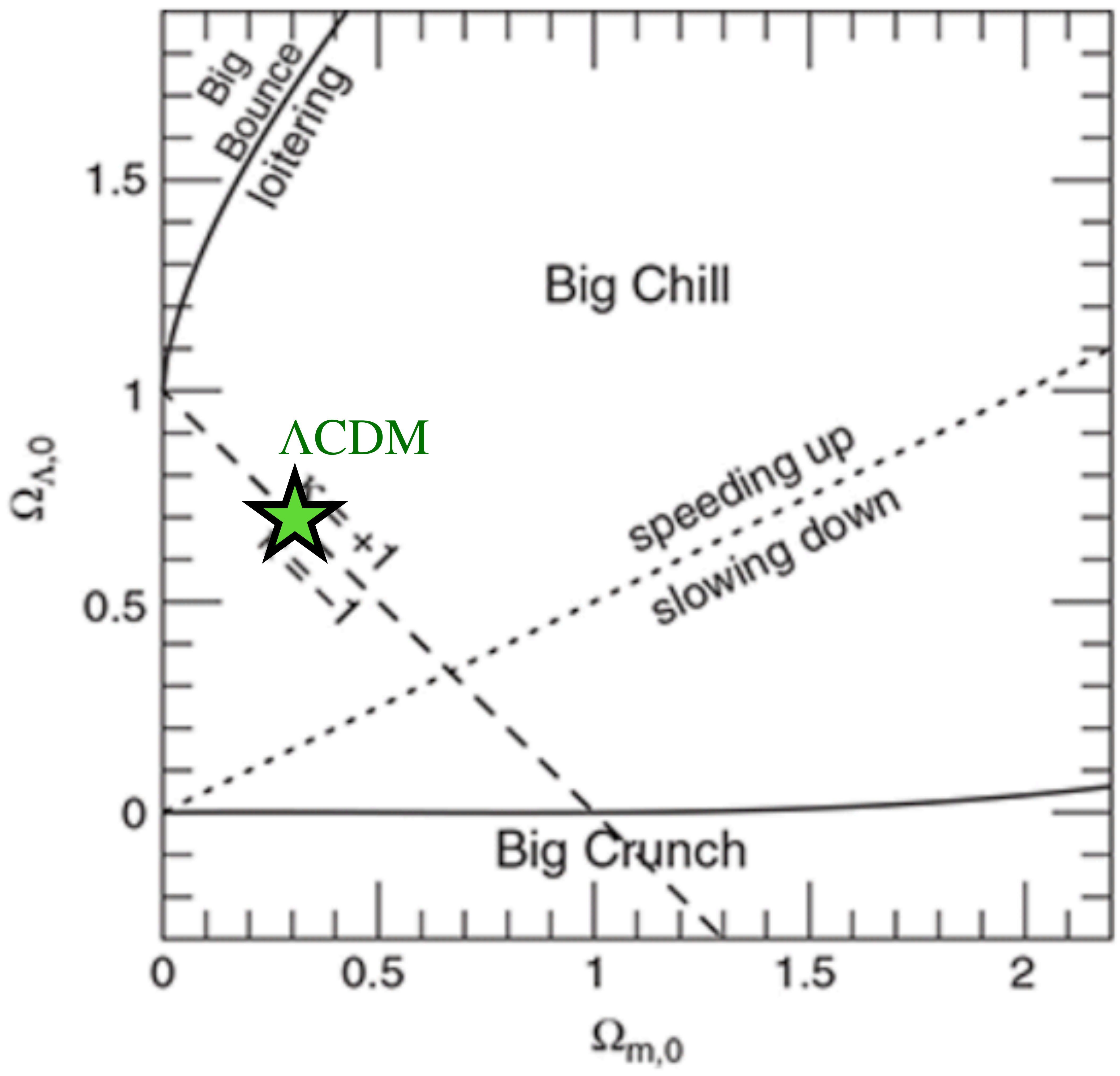


ΛCDM

All current observational evidence is well fit by a model with **flat** geometry and a cosmological constant $\Omega_\Lambda \gtrsim \Omega_m$.

This is often called "the" ΛCDM model.

Ryden calls it the "benchmark" model, others call it the "standard model of cosmology" or the "consensus" model.



Constraints on cosmological models

Together with the Hubble expansion, the major **constraints** on the ΛCDM model are the following.

The Cosmic Microwave Background

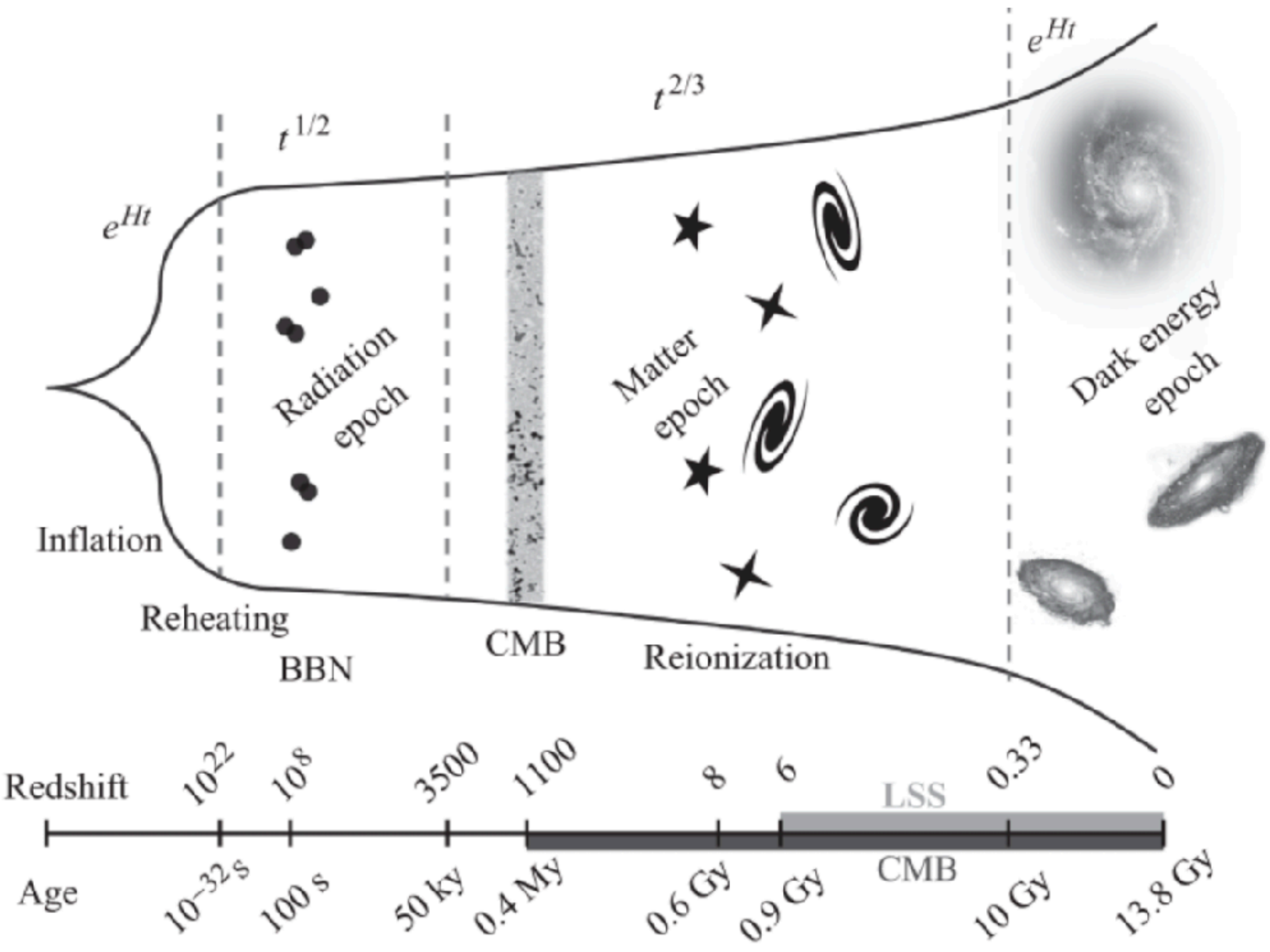
Inflation *(more like an extra ingredient, required by the CMB and cosmic structure, than a constraint)*

Primordial nucleosynthesis

Cosmic structure

We will spend the rest of the course exploring the observations and theory associated with these topics.

The big picture



Reproduced from Huterer Fig 1.2

Constraints on cosmological models

The Cosmic Microwave Background: the Universe is expanding; in the past the density of matter and radiation were much higher. At some time in the past, the Universe was a "plasma soup" of photons and ionised atoms (similar to conditions at the surface of the Sun, but *everywhere*).

We observe almost perfectly uniform flux of microwave photons from all directions. This is the relic of the time when the cooling, expanding Universe "switched" from being opaque to transparent.

Temperature fluctuations in the CMB: the CMB is not perfectly isotropic. Tiny differences in the CMB **temperature** from one part of the sky to another suggest the existence of tiny primordial fluctuations in the **density** of matter, and the statistics of those fluctuations provide *amazingly precise* constraints on the cosmological parameters. In particular, they provide compelling evidence for **flatness** and **non-baryonic dark matter**.

Constraints on cosmological models

Inflation: the ΛCDM model alone cannot explain observations of the CMB anisotropies — why was the Universe not perfectly smooth in the beginning?

Also, extrapolating an early radiation-dominated phase back to $t = 0$ leads to some troubling "coincidence problems".

All these problems are solved by an extra ingredient dark energy-like ingredient that acts only at very early times: **inflation**.

The idea sounds crazy, the motivation sounds a bit shaky, and we have no direct observations for inflation. Even so, inflation (like Λ and dark matter) is a simple model that works *really well* to explain what we see.

Really, inflation is so important that we should call the standard model **"inflationary** Λ**CDM"**.

Constraints on cosmological models

Primordial Nucleosynthesis: the universe went through a phase in which the "homogeneous" conditions became cold enough to allow neutrons to form, produce Helium nuclei and start the chain of reactions to build up heavier atomic nuclei.

This phase was very short, because efficient nucleosynthesis requires high density (e.g. in the Sun). Most heavy elements were made much later, in the cores of stars.

Predictions for the era primordial nucleosynthesis — for example, how much primordial Helium exists — are very sensitive to the density of **baryonic matter**, Ω_b, and the expansion rate at that time.

Constraints on cosmological models

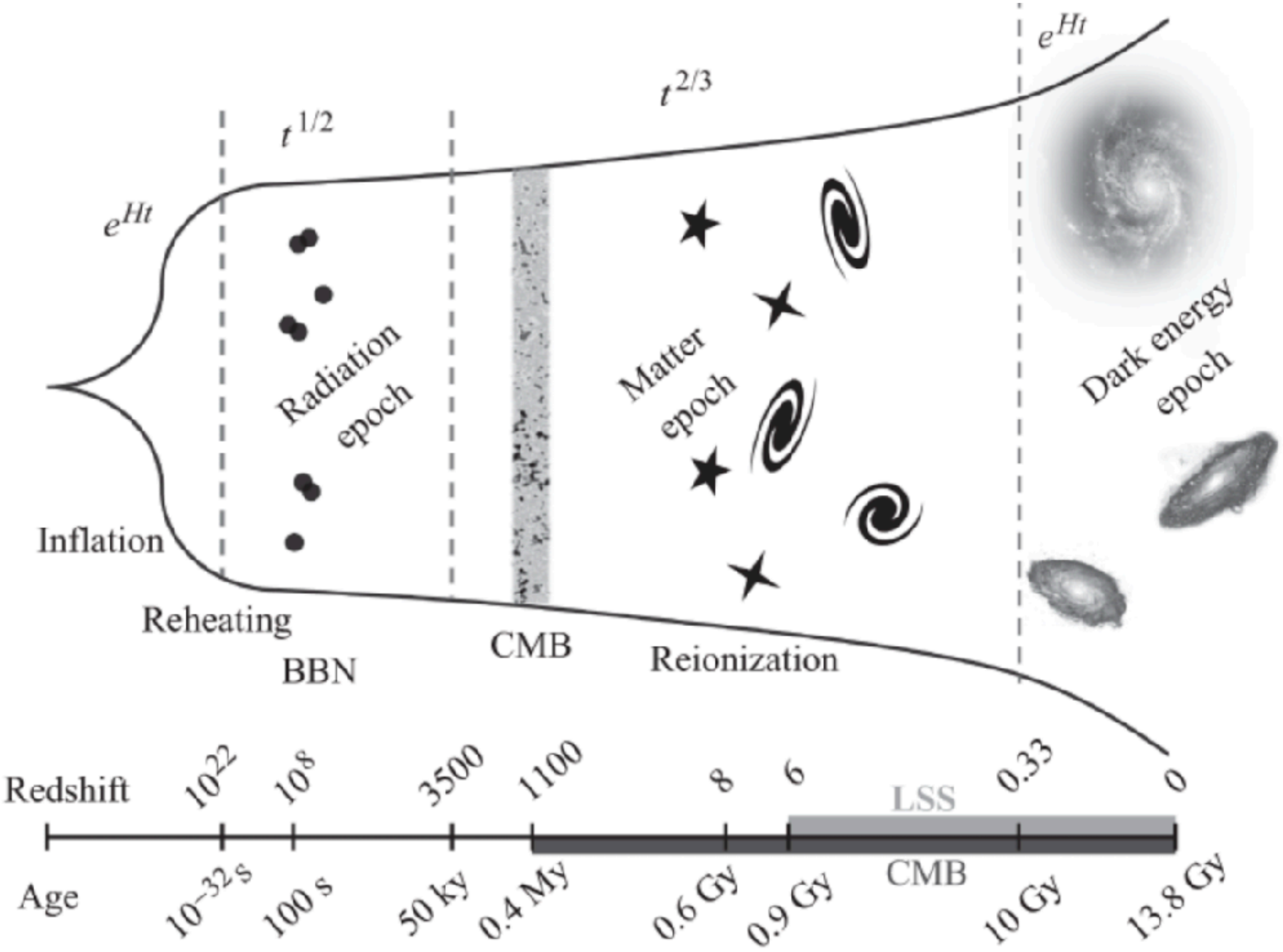
Cosmic Structure: the universe is full of inhomogeneous **structure on small scales** ($\lesssim 100\,\mathrm{Mpc}$). Where did that come from?

Inflation provides the **initial conditions for the formation of cosmic structure:** patches of very slightly higher and lower density than average, which are amplified by gravity (and plasma physics, up to the time of the CMB).

Starting from observations of the CMB, the theory of **cosmic structure formation** tries to explain statistical features of the large-scale **distribution of galaxies**, in particular, the **cosmic web**.

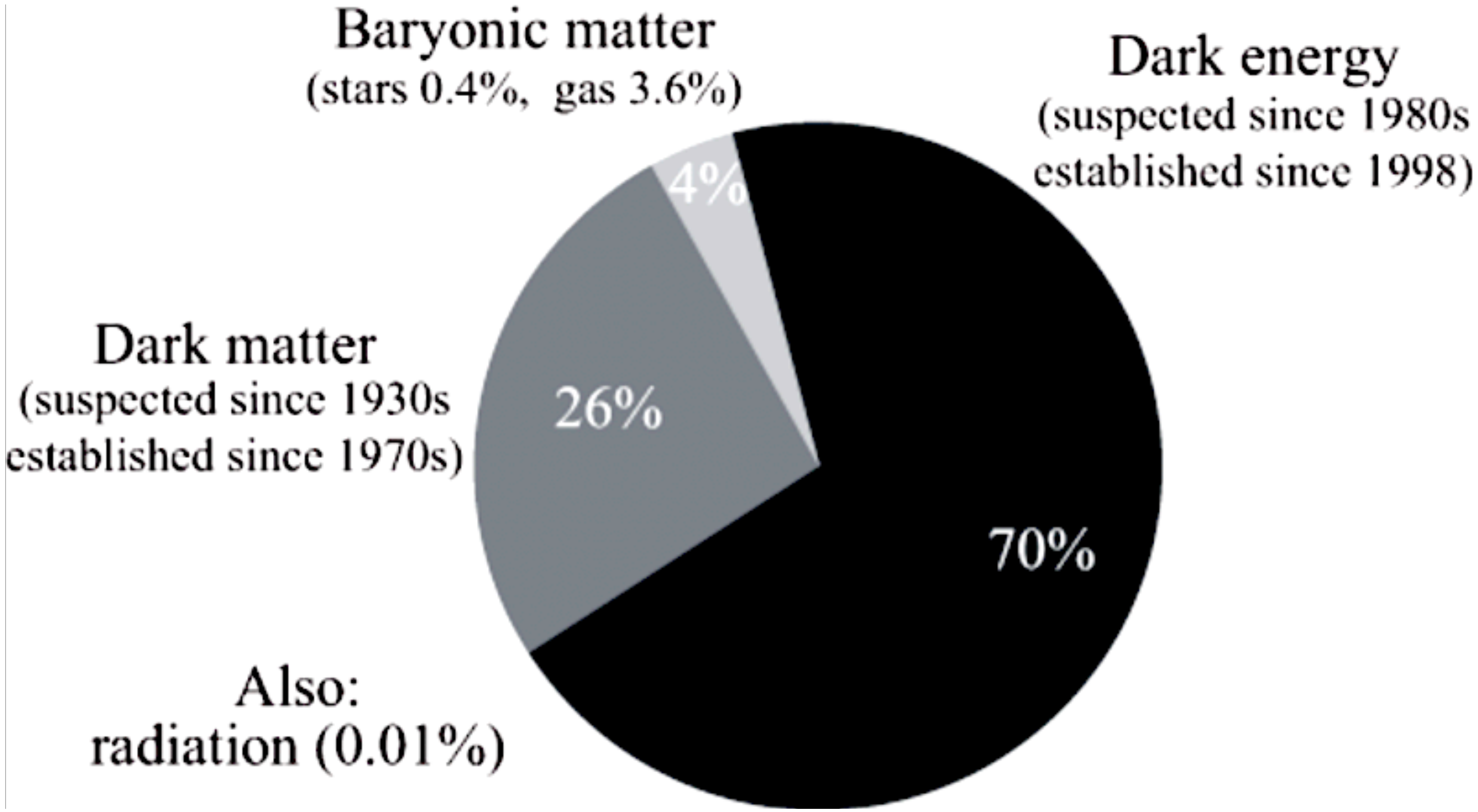
The existence of those features, and how they change with time/redshift, is another important source of constraints on the cosmological parameters.

The big picture

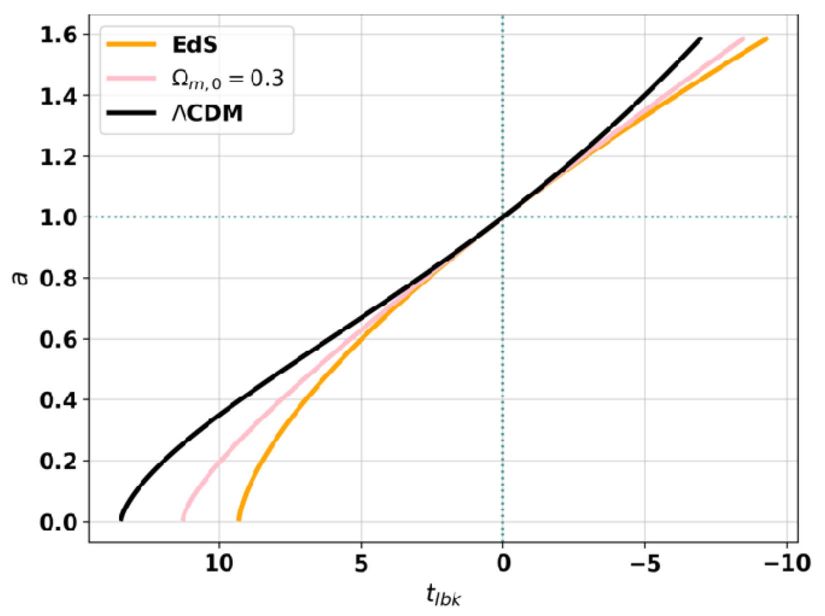


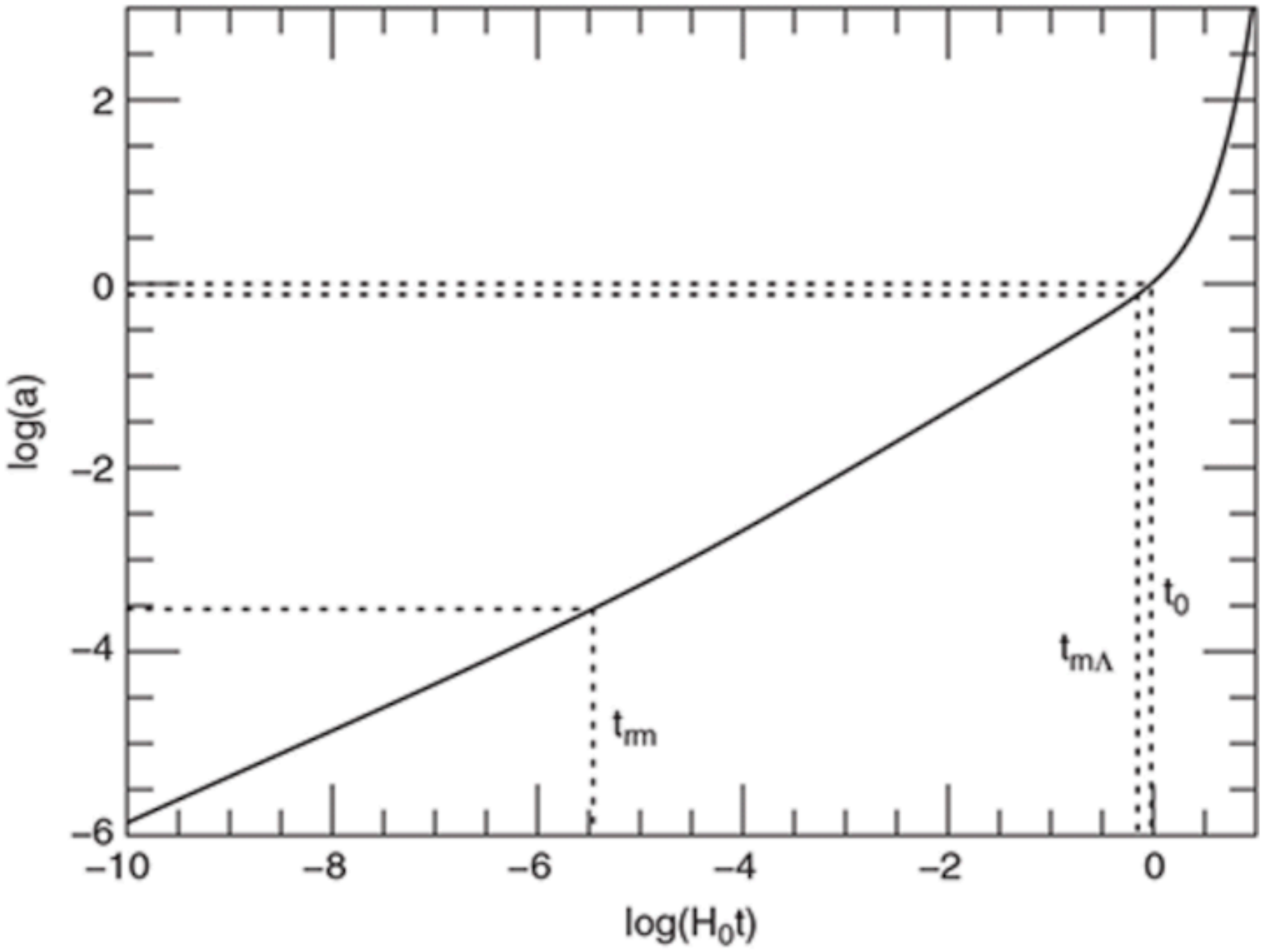
Reproduced from Huterer Fig 1.2

Components of the ΛCDM model



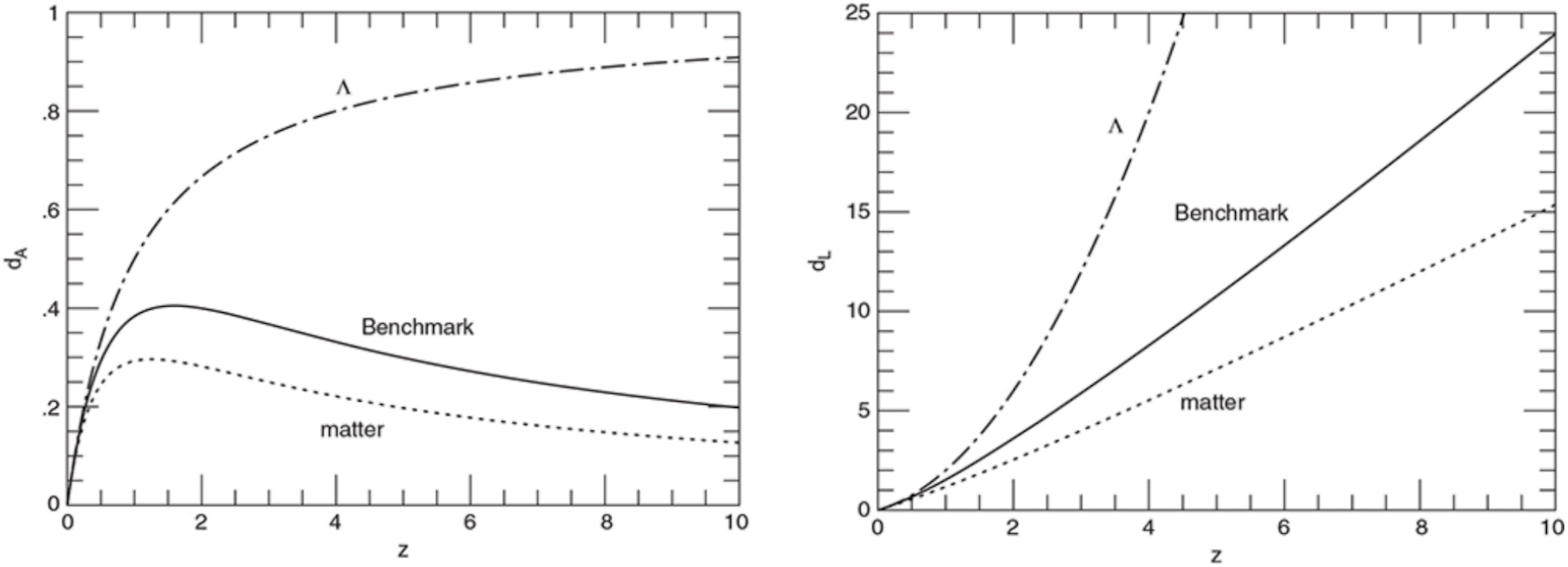
ΛCDM


EdS
$\Omega_{m,0} = 0.3$
ΛCDM
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
a
10
5
0
−5
−10
t_{lbk}


2
0
−2
−4
−6
log(a)
−10
−8
−6
−4
−2
0
log(H_0t)
t_{rm}
$t_{m\Lambda}$
t_0

ΛCDM

Reproduced from Ryden



ΛCDM

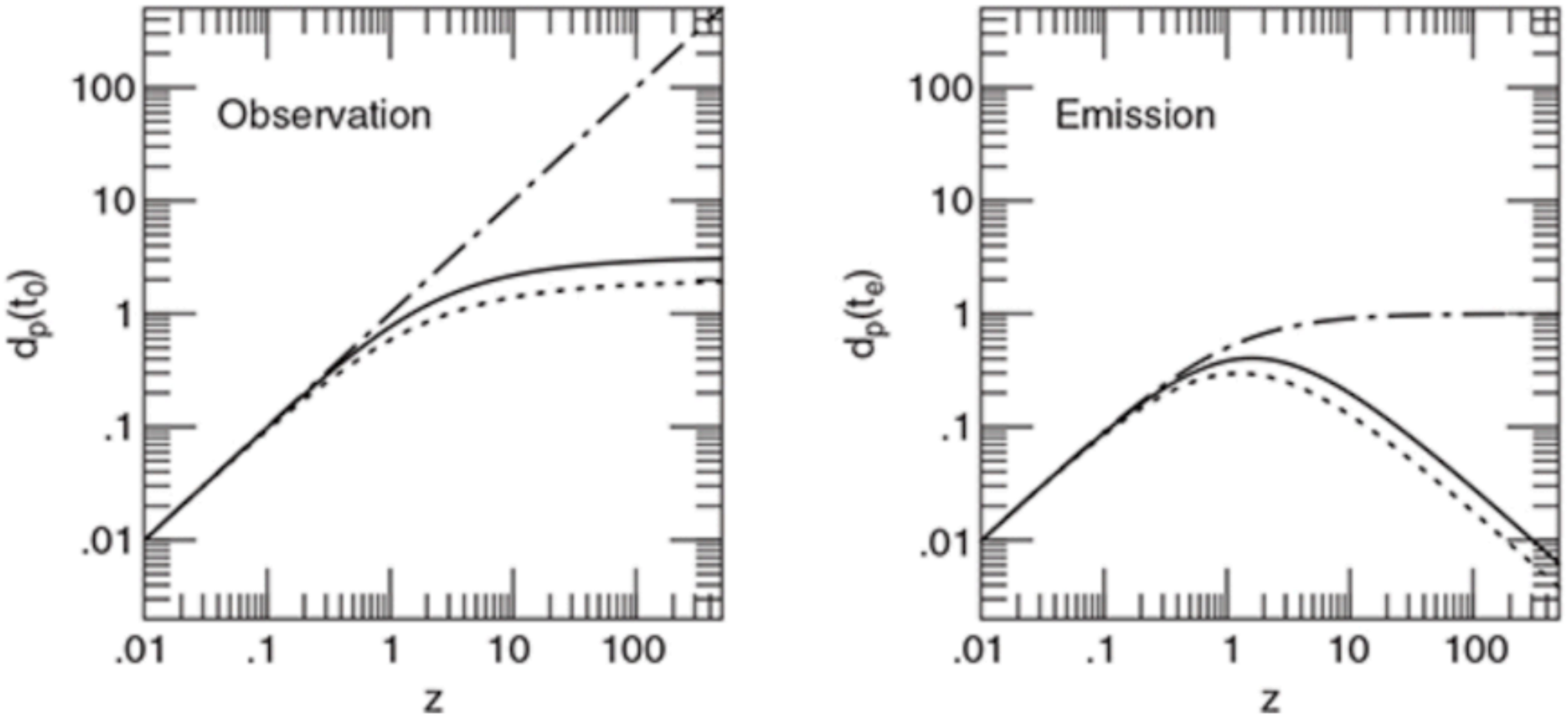
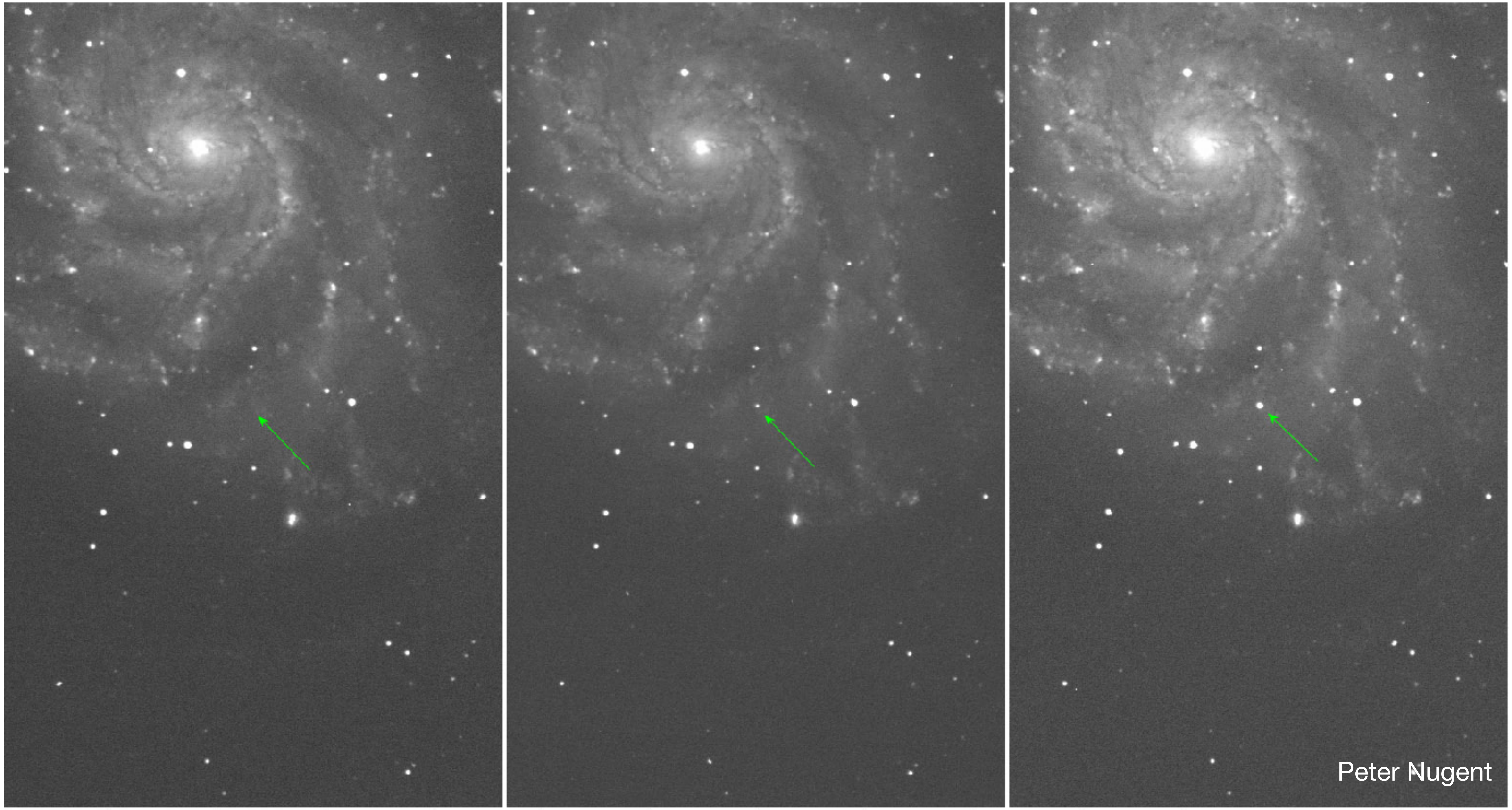


Figure 5.9 The proper distance to a light source with redshift z, in units of the Hubble distance, c/H_0. The left panel shows the distance at the time of observation; the right panel shows the distance at the time of emission. The bold solid line indicates the Benchmark Model. For comparison, the dot-dash line indicates a flat, lambda-only universe, and the dotted line a flat, matter-only universe.

Evidence for Accelerated Expansion

Supernovae

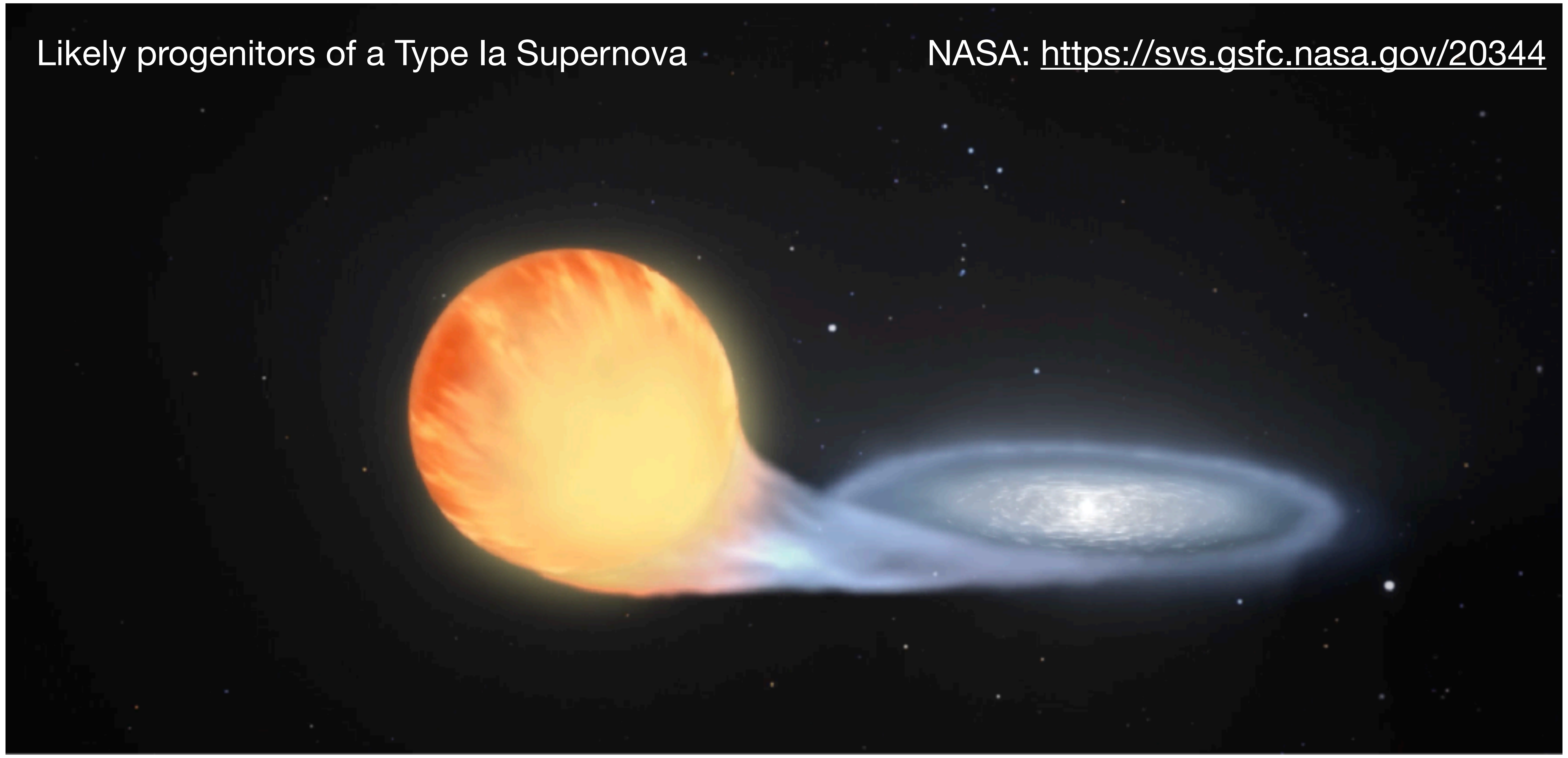


Supernovae

Two main types:

Type II: **core collapse** (individual massive stars, more common)

Type Ia: **thermonuclear** (mass transfer between binary stars, less common)



Astronomical magnitudes

Since the supernovae measurements are about astronomical observations, it's easiest to talk about them using the language of observational astronomy.

In particular, we need to learn what a "magnitude" is in astronomm.

The bottom line is:

Magnitude *is a* ***logarithmic measure of flux*** *(**<u>apparent</u>** magnitude) or luminosity (**<u>absolute</u>** magnitude) on a somewhat arbitrary and confusing scale.*

Astronomical magnitudes

Astronomers use the **magnitude scale** to quantify the brightness of objects. By definition, the **apparent magnitude** m of an object (star or galaxy) is related to its **flux** f by

$$m \equiv -2.5 \log_{10} \frac{f}{f_0}.$$

f_0 is the **zero point** (because $f = f_0 \implies m = 0$). Effectively it is the "unit" in which we measure flux. It's up to us to choose f_0 — for this choice it is not important how it is determined. For **bolometric fluxes** (i.e. the total energy received at all wavelengths), $f_0 = 2.51802 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}}$.

The "2.5" is a scaling factor, such that when the flux increases by a factor of 10, the magnitude increases by 2.5 (i.e. +1 magnitude means $f = 10^{-2/5} f_0 \simeq 2.512 f_0$).

Astronomical magnitudes

$$m \equiv -2.5 \log_{10} \frac{f}{f_0}.$$

The minus sign means that **things that look brighter** have **smaller apparent magnitudes**. For example, say the **magnitude of A is 0 and the magnitude of B is 1:**

Then $m_A - m_B = 0 - 1 = -1$

$$\implies \left[-2.5 \log_{10}\left(\frac{f_A}{f_0}\right)\right] - \left[-2.5 \log_{10}\left(\frac{f_B}{f_0}\right)\right] = -2.5 \left[\log_{10}\left(\frac{f_A}{f_0}\right) - \log_{10}\left(\frac{f_B}{f_0}\right)\right] = -2.5 \log_{10}\left(\frac{f_A}{f_B}\right) = -1$$

$\implies \log_{10}\left(\frac{f_A}{f_B}\right) = 1 \implies f_A = 10 f_B$, so **A is <u>brighter</u> than B.**

Astronomical magnitudes

You are **right** to think magnitudes are weird and not very intuitive!

Why the log scale? Why the 2.5? Why the strange zero-point? Why the minus sign?

As you can read in your textbooks, a magnitude scale for stars visible to the human eye was established long ago (by Ptolemy, following the earlier Greek astronomer Hipparchus). That scale ran from 0 (brightest visible star) to ~ 6 (faintest visible star). The modern mathematical definition tries to match that scale for "historical reasons".

The human eye is a roughly a logarithmic detector, hence the log.

$m_{faintest} - m_{brightest} = 6 \implies f_{\text{brightest}} = 10^{6/2.5} f_{\text{faintest}} \sim 250 f_{\text{faintest}}$ must be roughly our range of brightness perception.

The zeropoint is whatever is needed to make the brightest star we can see have zero magnitude.

Absolute magnitude

The apparent magnitude of a star or galaxy is a measure of how bright it *looks*. The **absolute magnitude,** M, is the same idea but measuring how bright the star or galaxy *actually is*, i.e. a ratio of luminosities rather than fluxes.

$$M \equiv -2.5 \log_{10} \frac{L}{L_0}$$

Of course, **distance** is the difference between how bright something looks and how bright it really is.

$$m - M = -2.5 \left[\log_{10} \frac{f}{f_0} - \log_{10} \frac{L}{L_0} \right] = -2.5 \left[\log_{10} \frac{f}{L} - \log_{10} \frac{f_0}{L_0} \right]$$

$$= -2.5 \left[2 \log_{10} D_{L,0} - 2 \log_{10} D_L \right] = 5 \log_{10} D_L - 5 \log_{10} D_{L,0} = 5 \log_{10} D_L - 5$$

if we defined $D_{L,0} = 10$ distance units. Since our favourite astronomy distance unit is the parsec, we take $D_{L,0} = 10\,\mathrm{pc}$, so an object a distance of 10 pc has $m - M = 0 \implies m = M$.

The absolute magnitude of an astronomical object is its apparent magnitude when observed at a distance of 10 pc.

Absolute magnitude

$$m - M = 5\log_{10}\left(\frac{D_L}{\mathrm{pc}}\right) - 5$$

For example, at 1 AU the Sun has an apparent (bolometric) magnitude of $m_\odot \sim -27$.

Its absolute magnitude is therefore:

$$M_\odot = m_\odot - 5\log_{10} 1\,\mathrm{AU\,in\,pc} + 5 = -22 - 5\log_{10} 4.85 \times 10^{-6} = -20 - 5(-5.314) \sim 4.6.$$

Standard candles

$$m - M = 5\log_{10}\left(\frac{D_L}{\mathrm{pc}}\right) - 5$$

A standard candle is a particular type of object for which we know the absolute magnitude M. For such objects, m is a measure of distance.

For objects co-moving with the expansion, the redshift z is also a measure of distance. These two observables — redshifts and apparent magnitudes — will therefore be related by some function that depends on $D_L(z)$, and hence on the cosmological parameters.

Switching to cosmology units (Mpc),

$$m = M + 5\log_{10}\left(\frac{D_L(z)}{\mathrm{Mpc}}\right) + 25$$

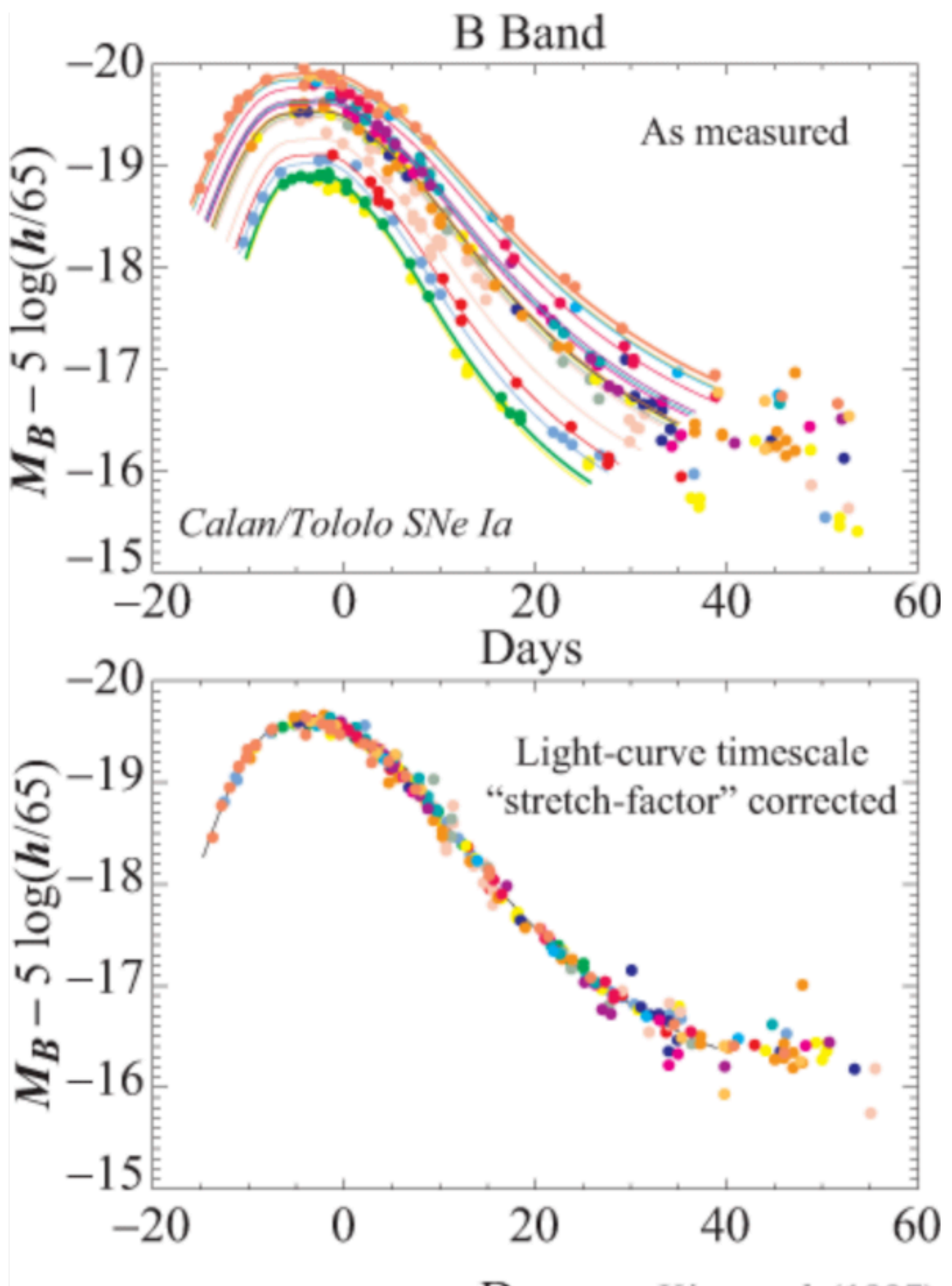
Lightcurve standardization

When a supernova explodes, it brightens rapidly and then fades on a timescale of $\sim$ 1 month.

The *intrinsic* luminosities of type Ia supernovae vary by orders of magnitude from one supernova to another. They are **not** all the same brightness!

However, the intrinsic luminosity is very tightly correlated with the **shape of their lightcurve**.

Broader light-curves (slower decay times) correspond to intrinsically **brighter** supernovae (known as the Phillips relation).

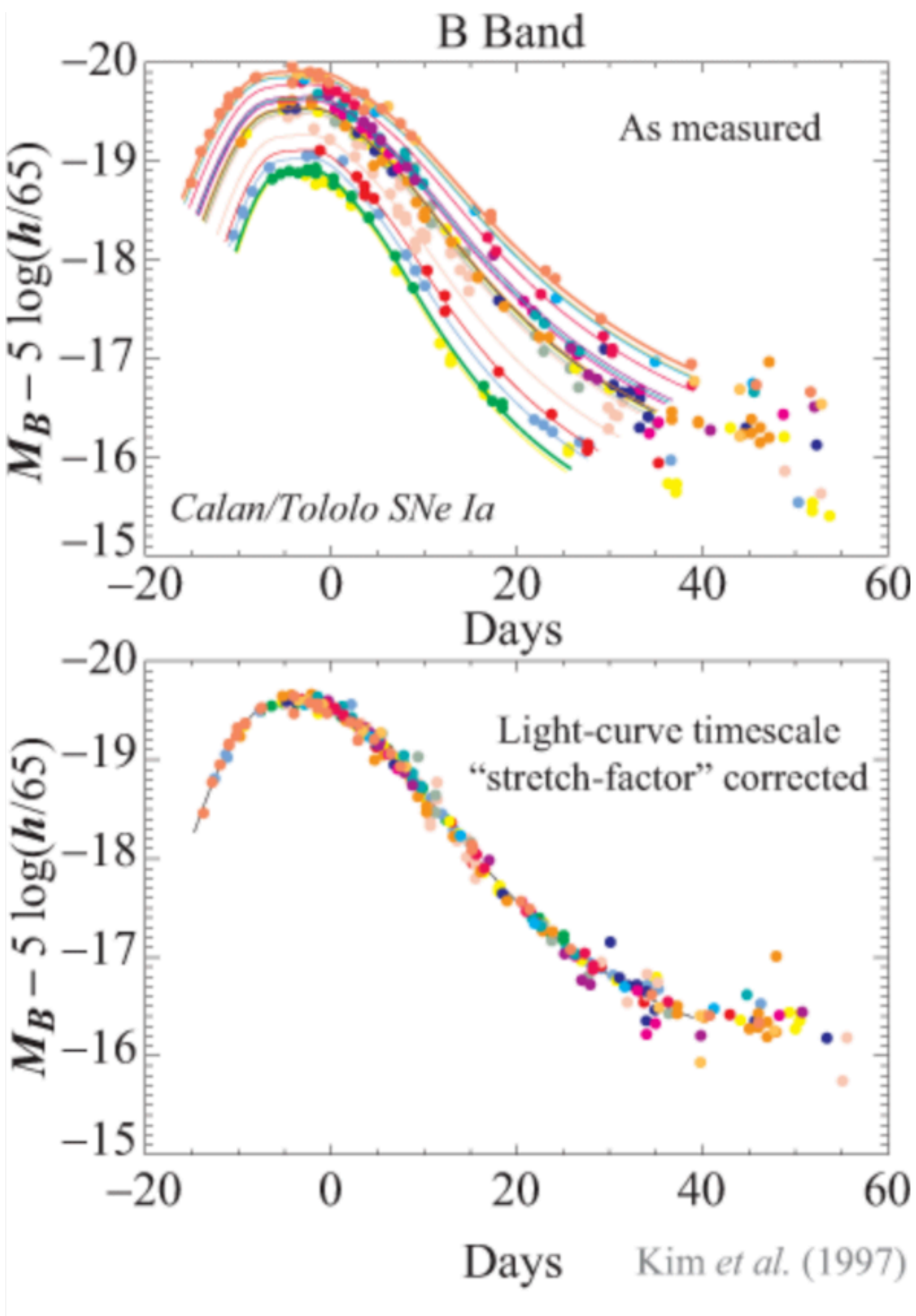


Kim *et al.* (1997)

Lightcurve standardization

We also have to adjust the measured magnitudes of the supernovae for *intrinsic extinction* — the dimming effect of **dust** in their host galaxy.

Because of these corrections, we don't really know the "true" brightness of supernovae, but we do know their **relative** brightness quite accurately.



Supernova distances

$$m = M + 5\log_{10}\left(\frac{D_L(z)}{\text{Mpc}}\right) + 25$$

In the last lecture we had $D_L(z) = S_k(r)(1+z)$ and the low-redshift ($z \ll 1$) approximation for $r(z)$:

$$r(z) \approx \frac{cz}{H_0}\left[1 - \frac{1+q_0}{2}z\right]$$

For an approximately flat universe, $S_k(r) \sim r$, we have

$$D_L(z) \approx \frac{cz}{H_0}\left[1 + \frac{1-q_0}{2}z\right]$$

The luminosity distance clearly depends on H_0; from redshift measurements alone we can constrain $H_0 D_L$ rather than D_L directly.

Supernova distances

$$m = M + 5\log_{10}\left(\frac{D_L(z)}{\text{Mpc}}\right) + 25$$

$$m = M + 5\log_{10}\left(\frac{H_0 D_L}{\text{Mpc}}\right) - 5\log_{10}\left(\frac{H_0}{\text{Mpc}}\right) + 25$$

With a little bit of work (see Hutter 12.1.4) this becomes

$$m = 5\log_{10}(z) - \mathscr{M}$$

Where $\mathscr{M}$ is a factor that depends on M (which we don't know) and H_0 and q_0 (which are the things we really want to know).

Since we don't know M for supernovae, we can't even use them to measure H_0. Supernova magnitudes don't tell us the absolute distance, only the relative distance.

The distance ladder

To find M, we need to use very nearby galaxies with distances measured by other redshift-independent methods, like the **period-luminosity relation of Cepheid variable stars**.

We can determine M for any supernovae in these nearby galaxies, and combine that with the measurement of $\mathscr{M}$ from supernovae to find H_0 and q_0.

Thus any uncertainties in the chain of very nearby distance calibrations will increase the total uncertainty on H_0 and q_0 (or equivalently, on what supernovae can tell us about $\Omega_{\mathrm{m},0}$ and $\Omega_{\Lambda,0}$).

Since the discovery of accelerated expansion in the 1990s, a lot of effort has gone into understanding the uncertainties in the distance ladder and hence the supernova measurement of H_0.

Recently this has revealed an interesting “tension” with the value of H_0 measured from the cosmic microwave background.

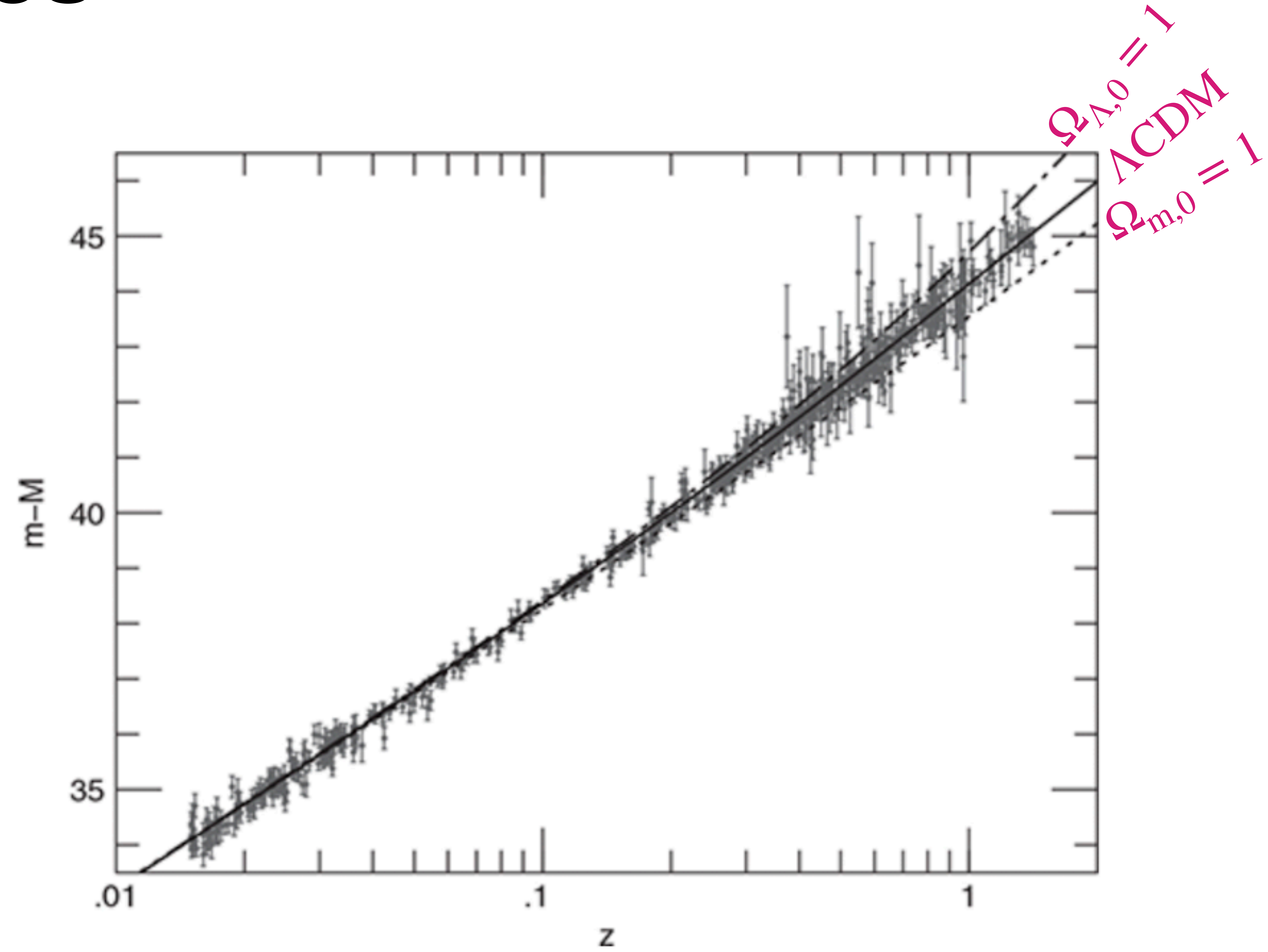
Supernova distances

The figure shows the distance moduli (effectively luminosity distances) of type Ia supernovae as a function of redshift.

The lines show different Friedmann models; the best fit is for $\Omega_{m,0} \sim 0.3$ and $\Omega_{\Lambda,0} \sim 0.7$.

$\Omega_{m,0} = 1$ and $\Omega_{\Lambda,0} = 1$ models are not consistent with the data.

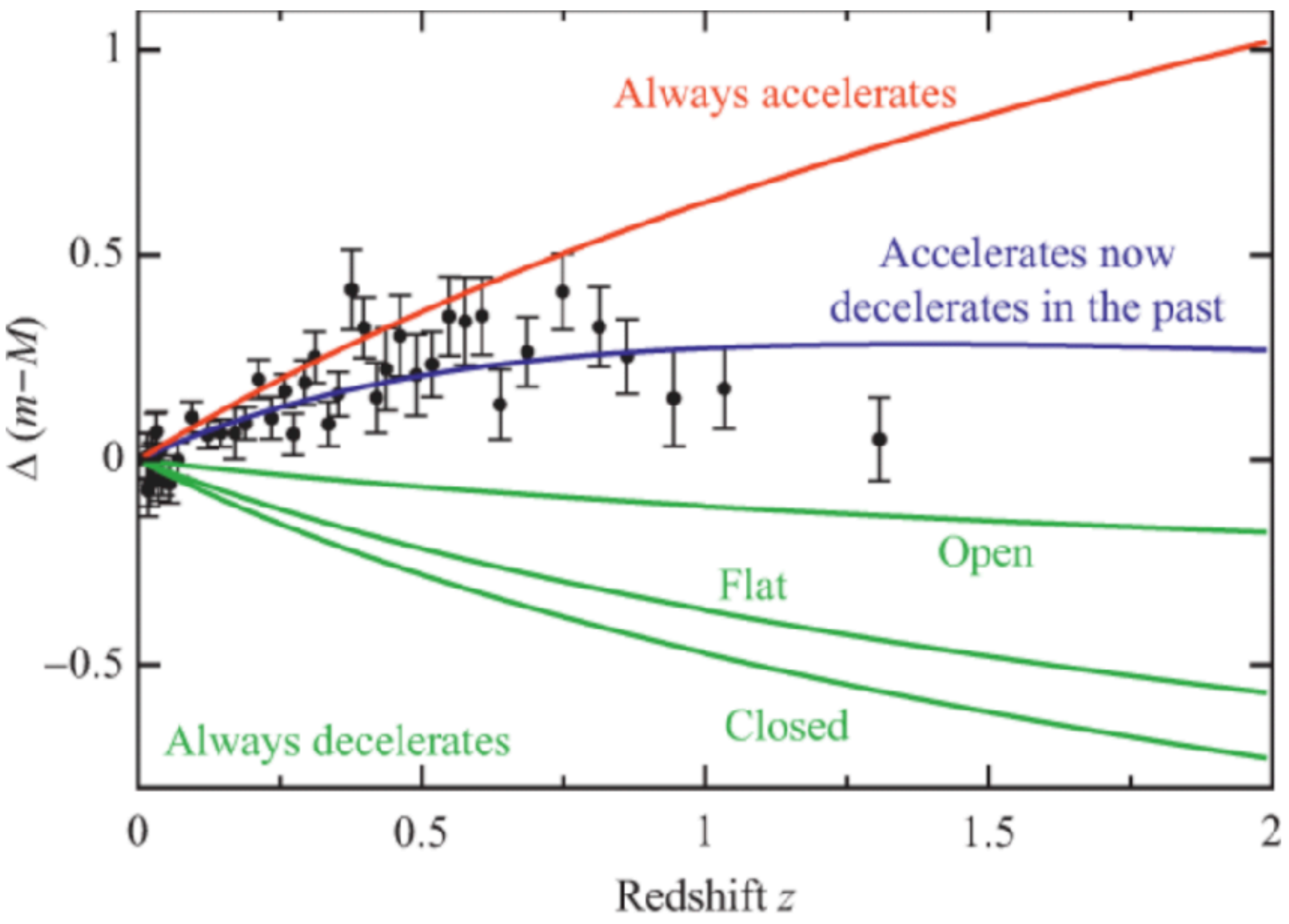
Notice all the models look the same up to $z \sim 0.1$.



Supernova distances

This version (Huterer's figure 12.2) takes the ratio with respect to an arbitrary reference model to make the differences more clear.

The points are the re-binned "Pantheon" supernova dataset from Scolnic et al. (2012)

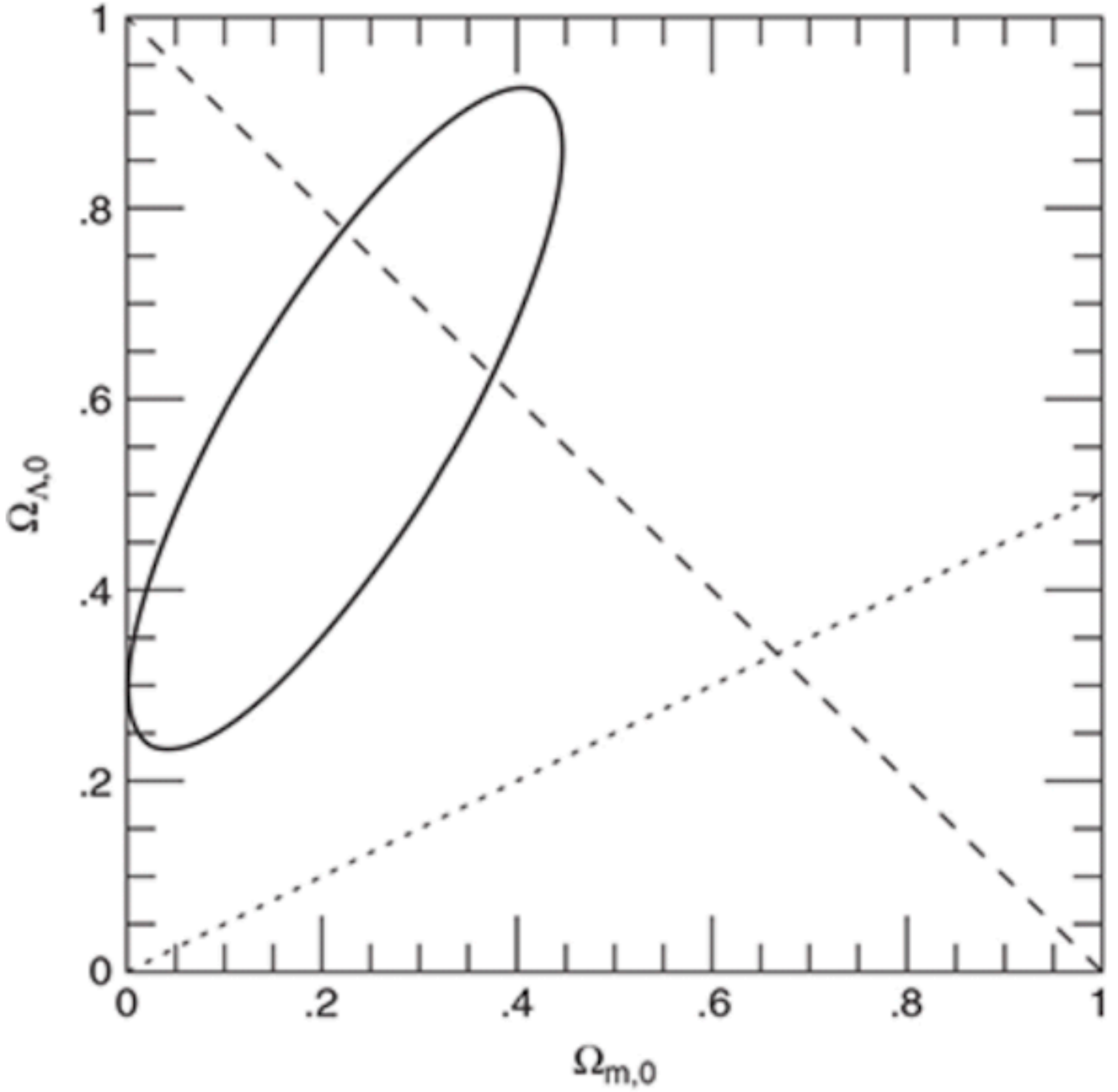


Cosmological constraints from supernovae

The elliptical contour on the figure shows the region of 95% confidence on the combination of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ from supernova distances.

The data are consistent with a flat geometry, but also with positive or negative curvature.

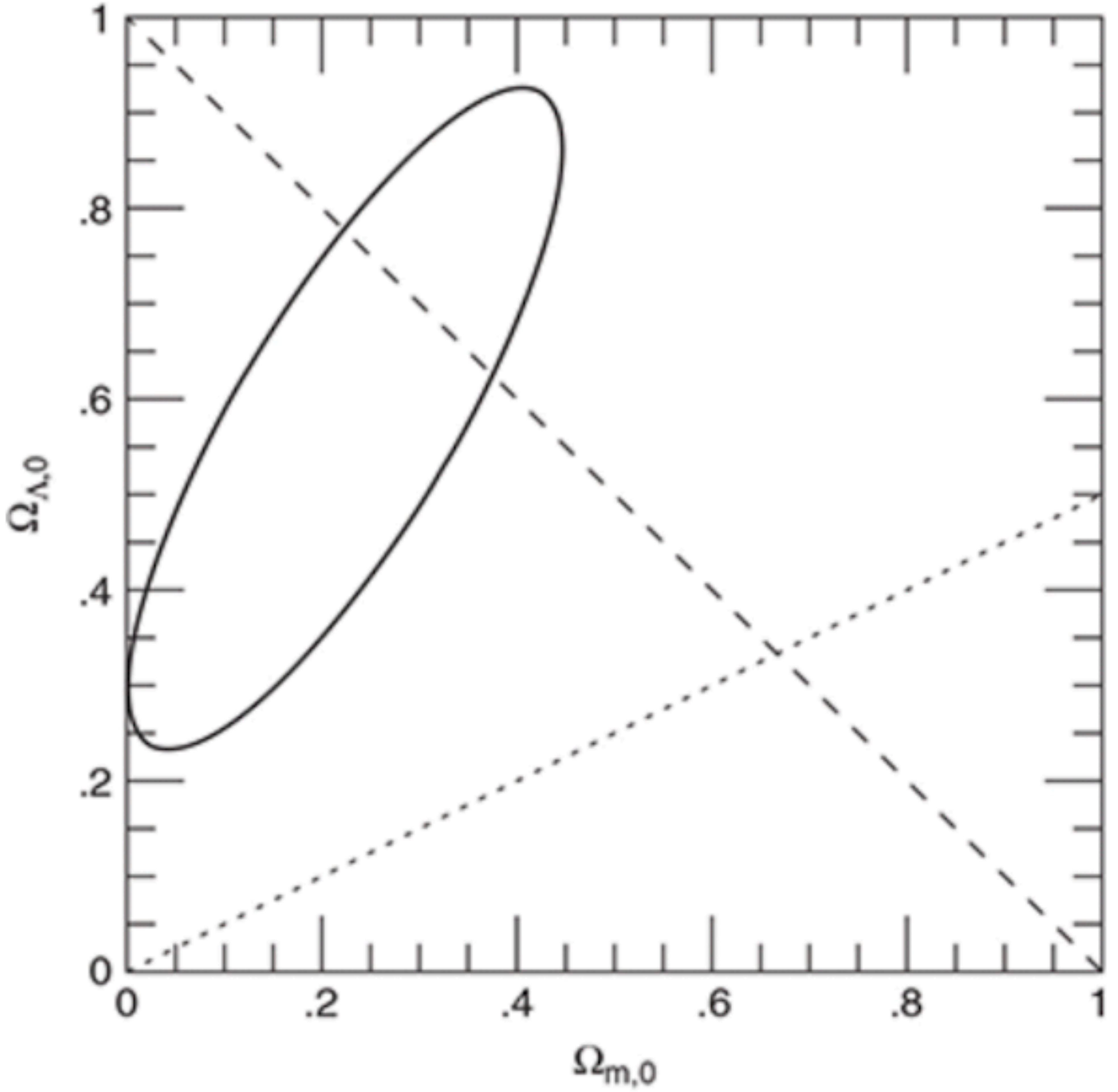
In all cases, they clearly prefer acceleration at the present day (above the dotted line): $\Omega_{m,0} < 0.5$ and $\Omega_{m,0} < \Omega_{\Lambda,0}$.



Degeneracies

The supernova constraints on $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ are **degenerate**: the data can be fit with different combinations of the two parameters.

To make progress, we need other experiments that are sensitive the combination of these two parameters in *different* ways.

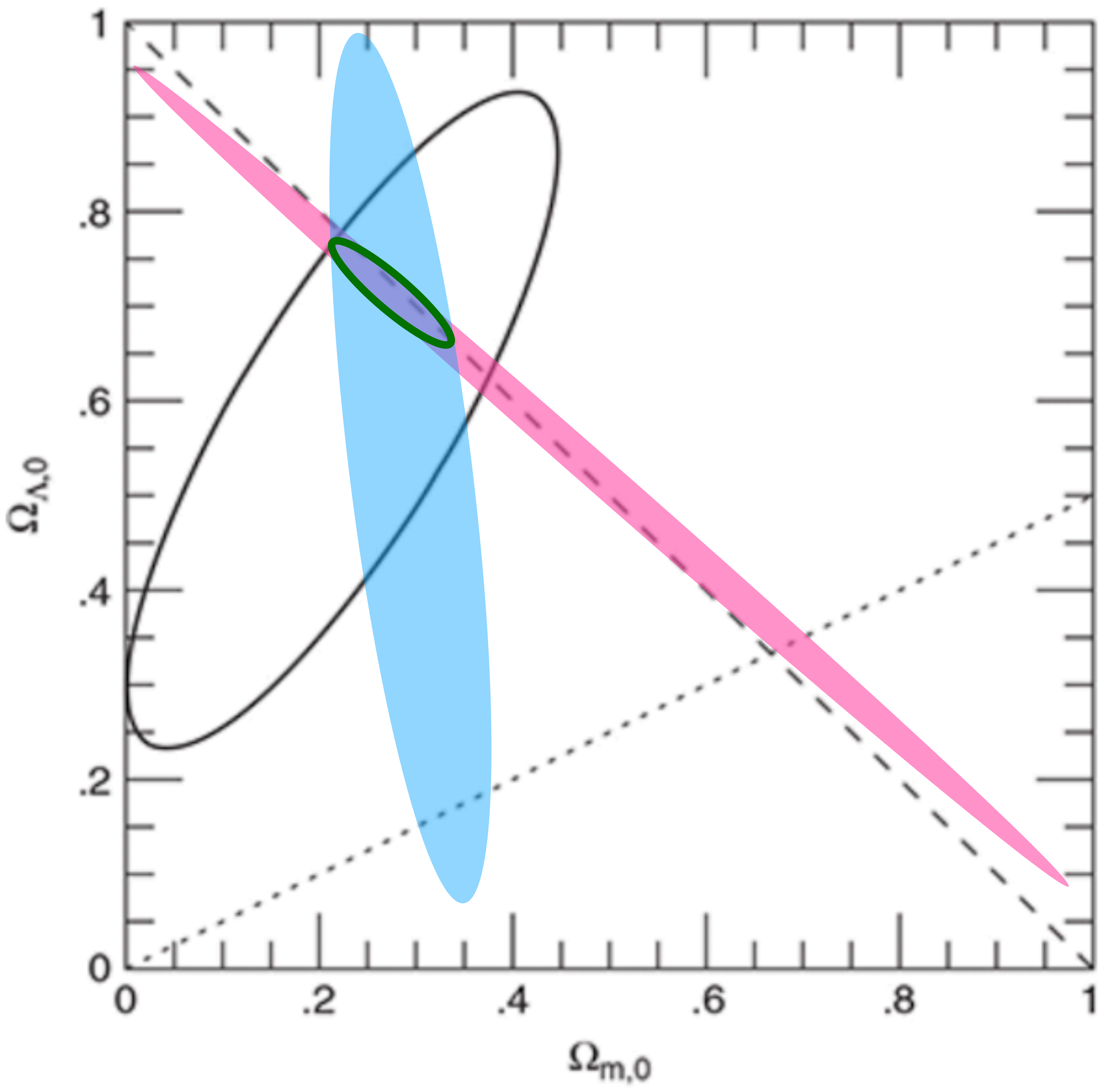


Other constraints

For example, imagine we had an independent constraint on $\Omega_{m,0}$ that didn't depend strongly on $\Omega_{\Lambda,0}$, or if we knew that that $\kappa \simeq 0$…

This is the goal of modern cosmological observations — and these different constraints now pin down the values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ very precisely.

We will look at these constraints in later lectures.



Vacuum energy

The most "obvious" physical explanation for the cosmological constant (or dark energy) is a **vacuum energy** arising from the probabilisitic nature of energy density in quantum mechanics.

Each type of "particle" is an excitation of a field. Spontaneous excitations of these fields — corresponding to the creation and annihilation of particle-antiparticle pairs — are thought to happen probabilistically, limited by the uncertainty principle $\Delta E \Delta t \geq \hbar$.

Since this process happens in a homogeneous way, the "zero point" energy of these vacuum fluctuations could contribute a cosmological-constant-like energy density.

Vacuum energy

A naive calculation of the vacuum energy density gives $\varepsilon_{\text{vac}} \sim m_{\text{planck}}^4 \sim 10^{76}\,\text{GeV}^4$ (see Huterer 12.3.2).

The required energy density for the "observed" Λ is $\sim 8 \times 10^{-11}\,\text{eV}^4 = 8 \times 10^{-47}\,\text{GeV}^4$

This is a serious discrepancy: ~ 122 orders of magnitude.

Up-to-date particle physics theories can reduce this to ~ 50 orders of magnitude, but that's still a serious discrepancy!

Conclusion: if a vacuum energy really exists, we do not understand why it is so small. At present, vacuum energy as currently understood is not a good explanation for the effect we associate with the cosmological constant.

Alternative dark energy models

The cosmological evidence points to a component of the energy density that looks at lot like a cosmological constant with $w = -1$.

However we don't have any good reason yet to assume it is truly constant. We should be open-minded about small variations, i.e. we only know $w \approx 1$, with some uncertainty, and the value of w for this component could change with time.

A simple parameterisation that allows for these possibilities with a smooth linear change in w is:

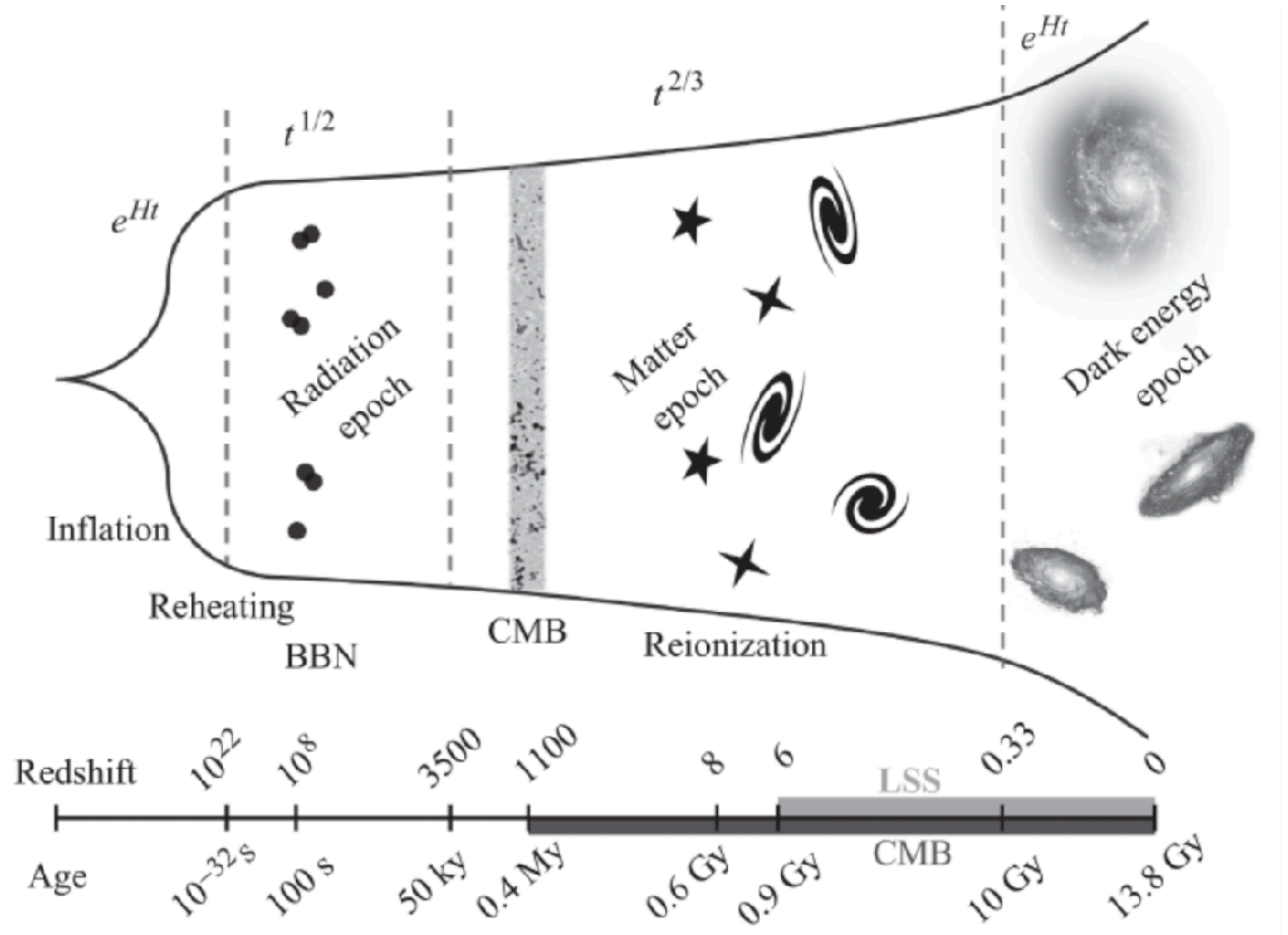
$$w(a) = w_0 + w_a(1 - a)$$

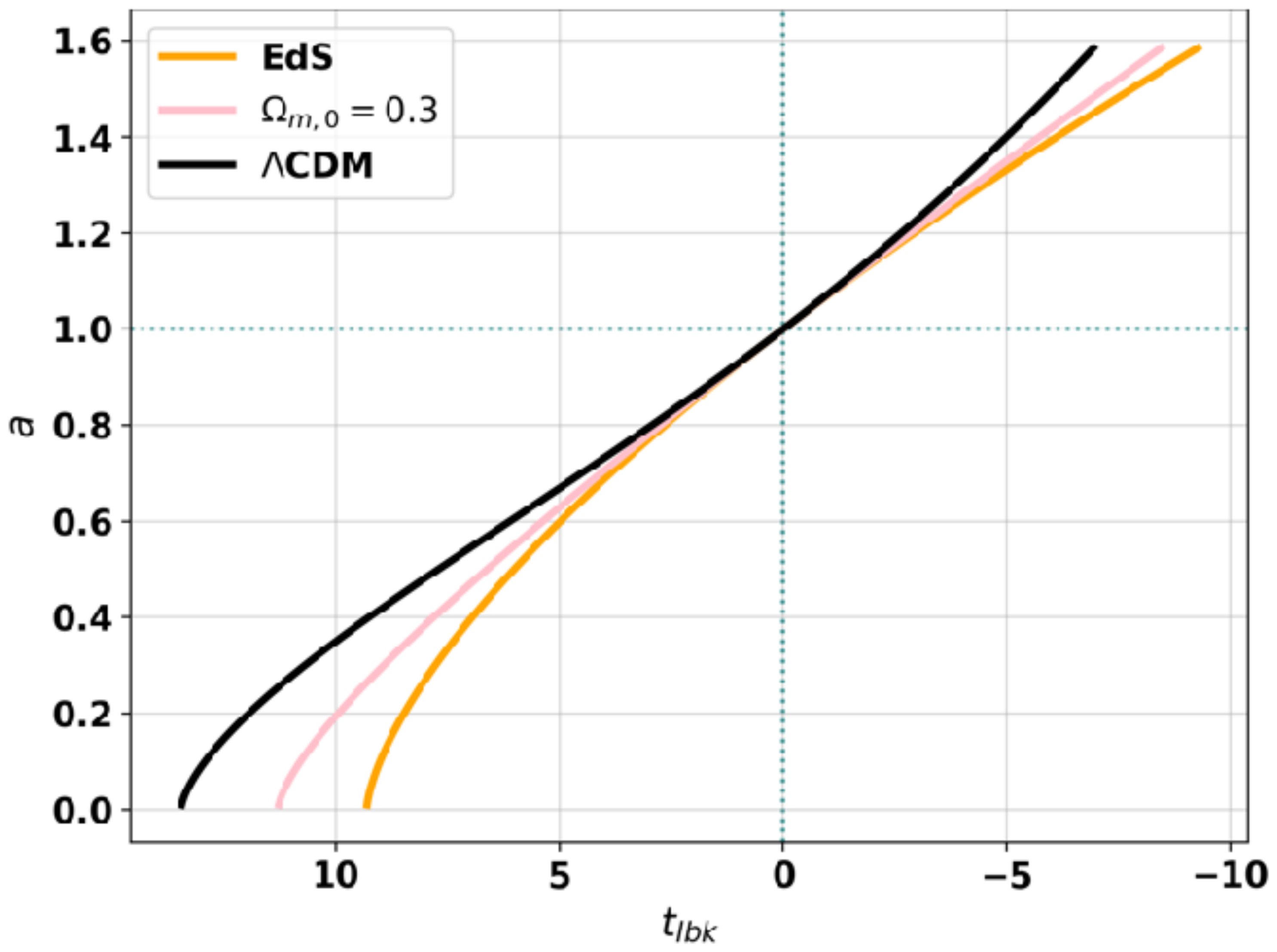
In this case, the parameters we need to measure are $\Omega_{\mathrm{DE},0}$, $\Omega_{\mathrm{m},0}$ (if $k = 0$ we know this exactly), w_0 and w_a. Such models are sometimes called wCDM. We may come back to this later.

Dark energy and acceleration

A dark energy/cosmological constant universe with the standard ΛCDM parameters is somewhat older than matter-only universe. It is this age difference that has the biggest effect on predictions for structure formation and galaxies, rather than the late-time acceleration.

It is a somewhat troubling fact that human beings showed up just at the very special (and fairly short) time when the expansion is switching from matter-driven to dark energy-driven…





Other probes of dark energy

To make progress, apart from more supernova data, we need other experiments (cosmologists say "probes") that are sensitive to possible small variations around $w = -1$.

These include:

Large-scale features in the distribution of galaxies (we will look at these in a later lecture)

Weak gravitational lensing

The growth of galaxy clusters

Dark Matter

Dark matter

Many lines of observational evidence point to $\Omega_{m,0} \sim 0.3$, much higher than $\Omega_{b,0} \lesssim 0.005$, the density of ordinary **baryonic matter.**

This suspected **non-baryonic gravitating matter** is called **dark matter**.

We don't know what it is yet, but we have a lot of empirical evidence for how it "works".

Any particle candidate or alternative theory for dark matter has to explain a lot of well-measured phenomena. So far, the assumption of an unknown, massive and weakly-interacting particle satisfies all those requirements.

Dark matter is very important for explaining the existence and properties of **structure** in the universe (i.e. the departure from a homogeneous mass distribution on small scales).

Baryonic Matter

Dark matter is distinct from “ordinary” **baryonic matter** — mostly massive particles like protons (rest mass 938.8 MeV) and neutrons (rest mass 939.6 MeV), which are “made” of quarks.

Baryonic matter mostly moves at non-relativistic speeds, and interacts through all the fundamental forces (gravity, electromagnetic, weak, strong).

Most baryonic matter is in the form of <u>**diffuse**</u> **hydrogen gas,** with about 1/4 in **diffuse helium**. This diffuse gas (i.e. outside galaxies) has very low density and is mostly very hard to observe.

A small fraction of baryons are concentrated in heavier elements and in the “cold” clumps of **stars and dense gas** we call galaxies.

Leptons

We also know about elementary particles called **leptons** — electrons and neutrinos and their relatives.

These have very small mass and interact electromagnetically (electrons) or via the weak force (neutrinos).

Electrons are mostly “ignored” from the point of the view of the mass budget (they are lumped together with the baryons, because they interact strongly with protons; the Universe has no net charge)

Neutrinos are a bit more interesting for cosmology, but they don’t contribute much mass. They are one form of very weakly interacting **non baryonic** dark matter, but not the most important one (as far as we know now).

Stellar mass density

We can estimate the cosmic density of baryons, starting with what we can most easily see — stars.

Most of the baryonic mass of the Solar System is in the Sun. We can measure this mass from the gravitational accelerations it creates. In general, we cannot estimate the masses of other stars that way.

If we want to find the combined mass of all the stars in a galaxy, we can instead add up their total **luminosity** and use knowledge from stellar physics of the relationship between stellar mass and luminosity: the **stellar mass-to-light ratio**.

In the wavelengths of visible light, $2 \lesssim M/L_\star \lesssim 10$, depending on the chemistry and age of the star.

Stellar mass density

In detail it is quite hard to estimate the mass of stars in a galaxy from its luminosity (e.g. corrections are needed for faint stars like white dwarfs and brown dwarfs, and 'stellar remnants' like black holes).

Estimates are usually accurate to $\pm 20\,\%$ or so, at best.

To calculate $\Omega_{\star,0}$ further requires some knowledge of how many galaxies there are of different luminosity per unit volume of the universe.

Approximately, $\Omega_{\star,0} \simeq 0.005$ ($\rho_{\star,0} \sim 4 \times 10^8\,\mathrm{M}_\odot\,\mathrm{Mpc}^{-3}$)

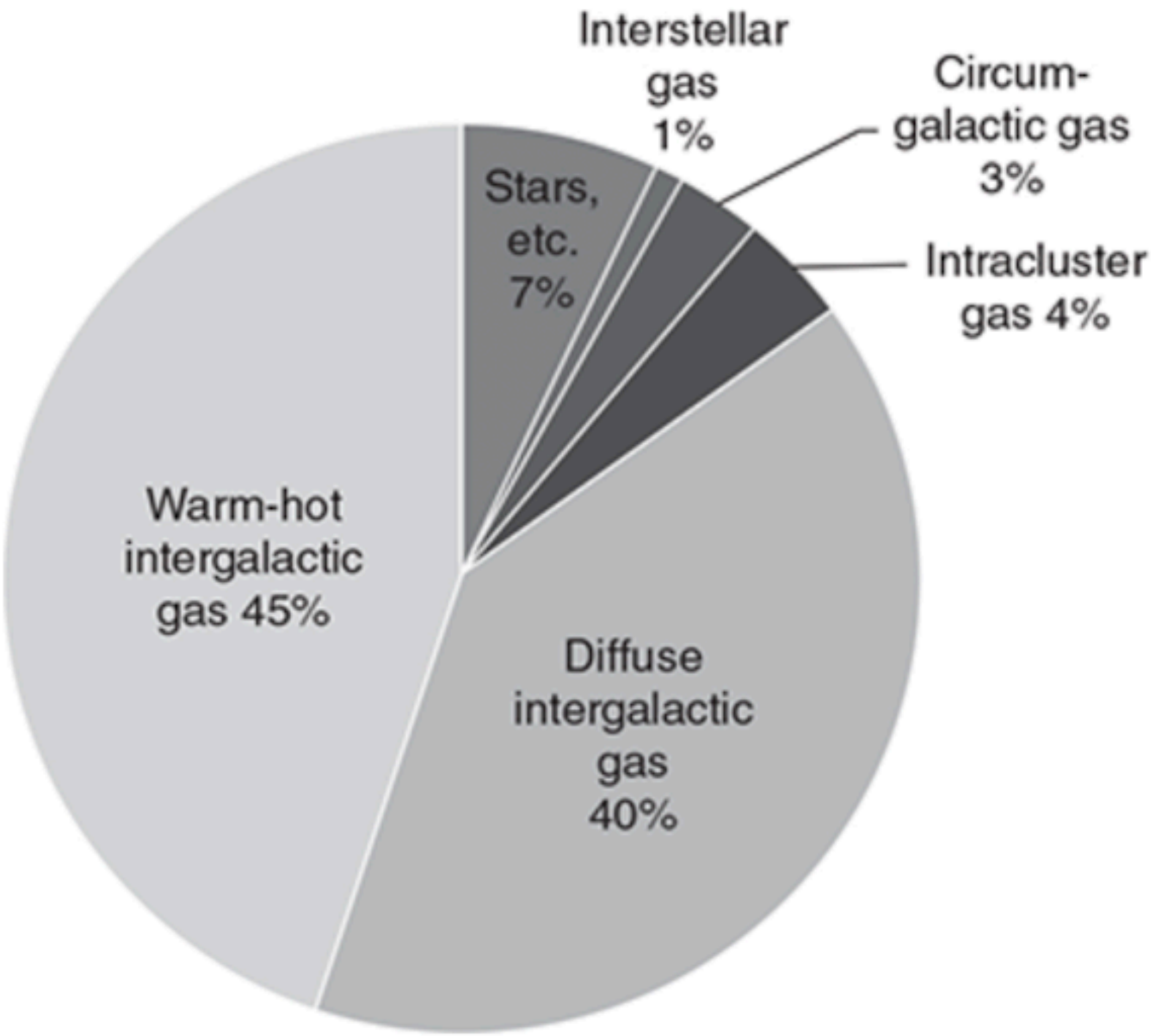
Clearly $\Omega_{\star,0} \ll \Omega_{m,0} \sim 0.3$.

Gas in galaxies

We know stars are only a small fraction of the baryons. Galaxies like the Milky Way have a lot of gas, probably as much or more than their mass in stars.

In galaxies like the Milky Way, the gas we can easily detect is mostly **cold ($\sim 10-100\,\mathrm{K}$) atomic hydrogen**, which can be observed readily with radio telescopes (21 cm emission).

However, we also know that the galaxy is surrounded by a lot of **diffuse ionized** gas, with temperature is $\sim 10^5\,\mathrm{K}$. This is much harder to detect. Measuring its density is even harder.



Ryden Fig. 12.1

Gas in galaxy clusters

Galaxy clusters are good place to measure the combined mass of stars and gas.

The ionized gas in galaxy clusters is very hot ($\sim 10^7 - 10^8$ K) because it is in **hydrostatic equilibrium** in a much deeper gravitational potential.

Interactions between high-energy electrons and protons in the gas lead to **X-ray emission**, which reveals both the total extent and (indirectly) the density of the gas.


ESA/XMM-Newton/SDSS/J. Sanders et al. 2019
X-ray image of the Perseus cluster

Gas in galaxy clusters

The gas is in **hydrostatic equilibrium** with the gravitational potential of the cluster: pressure balances gravity.

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho_{\text{gas}}(r)$$

For an ideal gas, $P(r) = \dfrac{\rho_{\text{gas}}(r)\, k_B T(r)}{\mu}$ with $T(r)$ the temperature, k_B Boltzmann's constant and μ the mean molecular weight of the gas. From these two equations, we can obtain an expression for $M(r)$ in terms of the observable temperature and density gradients of the gas (see Ryden 7.3, eq. 7.41).

Gas in galaxy clusters

In clusters, it is clear that $M_{\rm gas} \sim 10\, M_\star$. Very roughly, this implies $\Omega_{b,0} \sim 0.05$. However, these estimates are not easy, and clusters might be special places.

As we will see next time, there are even stronger constraints on $\Omega_{b,0}$ from observations of the cosmic abundance of heavy elements (primordial nucleosynthesis) and the baryon-to-photon ratio at the time of the CMB emission.

All together, the current best value is $\Omega_{b,0} \simeq 0.048$.

Missing mass from orbital motion

Having counted up the baryons in galaxies and galaxy clusters, we can make a separate estimate of the total **gravitating mass** of the same objects.

Those estimates are much bigger than the baryonic mass, which implies a lot of **“missing mass”** in galaxies.

The idea of finding missing mass through its gravitational effect is a long-running theme of astronomy. Hundreds of years ago, unexplained **perturbations to the orbits** of known planets were used to infer the presence of ‘missing mass’ in the outer Solar system (i.e. to find more planets).

Fundamentally the same idea applies to the motions of stars in galaxies, and galaxies in galaxy clusters. Their **orbital speeds are determined by the depth of the gravitational potential**.

Rotation curves

In "disk" galaxies like the Milky Way, most of the stars and gas are on **circular orbits**.

For such orbits, Newtonian dynamics relates the **circular orbit speed,** V_c**,** to the mass enclosed by the radius of the orbit:

$$V_c^2 = \frac{GM(<R)}{R}$$

Measuring V_c at different R therefore yields the **total mass profile of the system**, and hence the density profile e.g. $M(<R) = \frac{4}{3}\pi R^3 \bar{\rho}(<R)$.

Rotation curves

In the 1970s, measurements of the rotation curves of M31 and other spiral galaxies by Vera Rubin and Kent Ford (and others) provided compelling evidence for missing mass.

The density of visible baryonic mass in these galaxies decreases **exponentially** outwards from the center (in the Milky Way, the characteristic scale length is $\sim 3\,\mathrm{kpc}$).

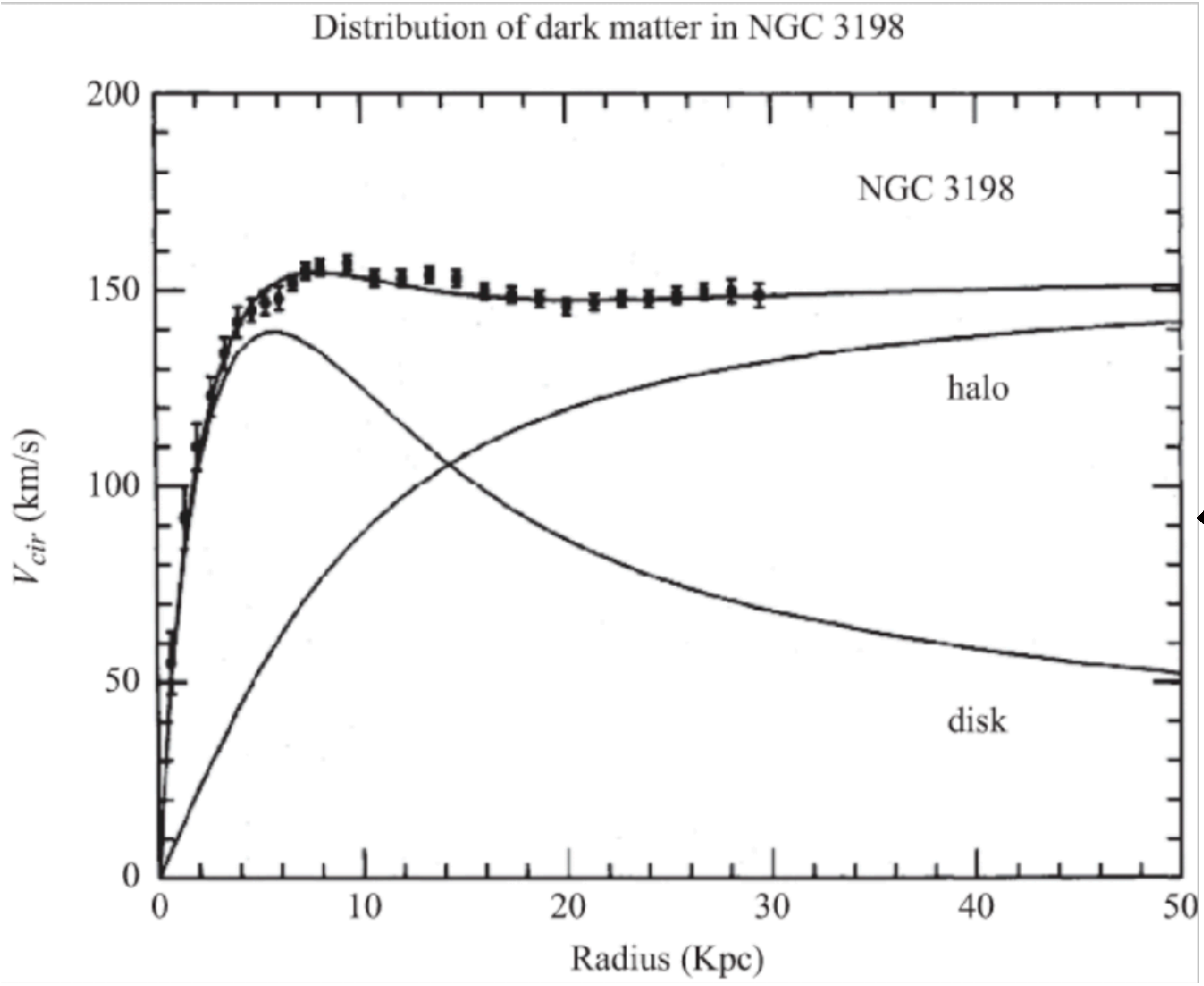
At large distances (several scale lengths), $M_b(<R) \sim M_{b,total} \sim$ constant, implying a **Keplarian** rotation curve $V_c \propto 1/\sqrt{R}$ (i.e. declining outwards)

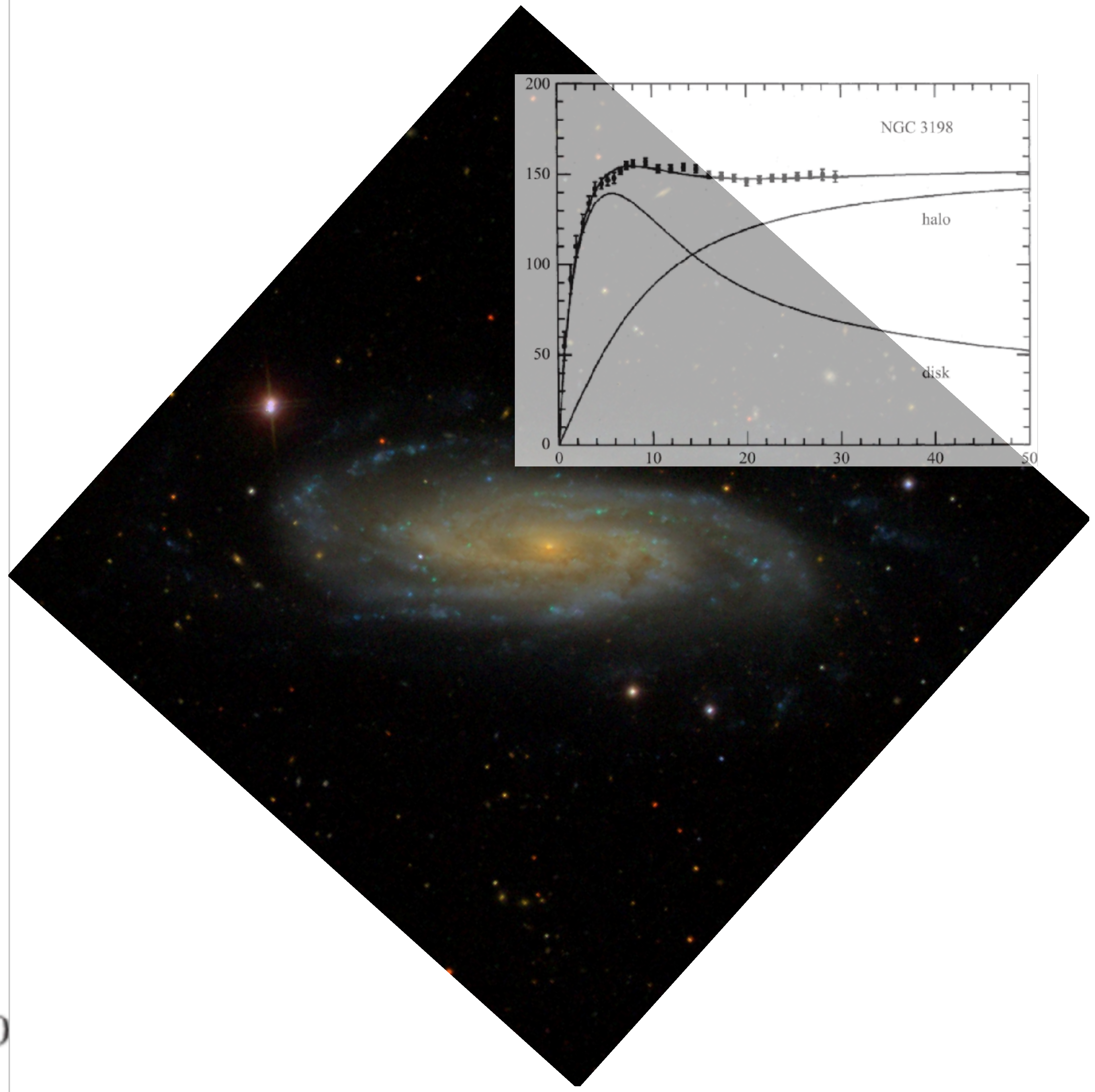
However, observed rotation curves are $V_c \sim$ constant!

This implies there must be **a lot** of missing mass at large R, that isn't normal stars or gas. Rotation curves imply $M_{\rm tot}/L_\star \sim 50-100$ for typical Milky Way-like galaxies.

Rotation curves

Reproduced from Huterer Fig. 11.1
Originally from Albada et al. 1985 (c) AAS





Other kinematic measurements

Rotation curves are quite hard to interpret in practice.

Also, they only directly constrain $M(<R_{\max})$, where $R_{\max}$ is the largest radius for which the rotation speed of the baryons can be measured.

An alternative is to give up on circular orbits at a fixed radius and instead measure the **distribution of speeds** for objects that are much more weakly bound to galaxies (like **satellite galaxies** and **globular clusters**). These have orbits that extend to larger radii, although there are fewer of them to measure.

If they really are in equilibrium, then the average speed of those objects contains information about the strength of the gravitational potential that accelerates them.

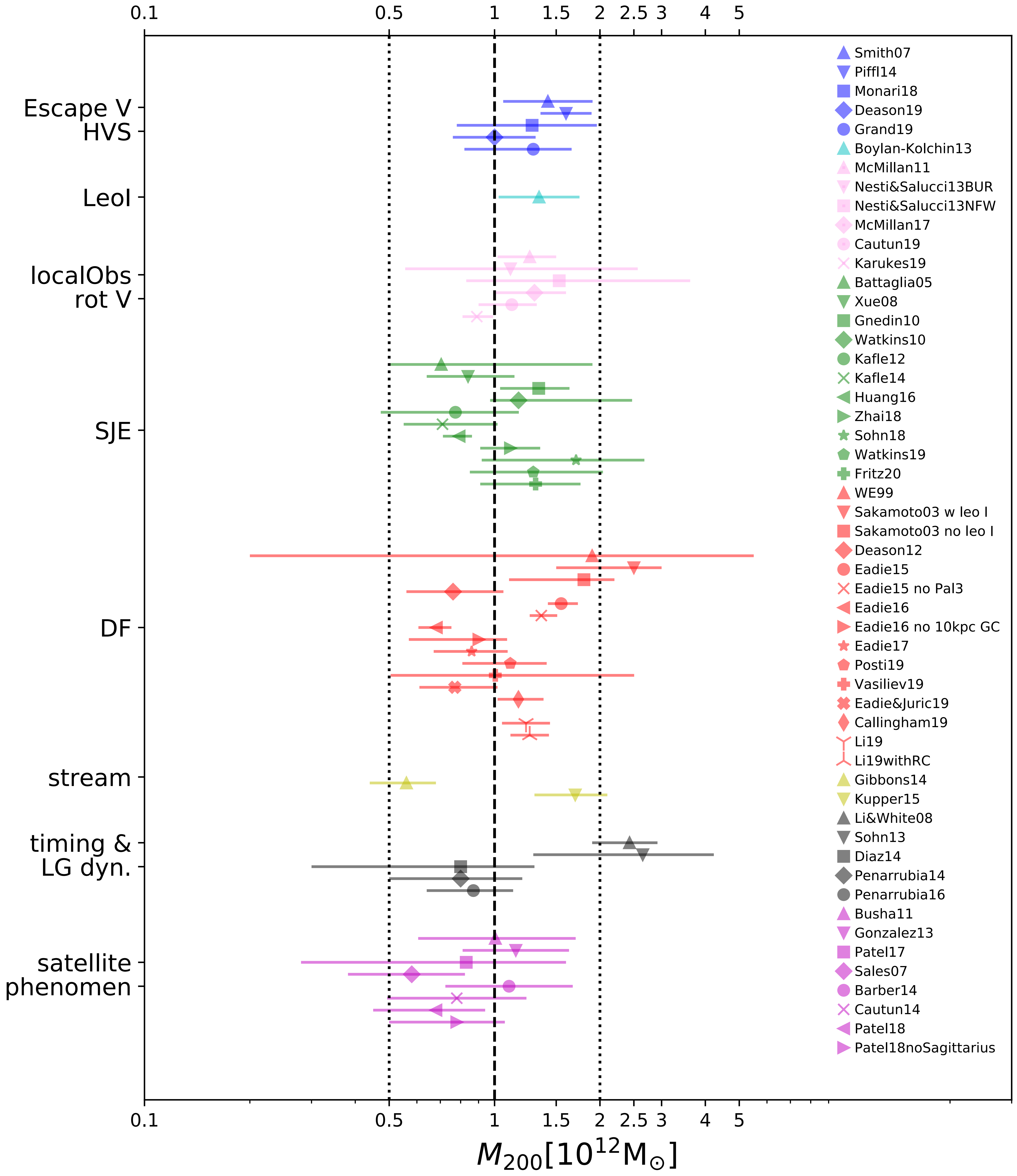
These measurements support the idea that galaxies are embedded in **halos** of some invisible mass.

Mass of the Milky Way

From Wang et al. (2020)
https://ui.adsabs.harvard.edu/abs/2020SCPMA..63j9801W

In the Milky Way, we can use many different kinds of dynamical methods to estimate the total mass of the dark matter halo.

These methods agree to roughly a factor of 2. It is unlikely that the methods used to measure halo masses for other galaxies are more accurate.



The virial theorem

The first evidence the galaxies are embedded in **dark matter halos** was that from Fritz Zwicky's study of the Coma galaxy cluster in the 1930s.

The fundamental idea is the **Virial Theorem**.

A gravitating system of masses in equilibrium obeys the relation $2T + W = 0$, where T is the (time-averaged) total kinetic energy of the masses and W is the total (time-averaged) potential energy.

This is a "very important result" in classical dynamics, with many applications. You can read about the Virial Theorem in Ryden (7.3).

The most important thing for us is the idea that the average kinetic and potential energies of masses in a **stable system** obey the virial theorem (otherwise the system would not be stable).

The virial theorem

Galaxy clusters are stable systems (or at least, the timescale of any change in the system is long compared to the time it takes a galaxy to go from one side of the cluster to the other).

For a system of equal masses moving in a gravitational potential, the virial theorem implies

$$\frac{1}{2}M\langle v^2\rangle = \frac{1}{2}\frac{GM^2}{\alpha r_h}$$

where $M = \sum_i m_i$ is the total mass, $\langle v^2\rangle$ is the mean-square speed of the masses, r_h is the radius that encloses half the mass, and α is a numerical factor ($\sim$ 1) that hides the specific details of how the mass is distributed and the choice of r_h as the characteristic radius.

$$\implies M = \frac{\langle v^2\rangle r_h}{\alpha G}$$

The virial theorem

$$M = \frac{\langle v^2 \rangle r_h}{\alpha G}$$

We can't measure $\langle v^2 \rangle$ directly because we only observe the velocities of galaxies along the line of sight. The **line-of-sight velocity dispersion** is:

$$\sigma_{\rm los} = \sqrt{\langle \left(v_{\rm los} - \langle v_{\rm los} \rangle \right)^2 \rangle}$$

where v_r are the line-of-sight velocities of the galaxies. This is related to the 3-d mean square speed as $\langle v^2 \rangle = 3\sigma_{\rm los}^2$.

For a typical very massive cluster like Coma, $\sigma_r \sim 1000\,{\rm km\,s^{-1}}$, $\alpha \approx 0.45$ and $r_h \sim 1.5\,{\rm Mpc}$, so $M \sim 2 \times 10^{15}\,{\rm M_\odot}$.

Strong lensing

Gravitational lensing effects in galaxy clusters also imply significant missing mass, consistent with the virial theorem and gas temperature.

Natarajan et al.
arXiv:2403.06245v1

Gallery of HST detected cluster arcs seen in the cluster Abell 370 (top-left); SDSS J1038+4849 (bottom-left); PSZ1 G311.65–18.48 (top-right) and Abell 611 (bottom-right).

Purple color shows the **weak lensing** mass reconstruction.

NASA, ESA, E. Jullo (Jet Propulsion Laboratory), P. Natarajan (Yale University), and J.-P. Kneib (Laboratoire d'Astrophysique de Marseille, CNRS, France);

Acknowledgment: H. Ford and N. Benetiz (Johns Hopkins University), and T. Broadhurst (Tel Aviv University)

Purple color shows the weak lensing mass reconstruction.

Red color shows X-ray emission.

The **Bullet Cluster**.

NASA / STScI

Non-baryonic dark matter

A simple idea (held for quite some time up to the 1980s) was that the dark matter might just be "dead" stars, black holes or some other kind of hard-to-see baryons.

However, this idea is not consistent with the CMB evidence we will look at later, which (very) strongly prefers the dark matter (a) to exist in the very early universe, before stars could form and (b) to be non-interacting and effectively **collisionless** — when you squeeze it, it should not develop a thermal pressure by interacting with itself or with baryons or photons.

The favourite working hypothesis is a "WIMP": weakly interacting massive particle.

Big lumps of mass (e.g. sub-stellar black holes or similar) called "Massive Compact Halo Objects" (MACHOS) were once considered a possibility, but have been largely ruled out by **microlensing** observations.

CMB evidence for Dark Matter

Although the evidence for Dark Matter in galaxies is strong, it does not completely rule out (for example) alternative theories of gravity on galactic scales.

By far the most significant and compelling evidence for dark matter comes from the **very tiny temperature fluctuations observed in the cosmic microwave background radiation**, and the closely related "initial conditions problem" for galaxy formation.

Finally, there is an important limit on the total density of baryonic matter set by primordial nucleosynthesis.

This early-universe evidence is much more closely linked to the ideas of General Relativity and the Friedmann equation. It also is very clear that the dark matter **cannot** be made of baryons (except under some very special circumstance to do with tiny black holes).

The truly remarkable thing is that the CMB evidence is completely consistent with the present-day "galaxy-scale" phenomena — they are tied together by a very simple theory of cosmic structure formation.

The nature of dark matter

DM is generally imagined to be a **massive (non-relativistic) particle** that interacts only **extremely weakly** or **not at all** with other particles through the electromagnetic and nuclear forces (i.e. it doesn't scatter light and it doesn't form atoms).

It only interacts through **gravity.** Its energy density curves spacetime in the same way as other matter; its equation of state parameter is $w \approx 0$).

The **neutrinos** are a form of dark matter, but they are **light** (**relativistic**).

Such particles cannot explain most of the observations associated with dark matter, because they move too quickly to form the seeds of the deep gravitational potentials we associated with the CMB temperature fluctuations and the dark matter halos around galaxies.

Cold, hot and warm dark matter

These terms refer to the **thermal velocity** of the imagined particle at the time it decouples from the radiation background. We can come back to this after discussing the CMB observations. For now we can think of it in simpler terms:

Cold $\implies$ massive particle (GeV), moves very slowly.

Hot $\implies$ light particle (eV), moves very fast.

Of course, there is a continuum in between. The colder the dark matter, the more **small scale structure** forms in the early universe and the easier it is to make galaxies, because fast-moving, non-interacting particles can "smooth out" any inhomogeneous lumps in the initial conditions before they can grow..

"Hot" dark matter has been ruled out for a long time, so current efforts focus on deciding if the dark matter is truly "cold" or a little bit "warm".

There are other, more exotic possibilities for dark matter, but whatever it is, it must "work" a lot like CDM.

Exotic Dark Matter

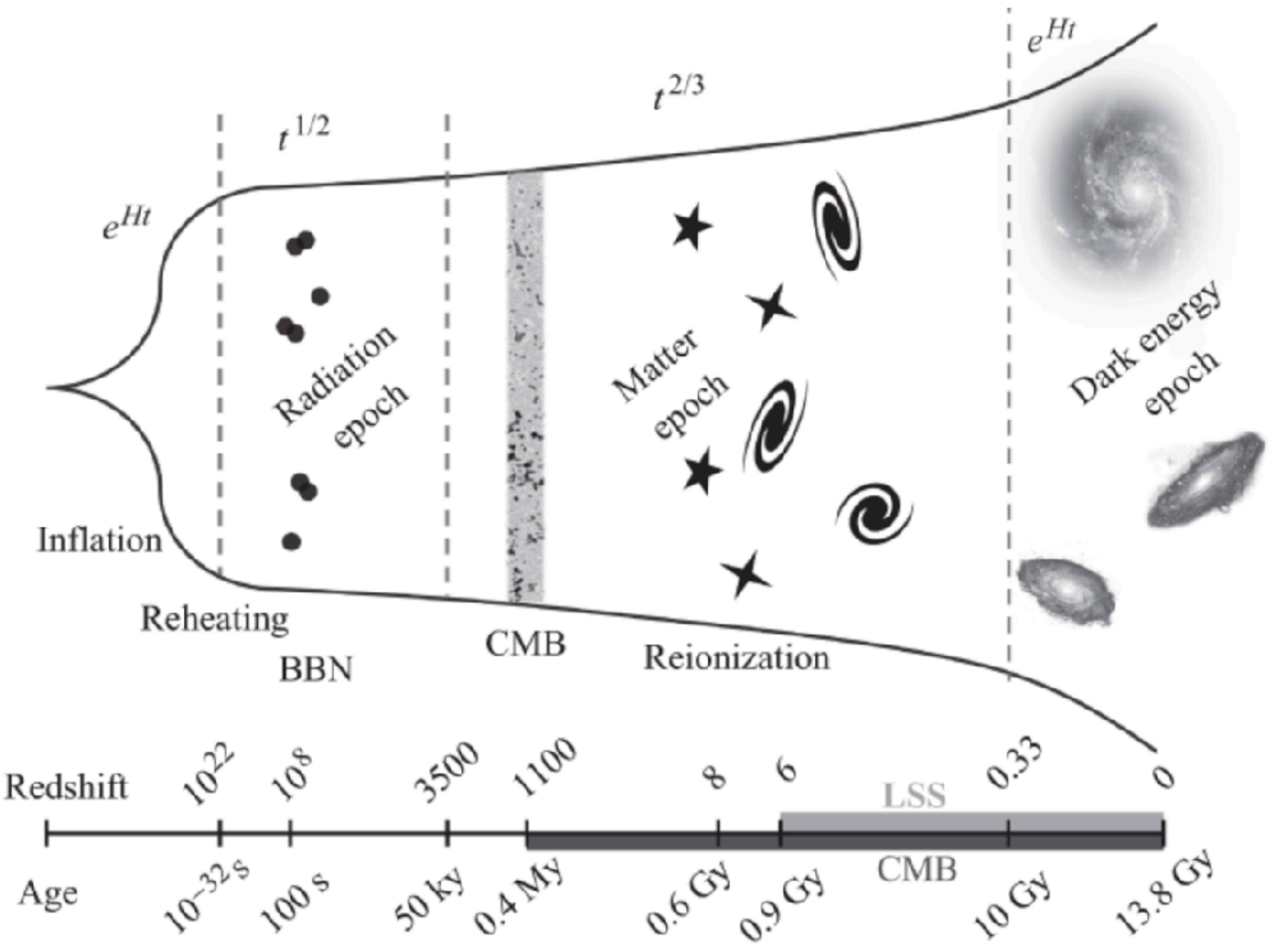
There are other, more exotic possibilities for dark matter, apart from “some kind of particle”. However, whatever it is, it must “work” a lot like CDM.

Some examples:

“Fuzzy” Dark Matter: for example, **axions**. These very light particles have a super-long de Broglie wavelength, so exhibit wave-like behaviour on cosmic scales.

Primordial black holes: There is still some room to generate a population of very small **baryonic** black holes, but this would require a different understanding of how the “seeds” of cosmic structure are generated by inflation.

The big picture



Reproduced from Huterer Fig 1.2

Summary: current parameter values

From Huterer

Parameter name	Symbol	Measured value	Fiducial value
Spatial curvature	Ω_k	0.001 ± 0.002	0
Matter density rel. to critical	Ω_{M}	0.310 ± 0.007	0.30
Baryon density	$\Omega_{\mathrm{B}} h^2$	0.0224 ± 0.0002	0.0224
Hubble constant	H_0	$(67.9 \pm 0.7)\,\mathrm{km/s/Mpc}$	$67\,\mathrm{km/s/Mpc}^{**}$
$P(k)$ amplitude at $k_{\mathrm{piv}} = 0.05$	A_s	$(2.10 \pm 0.03) \times 10^{-9}$	2.1×10^{-9}
Scalar spectral index	n_s	0.966 ± 0.005	0.966
Age of universe	t_0	$(13.76 \pm 0.08)\,\mathrm{Gyr}$	derived
Amplitude of mass fluctuations	σ_8	0.810 ± 0.007	derived
CMB temperature	T_0	$(2.7255 \pm 0.0006)\,\mathrm{K}$	$2.725\,\mathrm{K}$
Photon density	$\Omega_\gamma h^2$	derived from T_0	2.47×10^{-5}
Assumed-massless neutrino density	$\Omega_{\nu,\mathrm{rel}} h^2$	derived from T_0	1.68×10^{-5}
Equation of state of dark energy	w	-1.04 ± 0.06	-1

Summary

"Standard" ΛCDM **model:** Flat, $\Omega_{\Lambda,0} \simeq 0.7$, $\Omega_{m,0} \simeq 0.3$, $\Omega_{b,0} \simeq 0.0224\, h^{-2} \sim 0.05$ ($\implies \Omega_{DM} \sim 0.25$)

Accelerated expansion: evidence from supernovae (luminosity — redshift relation) plus H_0; uncertainty depends on calibration of distance ladder.

Cosmological Constant / Dark Energy: seems to work like vacuum energy, but predictions on that basis are much, much larger than the value needed to explain cosmological observations.

Dynamical estimates of mass: $V_c^2 = \dfrac{GM(< R)}{R}$ (rotation curves), $M \propto \dfrac{\langle v^2 \rangle r_h}{G}$ (virial theorem)

Many lines of evidence point to the fact that $\Omega_m \sim 0.3$ is much higher than Ω_b, the density of ordinary baryonic matter: **Rotation Curves, Virial Theorem, Cluster Gas, Lensing.** The most significant evidence comes from analysis of temperature fluctuations in the cosmic microwave background.

The working hypothesis for dark matter is a weakly (or non-) interacting massive particle.

Next time

The cosmic microwave background

The plasma era and recombination

Cosmological parameters from measurements of CMB anisotropies

For next time:

In Ryden: Ch. 8

In Huterer: 13.1-13.3; 13.14 (skim)