

數值分析 Direct Methods

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A. test data

sample points = 11 ~ 16 , degree = 7 ~ 12

```
sample points = 11 , degrees = 7      sample points = 12 , degrees = 8
x      y                               x      y
2.00 255.00                            2.00 511.00
2.20 456.47                            2.20 1005.22
2.40 785.54                            2.40 1886.29
2.60 1304.54                          2.60 3392.81
2.80 2098.34                          2.80 5876.36
3.00 3280.00                          3.00 9841.00
3.20 4997.33                          3.20 15992.44
3.40 7440.39                          3.40 25298.33
3.60 10850.04                        3.60 39061.14
3.80 15527.47                        3.80 59005.39
4.00 21845.00                        4.00 87381.00
4.20 127084.49                       4.20 127084.49

sample points = 13 , degrees = 9      sample points = 14 , degrees = 10
x      y                               x      y
2.00 1023.00                            2.00 2047.00
2.20 2212.49                            2.20 4868.49
2.40 4528.10                            2.40 10868.44
2.60 8822.32                            2.60 22939.03
2.80 16454.82                          2.80 46074.50
3.00 29524.00                          3.00 88573.00
3.20 51176.81                          3.20 163766.80
3.40 86015.32                          3.40 292453.10
3.60 140621.09                        3.60 506236.94
3.80 224221.49                        3.80 852042.68
4.00 349525.00                        4.00 1398101.00
4.20 533755.88                        4.20 2241775.69
4.40 799922.09                        4.40 3519658.18
4.60 5420428.57                       4.60 5420428.57

sample points = 15 , degrees = 11      sample points = 16 , degrees = 12
x      y                               x      y
2.00 4095.00                            2.00 8191.00
2.20 10711.67                          2.20 23566.67
2.40 26085.25                          2.40 62605.60
2.60 59642.47                          2.60 155071.43
2.80 129009.59                        2.80 361227.86
3.00 265720.00                        3.00 797161.00
3.20 524054.77                        3.20 1676976.28
3.40 994341.53                        3.40 3380762.22
3.60 1822453.98                      3.60 6560835.31
3.80 3237763.18                      3.80 12303501.09
4.00 5592405.00                      4.00 22369621.00
4.20 9415458.90                      4.20 39544928.39
4.40 15486496.99                    4.40 68140587.77
4.60 24933972.41                    4.60 114696274.08
4.80 39365090.03                    4.80 188952433.12
5.00 305175781.00                    5.00 305175781.00
```

sample points = 11 , degree = 7

matrix A , AT , Y , B , D

```
matrix A
1      2      4      8     16     32     64     128
1      2.2    4.84  10.648 23.4256 51.5363 113.38 249.436
1      2.4    5.76  13.824 33.1776 79.6262 191.103 458.647
1      2.6    6.76  17.576 45.6976 118.814 308.916 803.181
1      2.8    7.84  21.952 61.4656 172.104 481.89 1349.29
1      3      9     27     81     243    729    2187
1      3.2   10.24  32.768 104.858 335.544 1073.74 3435.97
1      3.4   11.56  39.304 133.634 454.354 1544.8 5252.34
1      3.6   12.96  46.656 167.962 604.662 2176.78 7836.42
1      3.8   14.44  54.872 208.514 792.352 3010.94 11441.6
1      4     16     64     256    1024    4096    16384

matrix AT
1      1      1      1      1      1      1      1      1      1
2      2.2    2.4    2.6    2.8    3      3.2    3.4    3.6    3.8    4
4      4.84    5.76    6.76    7.84    9     10.24   11.56   12.96   14.44   16
8     10.648   13.824   17.576   21.952   27    32.768   39.304   46.656   54.872   64
16    23.4256   33.1776   45.6976   61.4656   81    104.858  133.634  167.962  208.514  256
32    51.5363   79.6262   118.814   172.104   243   335.544  454.354  604.662  792.352  1024
64    113.38    191.103   308.916   481.89    729   1073.74  1544.8   2176.78  3010.94  4096
128   249.436   458.647   803.181   1349.29  2187  3435.97  5252.34  7836.42  11441.6  16384

matrix Y
255
456.466
785.538
1304.54
2098.34
3280
4997.33
7440.39
10850
15527.5
21845

matrix B
11      33      103.4    336.6  1131.73  3907.99  13790.6  49525.8
33      103.4    336.6  1131.73  3907.99  13790.6  49525.8  180411
103.4    336.6  1131.73  3907.99  13790.6  49525.8  180411  664873
336.6  1131.73  3907.99  13790.6  49525.8  180411  664873  2.47385e+006
1131.73  3907.99  13790.6  49525.8  180411  664873  2.47385e+006  9.27831e+006
3907.99  13790.6  49525.8  180411  664873  2.47385e+006  9.27831e+006  3.50332e+007
13790.6  49525.8  180411  664873  2.47385e+006  9.27831e+006  3.50332e+007  1.33037e+008
49525.8  180411  664873  2.47385e+006  9.27831e+006  3.50332e+007  1.33037e+008  5.07685e+008

matrix D
68840.1
249240
914080
3.38782e+006
1.26658e+007
4.76979e+007
1.80731e+008
6.88402e+008
```

B. result 1 ~ 3

sample points = 11 ~ 16 , degree = 7 ~ 12

```
sample points = 11 , degrees = 7  sample points = 12 , degrees = 8

result1                                result1
-2.711865816812                        430.675469988361
10.166306130784                       -1180.627952882701
-8.603229429808                       1408.853806171802
6.533768124177                        -948.298813577740
-0.894537446357                       397.275749953587
1.385437784840                        -103.885788436264
0.956842712175                        18.192814952903
1.002052166858                        -0.596122272197
                                      1.064267757019

result2                                result2
4.383622138140                        18.858772540367
-7.320056314817                       -45.548912605778
9.679157578498                        53.389240658122
-3.979770162645                       -32.255573460911
2.697587555665                        14.023808783143
0.656087386157                        -2.222645881646
1.038348381421                        1.492082838233
0.998183868519                        0.957602478256
                                      1.001578213556

result3                                result3
1.000000132200                        1.000001773950
0.999999669204                        0.999995078293
1.000000351174                        1.000005913396
0.999999794961                        0.999995980921
1.000000071116                        1.000001690276
0.999999985345                        0.999999549494
1.000000001662                        1.000000074326
0.999999999920                        0.999999993059
                                      1.000000000281
```

sample points = 13 , degrees = 9 sample points = 14 , degrees = 10

result1

233.745919218572
-949.245018497404
1596.698389011255
-1481.599041021560
852.070409434340
-314.510852059448
76.933528085845
-10.482028353629
1.992541418810
0.962551538071

result2

-4630.019990393532
10368.065080417715
-8935.893189187193
3136.367504622750
286.619988173533
-674.936709016307
268.466446682484
-51.879881230146
6.433160244456
0.768462317391

result3

0.999992883622
1.000022591451
0.999968455685
1.000025429127
0.999986956320
1.000004415068
0.999999013823
1.000000140176
0.999999988494
1.000000000416

result1

-705.506346277973
2221.248718142731
-3147.670195246130
2659.478942674116
-1480.490986583696
570.466692912631
-151.796941471788
29.216658331166
-2.425529492947
1.246349305953
0.992046768646

result2

-3232.882992338235
6325.792142946454
-3375.257489415967
-1779.805874949936
3394.804256794100
-2113.897430406314
747.474855217001
-162.143588047411
22.950292134266
-0.672664298915
1.055404273878

result3

0.999534412853
1.001514787985
0.997804533039
1.001866931463
0.998968329729
1.000387186085
0.999900035106
1.000017535690
0.999997999367
1.000000134080
0.999999995991

sample points = 15 , degrees = 11 sample points = 16 , degrees = 12

result1

19598.456512387598
-29074.285605368132
-2638.845596212620
34013.328957662510
-31555.162207895559
14067.854769015496
-3201.051193112948
194.125594034068
85.287757032422
-22.252747290732
3.456633298450
0.900632270286

result2

72965.984945489108
-181572.704856701952
186759.738731314574
-99129.108501912706
25844.053224249110
-988.322889768108
-1163.031417171176
216.560335786751
26.240136333535
-12.245462556889
2.701126889147
0.923041935631

result3

0.993047614474
1.024749726277
0.960371766552
1.037674497181
0.976367370863
1.010272138622
0.996842409708
1.000686588776
0.999896481258
1.000010309780
0.999999389408
1.000000016296

result1

279096.807447136729
-762489.794880006695
886115.170569857000
-560495.246381297708
200029.179960520560
-33736.863962356751
-1222.142652290462
1408.572168770051
-70.084938585404
-61.906739656464
16.604572300337
-0.531563224978
1.057458747868

result2

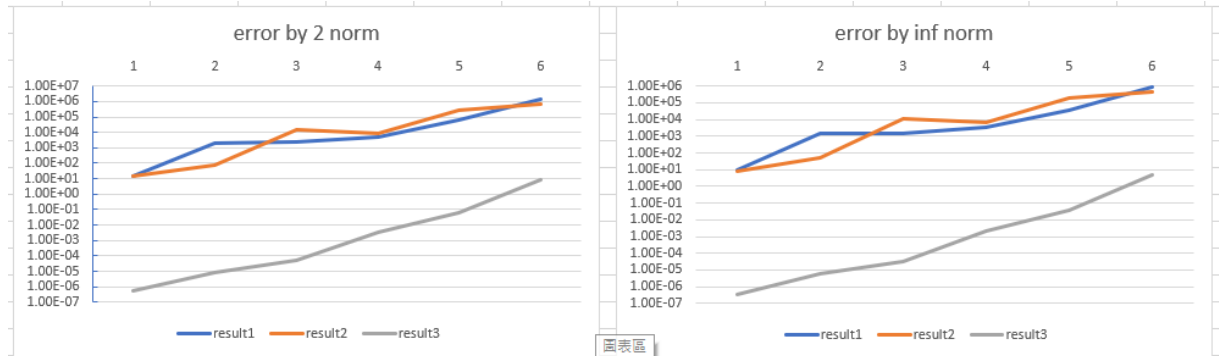
-117993.707094098703
33242.122379072891
323906.838966273761
-462378.217627752165
279712.447780144226
-81978.832922057045
6919.442278715157
2321.288108390926
-527.478090646466
-45.955694922448
29.624851434138
-2.678829418642
1.163363116579

result3

1.716509086335
-1.692586506566
5.598556149470
-3.720035824567
4.243144515980
-0.571663269850
1.550882598621
0.859268077812
1.026009477686
0.996608119519
1.000296307037
0.999984429349
1.000000372272

C. error by 2 norm and inf norm

sample points = 11 ~ 16 , degree = 7 ~ 12



points	11	12	13	14	15	16
result1	14.97943	2152.306	2555.632	4995.353	59976.71	1341564
result2	13.55727	80.72516	14806.55	9033.626	289255.5	647108.9
result3	5.45E-07	9.03E-06	4.89E-05	0.00347	0.065766	8.031248
result1	9.603229	1407.854	1595.698	3148.67	34012.33	886114.2
result2	8.679158	52.38924	10367.07	6324.792	186758.7	462379.2
result3	3.51E-07	5.91E-06	3.15E-05	0.002195	0.039628	4.720036

D. analysis

1. 根據 B, C 的結果，result 1 ~ 3 中，result3 最為穩定，即使 sample points = 16, degree = 12，error by 2 norm = 14.68955 而已，而其他兩個誤差都已經完全炸開。

2. 我認為會有這樣的結果是因為 A, Y 矩陣乘以 A^T 後 B, D 矩陣內的數值變得太大了，造成在 Gaussian elimination 中的 Forward elimination 裡的 r 可能會是一個很小的值除以一個很大的值，還有在兩個演算法中都有的 Backward substitution 也可能發生這樣的情形。當 degree 越大，這樣的情形越明顯，誤差也越大。

3. 根據 B 的結果，result1, result2 除了第一次的結果比較準之外，從第二次開始，他們兩個所算出來的值已經完全沒有意義了，誤差太大了，跟原本的天差地遠。