**2017 Fall Semester, Numerical Analysis Final Examination**

1. Given a linear system as below, solve the system by using Gaussian elimination: Please list all key computational steps (請列出主要計算步驟), (a) Compute the upper triangular system (***A*** and *b*) by using the forward elimination. (b) Compute the solution by using the backward substitution. (c) Do you need partial pivoting? Why? (10%)
2. Decompose matrix ***A*** of Problem 1 by using Doolittle’s LU-decomposition. List the computational steps for computing matrices ***L*** and ***U***. Then write down the resulted ***L*** and ***U*** matrices. (10%)
3. Decompose matrix ***A*** by using QR-decomposition. (a) Eliminate the 1st column, (b) then eliminate the 2nd column. Please modify and write down matrix ***A*** step by step. (c) Write down the following information in each elimination: {(i) the reflection vector ***v***, (ii) matrix ***A*** after the elimination, (iii) vector ***b*** after each elimination. (You don’t have to form the reflection matrix.) (15 %)
4. Solve the linear system of Problem 1 by using SOR method. Assume that ω=1.0 and and compute and show and . (10%)
5. Solve the linear system by using the Conjugate Gradient method using the same initial value of Problem 4. You have to perform the iteration at least twice. K=0, 1, 2. Please list the following information in each iteration: (a) α and β, (b) , (c) X(k). (15%)
6. Compute the max eigenvalue of matrix ***A*** of problem 1 by using the power method. You have to perform the iteration by at least 3 times, k=0, 1, and 2. Assume (a) Write down at the start of each iteration. (b) Write down λ at the end of each iteration. (10%)
7. Compute the eigenvalues and eigenvectors of matrix ***A*** of problem 1 by using the Jacobi method. Please iterate the rotation 2 times. (A) Write down matrix ***A*** after each iteration. (B) Write down matrix ***P*** after each iteration. (C) Write down the maximum off-diagonal entry (D) Write down the rotational angle. (15%)
8. Define the following terms and explain their usages in solving linear systems and eigenvalues: (15%)
   1. Quadratic form
   2. Condition number
   3. Rayleigh quotient.