OR Homework 1

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1 Problem 1

1.1 (a)

Set:

$I \in \{1, ..., 10\}$, $J \in \{S, L, H, M, O\}$.

Variables:

$x_i = 1$, if i is chosen, else $x_i = 0$. $\forall i \in I$.

$y_{i,j} = 1$, if i is chosen as j, else $y_{i,j} = 0$. $\forall i \in I$, $\forall j \in J$.

$w_{i,j}$ is a variable to linearize $x_i, y_{i,j}$. $w_j = \{0, 1\}.\forall i \in I$, $\forall j \in J$.

Constants:

$z_{i,j} = 1$, if i can be selected as j, else $z_{i,j} = 0$. $\forall i \in I$, $\forall j \in J$.

s_i, b_i, r_i, p_i are the rate of spiking, blocking, receiving, and passing abilities of player i. $\forall i \in I$.

$$
\begin{aligned}
\max \quad & \sum_{i \in I} w_{i,H} s_i + \sum_{i \in I} w_{i,M} s_i + \sum_{i \in I} w_{i,O} s_i + 3 \sum_{i \in I} w_{i,S} p_i \\
\text{s.t.} \quad & w_{i,H} \le x_i, w_{i,H} \le y_{i,H} \quad \forall i \in I \\
& w_{i,M} \le x_i, w_{i,M} \le y_{i,M} \quad \forall i \in I \\
& w_{i,O} \le x_i, w_{i,O} \le y_{i,O} \quad \forall i \in I \\
& w_{i,S} \le x_i, w_{i,S} \le y_{i,S} \quad \forall i \in I \\
& w_{i,L} \le x_i, w_{i,L} \le y_{i,L} \quad \forall i \in I \\
& \sum_{i \in I} w_{i,S} = 1, \sum_{i \in I} w_{i,L} = 1, \sum_{i \in I} w_{i,H} = 2, \sum_{i \in I} w_{i,M} = 2, \sum_{i \in I} w_{i,O} = 1 \\
& \sum_{i \in I} x_i z_{i,S} \ge 2, \sum_{i \in I} x_i z_{i,O} \ge 3 \\
& 1 - x_1 \ge x_3 \\
& x_1 + x_3 + x_4 + x_6 + x_8 \ge 2 \\
& \sum_{j \in J} y_{i,j} \ge x_i \quad \forall i \in I \\
& 1 \ge \sum_{j \in J} y_{i,j} \quad \forall i \in I \\
& z_{i,j} \ge y_{i,j} \quad \forall i \in I, \forall j \in J \\
& w_{i,j}, x_i, y_{i,j}, z_{i,j} \in \{0,1\} \quad \forall i \in I, \forall j \in J
\end{aligned} \tag{1}
$$

1.2 (b)

We only need to add two more constrains:

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i \in I} w_{i,H} r_i + \sum_{i \in I} w_{i,O} r_i + \sum_{i \in I} w_{i,L} r_i \ge 4 \times 3.5 \\
& \sum_{i \in I} w_{i,M} b_i + \sum_{i \in I} w_{i,I} b_i \ge 3 \times 3.5
\end{aligned} \tag{2}
$$

2 Problem 2

2.1 (a)

Set:

$I \in \{1, ..., 5\}$, $J \in \{1, ..., 8\}$.

10 groups $G_{u,v}$ as the groups that students work on day u and day v. $\forall u, v \in I, u \neq v$.

w_i as the maximum students needed in day i. $\forall i \in I$.

$$
\begin{aligned}
\min \quad & \sum_{u=1}^{4} \sum_{v=u+1}^{5} G_{u,v} \\
\text{s.t.} \quad & \sum_{v \in I} G_{i,v} \geq w_i \quad \forall i \in I, v \neq i \\
& w_i \geq D_{i,j} \quad \forall i \in I, j \in J \\
& G_{u,v} \geq 0 \quad \forall u, v \in I, u \neq v
\end{aligned} \tag{3}
$$

2.2 (b)

Set:

$I \in \{1, ..., 10\}$.

define interval 1 to 10 as:

slot 1 to 4: 1, 2, 3, 4, 5 (day 1 to 5)

slot 5 to 8: 6, 7, 8, 9, 10 (day 1 to 5)

80 groups $G_{w,x,y,z}$ as the groups that students work on interval w, x, y, z, $\forall w, x, y, z \in I, w \neq x \neq y \neq z$.

m_i as the maximum students needed in interval i, $\forall i \in I$.

$$
\begin{aligned}
\min \quad & \sum_{w=1}^{7}\sum_{x=w+1}^{8}\sum_{y=x+1}^{9}\sum_{z=y+1}^{10} G_{w,x,y,z} \\
\text{s.t.} \quad & \sum_{x=w+1}^{8}\sum_{y=x+1}^{9}\sum_{z=y+1}^{10} G_{w,x,y,z} \geq m_w \quad \forall w \in I, w \neq x \neq y \neq z \\
& m_w \geq D_{w,j} \quad \forall w \in \{1, ..., 5\}, j \in \{1, ..., 4\} \\
& m_w \geq D_{w-5,j} \quad \forall w \in \{6, ..., 10\}, j \in \{5, ..., 8\} \\
& G_{w,x,y,z} \geq 0 \quad \forall w, x, y, z \in I, w \neq x \neq y \neq z
\end{aligned}
\tag{4}
$$

3 Problem 3

Most notations are defined in the problem description, so we only define the ones that is not defined in it below.

Set:

$I \in \{0, ..., 40\}$ as each CSR's number

$x_{i,d,j} = 1$ if CSR i is assigned to shift j in day d, else $x_{i,d,j} = 0$. $\forall i \in I, \forall d \in D, \forall j \in J$.

$y_{d,t}$ is the number of CSR shortage in period t of day t. $\forall d \in D, \forall j \in J$.

$$
\begin{aligned}
\min \quad & \sum_{d\in D}\sum_{t\in T} y_{d,t} \\
\text{s.t.} \quad & \sum_{j\in J} x_{i,d,j} = 1 \quad \forall i\in I, d\in D \\
& y_{d,t} \geq C_{t,d} - \sum_{j\in J} A_{j,t} \sum_{i\in I} x_{i,d,j} \quad \forall d\in D, \forall t\in T \\
& y_{d,t} \geq 0 \quad \forall d\in D, t\in T \\
& \sum_{d\in D} x_{i,d,0} = 8 \quad \forall i\in I \\
& \sum_{d=k}^{k+6}\sum_{j\in J} x_{i,d,j} B_{j,night} \leq 1 \quad \forall i\in I, \forall k\in D - \{end \quad 6 \quad days\} \\
& \sum_{d=k}^{k+6}\sum_{j\in J} x_{i,d,j} B_{j,afternoon} \leq 2 \quad \forall i\in I, \forall k\in D - \{end \quad 6 \quad days\} \\
& \sum_{d=k}^{k+6}\sum_{j\in J} x_{i,d,0} \geq 1 \quad \forall i\in I, \forall k\in D - \{end \quad 6 \quad days\}
\end{aligned}
\tag{5}
$$

4 Problem 4

4.1 (a)

According to the answer solved by excel solver, an optimal solution is
$G_{1,2} = 0.0, G_{1,3} = 0.0, G_{1,4} = 0.0, G_{1,5} = 8.0, G_{2,3} = 2.0,$
$G_{2,4} = 5.0, G_{2,5} = 0.0, G_{3,4} = 4.0, G_{3,5} = 0.0, G_{4,5} = 0.0$
where the objective value equals 19.0.

So, the plan is to hire 8 students working in day 1 and 5, hire 2 students working in day 2 and 3, hire 5 students working in day 2 and 4, hire 4 students working in day 3 and 4. Then we can hire a minimum of 19 students to fulfill all the demands.

4.2 (b)

According to the answer solved by excel solver, an optimal solution is
$G_{1,2} = 2.0, G_{1,3} = 2.0, G_{1,4} = 0.0, G_{1,5} = 1.0, G_{2,3} = 2.0,$
$G_{2,4} = 0.0, G_{2,5} = 0.0, G_{3,4} = 0.0, G_{3,5} = 0.0, G_{4,5} = 4.0$
where the objective value equals 11.0.

So, the plan is to hire 2 students working in day 1 and 2, hire 2 students working in day 1 and 3, hire 1 student working in day 1 and 5, hire 2 students working in day 2 and 3, hire 4 students working in day 4 and 5. Then we can hire a minimum of 11 students to fulfill all the demands.

4.3 (c)

By comparing the two optimal solutions, there are differences between hiring 19 students and 11 students. Although the average numbers of students needed per time slot are identical in the above two parts, the second part is more well-distributed, which decreases the maximum students needed in each day in the constrain functions. Thus, causing a 8 student difference between when minimizing the student amounts to fulfill all the demands.