**2018 Fall Semester Numerical Analysis Midterm Examination**

每一題的計算過程都要列出，只寫出答案得0分，過程過於簡略要扣分。

1. 利用Taylor expansion計算函數值，*f(x)=sin(x), sin(0)=0, cos(0)=1.* (15%)
   1. Deduce the Taylor Expansion of *sin(x)* by using the value and derivatives of sine function at *x=0* until the 3rd derivative term.
   2. Compute *sin(0.1)* using the Taylor expansion.
   3. Assume that *sin(x) ≈ x* and *cos(x) ≈ 1*, if *x≈0*. Then, estimate the maximum truncation error of this computation.
2. Let and . Find the root by using the following methods. Iterate the computations twice for each method. (15%)
   1. Bisection method: . Newton’s method: using *x1* as the initial guess. Secant method: using *x0* and *x1* as the initial solutions.
3. (a) Given a set of sample points, shown in the following table, deduce the Newton polynomial. (b) Convert the Newton polynomial into the form which is used in Horner’s algorithm and compute *f(0.2)* using Horner’s algorithm. (c) What is the maximum truncation error? (15%)

Table 1, sample data set.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| xi | -0.5 | 0 | 0.5 | 1.0 | 1.5 |
| f(xi) | 2 | 2.5 | 0.5 | 1.5 | 4 |

1. Compute by using the data of problem 3 and the following two methods: (a) Trapezoid rule, (b) Simpson’s method. (10%)
2. Given , compute by using Gaussian quadrature integration method. (a) Using 4 sample points; (b) Divide [2, 4] into two intervals, [2, 3] and [3, 4] and evaluate the integration by using 3 sample points in these 2 intervals and combine the integral results. (10%)
3. Assume that the sample data set is given in Table 1, (10%)
   1. Compute the following 1st derivatives by using forward, central, and backward difference methods: .
   2. Compute the 2nd derivative *f”(1.0)* by using the central difference method.
4. Prove that the Newton’s root-finding method for Problem 2 possesses a quadratic converge rate by using the initial guess *x1*. (10%)
5. 將Problem 2的sample data分成兩段區間:{(-0.5,2), (0, 2.5), (0.5, 0.5)} 與{(0.5,0.5),(1.0,1.5),(1.5,4.0)}。建立一個2次的Spline, *S(x)*。(15%)
   1. 使用Lagrange方法分別在兩區間內建立2次多項式*S1(x)*與*S2(x)*。
   2. *S(x)*是否有C0-continuity? C1-continuity?