2018 Numerical Analysis Quiz 1

100 points

1. 畫出IEEE754儲存32-bit floating point numbers的表示方式。(含Sign, Exponent, and Mantissa). (10%)
2. 定義以下errors::
   1. Round-off error, truncation error, arithmetic error, absolute error, relative error. (15%)
3. Bisection method is linear-converged and Newton’s root-finding method is quadratic-converged. 何謂linear-converged?何謂quadratic-converged? (10%)
4. 寫出*f(x+h)*在*x*這個位置上的Taylor expansion. What is the truncation error of the Taylor expansion, if we expand *f(x+h)* until the term of the k-th derivative? (10%)
5. 請證明通過n+1個sample points且degree≦n的polynomial是唯一的。(10%)
6. 假設有一個Polynomial, 。若使用Horner’s algorithm來計算*p(x=a)*，則*p(x)*要改成何種形式？同時寫出Horner’s algorithm的Pseudo-codes. (15%)
7. The truncation error of trapezoid rule integration method is 。
   1. What is *h*? What does mean? (10%)
8. The truncation error of Simpson’s rule for integration is bounded by . Prove that if *f(x)* is a polynomial with degree≦3, Simpson’s rule produces zero truncation error for computing the integration of *f(x)*. (10%)
9. Assume that *p(x)* and *q(x)* are functions defined in the R-space. (10%)
   1. Inner-product of *p(x)* and *q(x)*.
   2. If *p(x)* is an orthogonal polynomial of degree *n*, give the definition of orthogonal polynomial of degree *n*.