

# 數值分析 2D Gaussian

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## Requirement A

由 matlab 所積分出來的值為 0.160429671237858

```
format long
f = @(x,y) (sin(2.*pi.*x) ./ (2.*pi.*x)) .* (sin(3.*pi.*y) ./ (3.*pi.*y))
integral2(f, -1, 1, -1, 1) % integral2(fun,xmin,xmax,ymin,ymax)
```

```
f = function_handle with value:
@(x,y)(sin(2.*pi.*x)./(2.*pi.*x)).*(sin(3.*pi.*y)./(3.*pi.*y))
ans =
0.160429671237858
```

## Requirement B

Interval = 1 ~ 16, N = 2 ~ 4

左邊是積出來的值，右邊是左邊減 matlab 的值取絕對值

```
C:\Users\HAO\Desktop\Code\C_++\main.exe
interval = 1      error
0.000000000000000 0.1604296712378580
0.0705961600230158 0.0898335112148422
0.6754878584756927 0.5150581872378347

interval = 4      error
0.0239020452178012 0.1365276260200567
0.3109421331106727 0.1505124618728147
0.1453198121355345 0.0151098591023235

interval = 9      error
0.1614597990686126 0.0010301278307546
0.1331509217570508 0.0272787494808072
0.1626207134166774 0.0021910421788194

interval = 16     error
0.1603404278701082 0.0000892433677498
0.1585793527233802 0.0018503185144778
0.1605158782109374 0.0000862069730794
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```

## Requirement C

Interval = 1 ~ 49, N = 2 ~ 4

可以透過增加 interval 的數量，讓積出來的值更快的接近正解，收斂得更快。

```
C:\Users\HAO\Desktop\Code\C_C++\main.exe
interval = 1      error
0.000000000000000 0.1604296712378580
0.0705961600230158 0.0898335112148422
0.6754878584756927 0.5150581872378347

interval = 4      error
0.0239020452178012 0.1365276260200567
0.3109421331106727 0.1505124618728147
0.1453198121355345 0.0151098591023235

interval = 9      error
0.1614597990686126 0.0010301278307546
0.1331509217570508 0.0272787494808072
0.1626207134166774 0.0021910421788194

interval = 16     error
0.1603404278701082 0.0000892433677498
0.1585793527233802 0.0018503185144778
0.1605158782109374 0.0000862069730794

interval = 25     error
0.1604278233213805 0.0000018479164775
0.1599079029343330 0.0005217683035250
0.1604458625994082 0.0000161913615502

interval = 36     error
0.1604294493078839 0.0000002219299741
0.1602193195591790 0.0002103516786790
0.1604343107645197 0.0000046395266617

interval = 49     error
0.1604296270909778 0.0000000441468802
0.1603271574236357 0.0001025138142223
0.1604313566459277 0.0000016854080697
```

## Requirement D

Interval = 1 ~ 49, N = 1 ~ 7

在增加 interval 的數量，並且也增加 sample points 的數量的情況下，可以看

出來誤差明顯減少，interval 和 sample points 的數量越多收斂越快。

```
C:\Users\HAO\Desktop\Code\C_C++\main.exe
interval = 1      error
4.000000000000000 3.8395703287621421
0.0705961600230158 0.0898335112148422
0.6754878584756927 0.5150581872378347
0.0239020452178012 0.1365276260200567
0.1980801594804246 0.0376504882425666
0.1539715517075571 0.0064581195303009
0.1612302308770341 0.0008005596391761

interval = 4      error
-0.000000000000000 0.1604296712378580
0.3109421331106727 0.1505124618728147
0.1453198121355345 0.0151098591023235
0.1614597990686126 0.0010301278307546
0.1603834903466695 0.0000461808911885
0.1604311556998681 0.0000014844620101
0.1604296354204032 0.0000000358174548

interval = 9      error
0.2606681459705137 0.1002384747326557
0.1331509217570508 0.0272787494808072
0.1626207134166774 0.0021910421788194
0.1603404278701082 0.0000892433677498
0.1604318928901975 0.0000022216523395
0.1604296336536001 0.0000000375842579
0.1604296717629064 0.0000000005250484

interval = 16     error
0.1698249387409648 0.0093952675031068
0.1585793527233802 0.0018503185144778
0.1605158782109374 0.0000862069730794
0.1604278233213805 0.0000018479164775
0.1604296949939063 0.0000000237560483
0.1604296710923769 0.0000000001454812
0.1604296712998537 0.0000000000619957

C:\Users\HAO\Desktop\Code\C_C++\main.exe
interval = 25     error
0.1639131265742928 0.0034834553364348
0.1599079029343330 0.0005217683035250
0.1604458625994082 0.0000161913615502
0.1604294493078839 0.0000002219299741
0.1604296731035603 0.0000000018657023
0.1604296712886030 0.0000000000507450
0.1604296712985844 0.0000000000607264

interval = 36     error
0.1622339337507622 0.0018042625129042
0.1602193195591790 0.0002103516786790
0.1604343107645197 0.0000046395266617
0.1604296270909778 0.0000000441468802
0.1604296715472142 0.0000000003093562
0.1604296712975978 0.0000000000597398
0.1604296712985469 0.0000000000606889

interval = 49     error
0.1615370336576569 0.0011073624197989
0.1603271574236357 0.0001025138142223
0.1604313566459277 0.0000016854080697
0.1604296594916174 0.0000000117462406
0.1604296713472395 0.0000000001093814
0.1604296712984085 0.0000000000605505
0.1604296712985445 0.0000000000606865

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```

## Requirement E

sample points 和 interval 哪個比較重要? sample points 的數量

情況一  $N = 3$  , interval = 1, 49 :

$N = 3$  , interval = 1 , error = 0.5150581872378347

$N = 3$  , interval = 49 , error = 0.0000016854080697

情況二 interval = 9 ,  $N = 2, 7$  :

$N = 2$  , interval = 9 , error = 0.0272787494808072

$N = 7$  , interval = 9 , error = 0.0000000005250484

可以看出來在 sample points 數量增加情況下，error 收斂了 8 個位數，而在

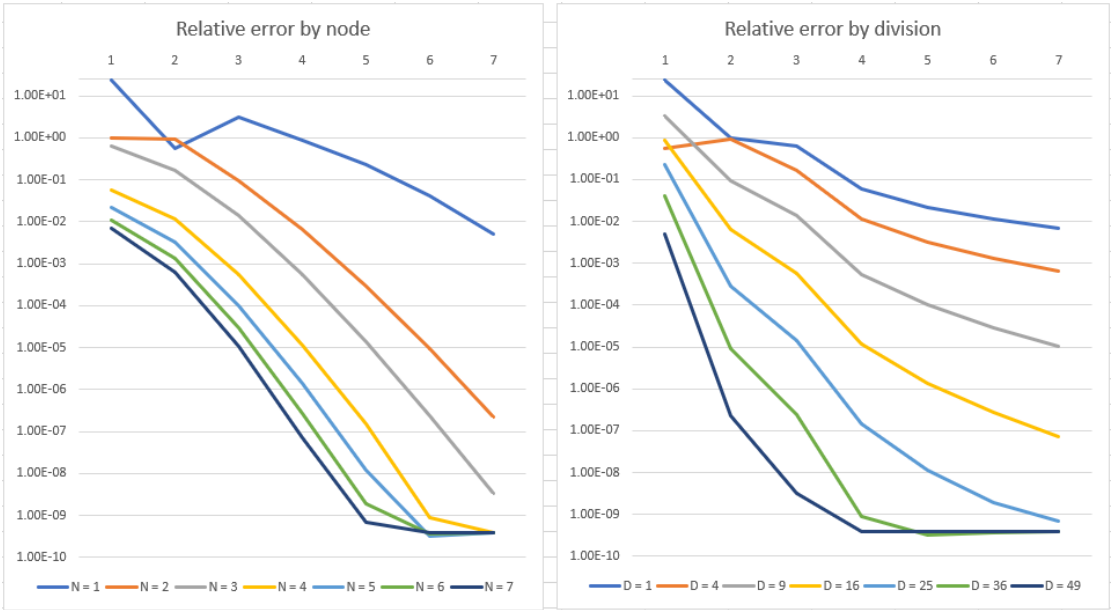
interval 數量增加情況下，error 只收斂了 5 個位數。

根據下面 Requirement F 的兩張圖中可以看出來，以 sample points 為主的折

線圖，它們的斜率都對應到 interval 為主的折線斜率還要更大(ex :  $N = 5$  對

上  $D = 25$ )。

Requirement F



interval	1	4	9	16	25	36	49
N = 1	23.93304	0.559956	3.210492	0.851012	0.234685	0.040255	0.00499
N = 2	1	0.938183	0.094184	0.006421	0.000288	9.25E-06	2.23E-07
N = 3	0.624813	0.170036	0.013657	0.000556	1.38E-05	2.34E-07	3.27E-09
N = 4	0.058563	0.011534	0.000537	1.15E-05	1.48E-07	9.07E-10	3.86E-10
N = 5	0.021713	0.003252	0.000101	1.38E-06	1.16E-08	3.16E-10	3.79E-10
N = 6	0.011246	0.001311	2.89E-05	2.75E-07	1.93E-09	3.72E-10	3.78E-10
N = 7	0.006902	0.000639	1.05E-05	7.32E-08	6.82E-10	3.77E-10	3.78E-10
node	1	2	3	4	5	6	7
D = 1	23.93304	1	0.624813	0.058563	0.021713	0.011246	0.006902
D = 4	0.559956	0.938183	0.170036	0.011534	0.003252	0.001311	0.000639
D = 9	3.210492	0.094184	0.013657	0.000537	0.000101	2.89E-05	1.05E-05
D = 16	0.851012	0.006421	0.000556	1.15E-05	1.38E-06	2.75E-07	7.32E-08
D = 25	0.234685	0.000288	1.38E-05	1.48E-07	1.16E-08	1.93E-09	6.82E-10
D = 36	0.040255	9.25E-06	2.34E-07	9.07E-10	3.16E-10	3.72E-10	3.77E-10
D = 49	0.00499	2.23E-07	3.27E-09	3.86E-10	3.79E-10	3.78E-10	3.78E-10