4.4 Discrete maximum entropy examples[4]

4.4.1 Discrete uniform

Example 2 (discrete uniform) *Suppose we know only that there are three possible (exchangeable) events,* $x_1 = 1$*,* $x_2 = 2$*,and* $x_3 = 3$*. The maximum entropy probability assignment is found by solving the Lagrangian*

$$\mathcal{L} \equiv \max_{p_i} \left[-\sum_{i=1}^{3} p_i \log p_i - (\lambda_0 - 1) \left(\sum_{i=1}^{3} p_i - 1 \right) \right]$$

First order conditions yield

$$p_i = e^{-\lambda_0} \quad \textit{for } i = 1, 2, 3$$

and

$$\lambda_0 = \log 3$$

Hence, as expected, the maximum entropy probability assignment is a discrete uniform distribution with $p_i = \frac{1}{3}$ *for* $i = 1, 2, 3$*.*

4.4.2 Discrete nonuniform

Example 3 (discrete nonuniform) *Now suppose we know a little more. We know the mean is* 2.5.[5] *The Lagrangian is now*

$$\mathcal{L} \equiv \max_{p_i} \left[-\sum_{i=1}^{3} p_i \log p_i - (\lambda_0 - 1) \left(\sum_{i=1}^{3} p_i - 1 \right) - \lambda_1 \left(\sum_{i=1}^{3} p_i x_i - 2.5 \right) \right]$$

First order conditions yield

$$p_i = e^{-\lambda_0 - x_i \lambda_1} \quad \textit{for } i = 1, 2, 3$$

and

$$\begin{aligned} \lambda_0 &= 2.987 \\ \lambda_1 &= -0.834 \end{aligned}$$

The maximum entropy probability assignment is

$$\begin{aligned} p_1 &= 0.116 \\ p_2 &= 0.268 \\ p_3 &= 0.616 \end{aligned}$$

---

[4] A table summarizing some maximum entropy probability assignments is found at the end of the chapter (see Park and Bera [2009] for additional maxent assignments).

[5] Clearly, if we knew the mean is 2 then we would assign the uniform discrete distribution above.