**Numerical Analysis Midterm Exam 2013 Spring Semester**

**(105 points)**

1. 請解釋下列error的來源：(10)
   1. Round-off error
   2. Truncation error
   3. Arithmetic error
2. , assume the precision of the computer is limited.請推導出error，含round-off error and arithmetic error. (10)
3. 請利用下列方法求根：(15)
   1. Bisection method, initial interval = [2, 3]. 做3次iteration.
   2. Newton’s method, the initial guess is x0 = 3. Iterate the computation 3 times.
4. 2D Newton’s method for root finding: (15) Compute the root of this system using the initial guess (1, 1). Please list all steps including the Jacobian matrix and its inverse and the method for computing the incremental values. Iterate the computation 3 times.
5. Given a data set: {(-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)}, (15)
   1. Compute the Lagrange polynomial.
   2. Compute the Newton’s (divided difference) polynomial.
   3. This data set is obtained from . Is the coefficient of x4 vanished (消失)? 解釋其原因。(利用某定理證明。)
6. Using the data set of the previous problem to compute the integration of f(x) from x=-1 to x=3: (15)
   1. Use trapezoid method, 過程要列。
   2. Use Simpson’s Method.
   3. Show that the Simpson’s method produces no numerical error. Please explain the reason. (利用其Truncation error性質)
7. Gaussian quadrature: (10)
   1. 何謂Orthogonal polynomial? Assume the inner product interval is [a, b].
   2. Gaussian quadrature法主要是用Lagrange polynomial 內插f(x).請推導出Gaussian quadrature的積分公式。
   3. 本方法的Truncation error為O(h2n+1), 請解釋此Truncation error的意義。
8. Function f(x) is defined in [0, 1]. Divide [0, 1] into N intervals by using N+1 sample points which are equal-spaced. (h=1/N). (15)
   1. Write down the central difference method for computing the 1st derivative of f(x). (3)
   2. Write down the central difference method for computing the 2nd derivative of f(x). (3)
   3. Deduce the truncation error of 8.B. (4)
   4. As N getting larger, the accuracy of the computing will increased. But after N reaches a certain value, the numerical error will increase dramatically. Please explain the reason. (5)