Operations Research, Spring 2022
Case Assignment 1

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1 Problem 1

Ranges:
$J = \{1, \ldots, n\}$ represent n jobs
$I = \{2, \ldots, 5\}$ represent machine 2 to 5

Coefficients:
P_j be the sum of processing time(hr) of job, $j \in J$
D_j be the due hours(hr) (due time - AM7:30) of job, $j \in J$

Variables:
W represent makespan
t_j be 1 if job j is tardy of 0 otherwise
f_j be the finish hours(hr) (finish time - AM7:30) of job, $j \in J$
x_{ij} be 1, if job $j \in J$ is assigned to machine $i \in I$, or 0 otherwise
z_{jk} be 1, if job $j =\in J$ is before job $k =\in J$ and they are in the same machine, or 0 otherwise

1.1 Step 1: Minimize the number of tardy jobs

$$\begin{aligned} \min \quad & \sum_{j \in J} t_j \\ \text{s.t.} \quad & f_j - D_j \leq M t_j \quad \forall j \in J \\ & \sum_{i \in I} x_{ij} = 1 \quad \forall j \in J \\ & z_{jk} + z_{kj} + 1 \geq x_{ij} + x_{ik} \quad \forall i \in I, j \in J, k \in J, j \neq k \\ & f_j + P_k - f_k \leq M(1 - z_{jk}) \quad \forall j \in J, k \in J, j \neq k \\ & f_j \geq P_j \quad \forall j \in J \\ & t_j \in \{0,1\} \quad \forall j \in J \\ & x_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \\ & z_{jk} \in \{0,1\} \quad \forall j \in J, k \in J, j \neq k \\ & M = \sum_{j \in J} P_j \end{aligned} \tag{1}$$

1.2 Step 2: Minimize the makespan

Let T be the minimum of $\sum_{j \in J} t_i$ which we get from solution of step 1.

$$
\begin{aligned}
\min \quad & W \\
\text{s.t.} \quad & W \geq f_j \quad \forall j \in J \\
& \sum_{j \in J} t_j \leq T \\
& f_j - D_j \leq M t_j \quad \forall j \in J \\
& \sum_{i \in I} x_{ij} = 1 \quad \forall j \in J \\
& z_{jk} + z_{kj} + 1 \geq x_{ij} + x_{ik} \quad \forall i \in I, j \in J, k \in J, j \neq k \\
& f_j + P_k - f_k \leq M(1 - z_{jk}) \quad \forall j \in J, k \in J, j \neq k \\
& f_j \geq P_j \quad \forall j \in J \\
& t_j \in \{0,1\} \quad \forall j \in J \\
& x_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \\
& z_{jk} \in \{0,1\} \quad \forall j \in J, k \in J, j \neq k \\
& M = \sum_{j \in J} P_j
\end{aligned}
\tag{2}
$$

1.3 Instance results

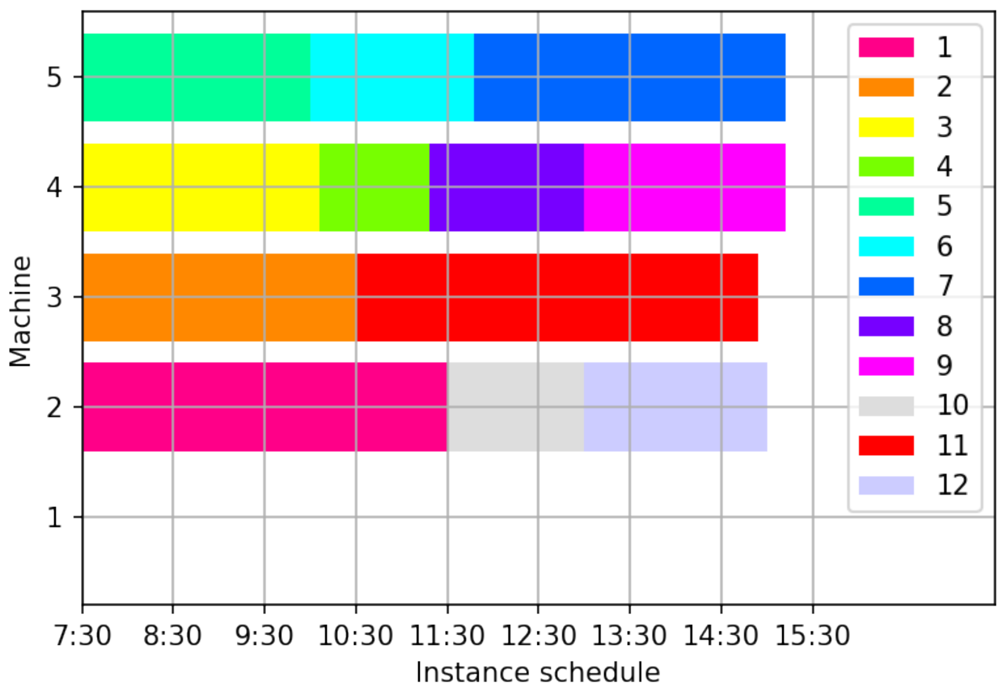

Figure 1: A schedule for the first instance in the first problem.

Job	Machine	Start	Finish	Processing Time	Tardiness
1	2	0	4	4	0
2	3	0	3	3	0
3	4	0	2.6	2.6	0
4	4	2.6	3.8	1.2	0
5	5	0	2.5	2.5	0
6	5	2.5	4.3	1.8	0
7	5	4.3	7.7	3.4	2.7
8	4	3.8	5.5	1.7	0
9	4	5.5	7.7	2.2	0
10	2	4	5.5	1.5	0
11	3	3	7.4	4.4	0
12	2	5.5	7.5	2	0
Makespan	7.7				

Table 1: Table for the first instance in the first problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

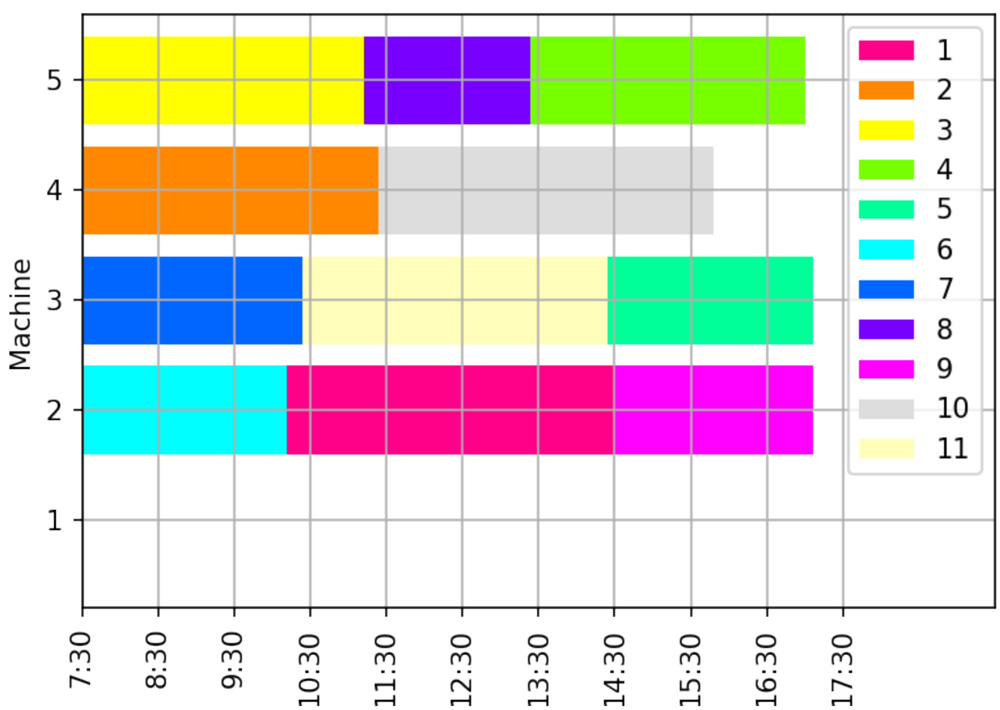

Figure 2: A schedule for the second instance in the first problem.

Job	Machine	Start	Finish	Processing Time	Tardiness
1	2	2.7	7	4.3	0
2	4	0	3.9	3.9	0
3	5	0	3.7	3.7	0
4	5	5.9	9.5	3.6	4.5
5	3	6.9	9.6	2.7	4.6
6	2	0	2.7	2.7	0
7	3	0	2.9	2.9	0
8	5	3.7	5.9	2.2	0
9	2	7	9.6	2.6	0
10	4	3.9	8.3	4.4	0
11	3	2.9	6.9	4	0
Makespan	9.6				

Table 2: Table for the second instance in the first problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

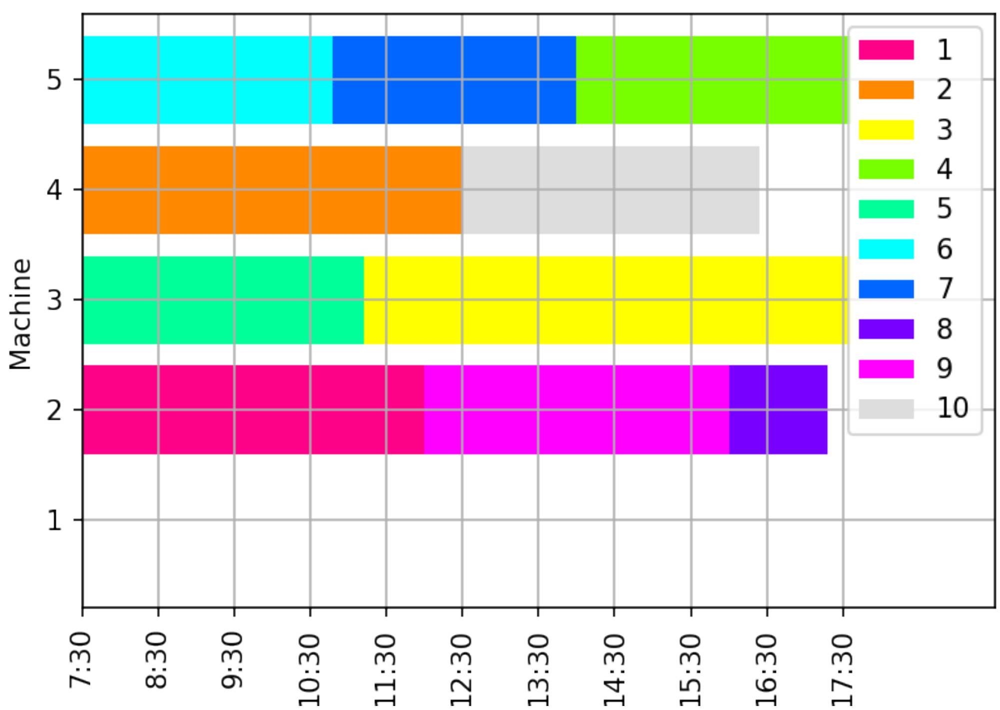

Figure 3: A schedule for the third instance in the first problem.

Job	Machine	Start	Finish	Processing Time	Tardiness
1	2	0	4.5	4.5	0
2	4	0	5	5	0
3	3	3.7	10.1	6.4	5.1
4	5	6.5	10.1	3.6	5.1
5	3	0	3.7	3.7	0
6	5	0	3.3	3.3	0
7	5	3.3	6.5	3.2	0
8	2	8.5	9.8	1.3	0
9	2	4.5	8.5	4	0
10	4	5	8.9	3.9	0
Makespan	10.1				

Table 3: Table for the third instance in the first problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

1.4 Conclusion

In problem 1, the jobs can't be split and the machines can complete every processes, so we have little constraints. After the scheduling process, we get the results in the Figures and Tables above.

For instance 1(Figure 1, Table 1), we make machine 2 doing job 1, job 10 and job 12 respectively; machine 3 doing job 2, job 11 respectively; machine 4 doing job 3, job 4, job 8 and job 9 respectively; machine 5 doing job 5, job 6 and job 7 respectively. By the schedule, only job 7 is tardy and we can end all jobs at 15:12.

For instance 2(Figure 2, Table 2), we make machine 2 doing job 6, job 1 and job 9 respectively; machine 3 doing job 7, job 11 and job 5 respectively; machine 4 doing job 2 and job 10 respectively; machine 5 doing job 3, job 8 and job 4 respectively. By the schedule, only job 4 and job 5 are tardy and we can end all jobs at 17:06.

For instance 3(Figure 3, Table 3), we make machine 2 doing job 1, job 9 and job 8 respectively; machine 3 doing job 5 and job 3 respectively; machine 4 doing job 2 and job 10 respectively; machine 5 doing job 6, job 7 and job 4 respectively. By the schedule, only job 3 and job 4 are tardy and we can end all jobs at 17:36.

2 Problem 2

Ranges:
$J = \{1, \ldots, n\}$ represent n jobs
$I = \{2, \ldots, 5\}$ represent machine 2 to 5
$L = \{1, 2\}$ represent the first and second piece of a job

Coefficients:
P_{jl} be the sum of processing time(hr) of job $j \in J$ piece $l \in L$
D_j be the due hours(hr) (due time - AM7:30) of job, $j \in J$

Variables:
W represent makespan
t_j be 1 if job j is tardy of 0 otherwise
f_{jl} be the finish hours(hr) (finish time - AM7:30) of job $j \in J$ piece $l \in L$
x_{ijl} be 1, if job $j \in J$ piece $l \in L$ is assigned to machine $i \in I$, or 0 otherwise
z_{jlkm} be 1, if job $j \in J$ piece $l \in L$ is before job $k \in J$ piece $m \in L$ and they are in the same machine, or 0 otherwise

2.1 Step 1: Minimize the number of tardy jobs

$$
\begin{aligned}
\min \quad & \sum_{j \in J} t_j \\
\text{s.t.} \quad & f_{j2} - D_j \leq M t_j \quad \forall j \in J \\
& \sum_{i \in I} x_{ijl} = 1 \quad \forall j \in J, \forall l \in L \\
& z_{jlkm} + z_{kmjl} + 1 \geq x_{ijl} + x_{ikm} \quad \forall i \in I, j \in J, k \in J, l \in L, m \in L \\
& f_{jl} + P_{km} - f_{km} \leq M(1 - z_{jlkm}) \quad \forall j \in J, k \in J, l \in L, m \in L \\
& f_{j2} \geq P_{j1} + P_{j2} \quad \forall j \in J \\
& f_{j1} + P_{j2} \leq f_{j2} \quad \forall j \in J \\
& t_j \in \{0, 1\} \quad \forall j \in J \\
& x_{ijl} \in \{0, 1\} \quad \forall i \in I, j \in J, l \in L \\
& z_{jlkm} \in \{0, 1\} \quad \forall j \in J, k \in J, l \in L, m \in L \\
& M = \sum_{j \in J} P_{j1} + \sum_{j \in J} P_{j2}
\end{aligned}
\tag{3}
$$

2.2 Step 2: Minimize the makespan

Let T be the minimum of $\sum_{j \in J} t_i$ which we get from solution of step 1.

$$\begin{aligned}
\min \quad & W \\
\text{s.t.} \quad & W \geq f_{j2} \quad \forall j \in J \\
& \sum_{j \in J} t_j \leq T \\
& f_{j2} - D_j \leq M t_j \quad \forall j \in J \\
& \sum_{i \in I} x_{ijl} = 1 \quad \forall j \in J, \forall l \in L \\
& z_{jlkm} + z_{kmjl} + 1 \geq x_{ijl} + x_{ikm} \quad \forall i \in I, j \in J, k \in J, l \in L, m \in L \\
& f_{jl} + P_{km} - f_{km} \leq M(1 - z_{jlkm}) \quad \forall j \in J, k \in J, l \in L, m \in L \\
& f_{j2} \geq P_{j1} + P_{j2} \quad \forall j \in J \\
& f_{j1} + P_{j2} \leq f_{j2} \quad \forall j \in J \\
& t_j \in \{0,1\} \quad \forall j \in J \\
& x_{ijl} \in \{0,1\} \quad \forall i \in I, j \in J, l \in L \\
& z_{jlkm} \in \{0,1\} \quad \forall j \in J, k \in J, l \in L, m \in L \\
& M = \sum_{j \in J} P_{j1} + \sum_{j \in J} P_{j2}
\end{aligned} \tag{4}$$

2.3 Instance results

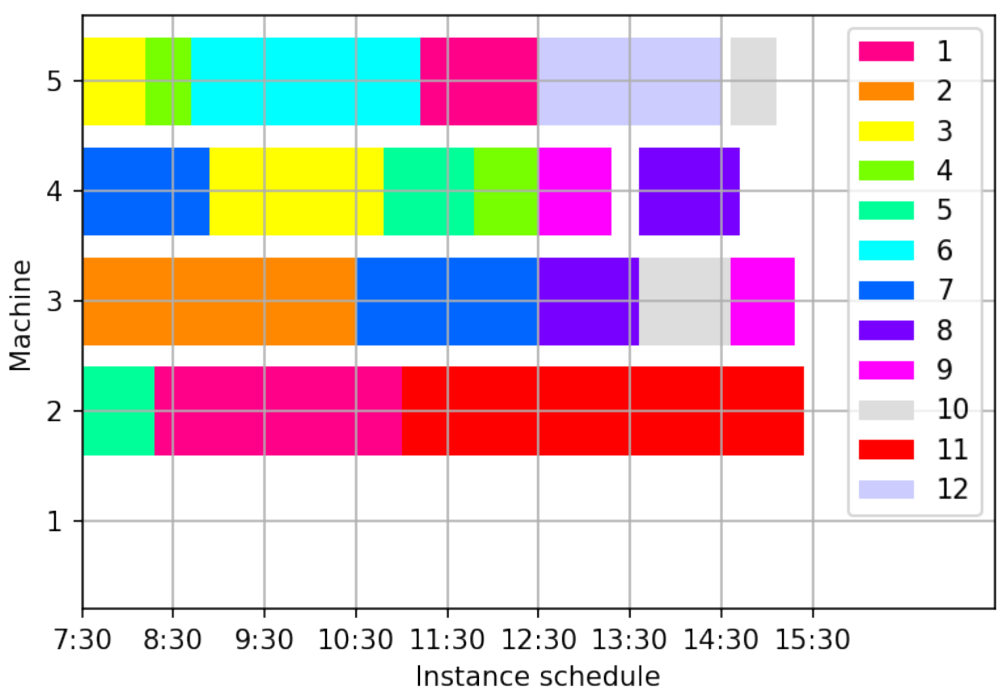

Figure 4: A schedule for the first instance in the second problem.

Job	Piece	Machine	Start	Finish	Processing Time	Tardiness
1	1	2	0.8	3.5	2.7	0
1	2	5	3.7	5	1.3	0
2	1	3	0	1.6	1.6	0
2	2	3	1.6	3	1.4	0
3	1	5	0	0.7	0.7	0
3	2	4	1.4	3.3	1.9	0
4	1	5	0.7	1.2	0.5	0
4	2	4	4.3	5	0.7	0
5	1	2	0	0.8	0.8	0
5	2	4	3.3	4.3	1	0
6	1	2	0	0	0	0
6	2	5	1.2	3.7	2.5	0
7	1	4	0	1.4	1.4	0
7	2	3	3	5	2	0
8	1	3	5	6.1	1.1	0
8	2	4	6.1	7.2	1.1	0
9	1	4	5	5.8	0.8	0
9	2	3	7.1	7.8	0.7	0
10	1	3	6.1	7.1	1	0
10	2	5	7.1	7.6	0.5	0
11	1	2	3.5	6.5	3	0
11	2	2	6.5	7.9	1.4	0
12	1	5	0	0	0	0
12	2	5	5	7	2	0
Makespan	7.9					

Table 4: Table for the first instance in the second problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

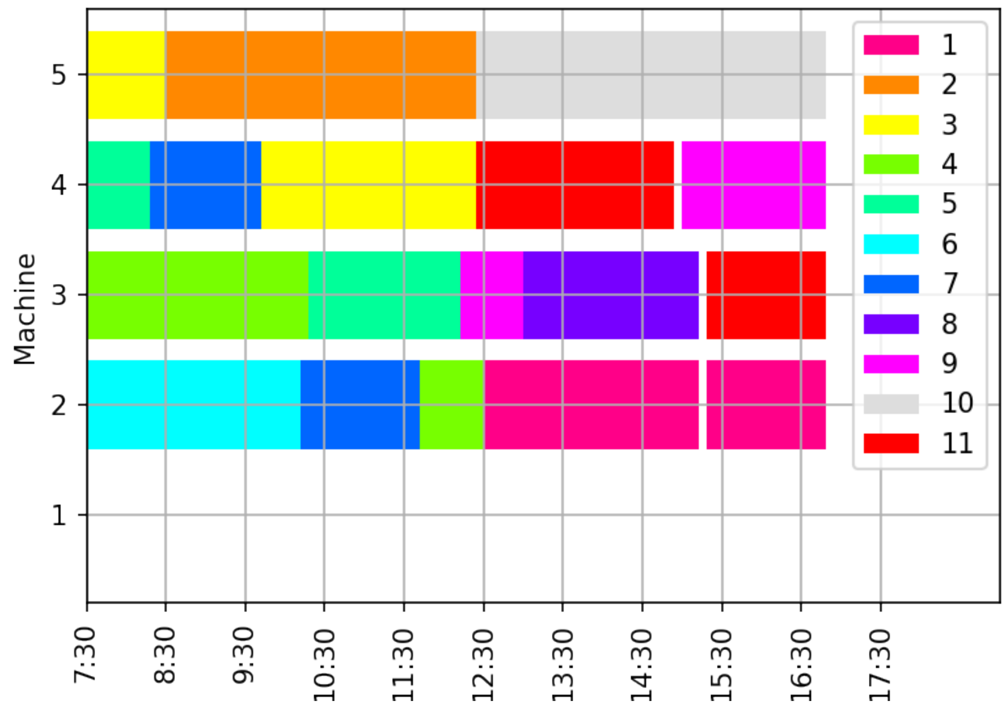

Figure 5: A schedule for the second instance in the second problem.

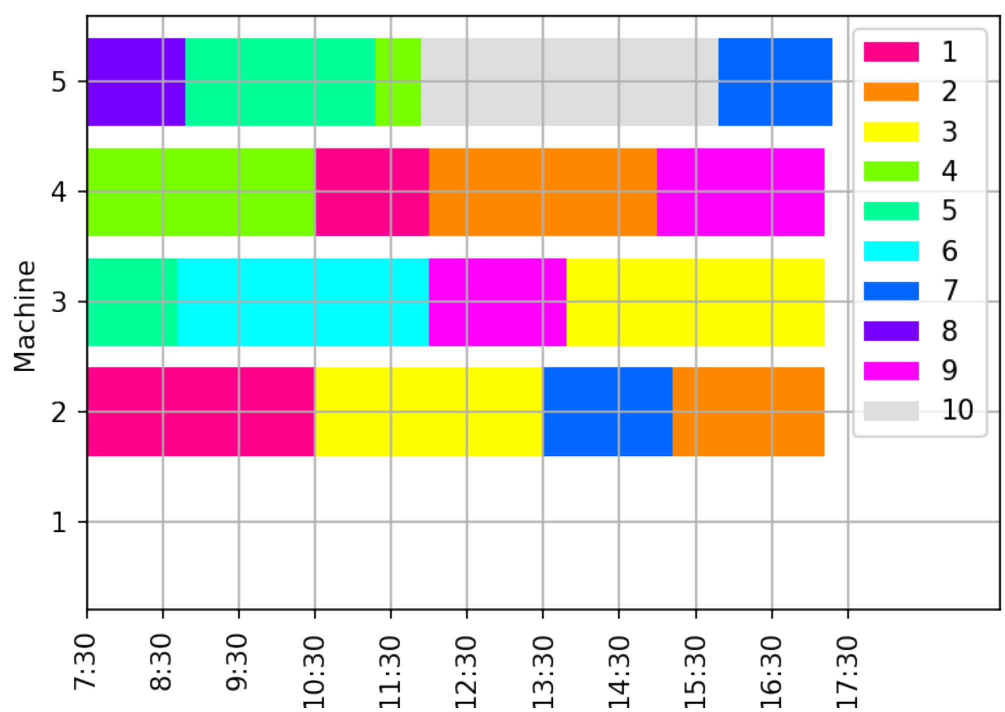

Figure 6: A schedule for the third instance in the second problem.

Job	Piece	Machine	Start	Finish	Processing Time	Tardiness
1	1	2	5.0	7.7	2.7	0
1	2	2	7.8	9.3	1.5	0
2	1	5	1	2.6	1.6	0
2	2	5	2.6	4.9	2.3	0
3	1	5	0	1	1	0
3	2	4	2.2	4.9	2.7	0
4	1	3	0	2.8	2.8	0
4	2	2	4.2	5	0.8	0
5	1	4	0	0.8	0.8	0
5	2	3	2.8	4.7	1.9	0
6	1	4	0	0	0	0
6	2	2	0	2.7	2.7	0
7	1	4	0.8	2.2	1.4	0
7	2	2	2.7	4.2	1.5	0
8	1	5	0	0	0	0
8	2	3	5.5	7.7	2.2	0
9	1	3	4.7	5.5	0.8	0
9	2	4	7.5	9.3	1.8	0
10	1	5	4.9	7.1	2.2	0
10	2	5	7.1	9.3	2.2	0
11	1	4	4.9	7.4	2.5	0
11	2	3	7.8	9.3	1.5	0
Makespan	9.3					

Table 5: Table for the second instance in the second problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

Job	Piece	Machine	Start	Finish	Processing Time	Tardiness
1	1	2	0	3	3	0
1	2	4	3	4.5	1.5	0
2	1	4	4.5	7.5	3.	2.5
2	2	2	7.7	9.7	2	4.7
3	1	2	3	6	3	1
3	2	3	6.3	9.7	3.4	4.7
4	1	4	0	3	3	0
4	2	5	3.8	4.4	0.6	0
5	1	3	0	1.2	1.2	0
5	2	5	1.3	3.8	2.5	0
6	1	4	0	0	0	0
6	2	3	1.2	4.5	3.3	0
7	1	2	6	7.7	1.7	0
7	2	5	8.3	9.8	1.5	0
8	1	3	0	0	0	0
8	2	5	0	1.3	1.3	0
9	1	3	4.5	6.3	1.8	0
9	2	4	7.5	9.7	2.2	0
10	1	5	4.4	7	2.6	0
10	2	5	7	8.3	1.3	0
Makespan	9.8					

Table 6: Table for the third instance in the second problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

2.4 Conclusion

In problem 2, the jobs can be split into two parts and the machines can complete every processes, so we split the jobs by the split time. We name the first half of job j as job j,1, and the last half of job j as job j,2, note that job j means the entire job which is not been splited. Then we schedule all the splited jobs by the method in problem 1. Note that job j,2 can only be done when job j,1 is complete, so there may be rest between jobs for every machines. After the scheduling process, we get the results in the Figures and Tables above.

For instance 1(Figure 4, Table 4), we make machine 2 doing job 5,1, job 1,1 and job 11 respectively; machine 3 doing job 2, job 7,2, job 8,1, job 10,1 and job 9,2 respectively; machine 4 doing job 7,1, job 3,2, job 5,2, job 4,2, job 9,1, job 8,2 respectively; machine 5 doing job 3,1, job 4,1, job 6, job 1,2, job 12 and job 10,2 respectively. By the schedule, no job is tardy and we can end all jobs at 15:24.

For instance 2(Figure 5, Table 5), we make machine 2 doing job 6, job 7,2, job 4,2, job 1,1 and job 1,2 respectively; machine 3 doing job 4,1, job 5,2, job 9,1, job 8 and job 11,2 respectively; machine 4 doing job 5,1, job 7,1, job 3,2, job 11,1 and job 9,2 respectively; machine 5 doing job 3,1, job 2 and job 10 respectively. By the schedule, no job is tardy and we can end all jobs at 16:48.

For instance 3(Figure 6, Table 6), we make machine 2 doing job 1,1, job 3,1, job 7,1 and job 2,2 respectively; machine 3 doing job 5,1, job 6, job 9,1, and job 3,2 respectively; machine 4 doing job 4,1, job 1,2, job 2,1 and job 9,2 respectively; machine 5 doing job 8, job 5,2, job 4,2, job 10 and job 7,2 respectively. By the schedule, only job 2 and job 3 are tardy and we can end all jobs at 17:18.

For most cases, we see that the makespan in problem 2 is smaller then problem 1. That is because to minimize the makespan, we need to assign the jobs averagely. When the jobs are splited into small pieces, it is more easily to do that, specifically when the processing time of the jobs have great variability.

3 Problem 3

Ranges:

$J = \{1, \ldots, n\}$ represent n jobs
$I = \{1, \ldots, 5\}$ represent machine 1 to 5
$L = \{1, 2\}$ represent the first and second piece of a job

Coefficients:

P_{jl} be the sum of processing time(hr) of job $j \in J$ piece $l \in L$
D_j be the due hours(hr) (due time - AM7:30) of job, $j \in J$
B_{jl} be 1 if job $j \in J$ piece $l \in L$ only process boiling or 0 otherwise.

Variables:

W represent makespan
t_j be 1 if job j is tardy of 0 otherwise
f_{jl} be the finish hours(hr) (finish time - AM7:30) of job $j \in J$ piece $l \in L$
x_{ijl} be 1, if job $j \in J$ piece $l \in L$ is assigned to machine $i \in I$, or 0 otherwise
z_{jlkm} be 1, if job $j \in J$ piece $l \in L$ is before job $k \in J$ piece $m \in L$ and they are in the same machine, or 0 otherwise

3.1 Step 1: Minimize the number of tardy jobs

$$
\begin{aligned}
\min \quad & \sum_{j \in J} t_j \\
\text{s.t.} \quad & f_{j2} - D_j \leq M t_j \quad \forall j \in J \\
& \sum_{i \in I} x_{ijl} = 1 \quad \forall j \in J, \forall l \in L \\
& z_{jlkm} + z_{kmjl} + 1 \geq x_{ijl} + x_{ikm} \quad \forall i \in I, j \in J, k \in J, l \in L, m \in L \\
& f_{jl} + P_{km} - f_{km} \leq M(1 - z_{jlkm}) \quad \forall j \in J, k \in J, l \in L, m \in L \\
& f_{j2} \geq P_{j1} + P_{j2} \quad \forall j \in J \\
& f_{j1} + P_{j2} \leq f_{j2} \quad \forall j \in J \\
& x_{1jl} \leq B_{jl} \quad \forall j \in J, l \in L \\
& t_j \in \{0, 1\} \quad \forall j \in J \\
& x_{ijl} \in \{0, 1\} \quad \forall i \in I, j \in J, l \in L \\
& z_{jlkm} \in \{0, 1\} \quad \forall j \in J, k \in J, l \in L, m \in L \\
& M = \sum_{j \in J} P_{j1} + \sum_{j \in J} P_{j2}
\end{aligned}
\tag{5}
$$

3.2 Step 2: Minimize the makespan

Let T be the minimum of $\sum_{j \in J} t_i$ which we get from solution of step 1.

$$
\begin{aligned}
\min \quad & W \\
\text{s.t.} \quad & W \geq f_{j2} \quad \forall j \in J \\
& \sum_{j \in J} t_j \leq T \\
& f_{j2} - D_j \leq M t_j \quad \forall j \in J \\
& \sum_{i \in I} x_{ijl} = 1 \quad \forall j \in J, \forall l \in L \\
& z_{jlkm} + z_{kmjl} + 1 \geq x_{ijl} + x_{ikm} \quad \forall i \in I, j \in J, k \in J, l \in L, m \in L \\
& f_{jl} + P_{km} - f_{km} \leq M(1 - z_{jlkm}) \quad \forall j \in J, k \in J, l \in L, m \in L \\
& f_{j2} \geq P_{j1} + P_{j2} \quad \forall j \in J \\
& f_{j1} + P_{j2} \leq f_{j2} \quad \forall j \in J \\
& x_{1jl} \leq B_{jl} \quad \forall j \in J, l \in L \\
& t_j \in \{0,1\} \quad \forall j \in J \\
& x_{ijl} \in \{0,1\} \quad \forall i \in I, j \in J, l \in L \\
& z_{jlkm} \in \{0,1\} \quad \forall j \in J, k \in J, l \in L, m \in L \\
& M = \sum_{j \in J} P_{j1} + \sum_{j \in J} P_{j2}
\end{aligned}
\tag{6}
$$

3.3 Instance results

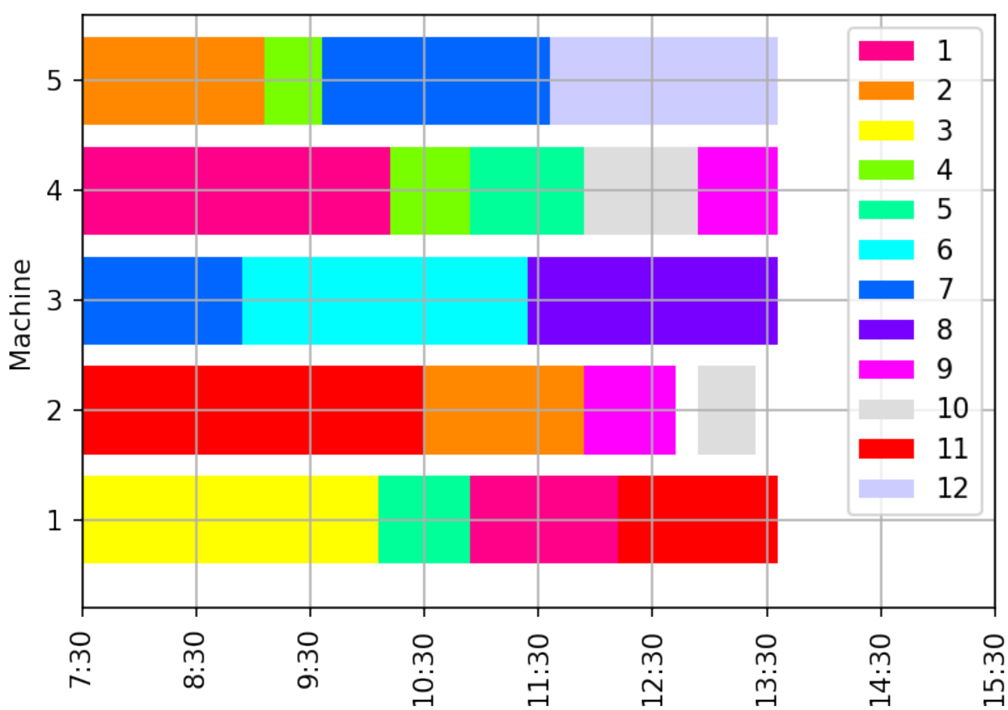

Figure 7: A schedule for the first instance in the third problem.

Job	Piece	Machine	Start	Finish	Processing Time	Tardiness
1	1	4	0	2.7	2.7	0
1	2	1	3.4	4.7	1.3	0
2	1	5	0	1.6	1.6	0
2	2	2	3	4.4	1.4	0
3	1	1	0	0.7	0.7	0
3	2	1	0.7	2.6	1.9	0
4	1	5	1.6	2.1	0.5	0
4	2	4	2.7	3.4	0.7	0
5	1	1	2.6	3.4	0.8	0
5	2	4	3.4	4.4	1	0
6	1	5	0	0	0	0
6	2	3	1.4	3.9	2.5	0
7	1	3	0	1.4	1.4	0
7	2	5	2.1	4.1	2	0
8	1	3	3.9	5	1.1	0
8	2	3	5	6.1	1.1	0
9	1	2	4.4	5.2	0.8	0
9	2	4	5.4	6.1	0.7	0
10	1	4	4.4	5.4	1	0
10	2	2	5.4	5.9	0.5	0
11	1	2	0	3	3	0
11	2	1	4.7	6.1	1.4	0
12	1	1	0	0	0	0
12	2	5	4.1	6.1	2	0
Makespan	6.1					

Table 7: Table for the first instance in the third problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

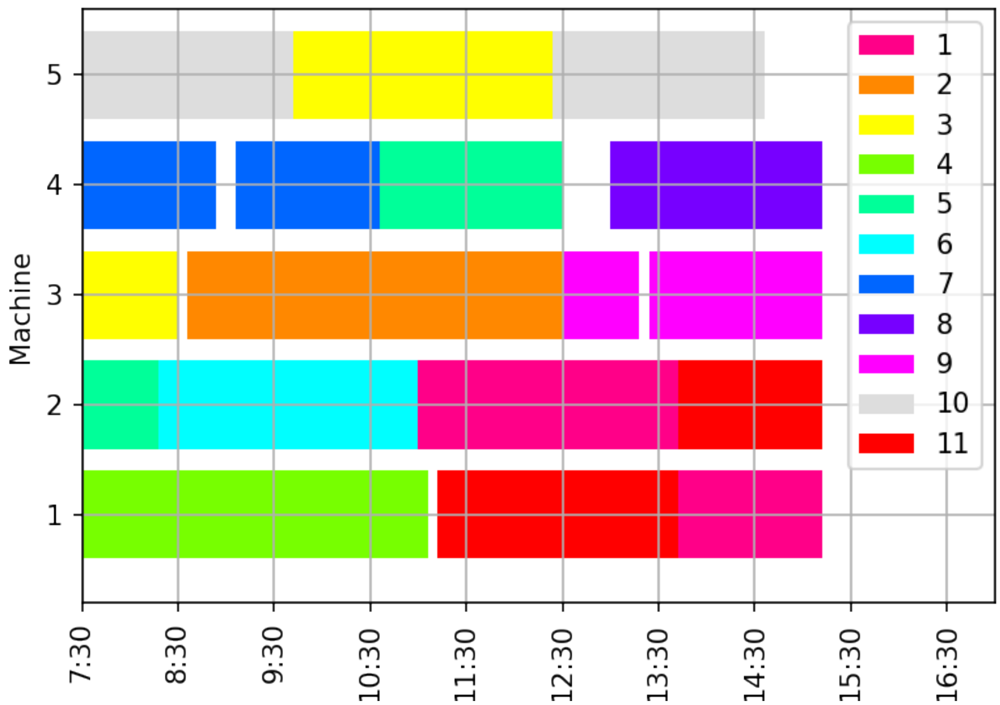

Figure 8: A schedule for the second instance in the third problem.

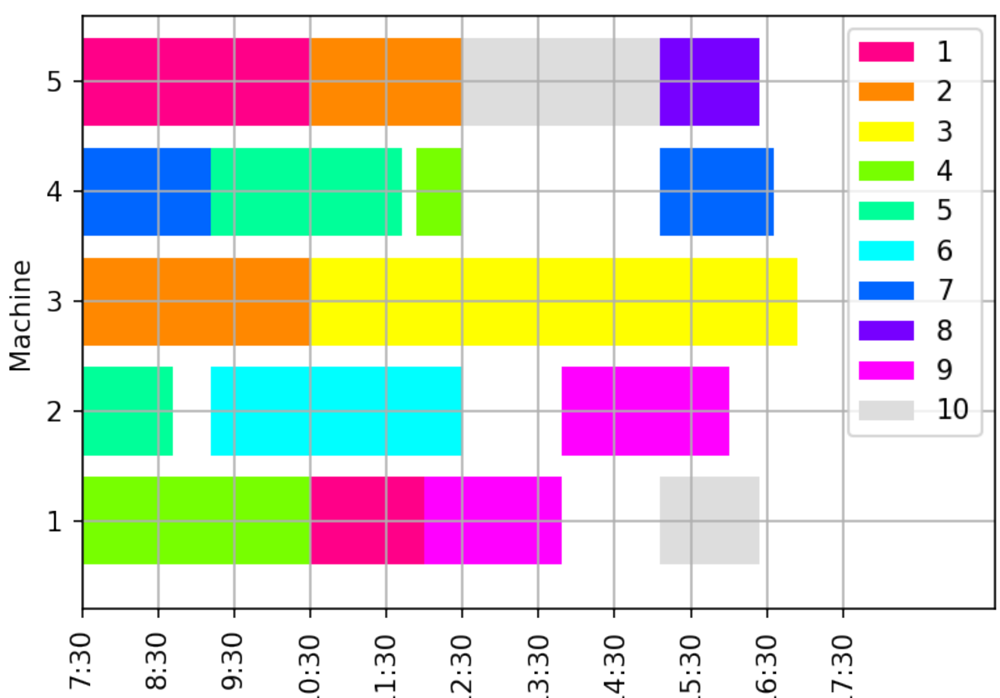

Figure 9: A schedule for the third instance in the third problem.

Job	Piece	Machine	Start	Finish	Processing Time	Tardiness
1	1	2	3.5	6.2	2.7	0
1	2	1	6.2	7.7	1.5	0
2	1	3	1.1	2.7	1.6	0
2	2	3	2.7	5	2.3	0
3	1	3	0	1	1	0
3	2	5	2.2	4.9	2.7	0
4	1	1	0	2.8	2.8	0
4	2	1	2.8	3.6	0.8	0
5	1	2	0	0.8	0.8	0
5	2	4	3.1	5	1.9	0
6	1	5	0	0	0	0
6	2	2	0.8	3.5	2.7	0
7	1	4	0	1.4	1.4	0
7	2	4	1.6	3.1	1.5	0
8	1	5	0	0	0	0
8	2	4	5.5	7.7	2.2	0
9	1	3	5	5.8	0.8	0
9	2	3	5.9	7.7	1.8	0
10	1	5	0	2.2	2.2	0
10	2	5	4.9	7.1	2.2	0
11	1	1	3.7	6.2	2.5	0
11	2	2	6.2	7.7	1.5	0
Makespan	7.7					

Table 8: Table for the second instance in the third problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

Job	Piece	Machine	Start	Finish	Processing Time	Tardiness
1	1	5	0	3	3	0
1	2	1	3	4.5	1.5	0
2	1	3	0	3.	3	0
2	2	5	3	5	2	0
3	1	3	3	6	3	1
3	2	3	6	9.4	3.4	4.4
4	1	1	0	3	3	0
4	2	4	4.4	5	0.6	0
5	1	2	0	1.2	1.2	0
5	2	4	1.7	4.2	2.5	0
6	1	1	0	0	0	0
6	2	2	1.7	5	3.3	0
7	1	4	0	1.7	1.7	0
7	2	4	7.6	9.1	1.5	0
8	1	3	0	0	0	0
8	2	5	7.6	8.9	1.3	0
9	1	1	4.5	6.3	1.8	0
9	2	2	6.3	8.5	2.2	0
10	1	5	5	7.6	2.6	0
10	2	1	7.6	8.9	1.3	0
Makespan	9.4					

Table 9: Table for the third instance in the third problem. (Start and finish time are in hours. (i.e.: actual start time = 7:30 + start))

3.4 Conclusion

In problem 3, the jobs can be split into two parts and the machines 1 can only do boiling, so we split the jobs by the same way in problem 2. Then we schedule all the splited jobs by the method in problem 1. Note that job j,2 can only be done when job j,1 is complete, so there may be rest between jobs for every machines. Since the ability of machine 1 is limited, the jobs which is assigned to machine 1 can only contain boiling. After the scheduling process, we get the results in the Figures and Tables above.

For instance 1(Figure 7, Table 7), we make machine 1 doing job 3, job 5,1, job 1,2 and job 11,2 respectively; machine 2 doing job 11,1, job 2,2, job 9,1 and job 10,2 respectively; machine 3 doing job 7,1, job 6 and job 8 respectively; machine 4 doing job 1,1, job 4,2, job 5,2, job 10,1 and job 9,2 respectively; machine 5 doing job 2,1, job 4,1, job 7,2 and job 12 respectively. By the schedule, no job is tardy and we can end all jobs at 13:36.

For instance 2(Figure 8, Table 8), we make machine 1 doing job 4, job 11,1 and job 1,2 respectively; machine 2 doing job 5,1, job 6, job 1,1 and job 11,2 respectively; machine 3 doing job 3,1, job 2, job 9,1 and job 9,2 respectively; machine 4 doing job 7,1, job 7,2, job 5,2 and job 8 respectively; machine 5 doing job 10,1, job 3,2 and job 10,2 respectively. By the schedule, no job is tardy and we can end all jobs at 15:12.

For instance 3(Figure 9, Table 9), we make machine 1 doing job 4,1, job 1,1, job 9,1 and job 10,2 respectively; machine 2 doing job 5,1, job 6 and job 9,2 respectively; machine 3 doing job 2,1 and job 3 respectively; machine 4 doing job 7,1, job 5,2, job 4,2 and job 7,2 respectively; machine 5 doing job 1,1, job 2,1, job 10,1 and job 8 respectively. By the schedule, only job 3 is tardy and we can end all jobs at 16:54.